Switch-Mode Power Concepts

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Seminar Agenda

Part I

- Energy Storage
- Switch Mode Conversion Principle
- The Buck Converter
- Selected Other Topologies

Part II

- Review Of Feedback
- Buck Converter Controller Design Example
- Overview Of Current Mode Control

Seminar Goal

- Goal: Teach You The Operating And Control Principles Of Switch Mode Converters
- Not A Goal: Teach Detailed Design
- Teaching Approach: "Follow The Energy"
  - Understand In Each Switching State Where Energy Is Flowing, Being Stored, And Being Released

Seminar Mechanics

- Part I: 90 Minutes (9:30 – 11:00)
- Break: 30 Minutes (11:00 – 11:30)
- Part II: 90 Minutes (11:30 – 13:00)

- Ask Questions At Any Time

Survey Forms!

Supplementary Material

This Presentation Plus Appendices

On The USB Flash Drive

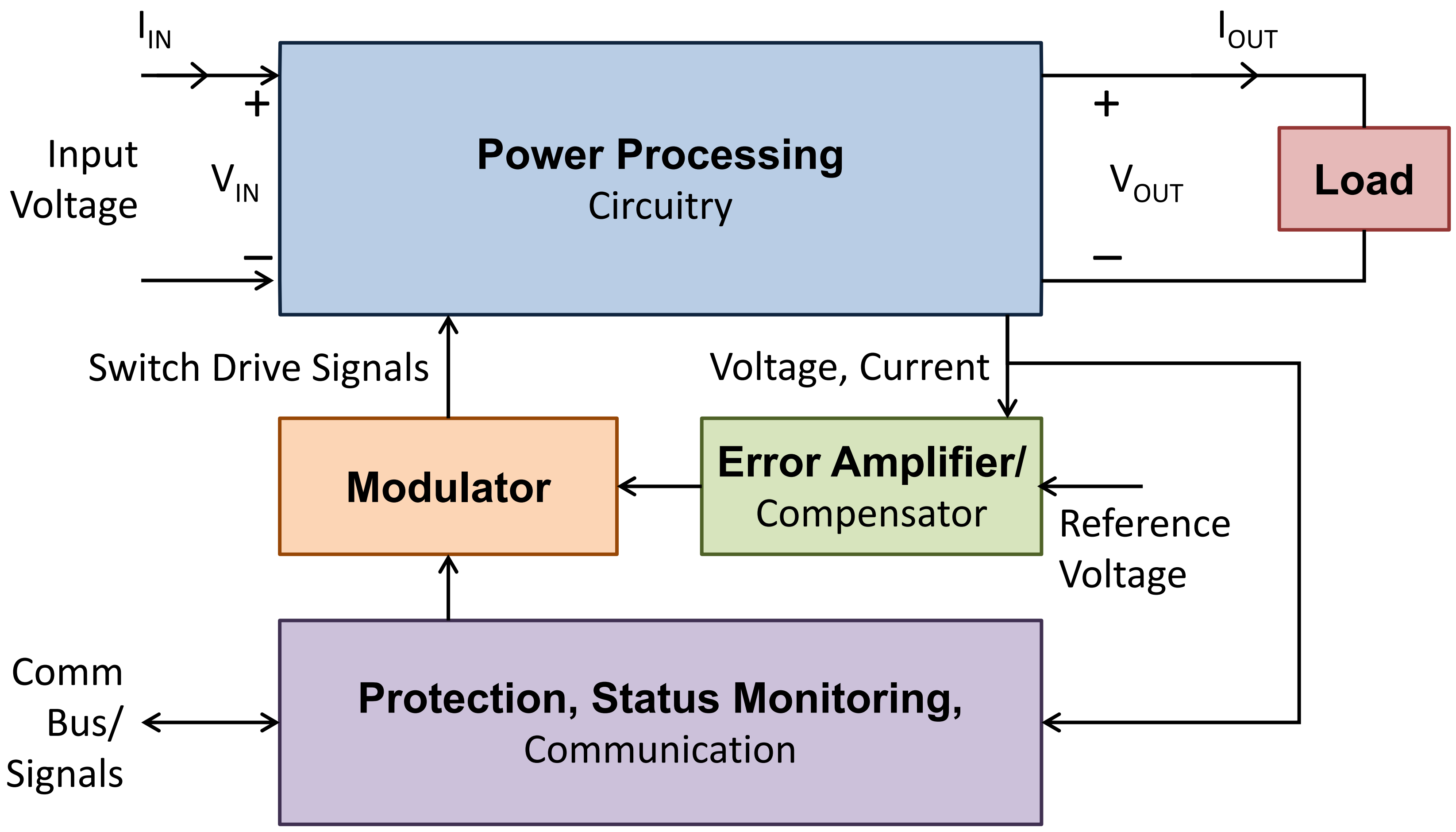
Lab Videos And Simulation Files

Available For Download At:
www.embeddedpowerlabs.com/APEC2023.html
Only Available Until 30 April 2023

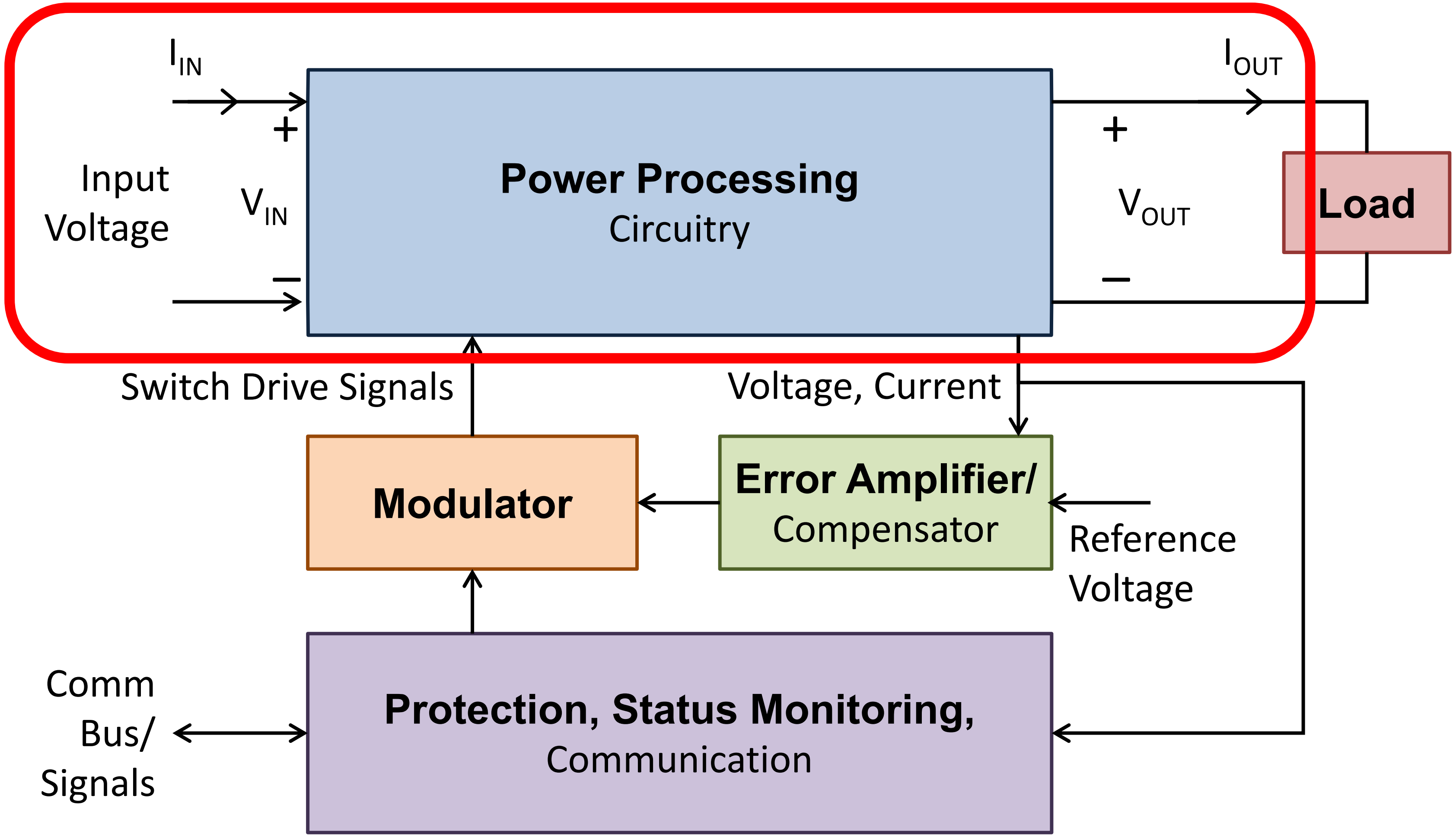
Simulation Files Can Be Run With Either Of Two Free Simulators:
SIMPLIS Elements or Microchip's MPLAB® Mindi™

Switch Mode Regulator Building Blocks

Switch Mode Converter

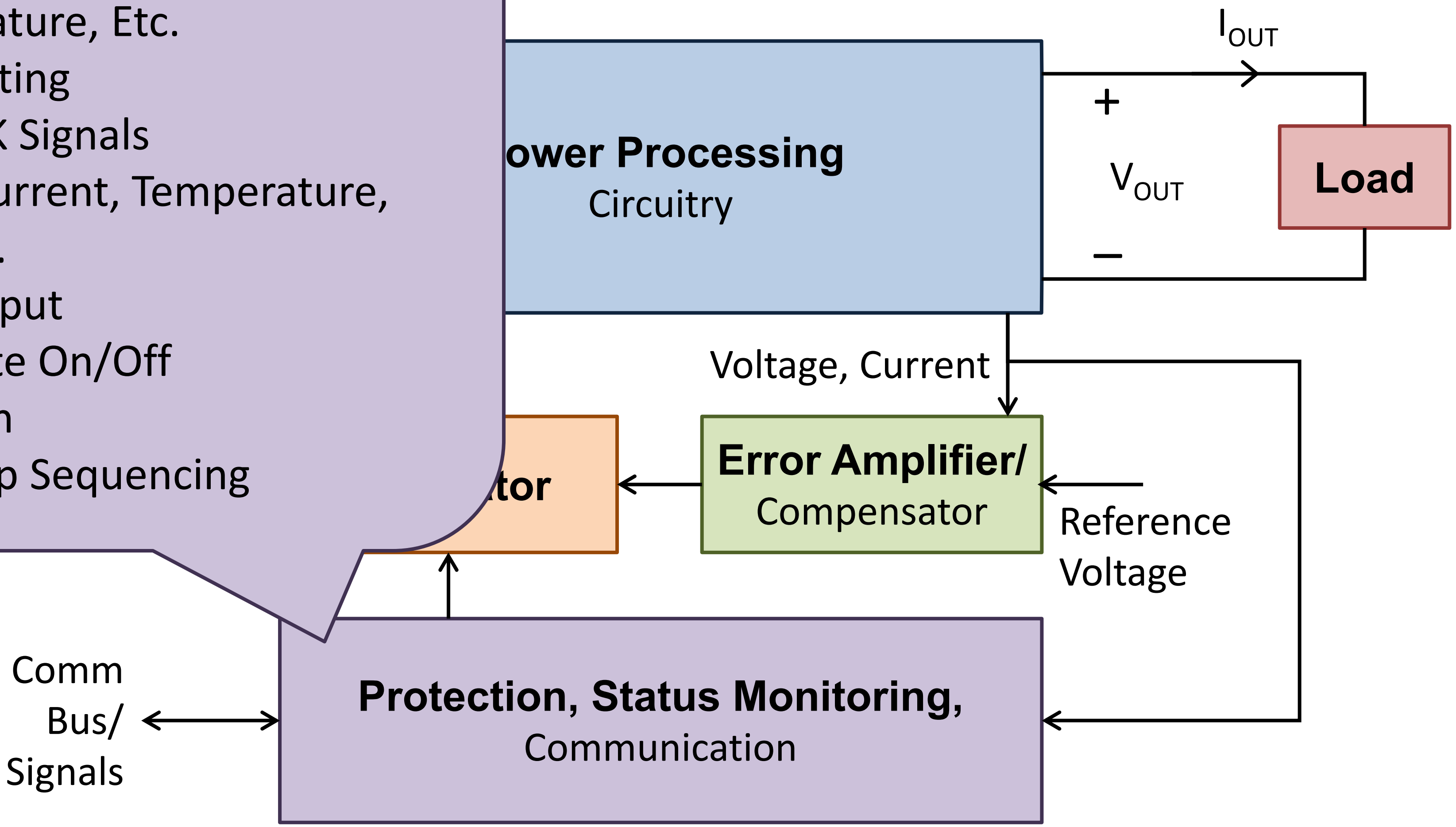


Switch Mode Converter



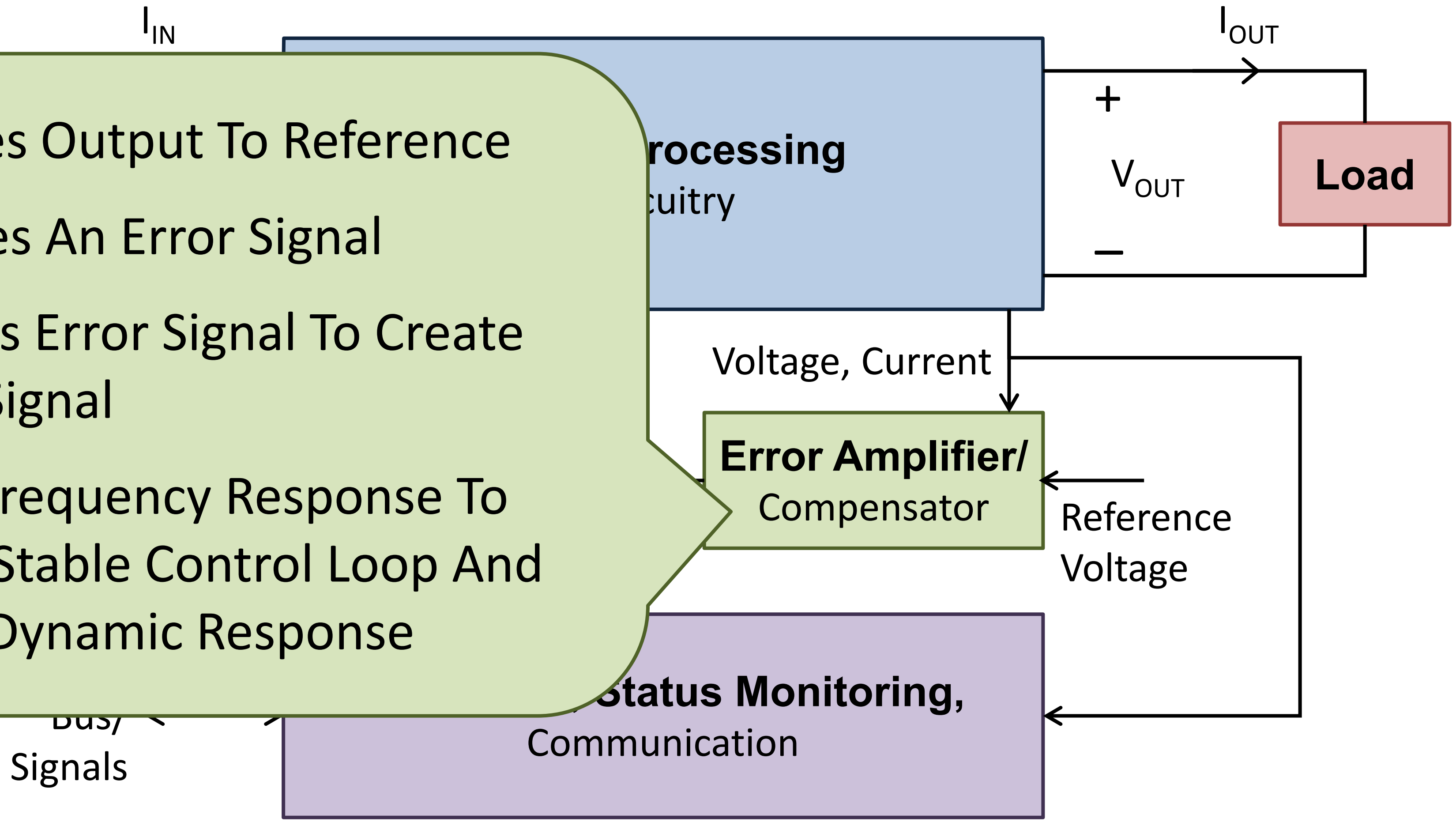
Switch Mode Converter

- Overcurrent, Overvoltage, Overtemperature, Etc.
- Status Reporting
  - OK/Not OK Signals
  - Voltage, Current, Temperature, Power, etc.
- Command Input
  - e.g. Remote On/Off
- Configuration
  - e.g. Startup Sequencing

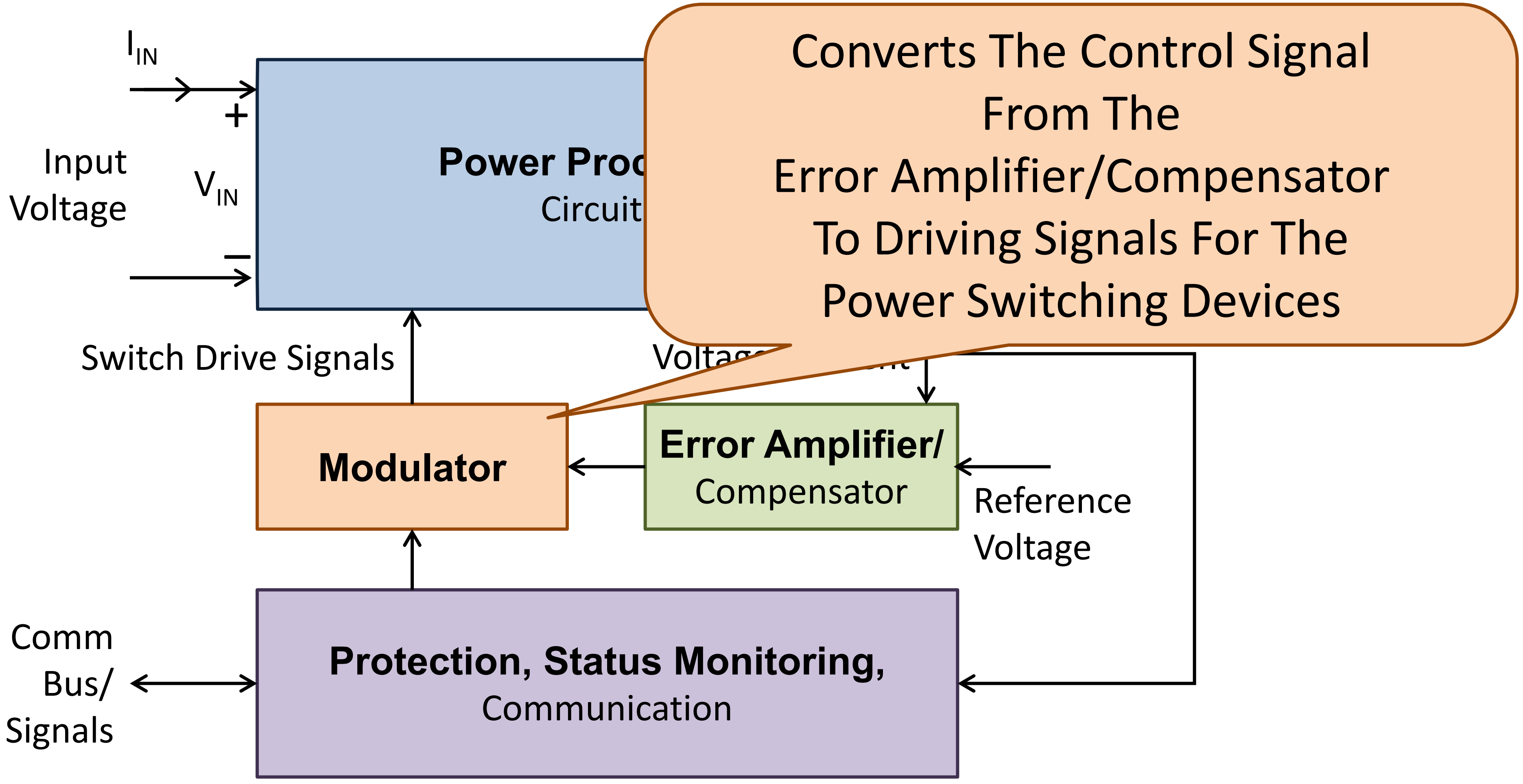


Switch Mode Converter

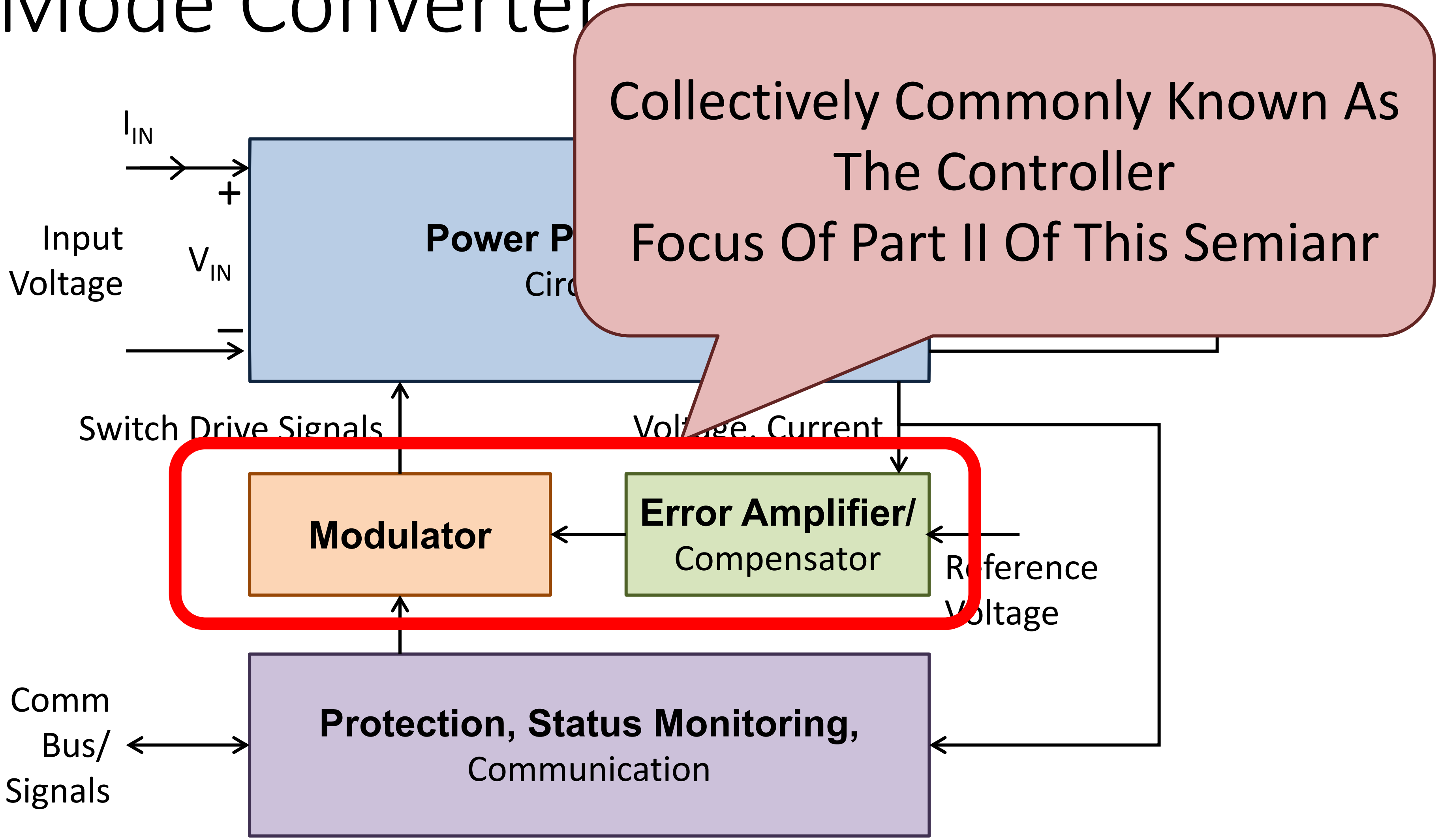
- Compares Output To Reference
- Generates An Error Signal
- Processes Error Signal To Create Control Signal
- Shapes Frequency Response To Achieve Stable Control Loop And Desired Dynamic Response



Switch Mode Converter

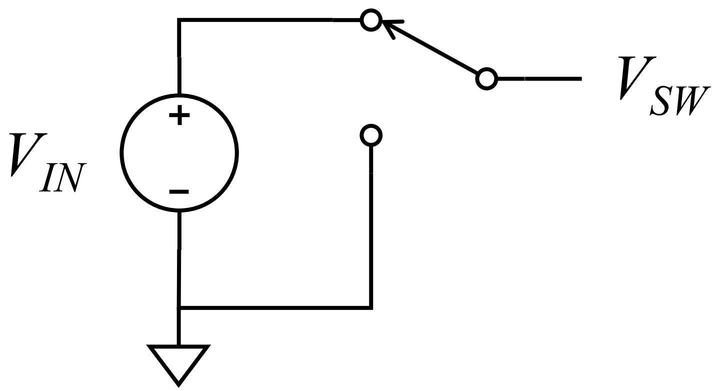


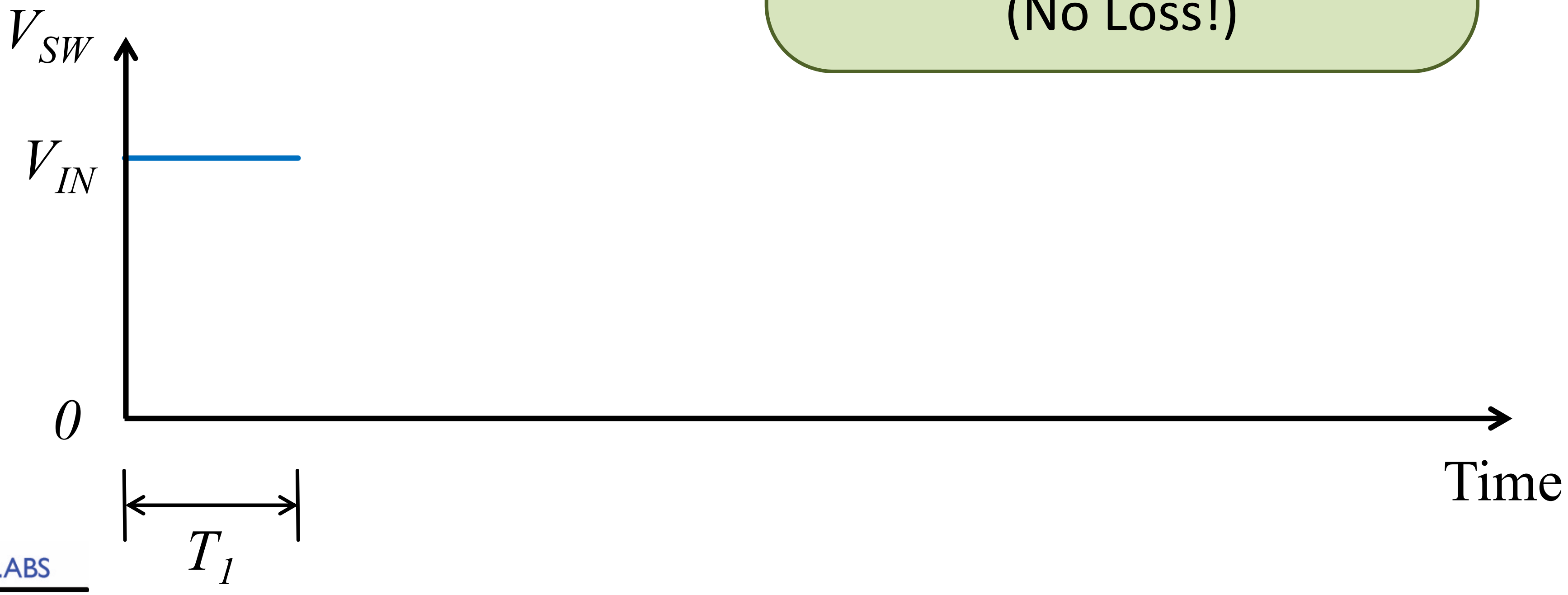
Switch Mode Converter



Switch Mode Concepts

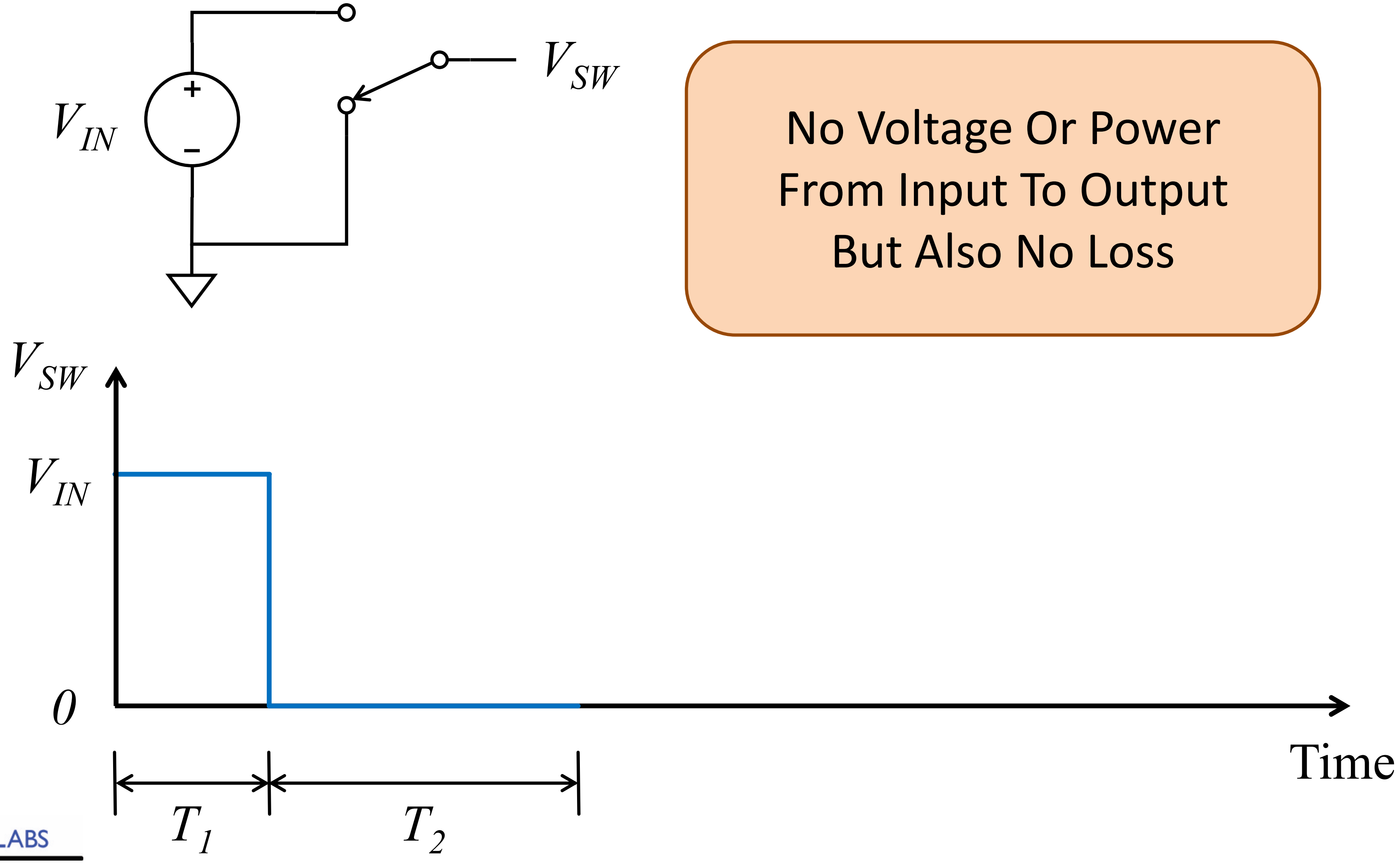
Switch Mode Concept





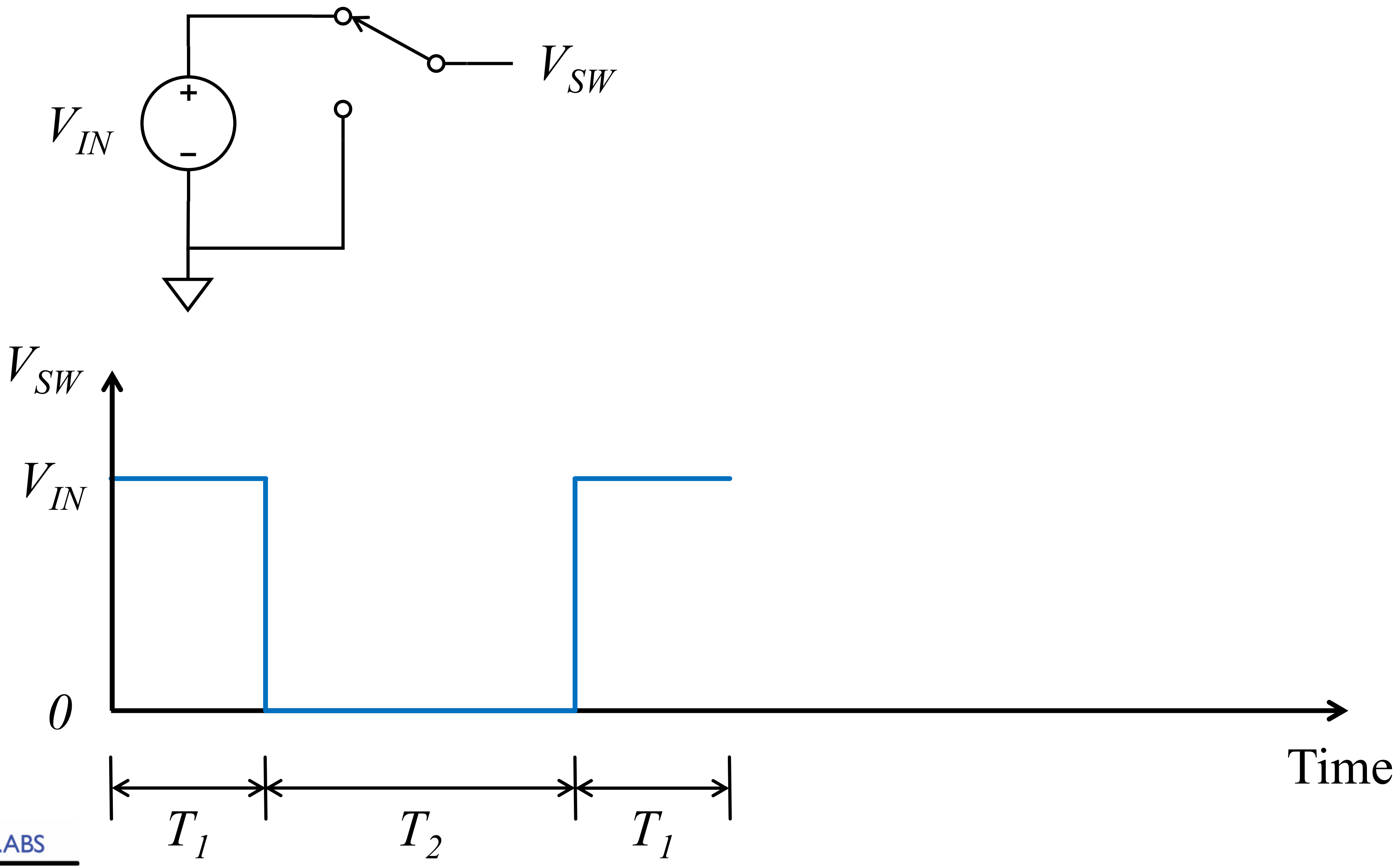
Delivering
Voltage And Power
From Input To Output
With 100% Efficiency
(No Loss!)

Switch Mode Concept

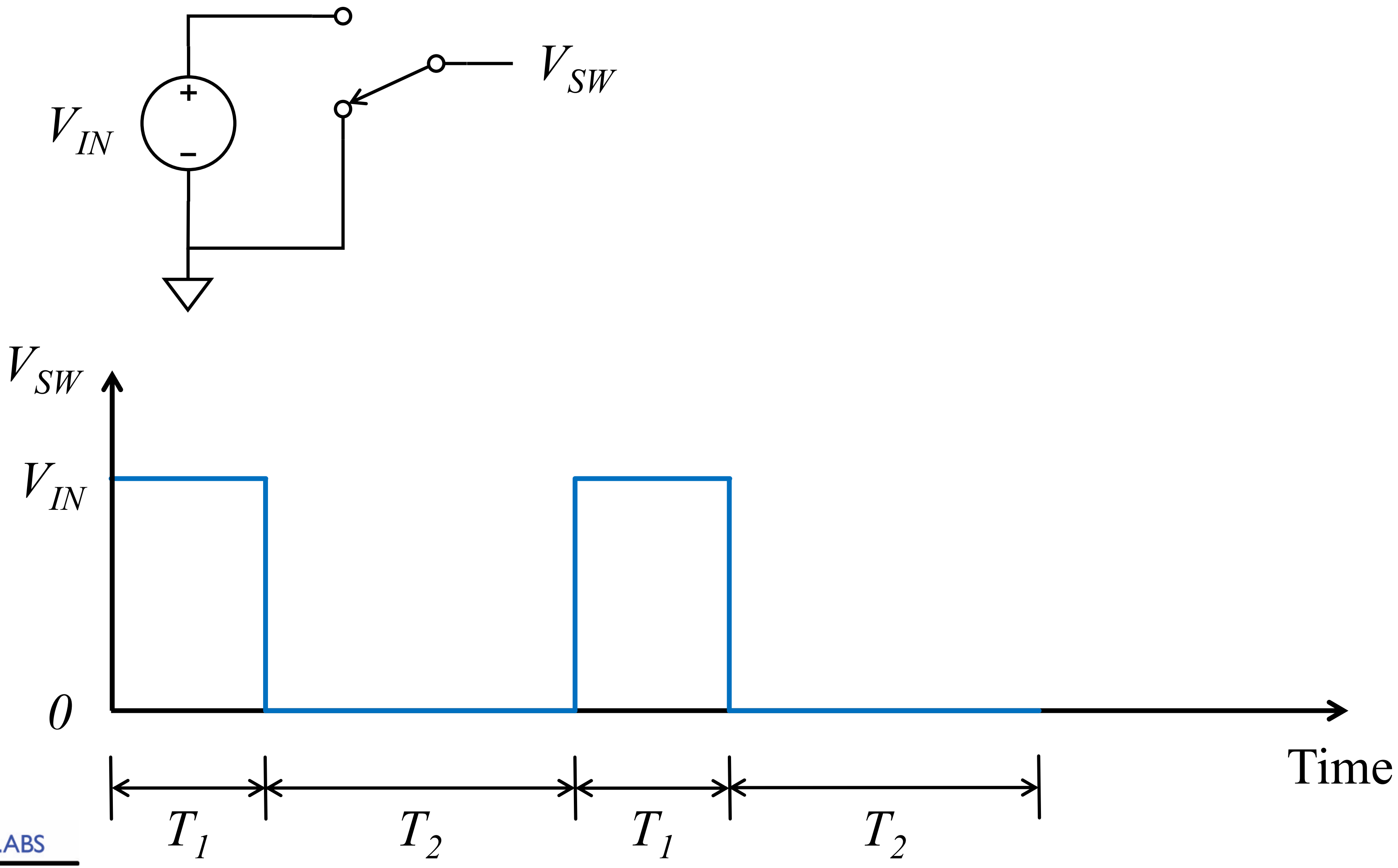


No Voltage Or Power
From Input To Output
But Also No Loss

Switch Mode Concept

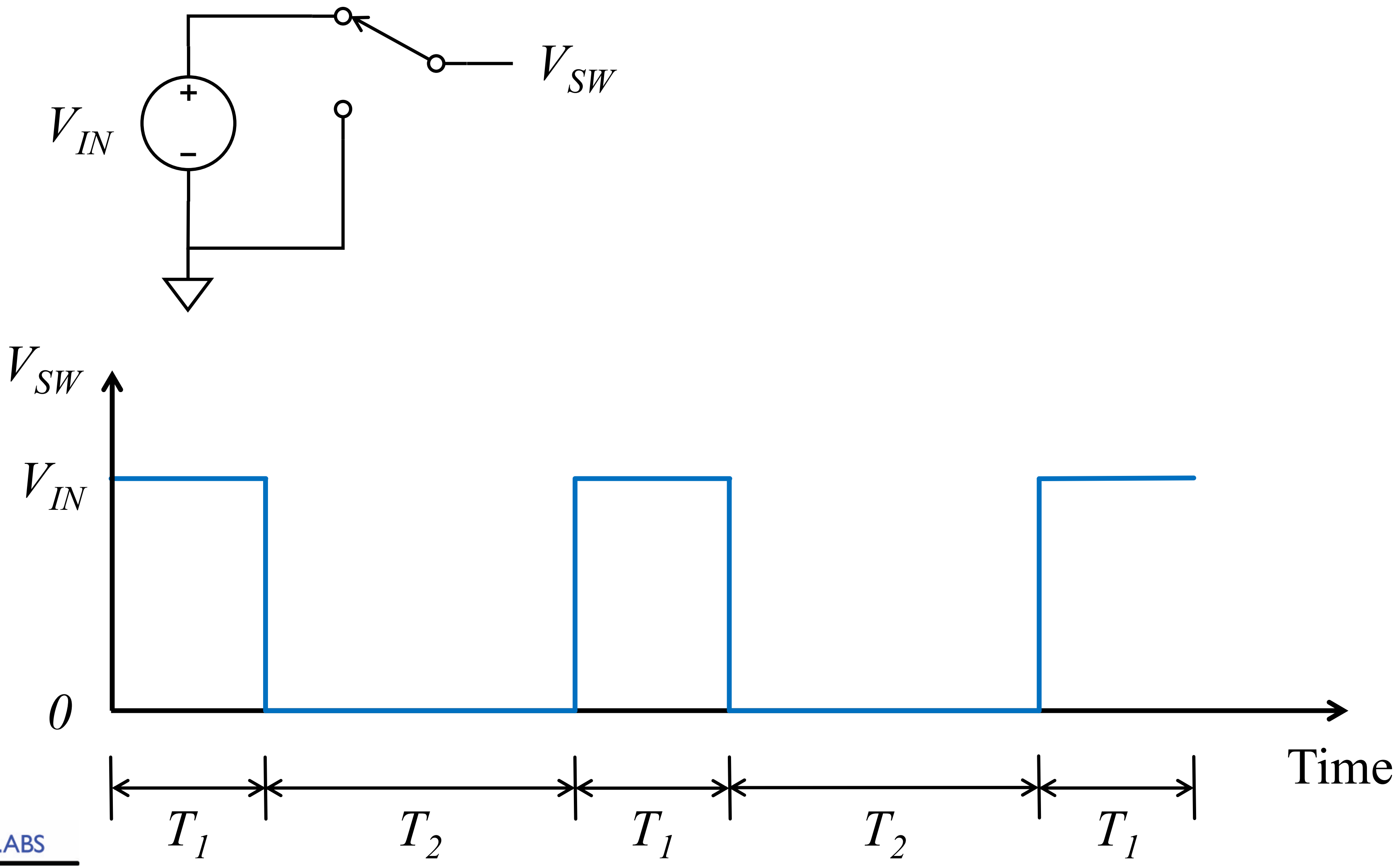


Switch Mode Concept



EMBEDDED POWER LABS

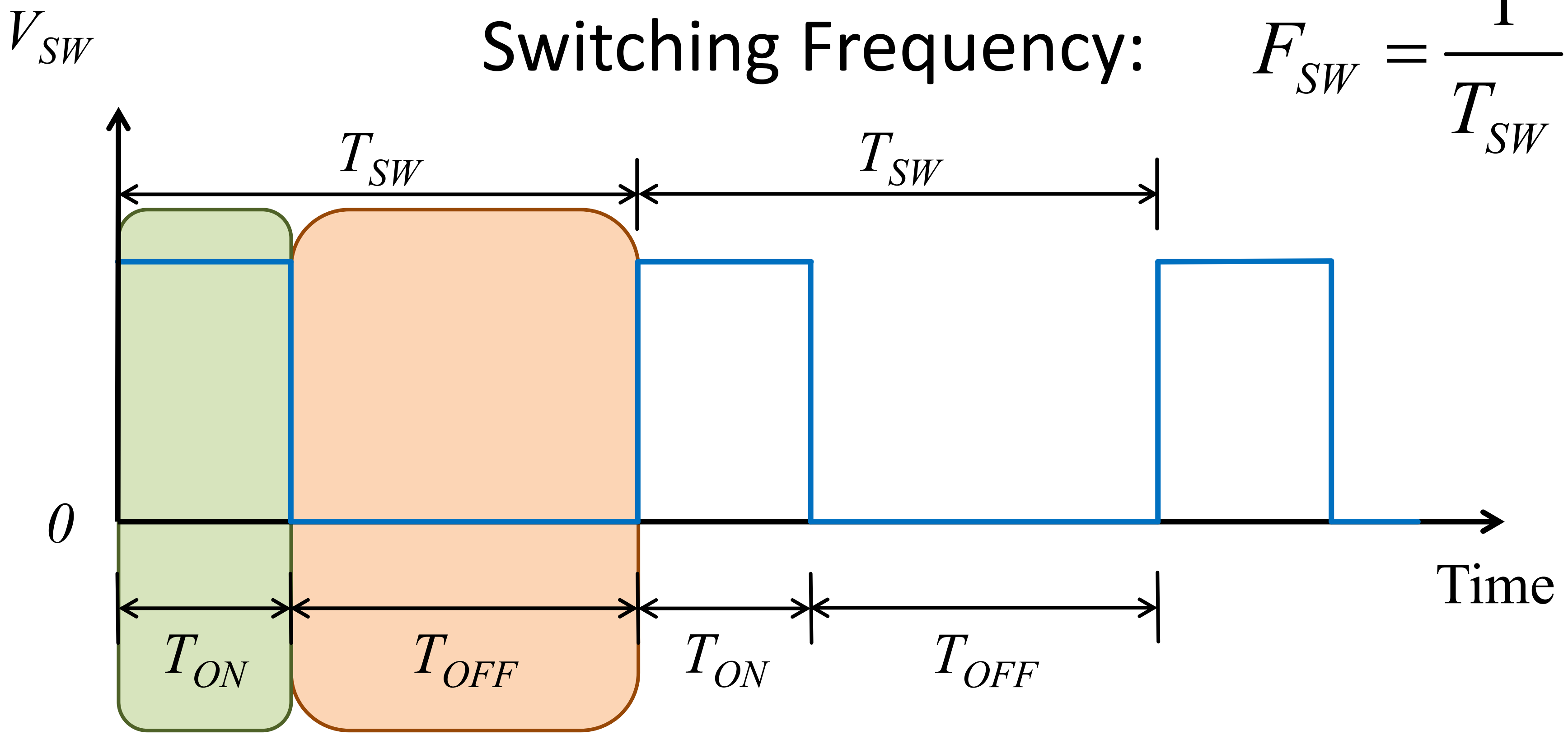
Switch Mode Concept



Terminology

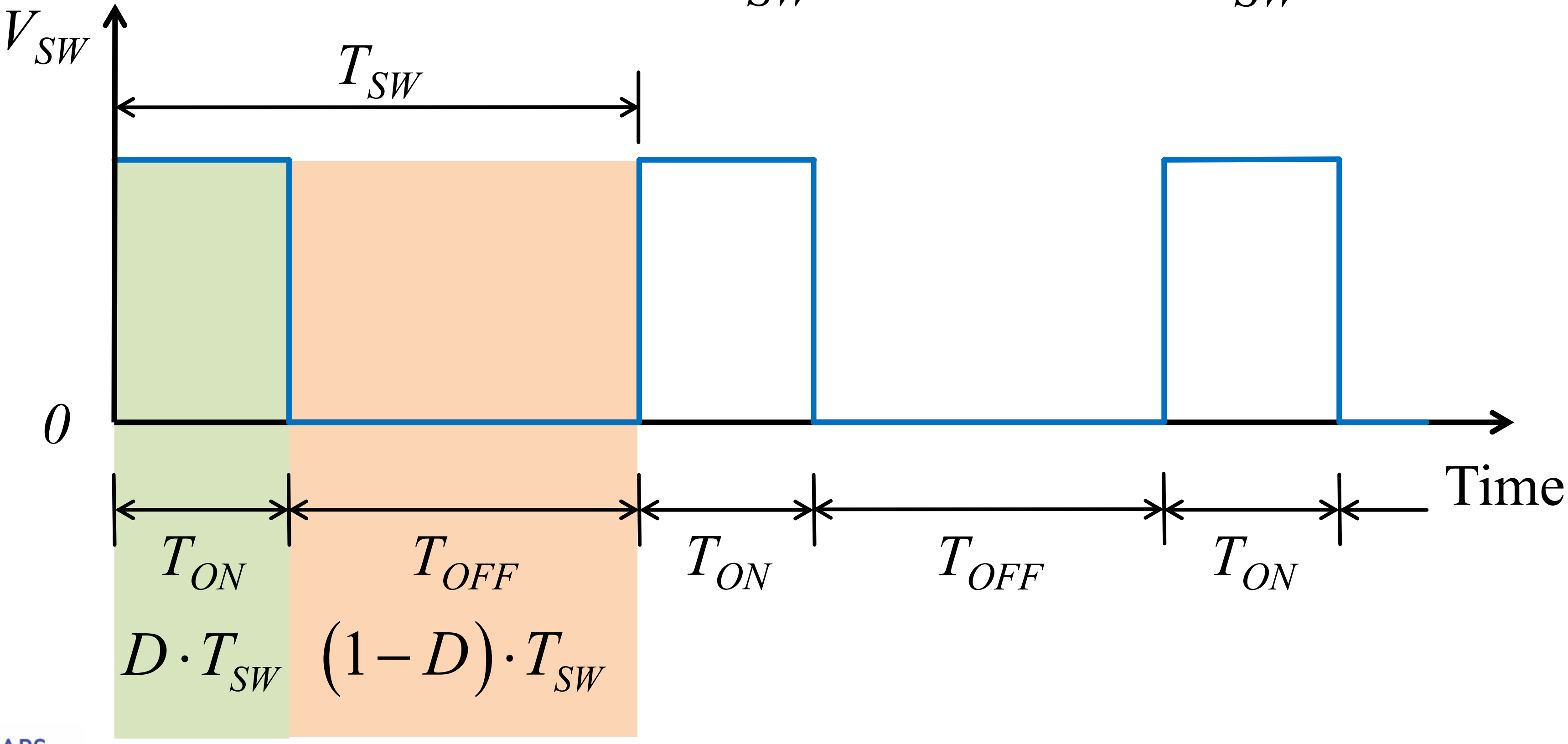
Switching Period: $T_{SW} = T_{ON} + T_{OFF}$

Switching Frequency: $F_{SW} = \frac{1}{T_{SW}}$



Terminology

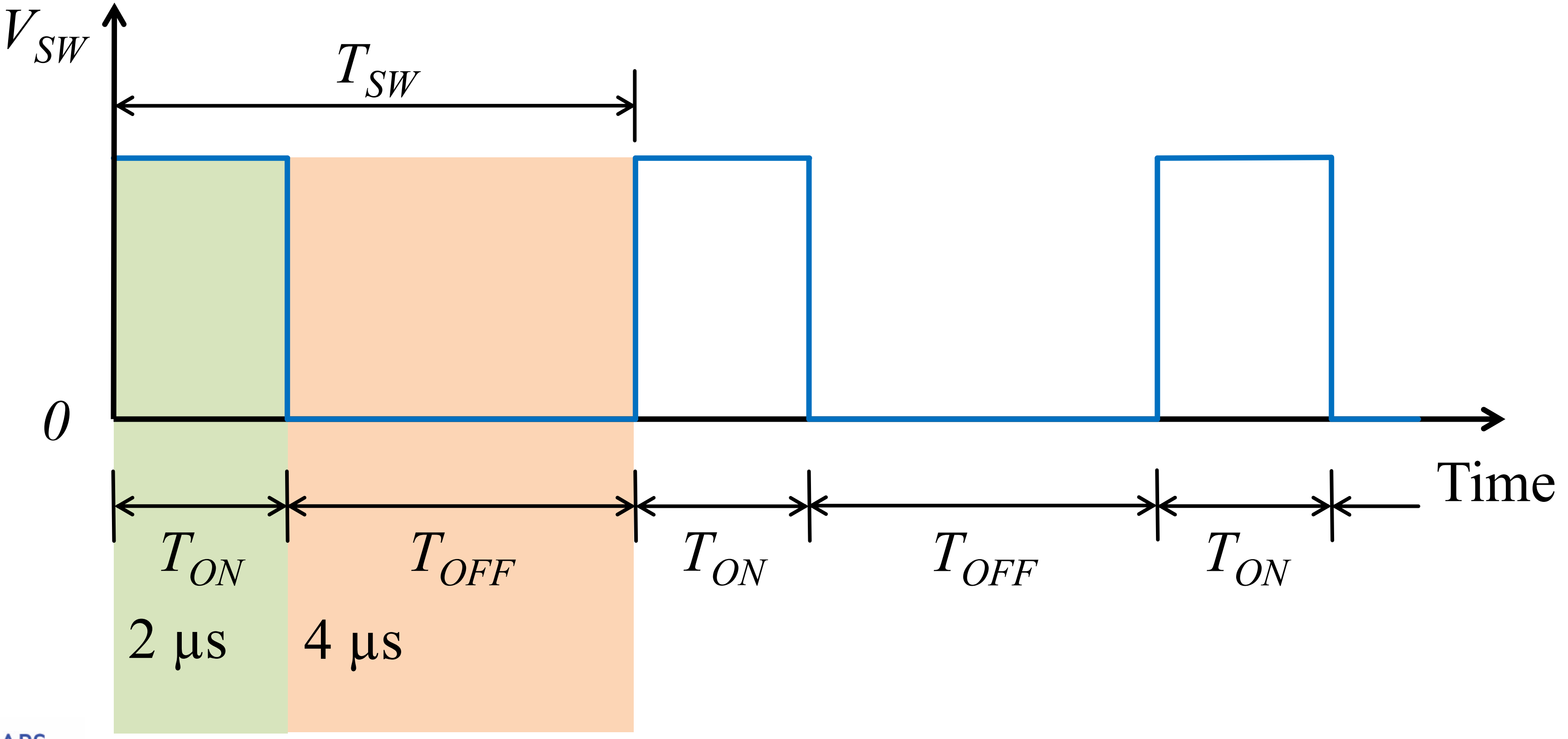
Duty Cycle: $D = \frac{T_{ON}}{T_{SW}}$ $D' = \frac{T_{OFF}}{T_{SW}} = 1 - D$



Example

$$T_{SW} = T_{ON} + T_{OFF} = 2\,\mu s + 4\,\mu s = 6\,\mu s$$

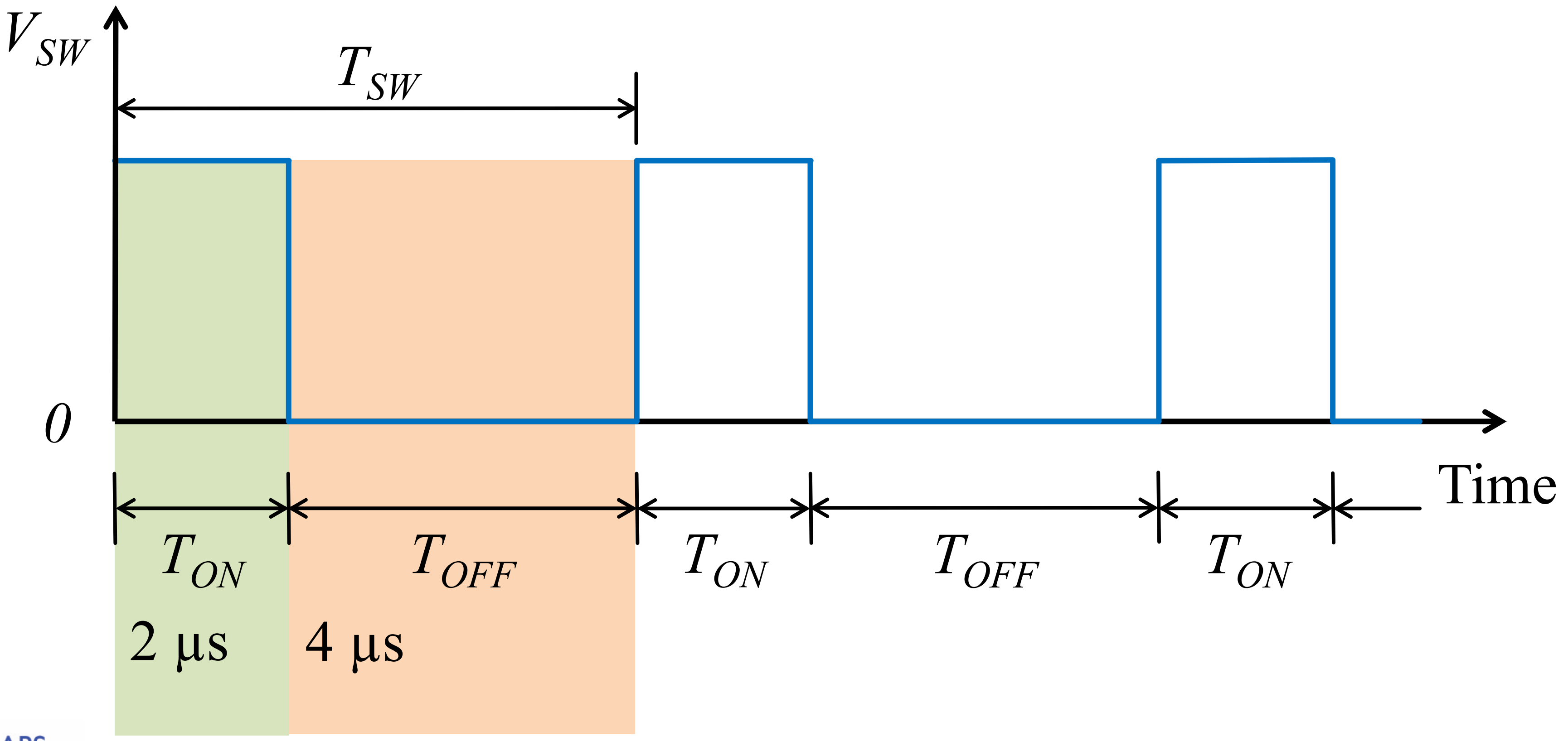
$$F_{SW} = \frac{1}{T_{SW}} = \frac{1}{6\,\mu s} = 166.7\,\text{kHz}$$



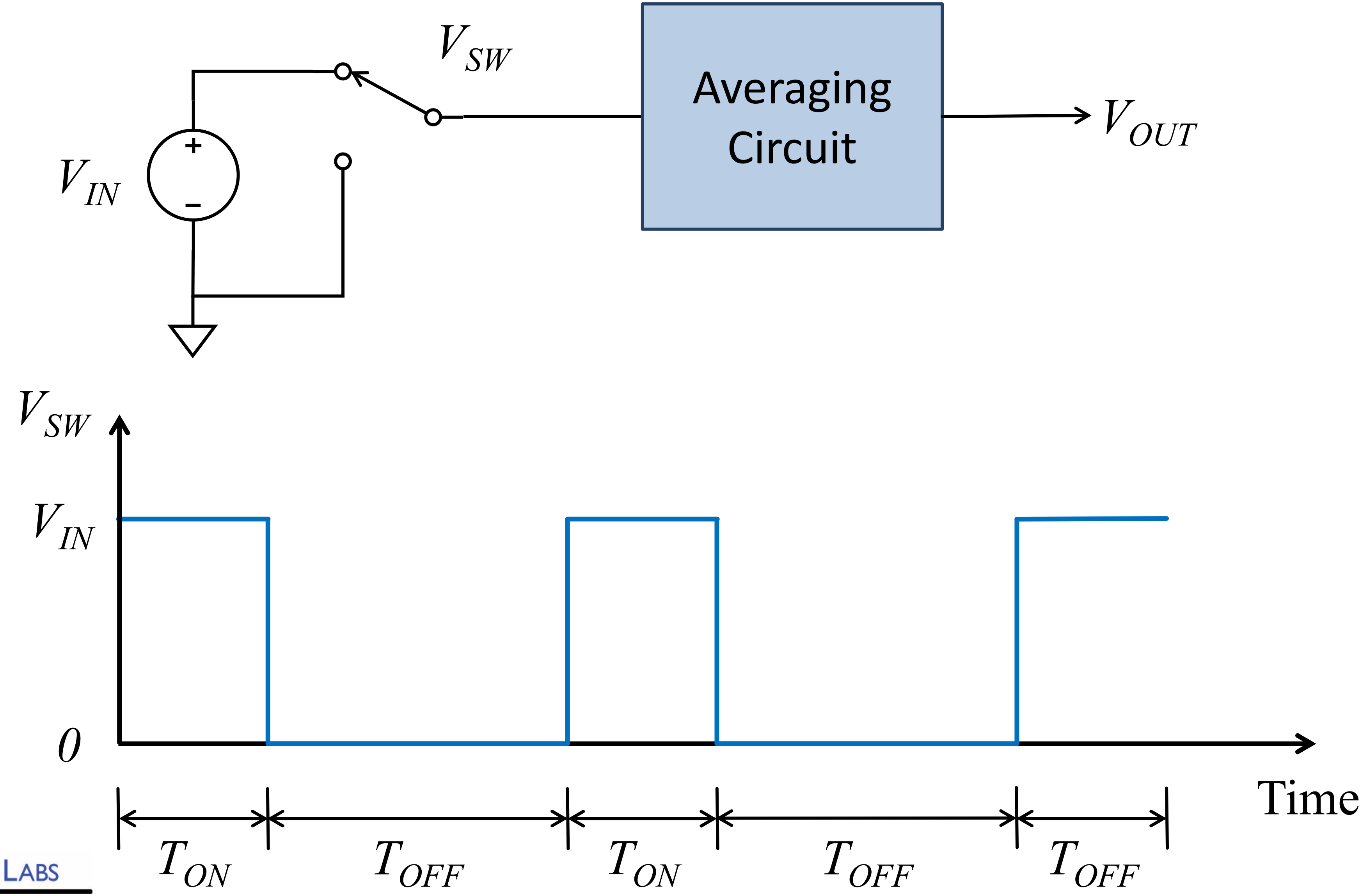
Example

$$D = \frac{T_{ON}}{T_{SW}} = \frac{2\,\mu s}{6\,\mu s} = \frac{1}{3} = 0.333$$

$$D' = \frac{T_{OFF}}{T_{SW}} = \frac{4\,\mu s}{6\,\mu s} = \frac{2}{3} = 0.667$$



Switch Mode Concept



EMBEDDED POWER LABS

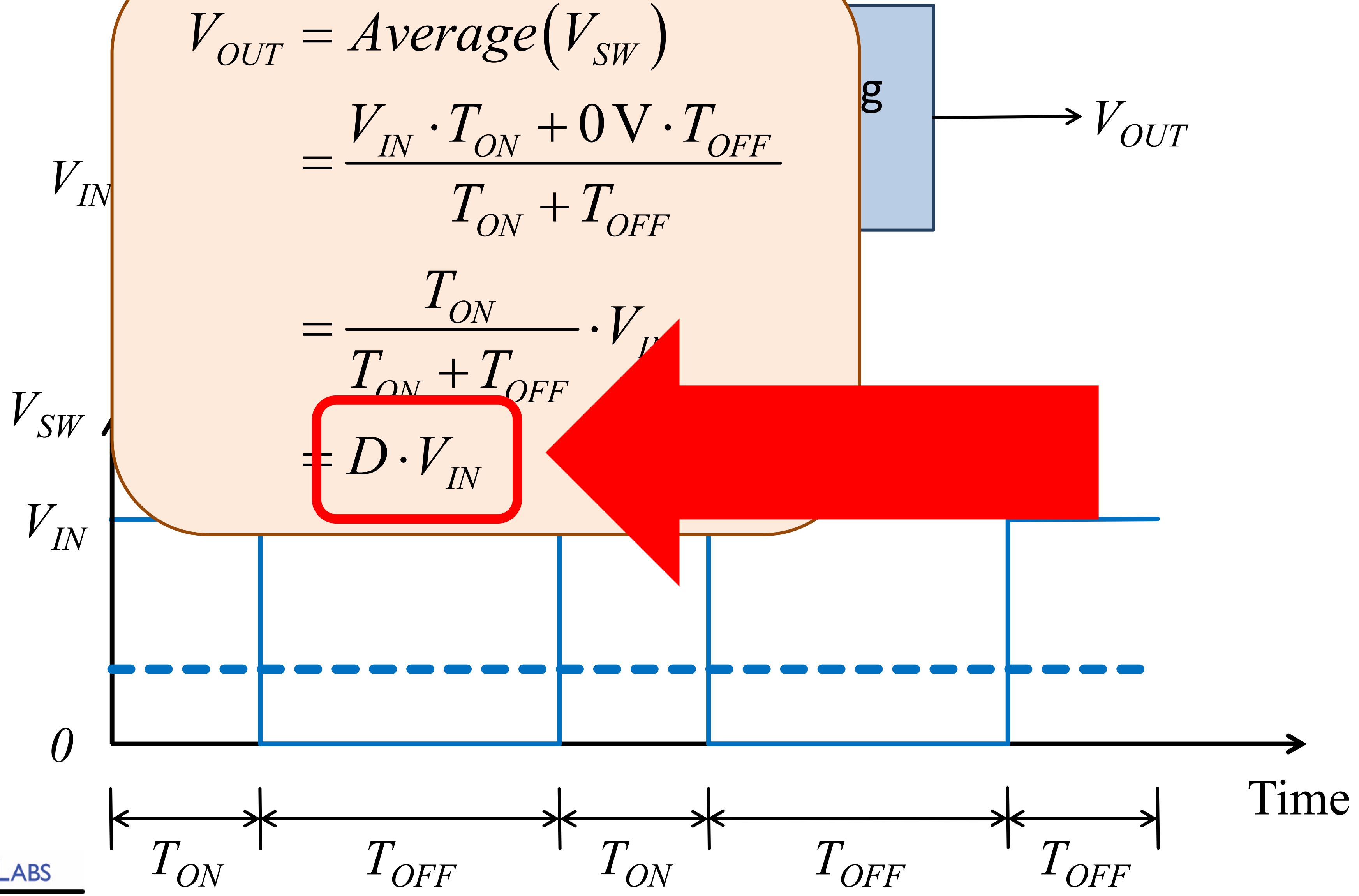
Switch Mode Concept

$$V_{OUT} = Average(V_{SW})$$

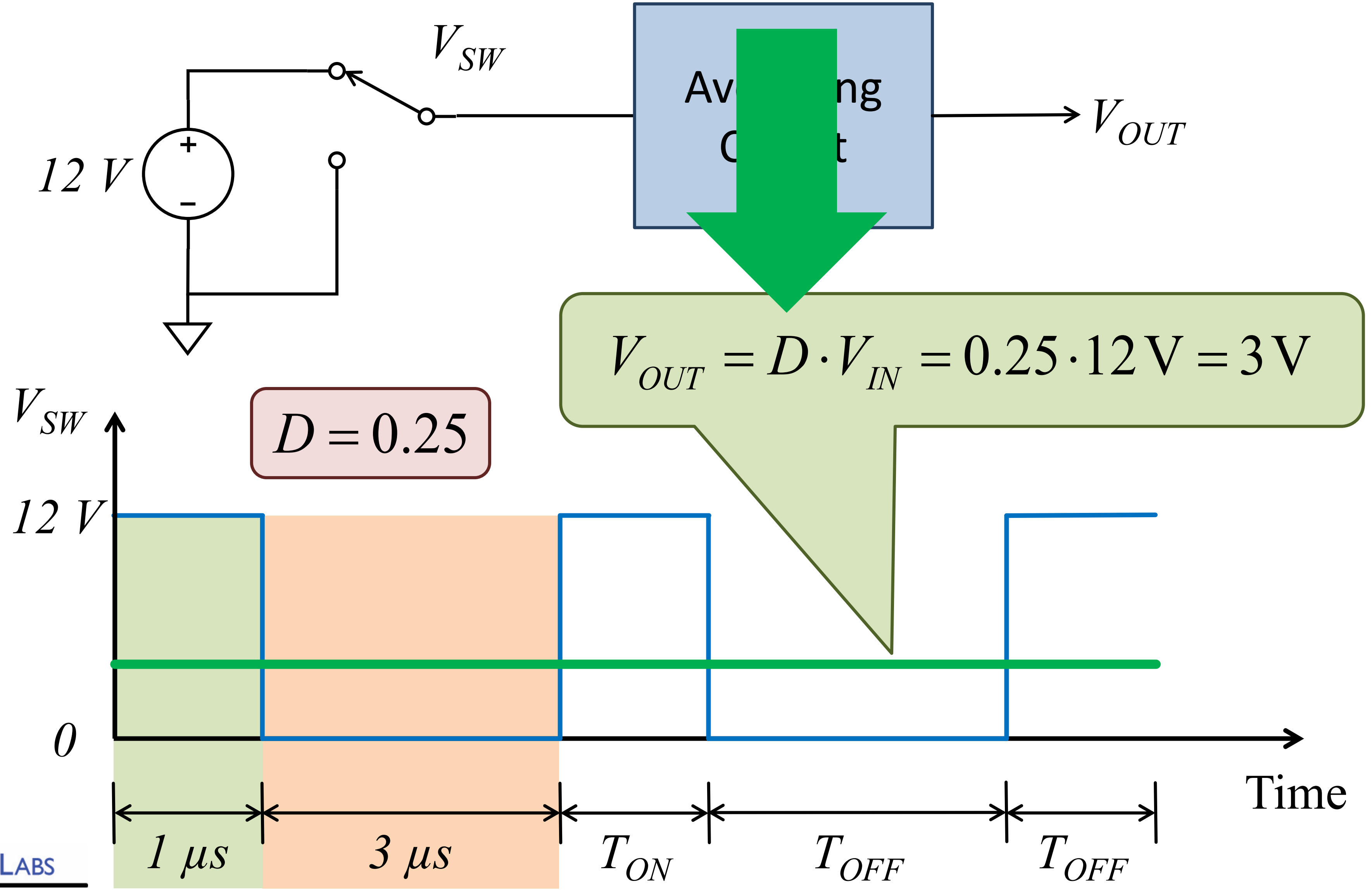
$$= \frac{V_{IN} \cdot T_{ON} + 0\,\mathrm{V} \cdot T_{OFF}}{T_{ON} + T_{OFF}}$$

$$= \frac{T_{ON}}{T_{ON} + T_{OFF}} \cdot V_{IN}$$

$$= D \cdot V_{IN}$$

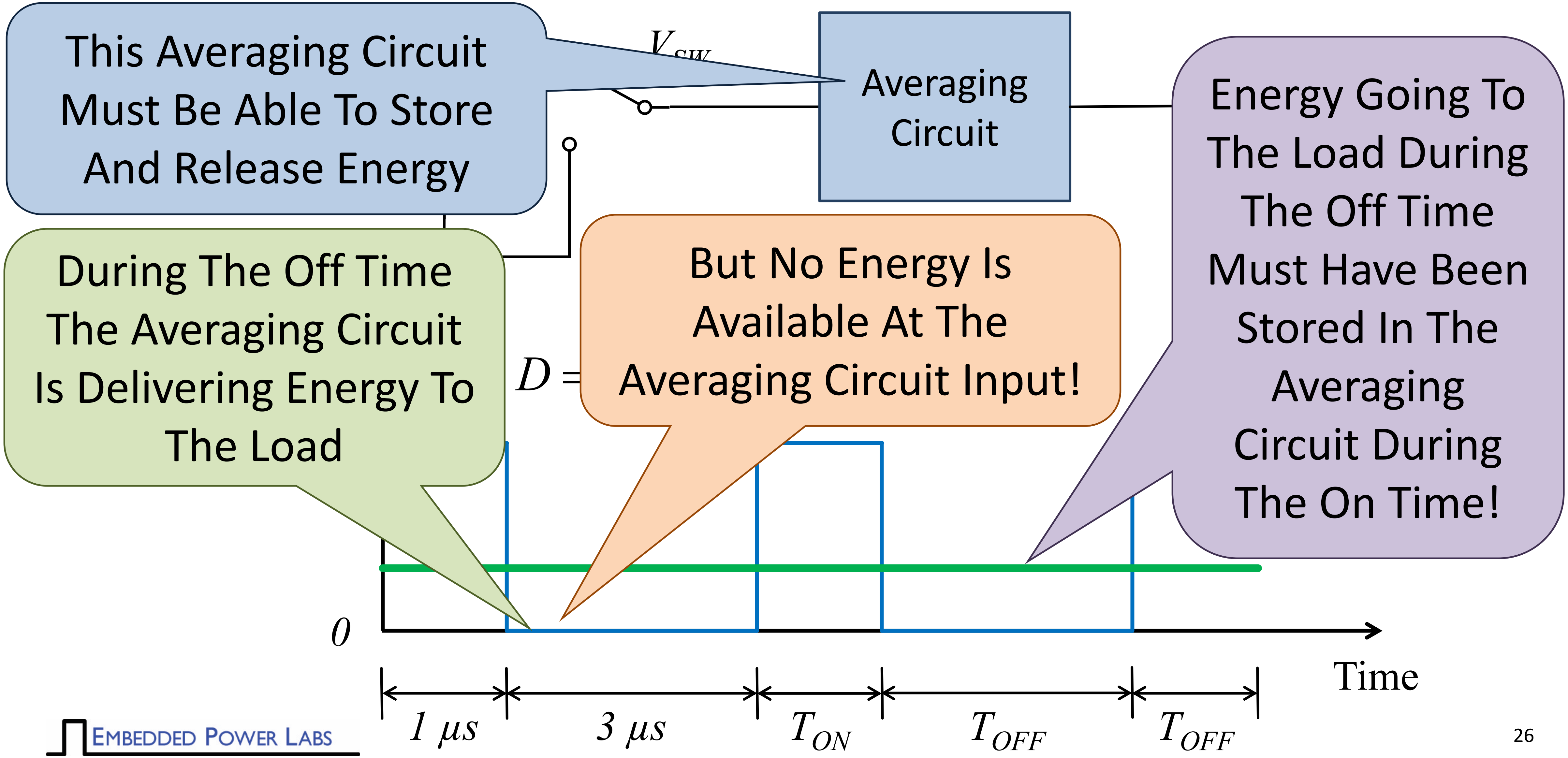


Example



V_{SW}
Av ng
C t
V_{OUT}
12 V
$V_{OUT} = D \cdot V_{IN} = 0.25 \cdot 12\,\text{V} = 3\,\text{V}$
V_{SW}
$D = 0.25$
12 V
0
Time
1 μs
3 μs
T_{ON}
T_{OFF}
T_{OFF}

Energy Storage Requirement



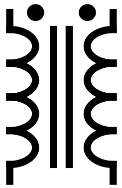
Power Conversion Components

Building Blocks

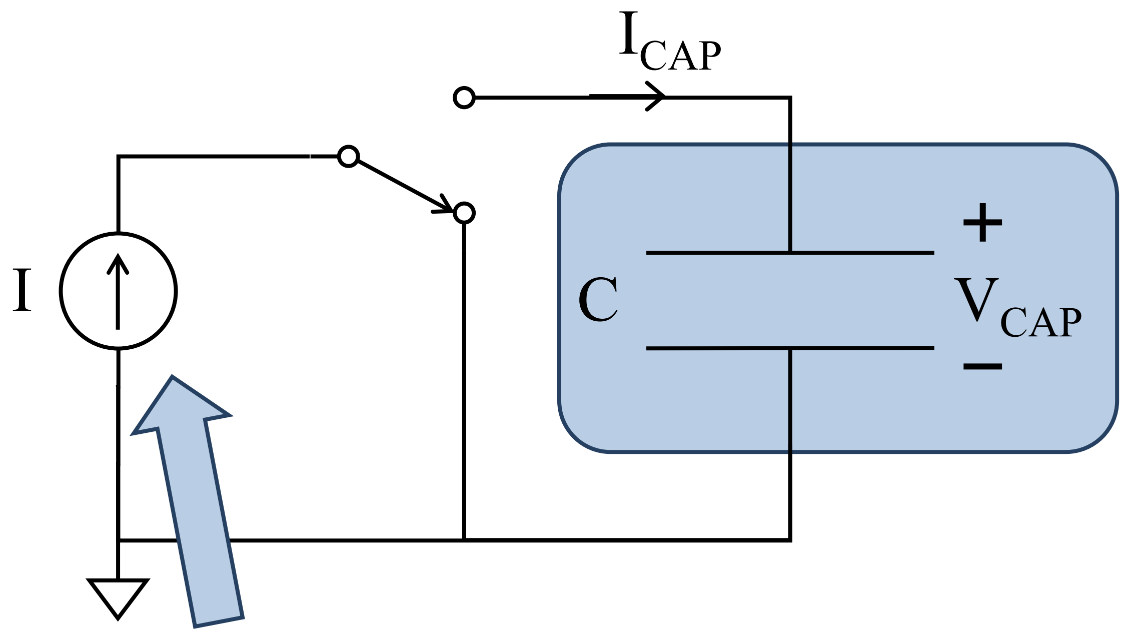
- Switches To Control The Flow Of Energy

- Containers To Store Energy

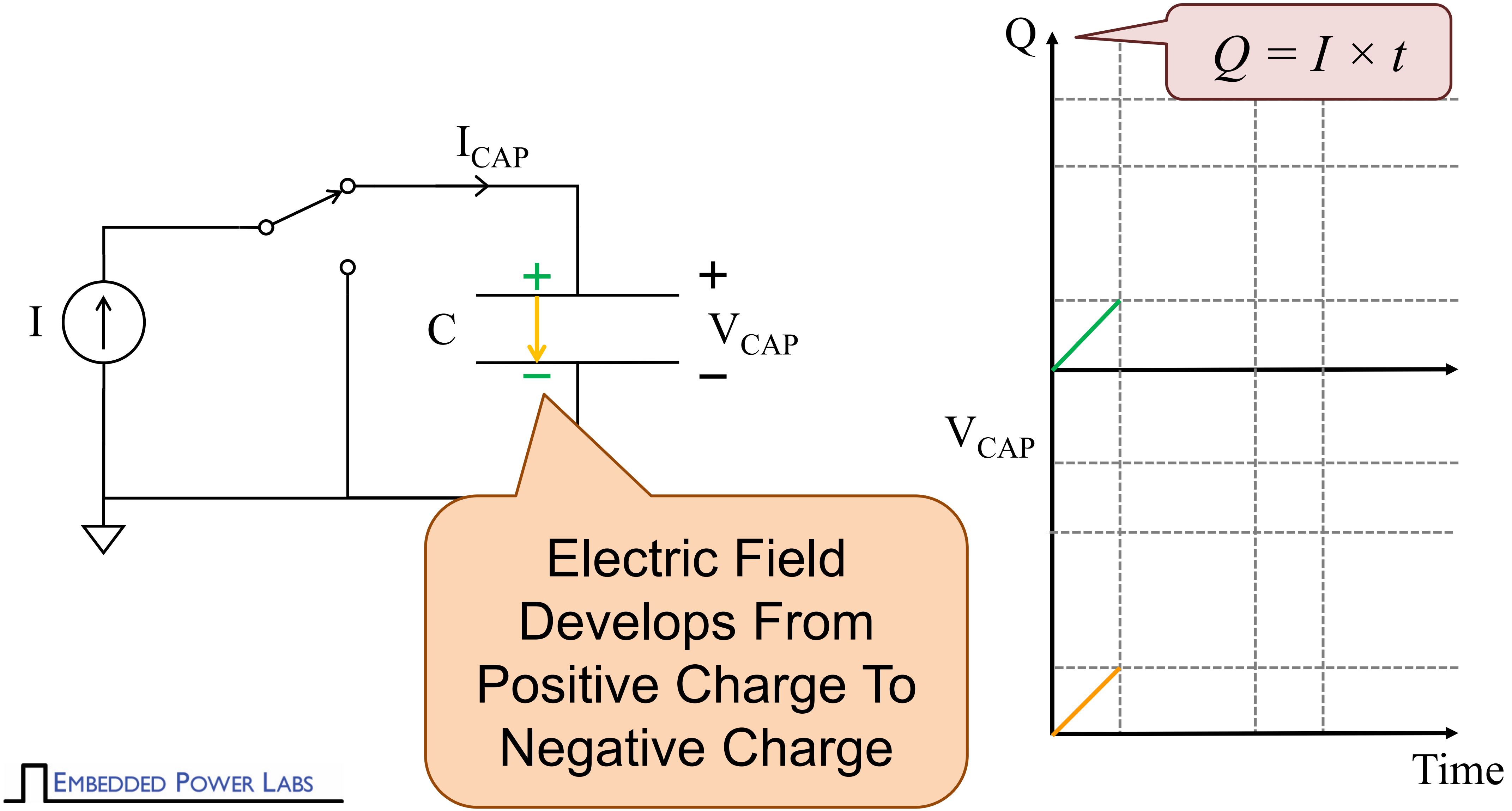
- Transformers



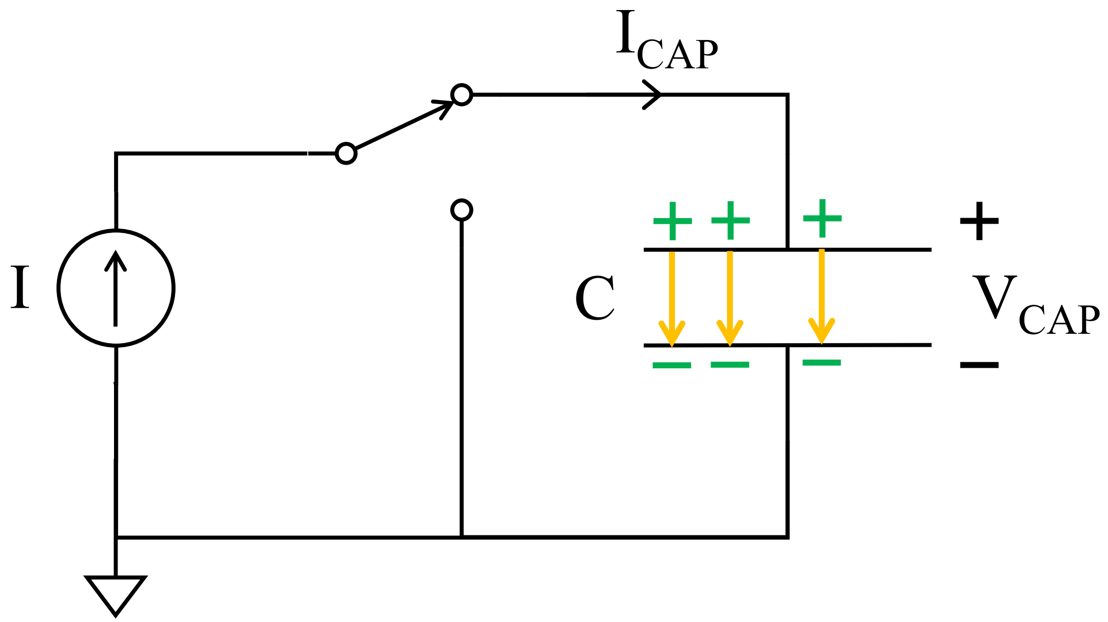
Energy Storage: Capacitor

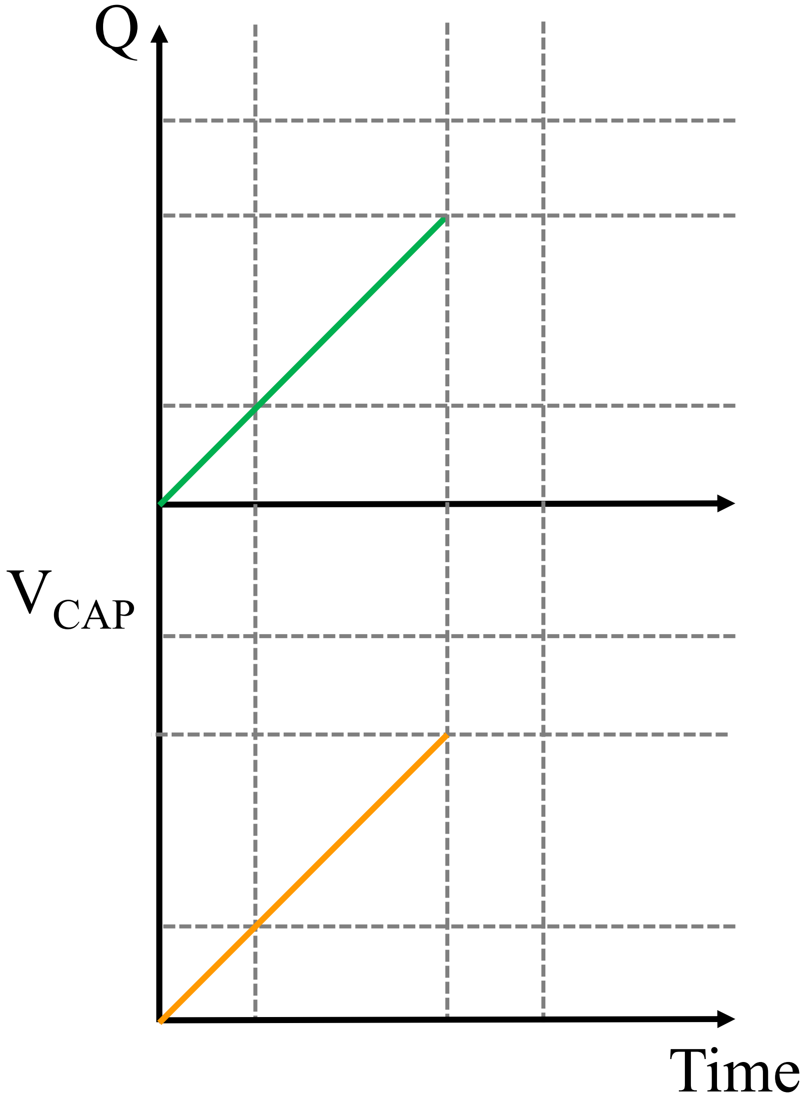


Energy Storage: Capacitor

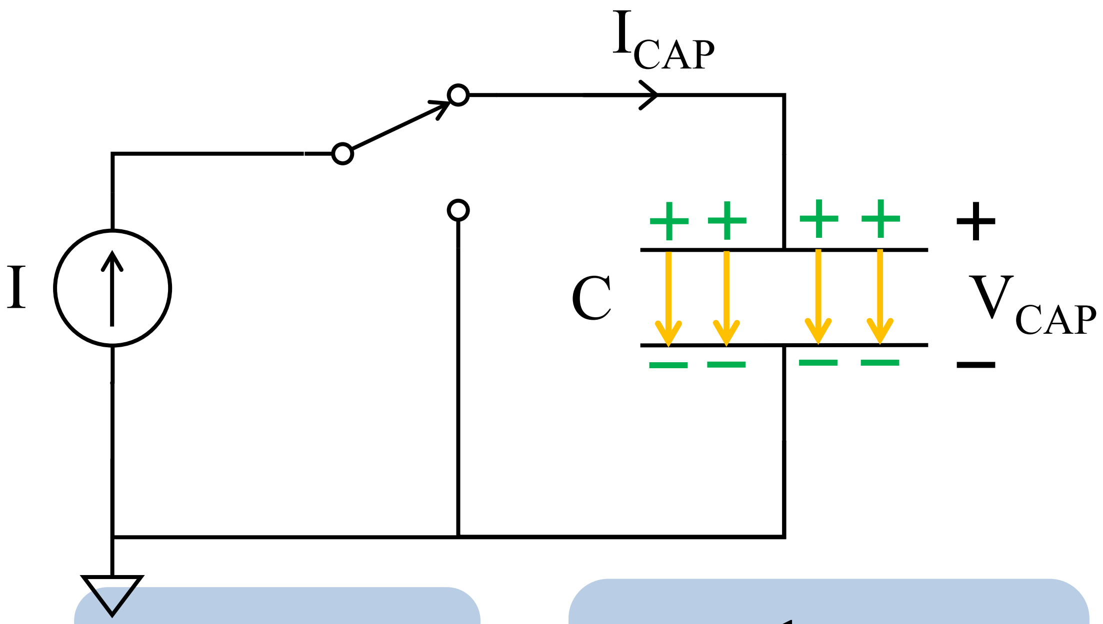


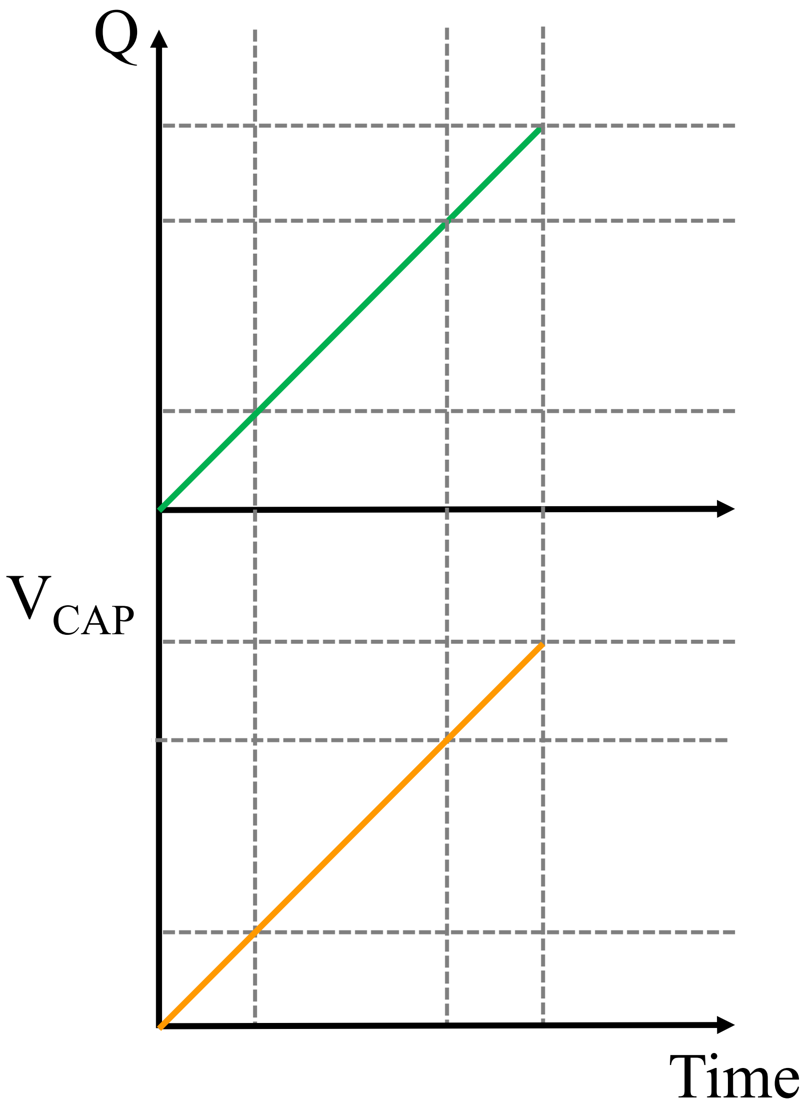
Energy Storage: Capacitor





Energy Storage: Capacitor



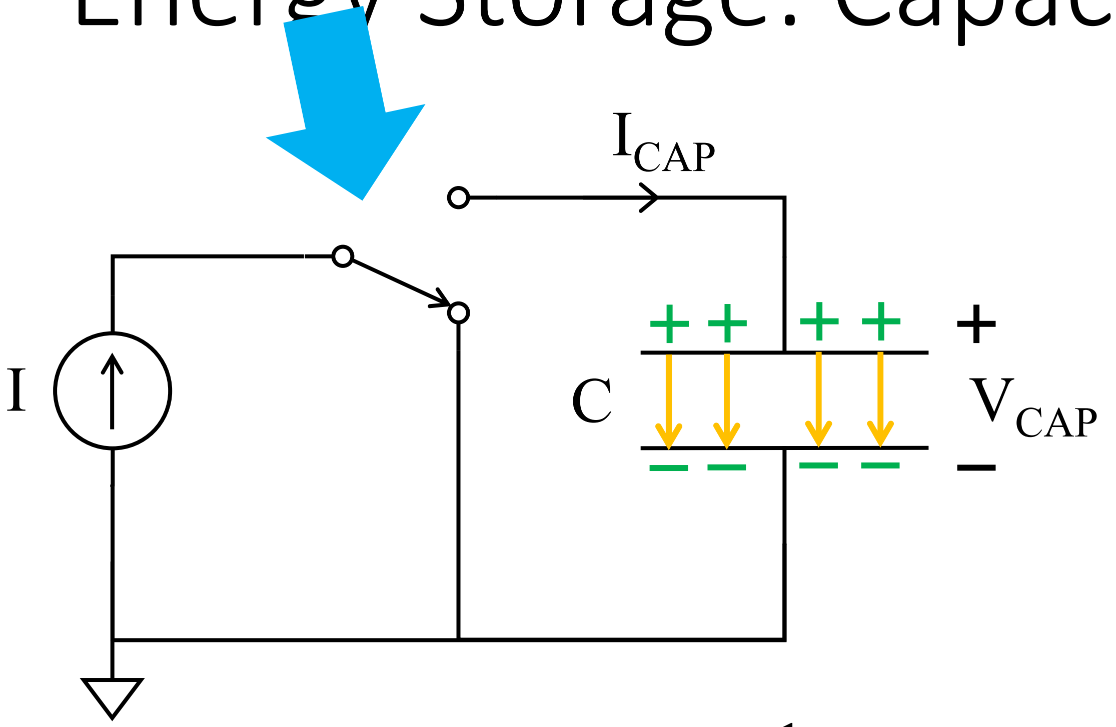


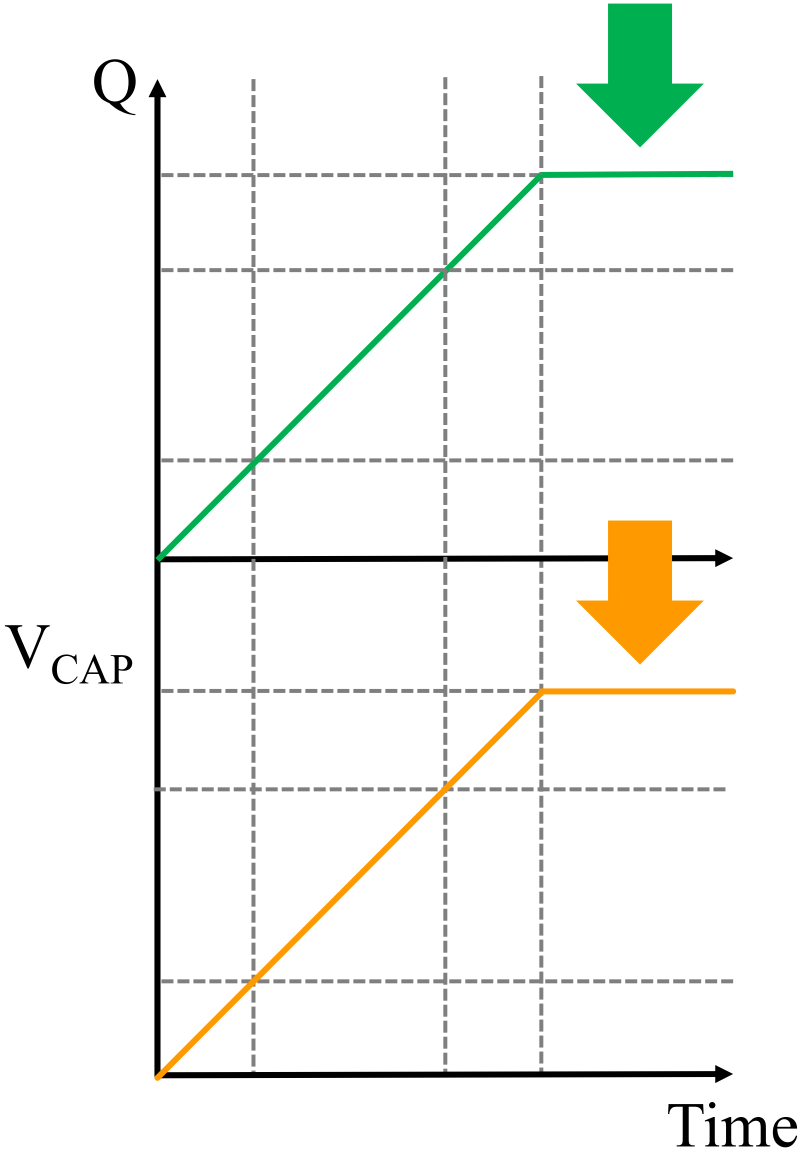
$$Q = C \cdot V$$

$$E = \frac{1}{2} \cdot C \cdot$$

$$I = \frac{dQ}{dt} = \frac{d\left(C \cdot V\right)}{dt} = C \cdot \frac{dV}{dt}$$

Energy Storage: Capacitor





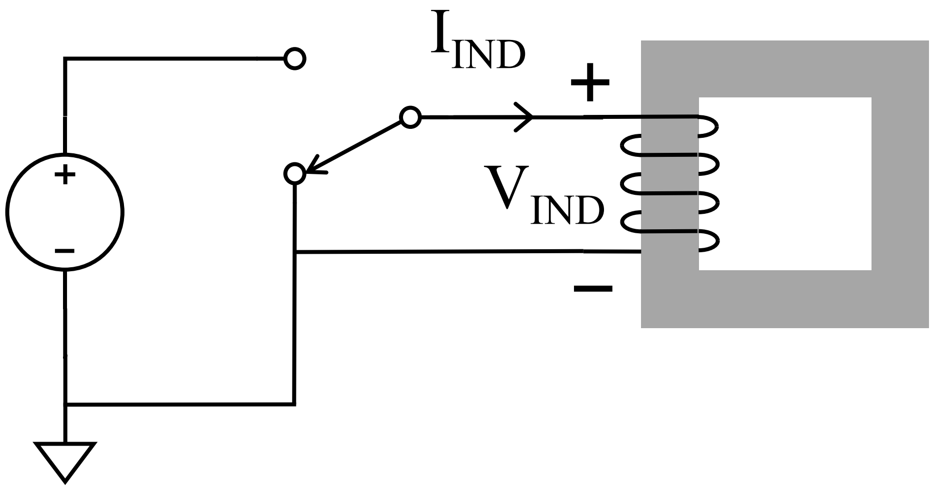
$$Q = C \cdot V$$

$$E = \frac{1}{2} \cdot C \cdot V^2$$

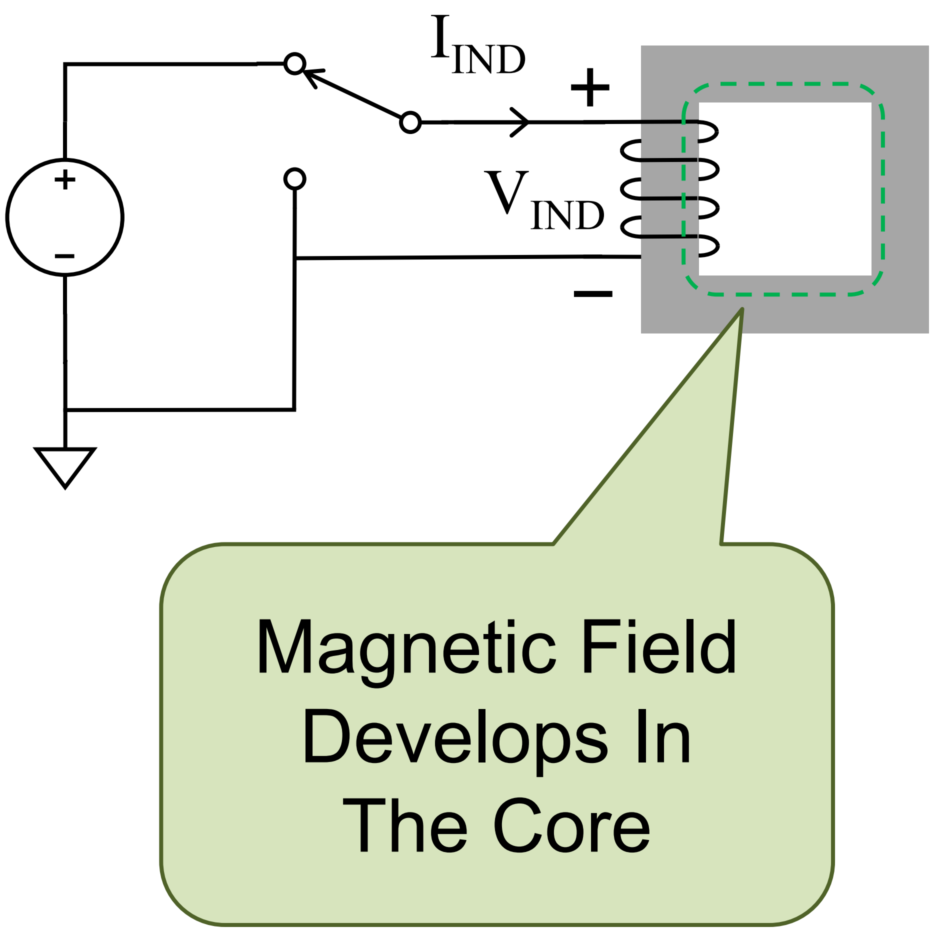
$$I = \frac{dQ}{dt} = \frac{d(C \cdot V)}{dt} = C \cdot \frac{dV}{dt}$$

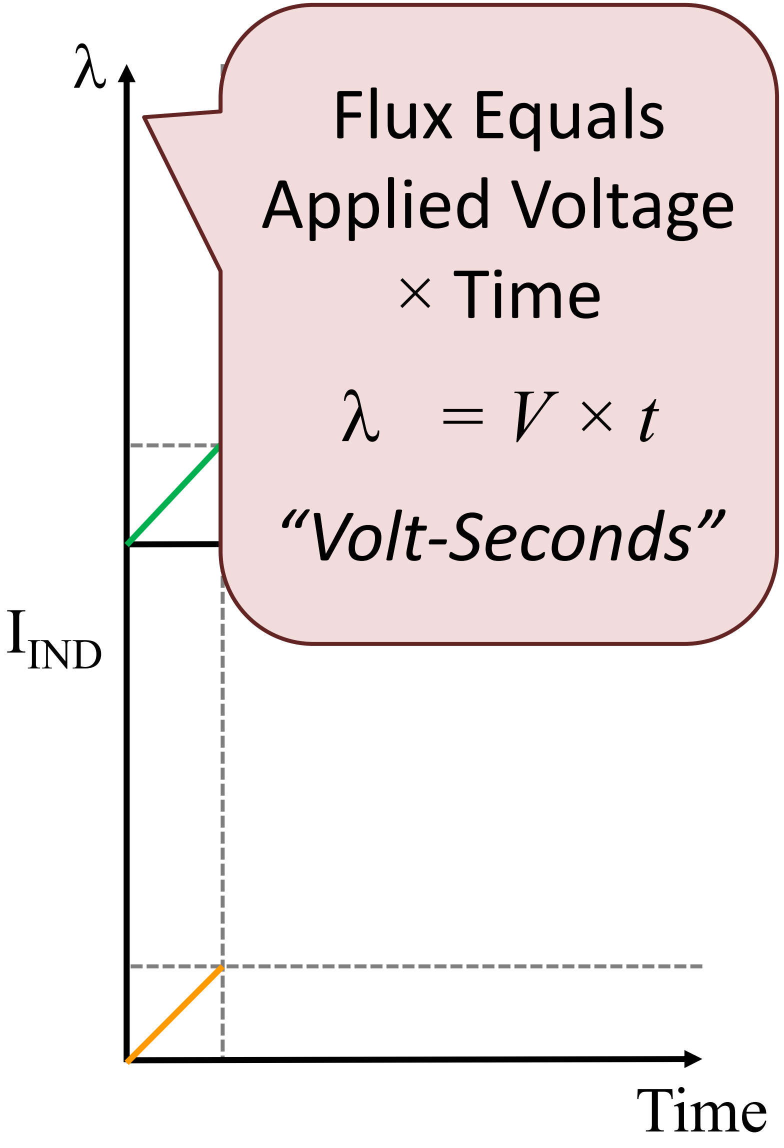
$$0 = \frac{0}{dt} = \frac{d(C \cdot V)}{dt} = C \cdot \frac{0}{dt}$$

Energy Storage: Inductor

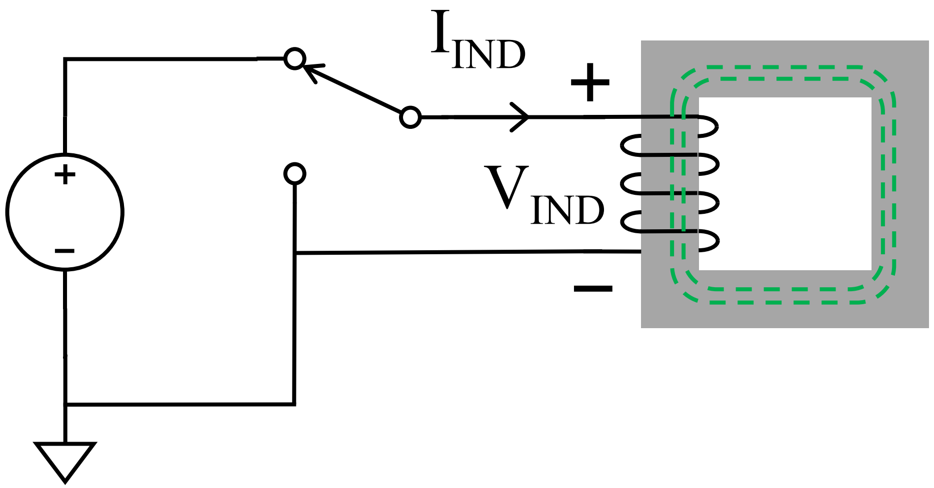


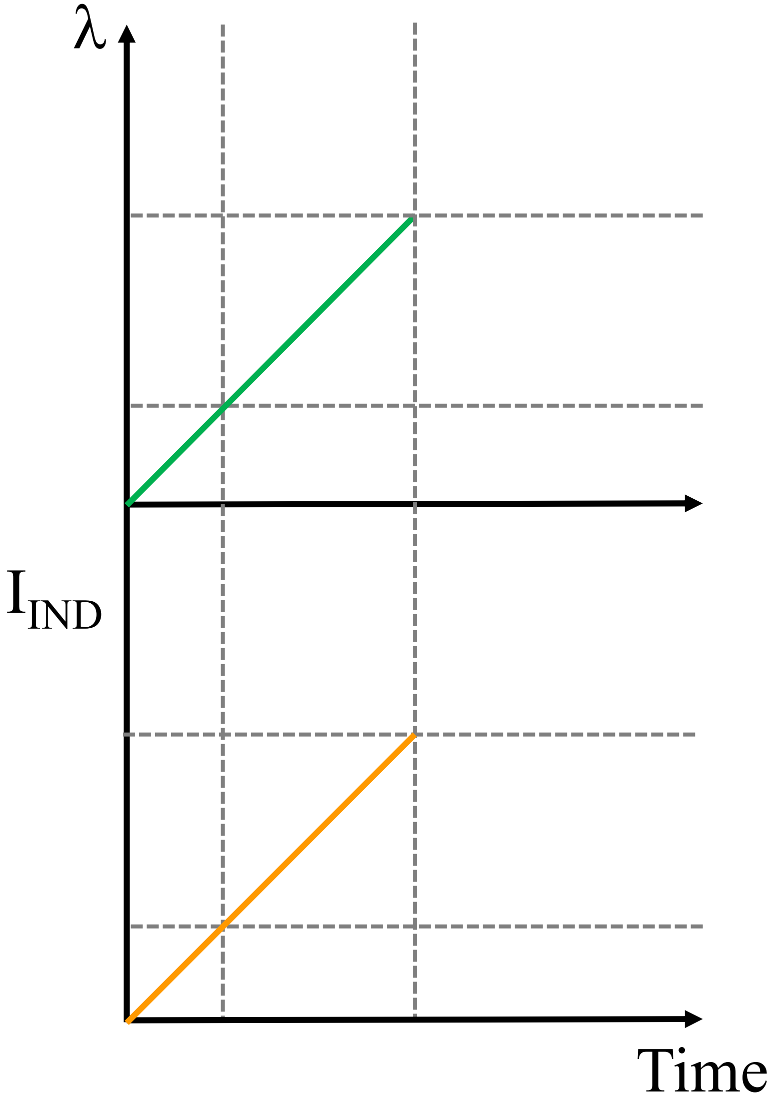
Energy Storage: Inductor



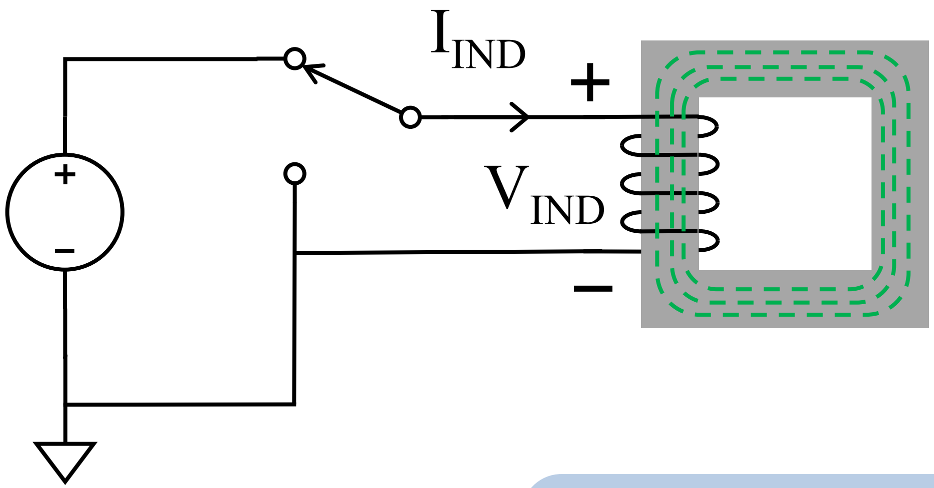


Energy Storage: Inductor





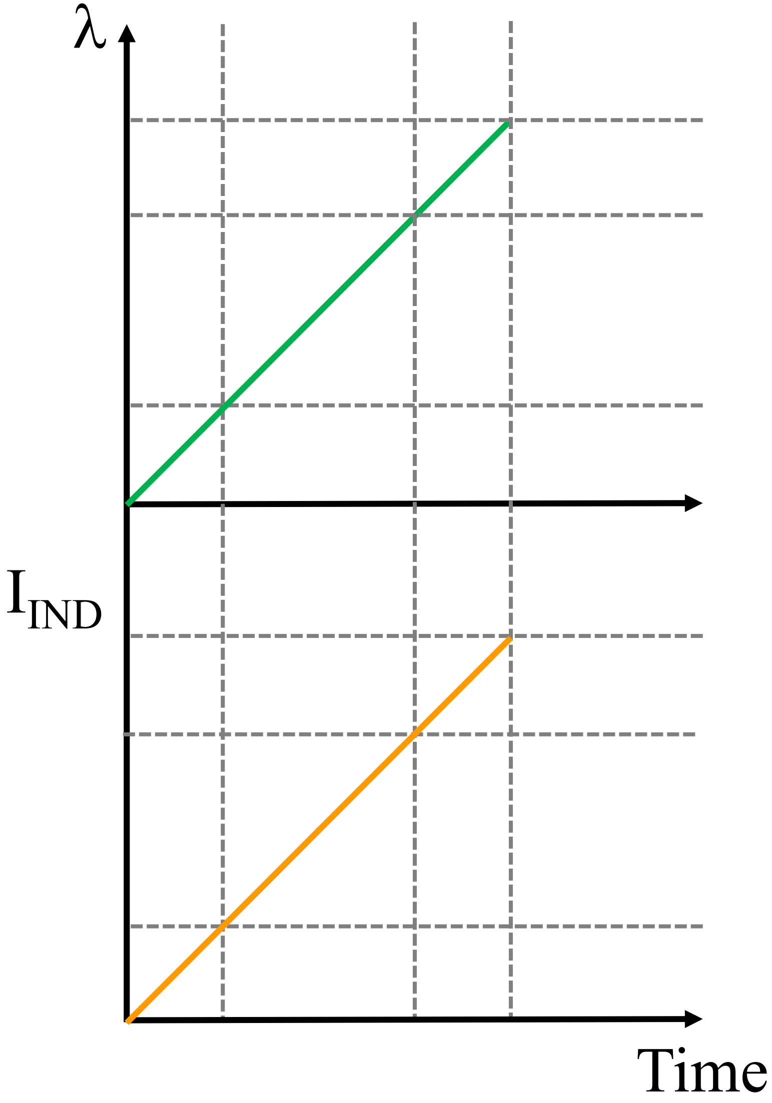
Energy Storage: Inductor



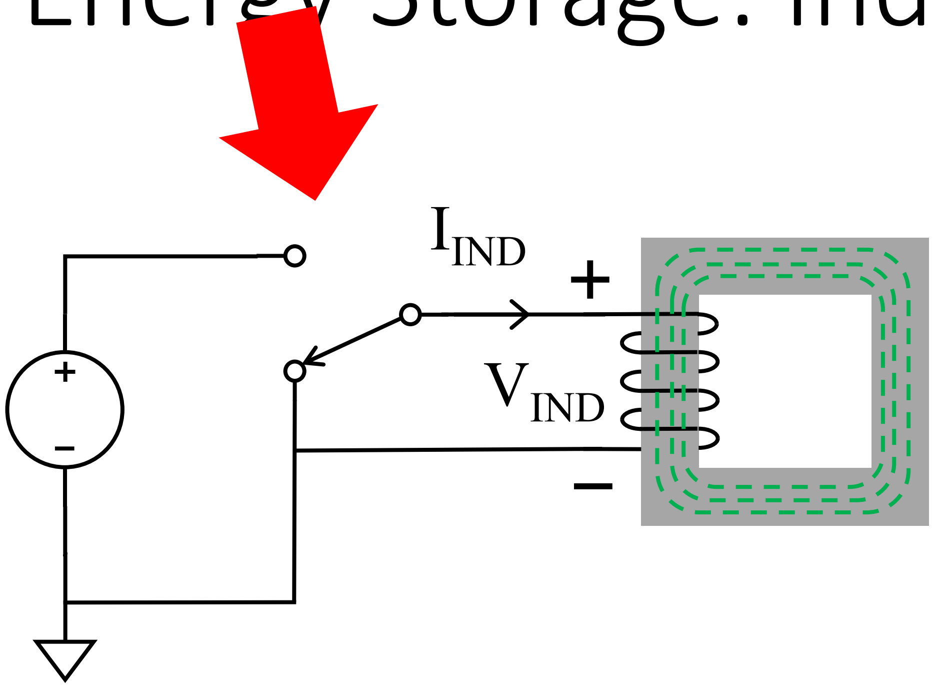
$$\lambda = L \cdot I$$

$$E = \frac{1}{2} \cdot L \cdot I^2$$

$$V = \frac{d\lambda}{dt} = \frac{d(L \cdot I)}{dt} = L \cdot \frac{dI}{dt}$$

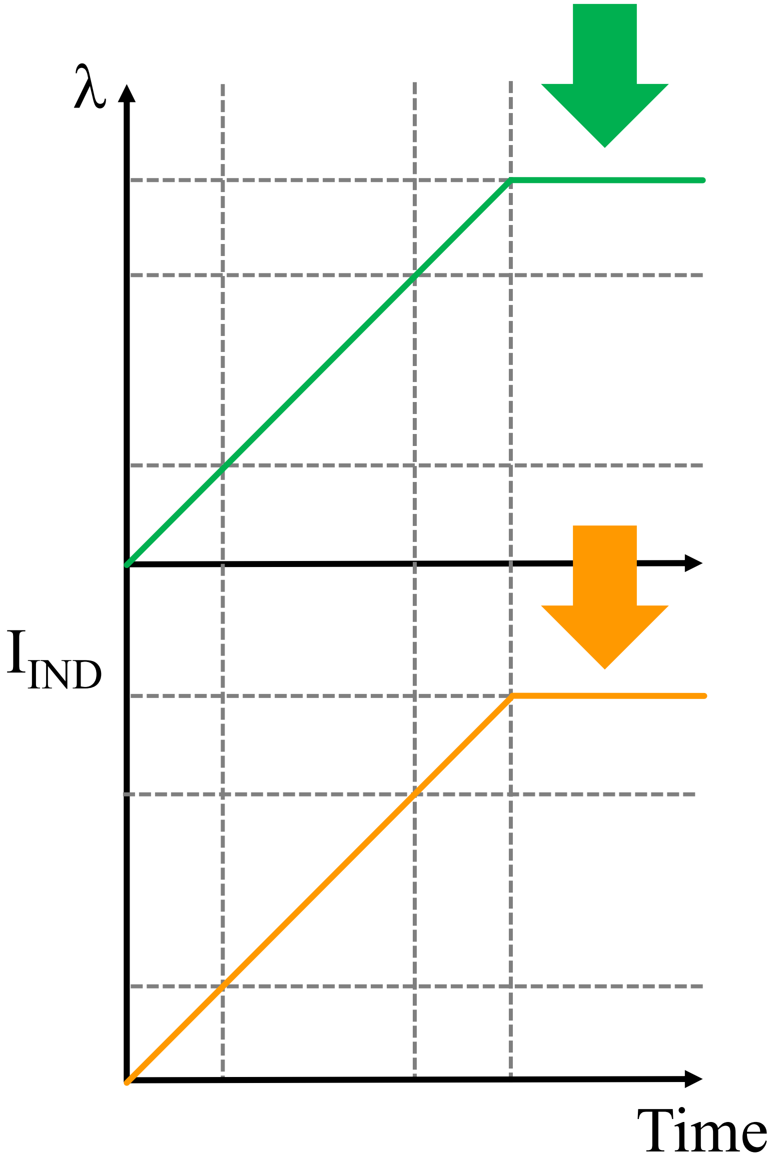


Energy Storage: Inductor

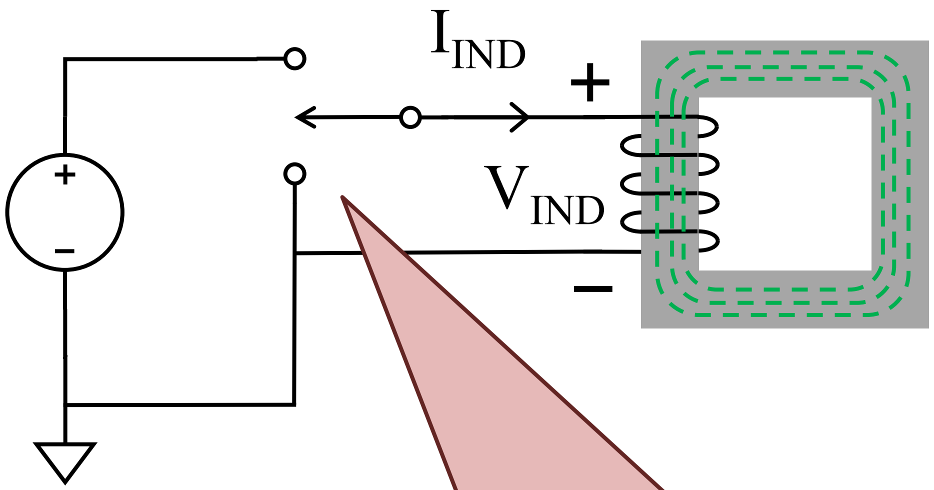


$$\lambda = L \cdot I \qquad E = \frac{1}{2} \cdot L \cdot I^2$$

$$\cancel{V}\,0 = \frac{\cancel{d\lambda}\,0}{dt} = \frac{d\left(L \cdot I\right)}{dt} = L \cdot \frac{\cancel{dI}\,0}{dt}$$



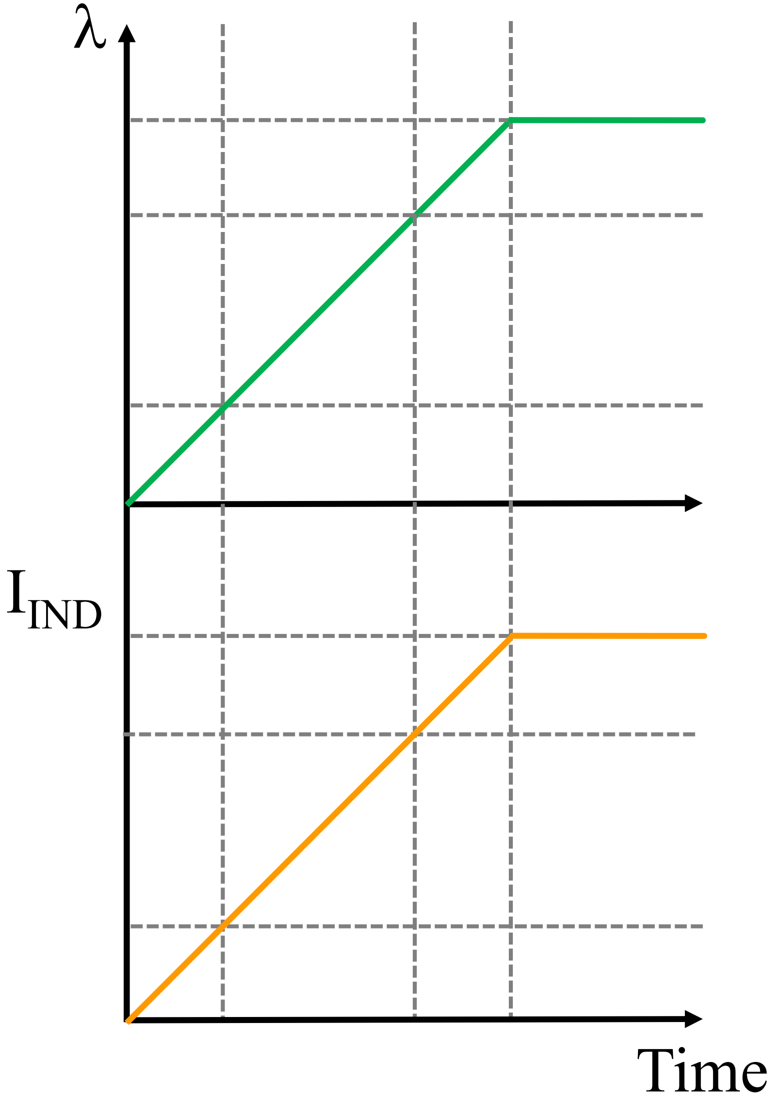
Energy Storage: Inductor



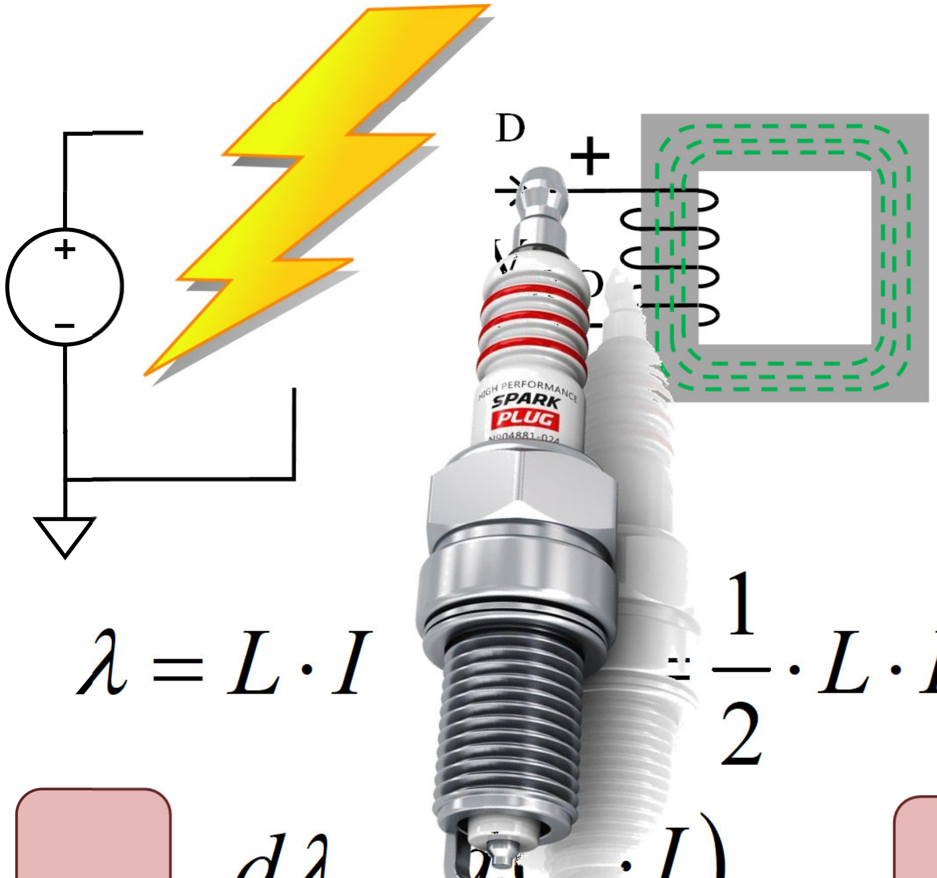
$\lambda = L \cdot$

$V = \frac{d\lambda}{dt}$

What Happens
If The Switch
Is Opened?

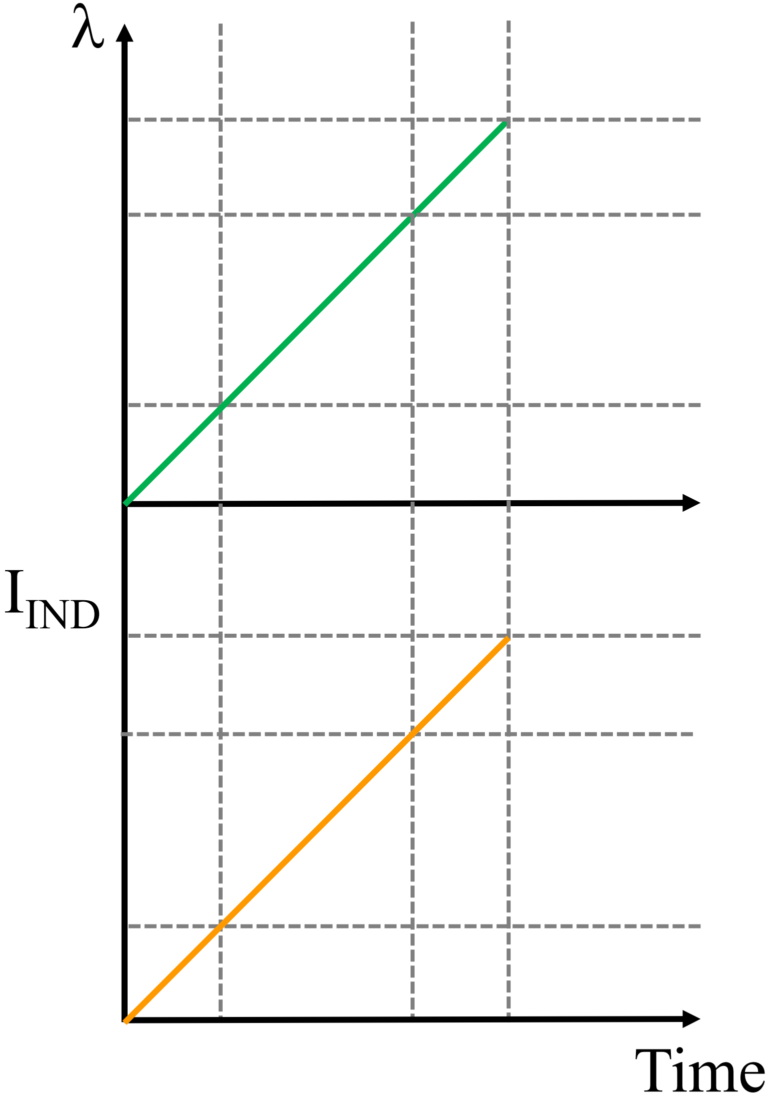


Energy Storage: Inductor

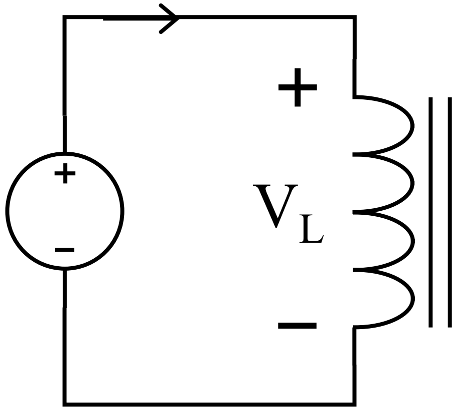


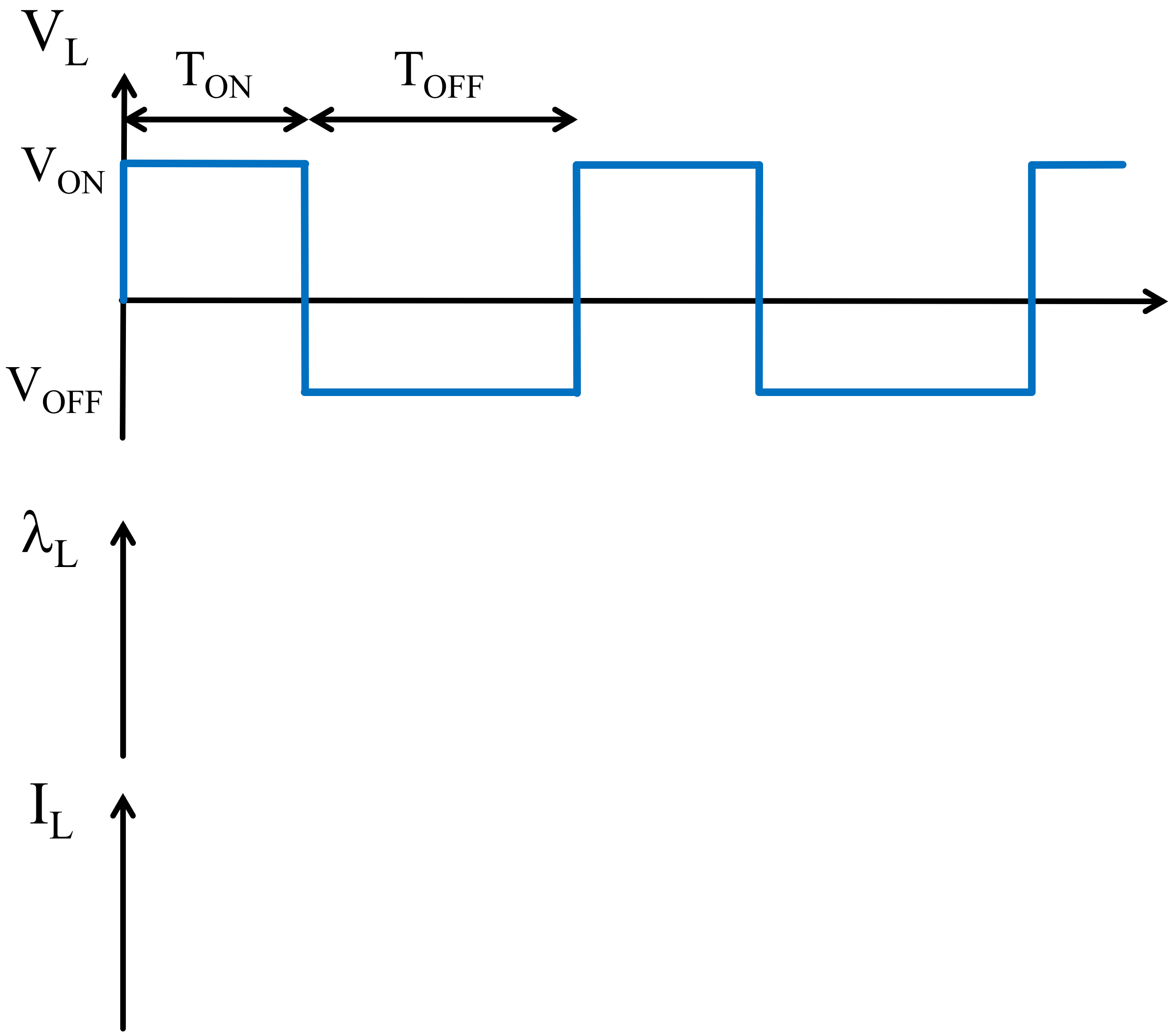
$$\lambda = L \cdot I \qquad \frac{1}{2} \cdot L \cdot I^2$$

$$V = \frac{d\lambda}{dt} = \frac{d(L \cdot I)}{dt} = L \cdot \frac{dI}{dt}$$

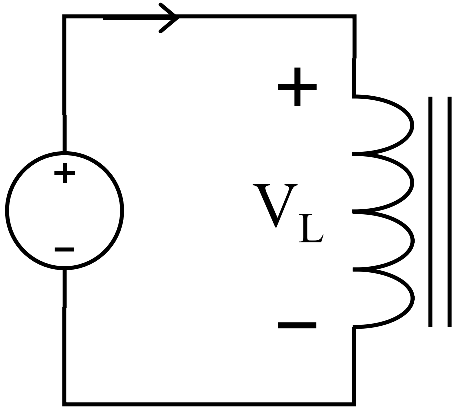


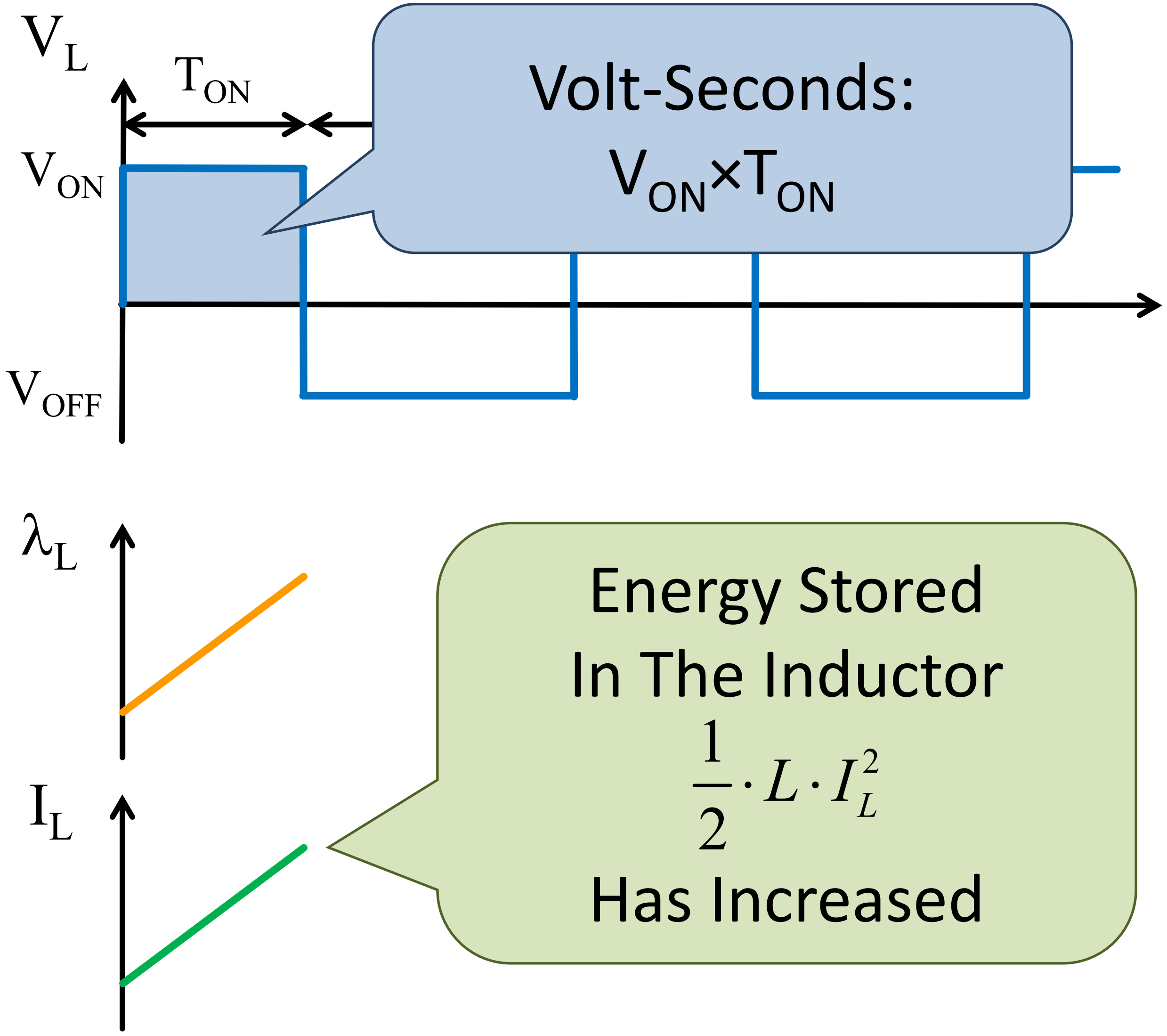
Inductor Ripple Current





Inductor Ripple Current

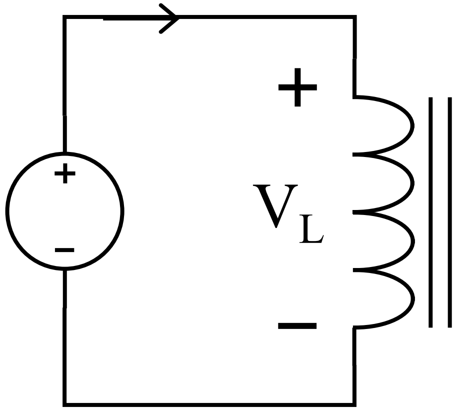


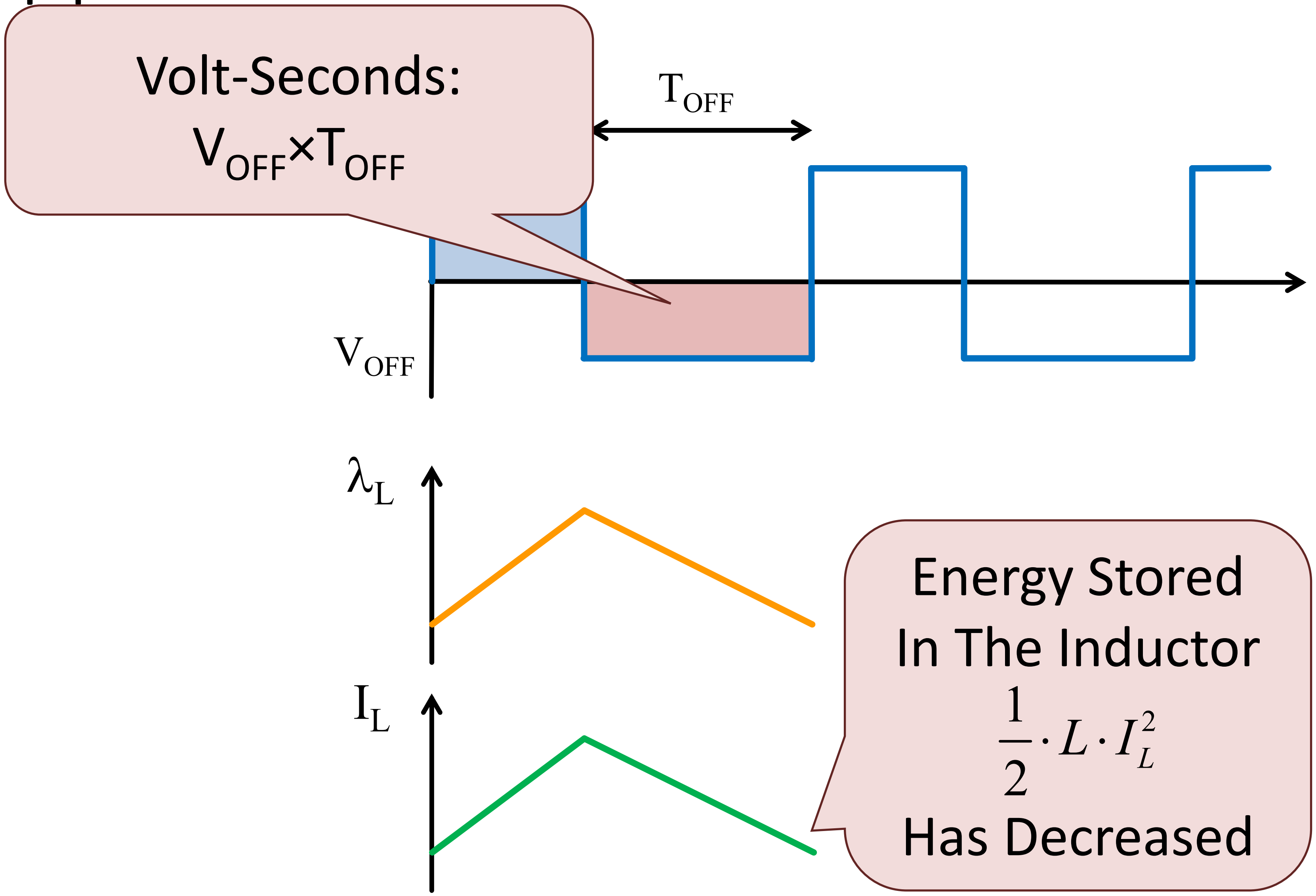


$$V_{ON}\cdot T_{ON}\Rightarrow \Delta\lambda_{ON}$$

$$\Delta\lambda_{ON}\Rightarrow L\cdot\Delta I_{ON}$$

Inductor Ripple Current

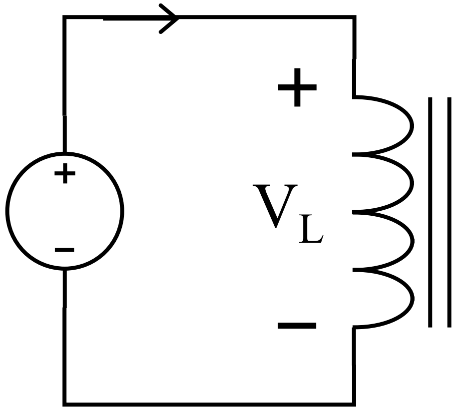


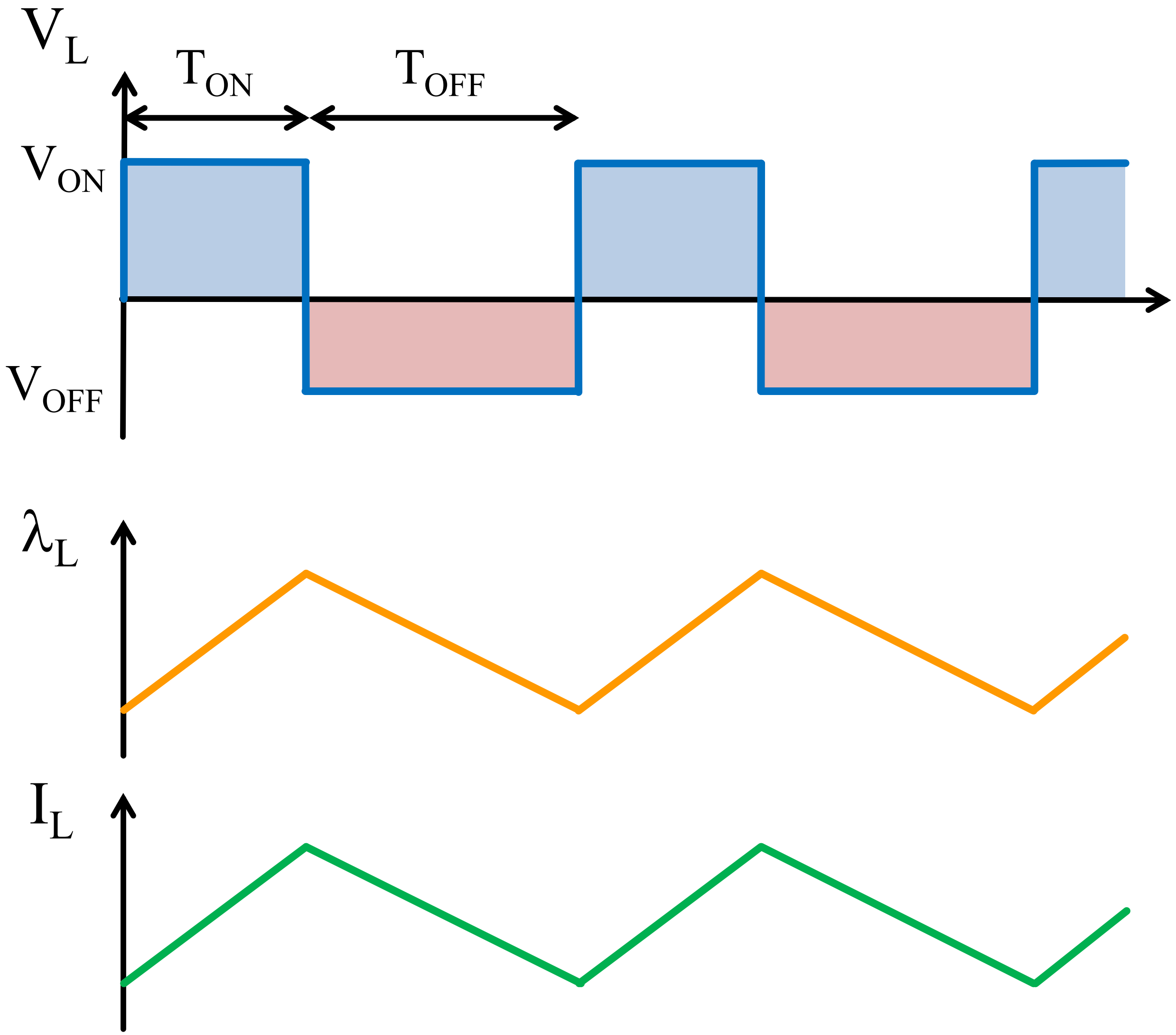


$$V_{OFF} \cdot T_{OFF} \Rightarrow \Delta\lambda_{OFF}$$

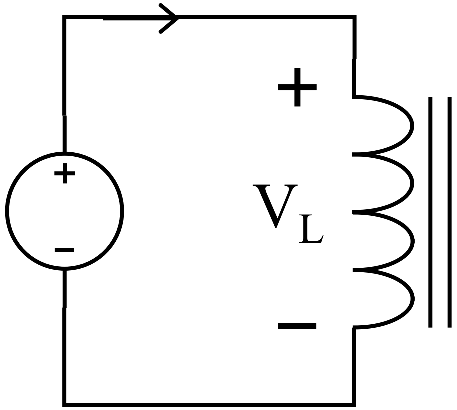
$$\Delta\lambda_{OFF} \Rightarrow L \cdot \Delta I_{OFF}$$

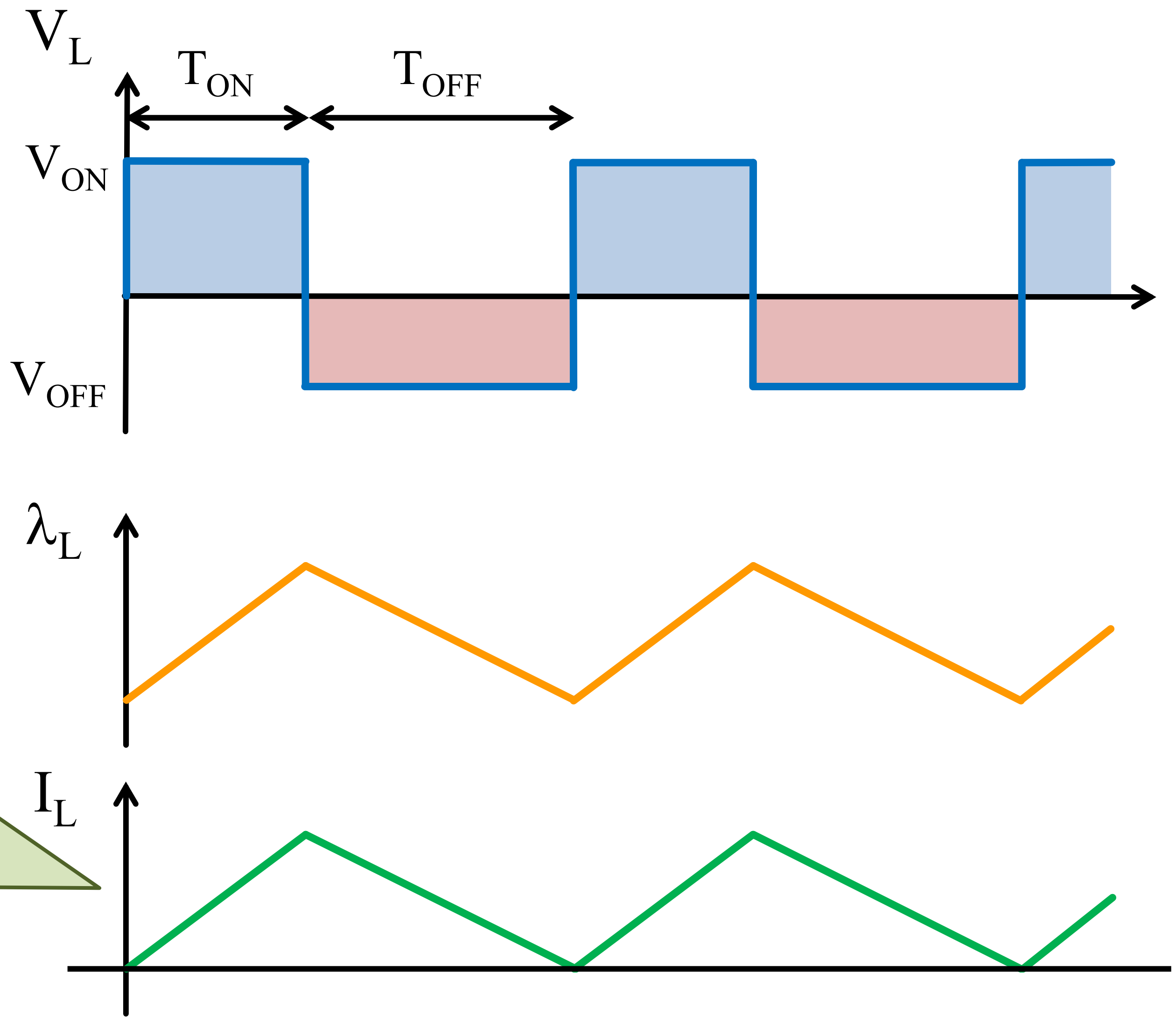
Inductor Ripple Current





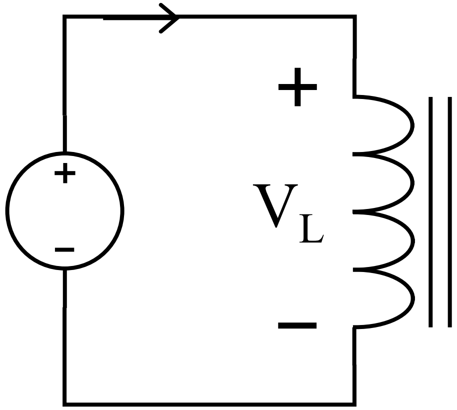
Inductor Ripple Current

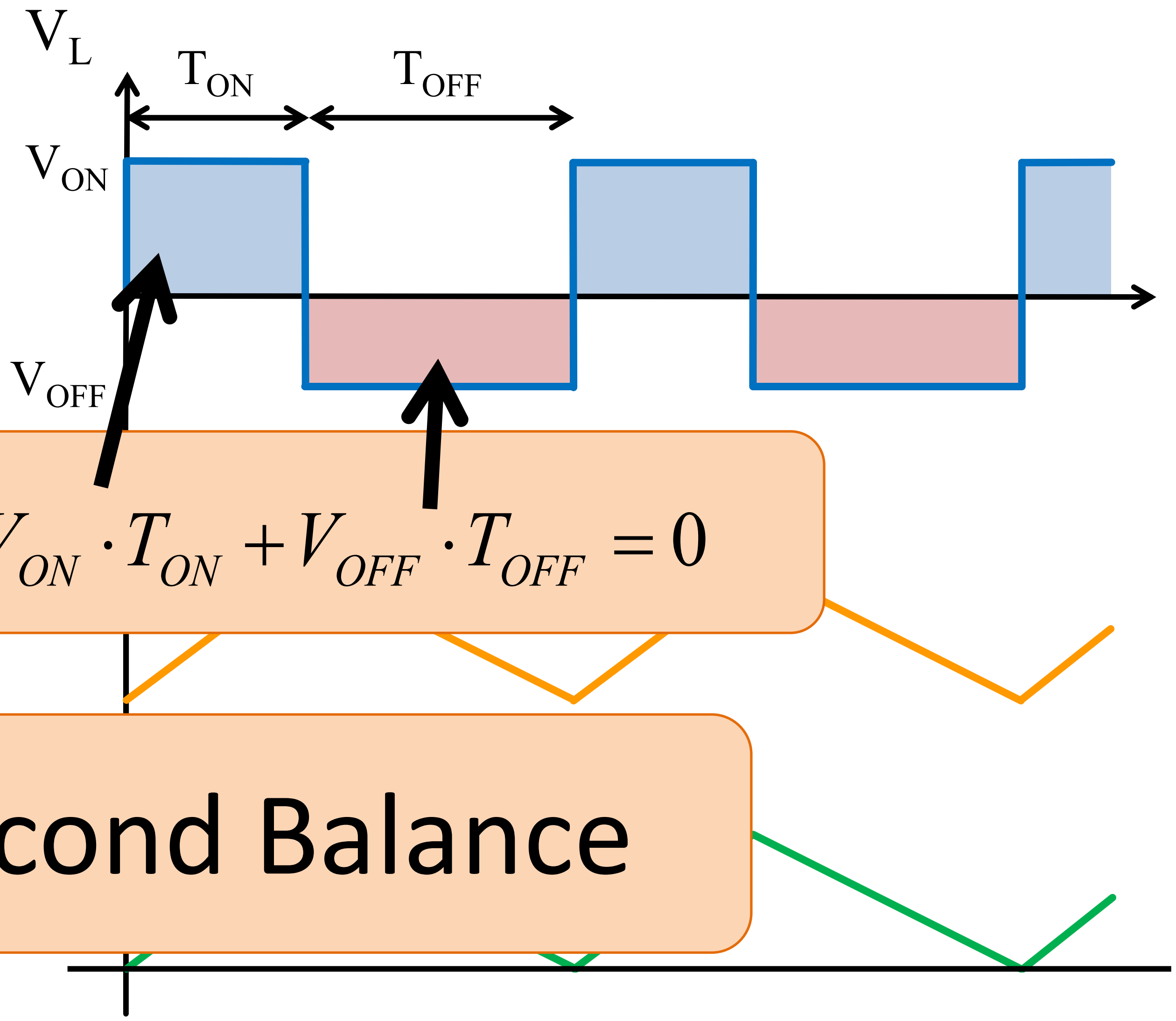




In
Steady State:
Starting Value
=
Final Value

Inductor Ripple Current

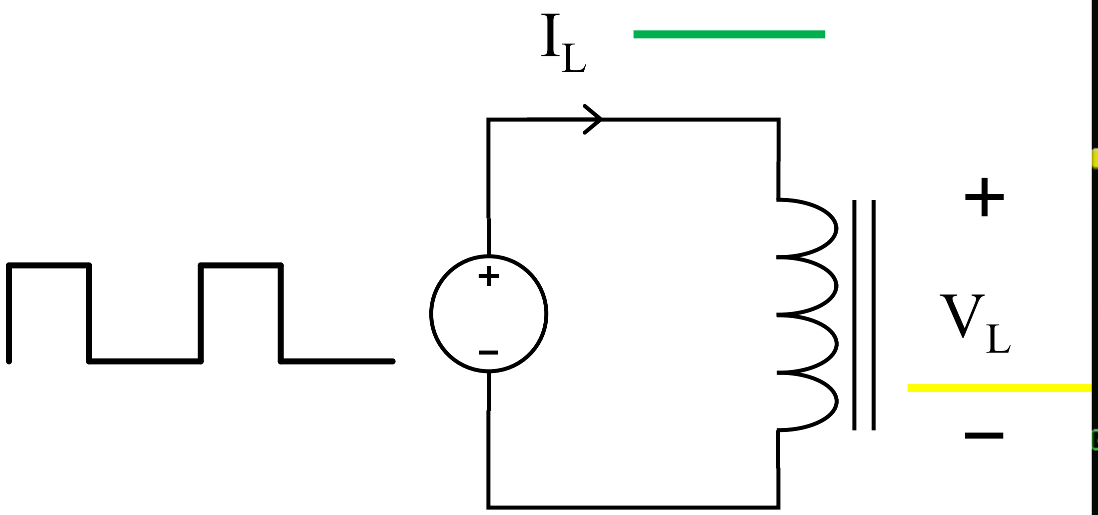


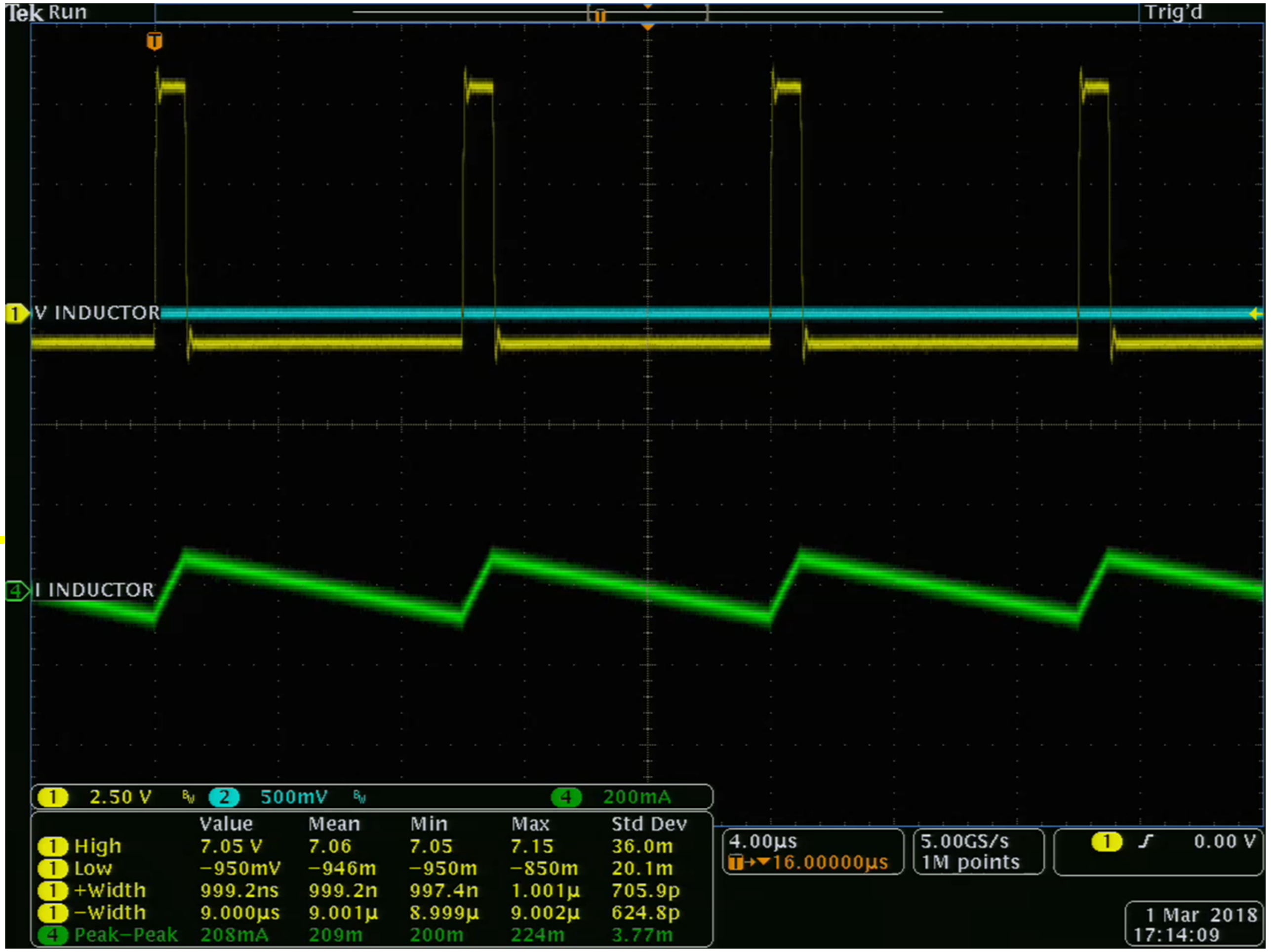


$$V_{ON} \cdot T_{ON} + V_{OFF} \cdot T_{OFF} = 0$$

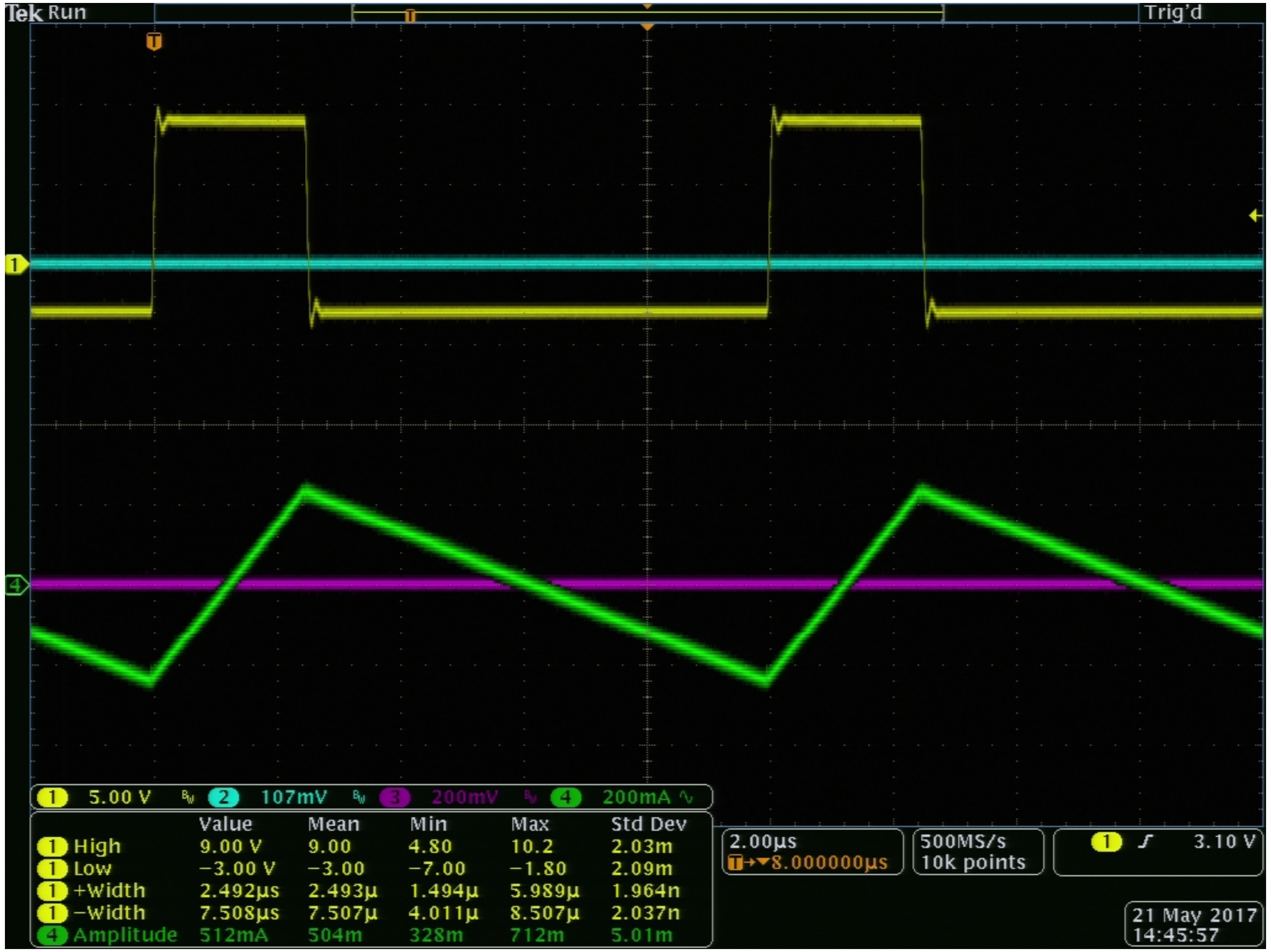
Inductor Volt-Second Balance

Video Lab 1:
Inductor Voltage And Current





Video Lab 1:
Inductor Voltage And Current



$$V_{PLUS} = 9.00\,\text{V}$$

$$T_{PLUS} = 2.5\,\mu\text{s}$$

$$V_{NEG} = -3.00\,\text{V}$$

$$T_{NEG} = 7.5\,\mu\text{s}$$

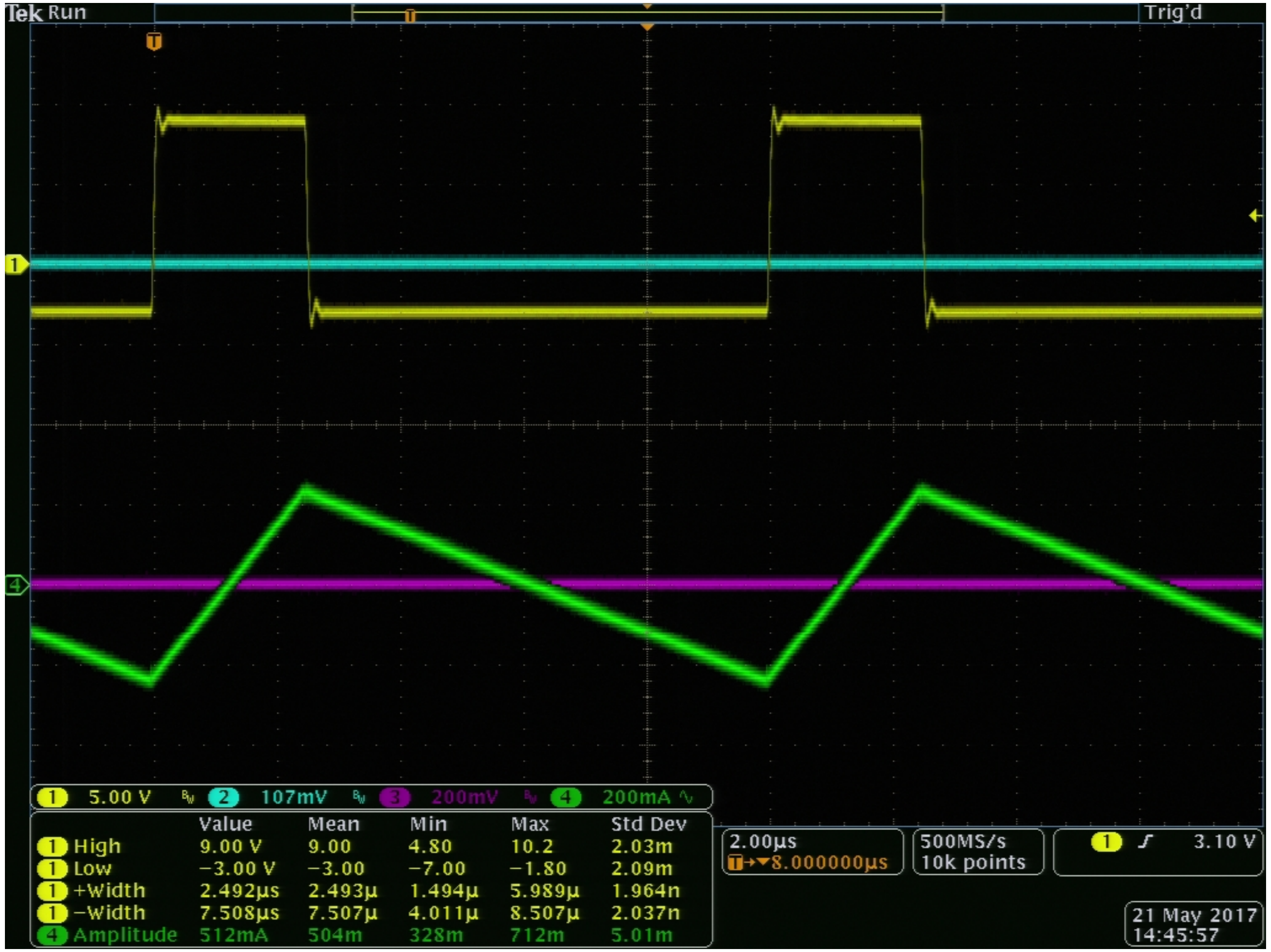
$$V_{PLUS} \cdot T_{PLUS} + V_{NEG} \cdot T_{NEG}$$

$$9.00\,\text{V} \cdot 2.5\,\mu\text{s} + \left(-3.00\,\text{V}\right) \cdot 7.5\,\mu\text{s}$$

$$22.5\,\text{V-}\mu\text{s} - 22.5\,\text{V-}\mu\text{s}$$

$$0\,\text{V-}\mu\text{s}$$

Video Lab 1:
Inductor Voltage And Current



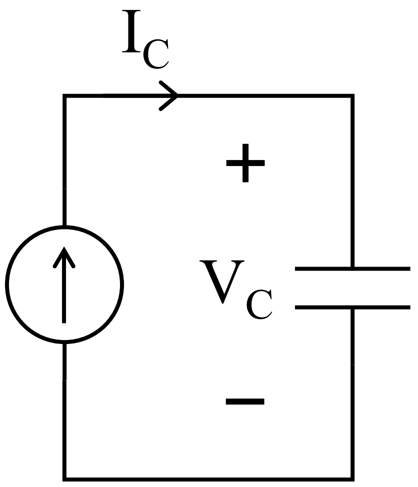
$$V = L \cdot \frac{di}{dt}$$

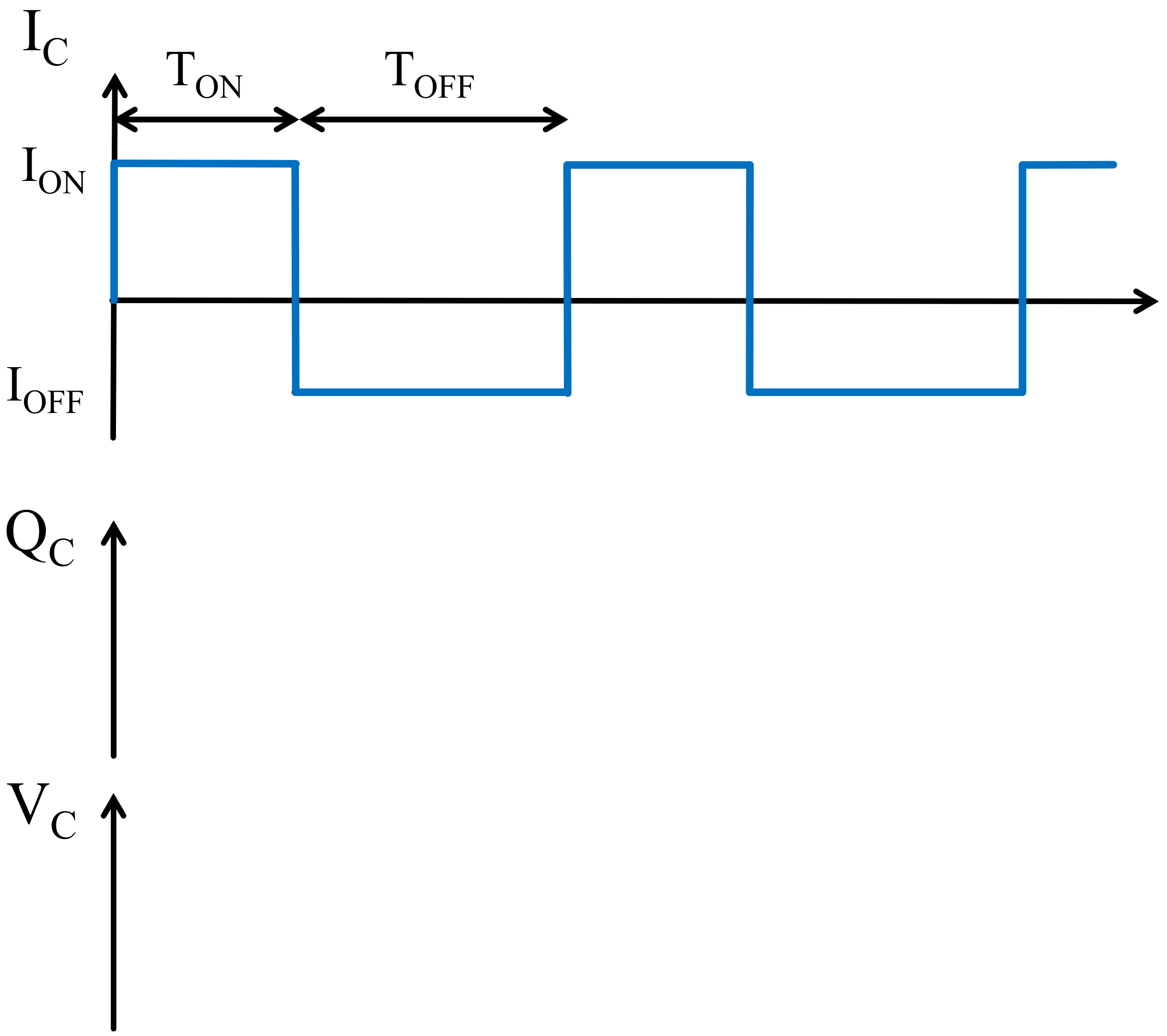
$$L = \frac{V \cdot dt}{di}$$

$$= \frac{9.00\,\text{V} \cdot 2.5\,\mu\text{s}}{512\,\text{mA}}$$

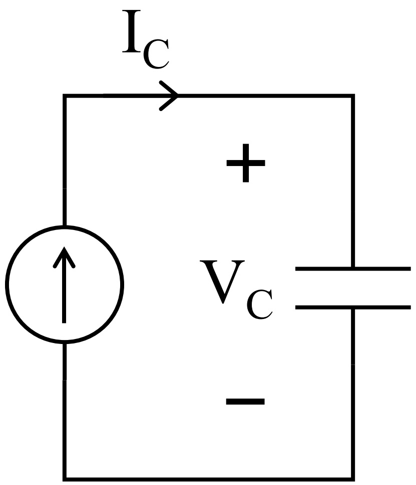
$$= 43.9\,\mu\text{H}$$

Capacitor Ripple Voltage



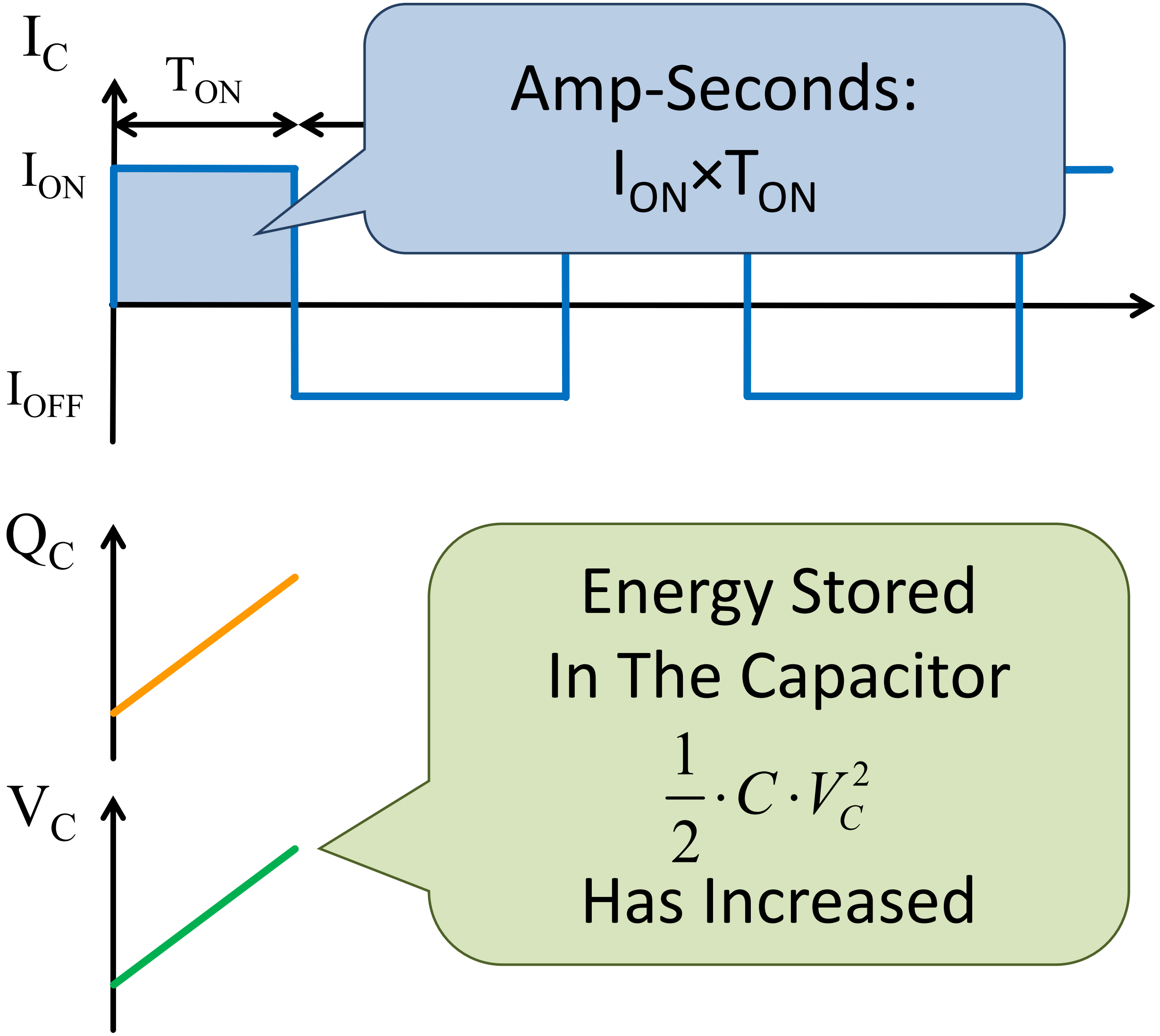


Capacitor Ripple Voltage

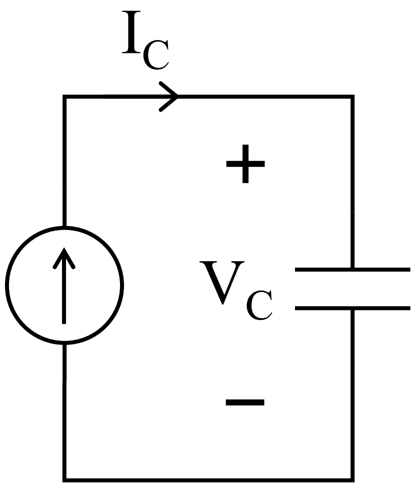


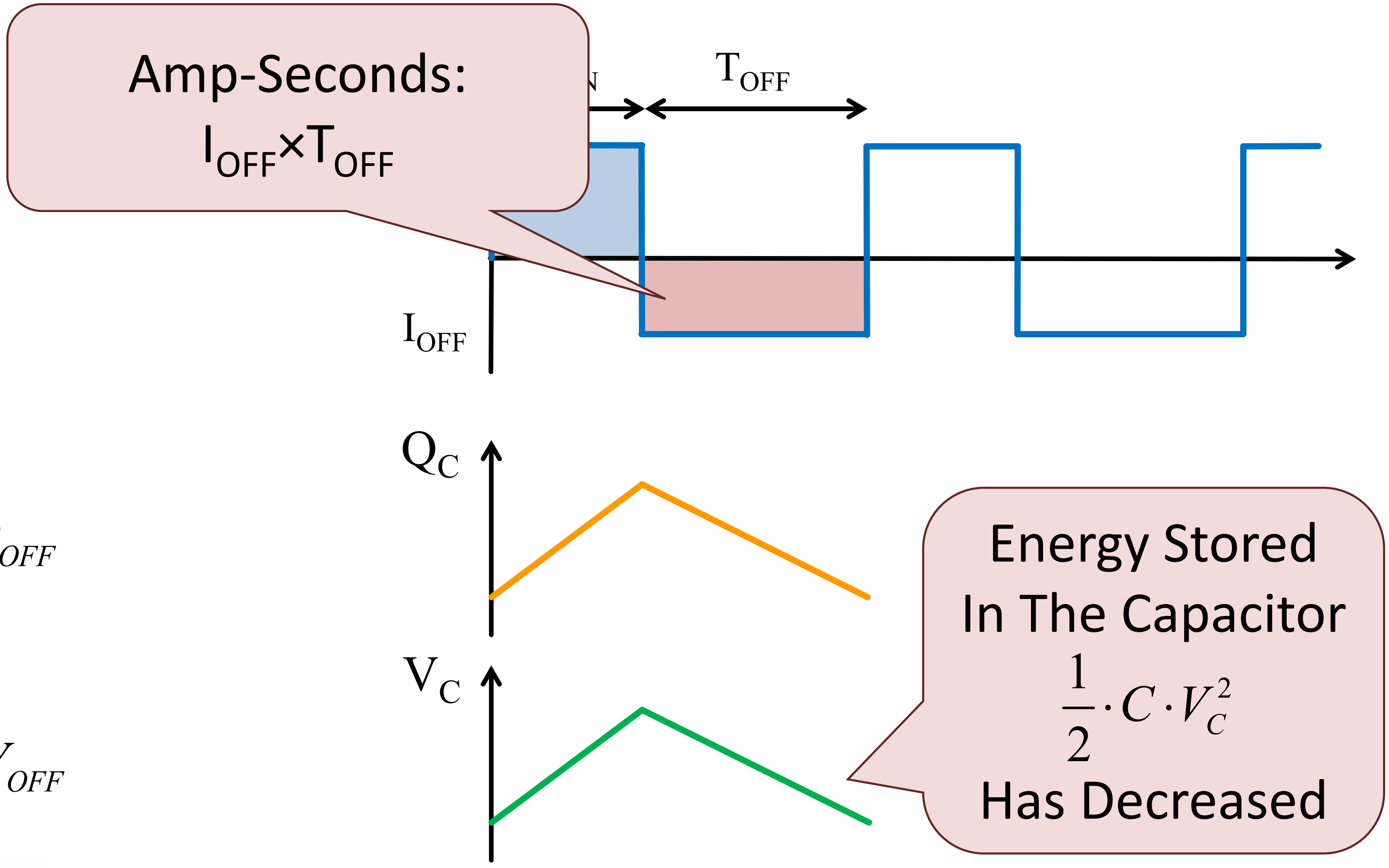
$$I_{ON} \cdot T_{ON} \Rightarrow \Delta Q_{ON}$$

$$\Delta Q_{ON} \Rightarrow C \cdot \Delta V_{ON}$$



Capacitor Ripple Voltage

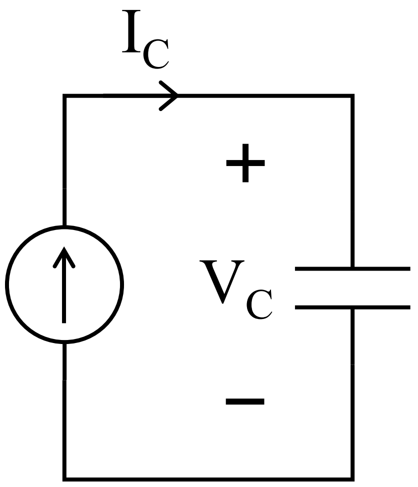


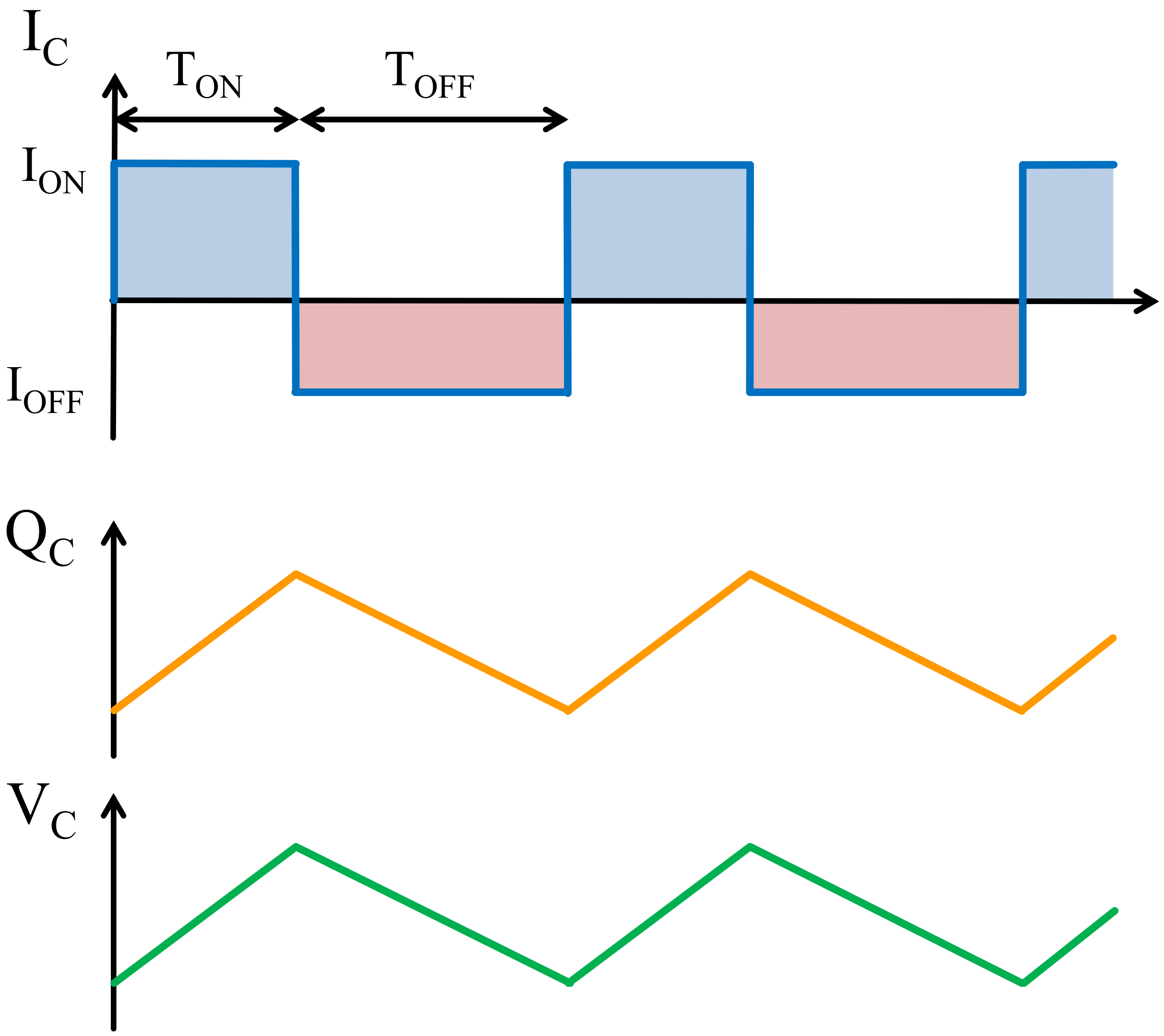


$$I_{OFF} \cdot T_{OFF} \Rightarrow \Delta Q_{OFF}$$

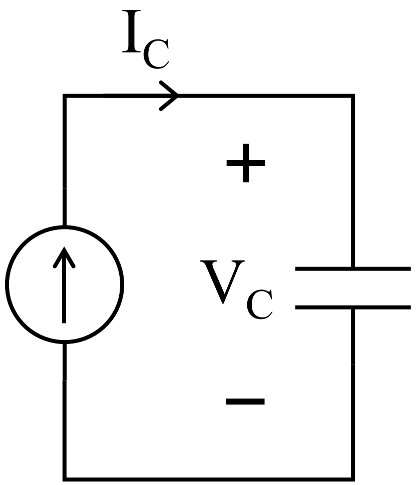
$$\Delta Q_{OFF} \Rightarrow C \cdot \Delta V_{OFF}$$

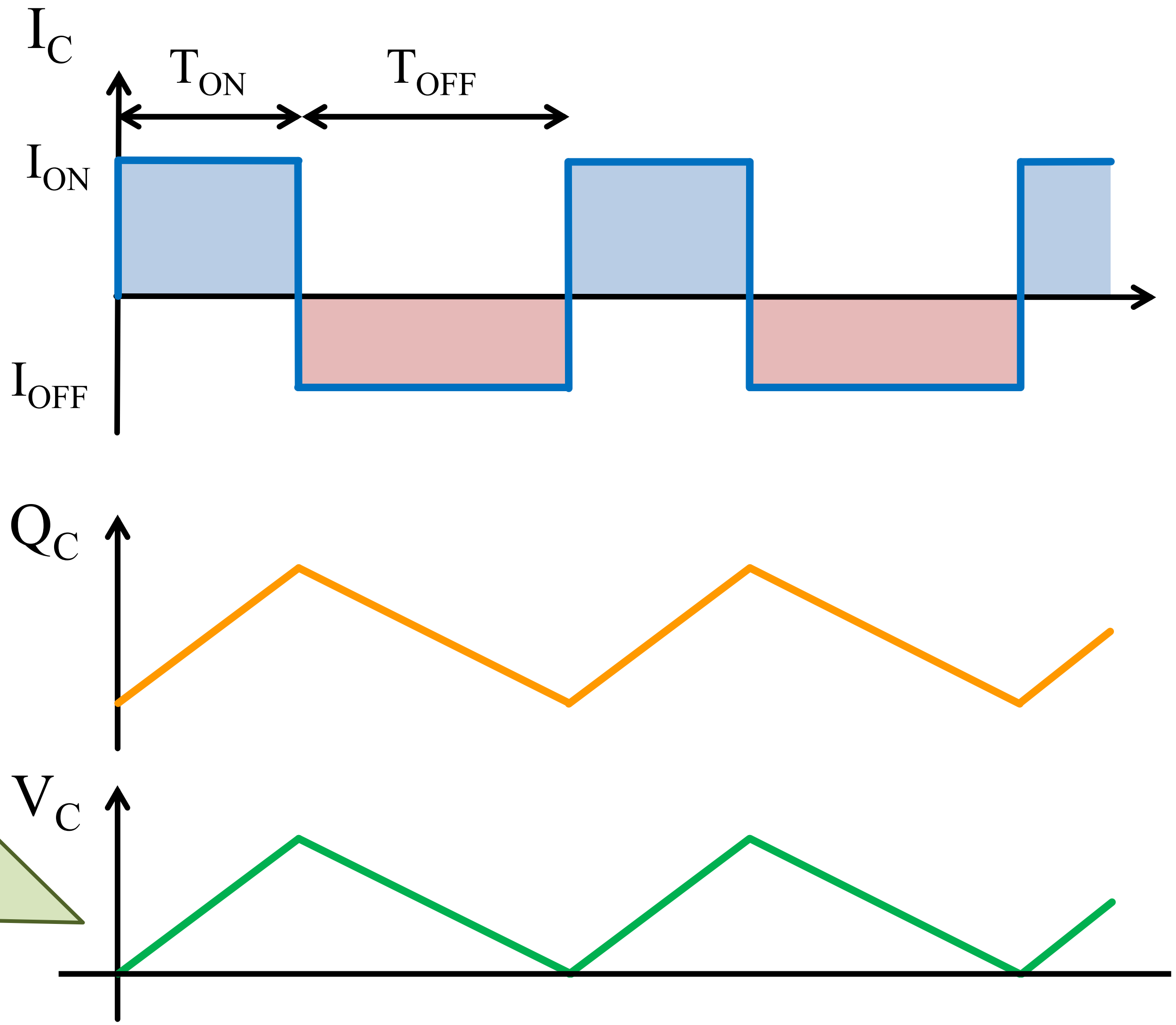
Capacitor Ripple Voltage





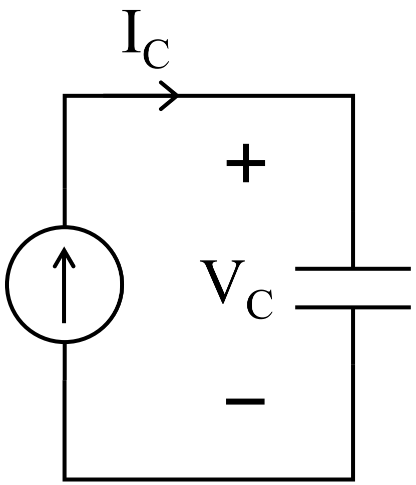
Capacitor Ripple Voltage

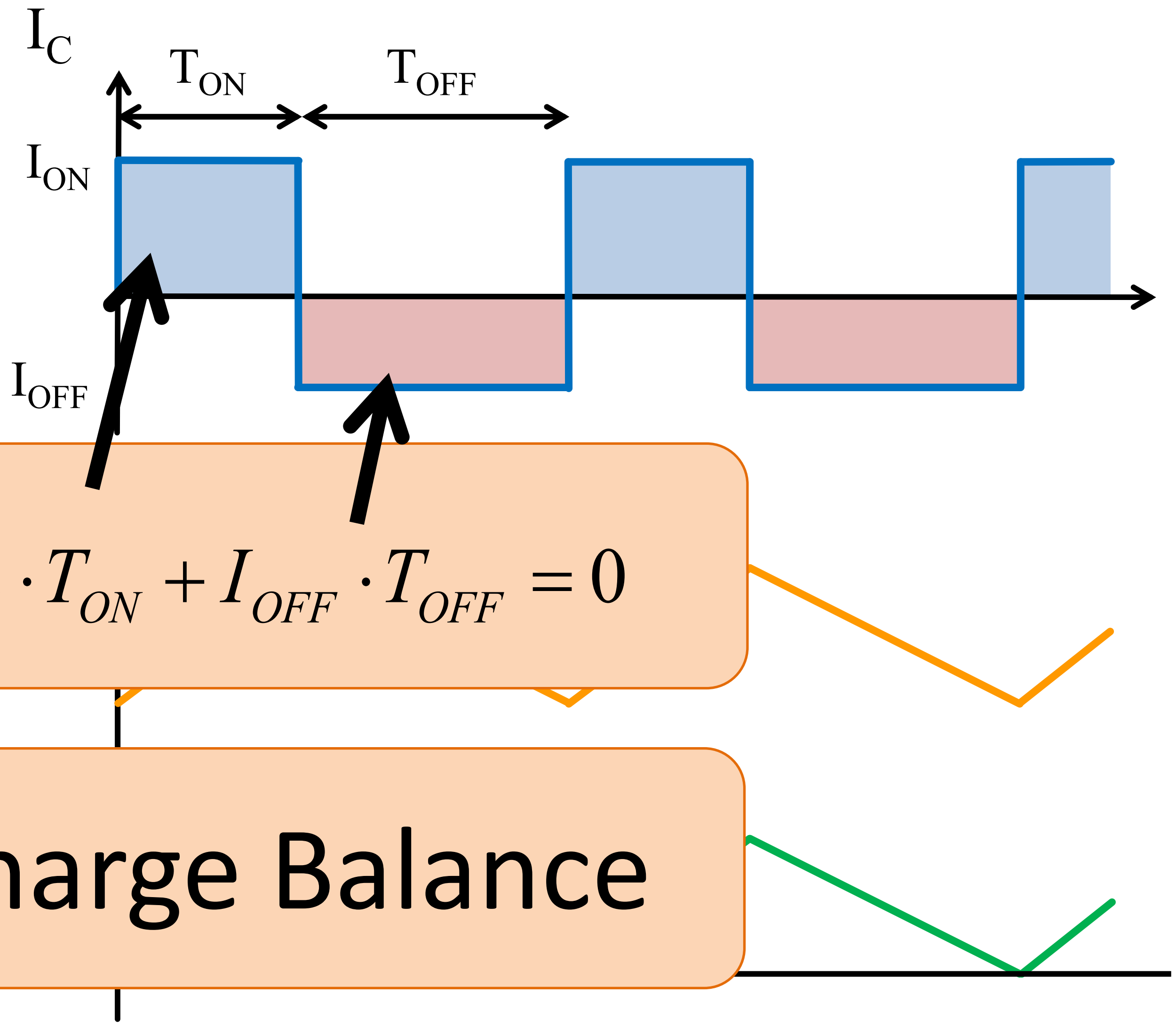




In
Steady State:
Starting Value
=
Final Value

Capacitor Ripple Voltage

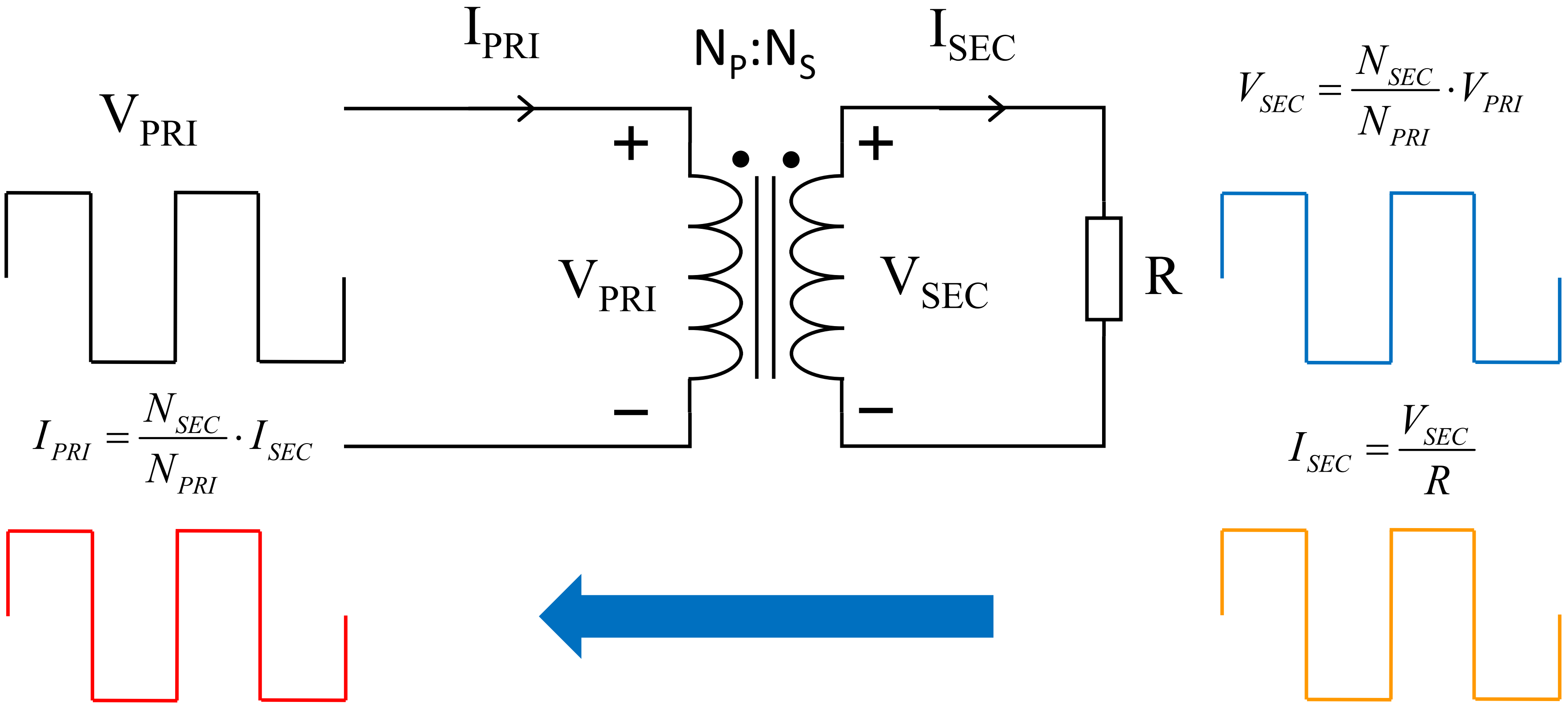




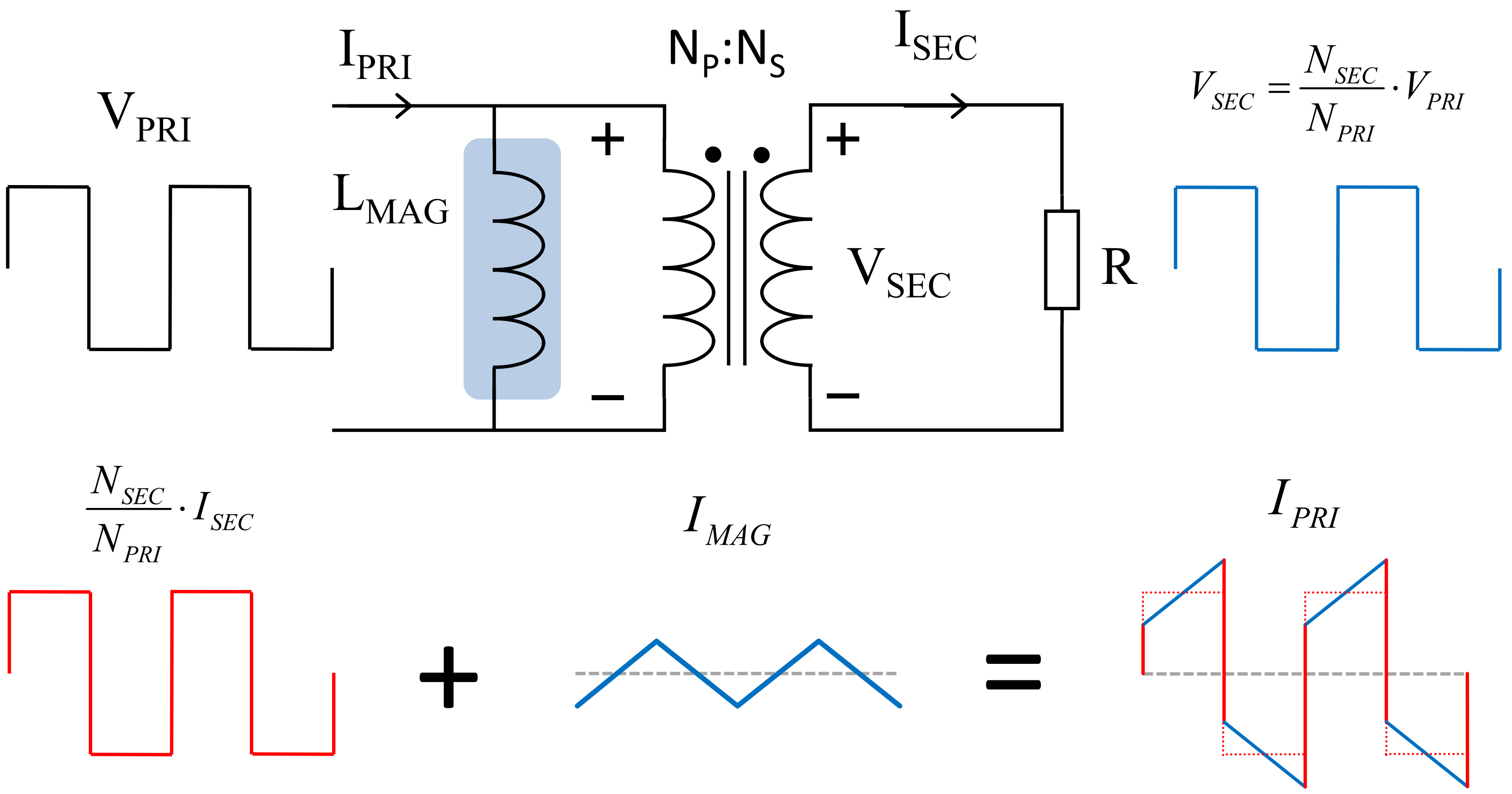
$$I_{ON} \cdot T_{ON} + I_{OFF} \cdot T_{OFF} = 0$$

Capacitor Charge Balance

Transformer

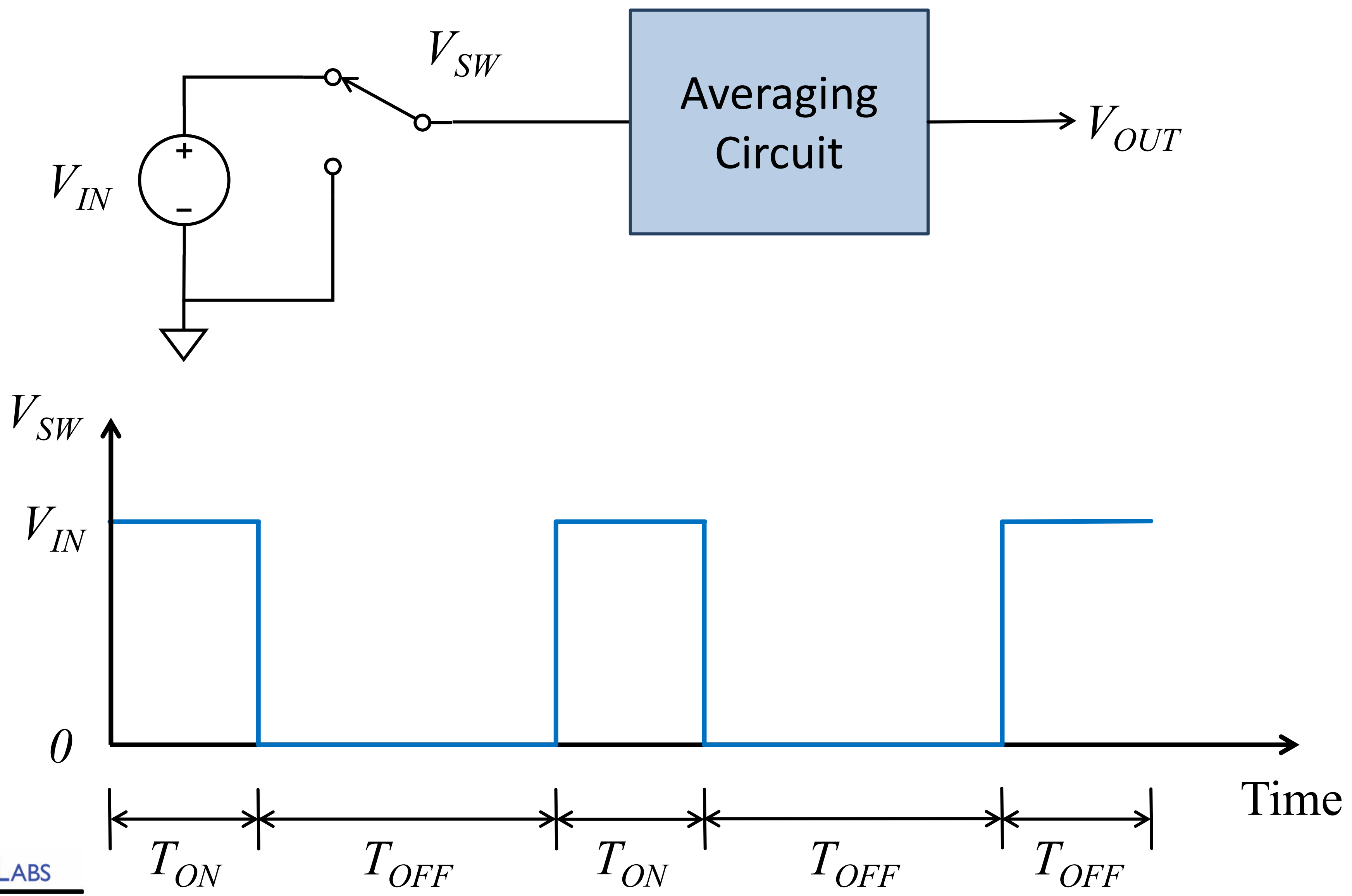


Transformer Magnetizing Current

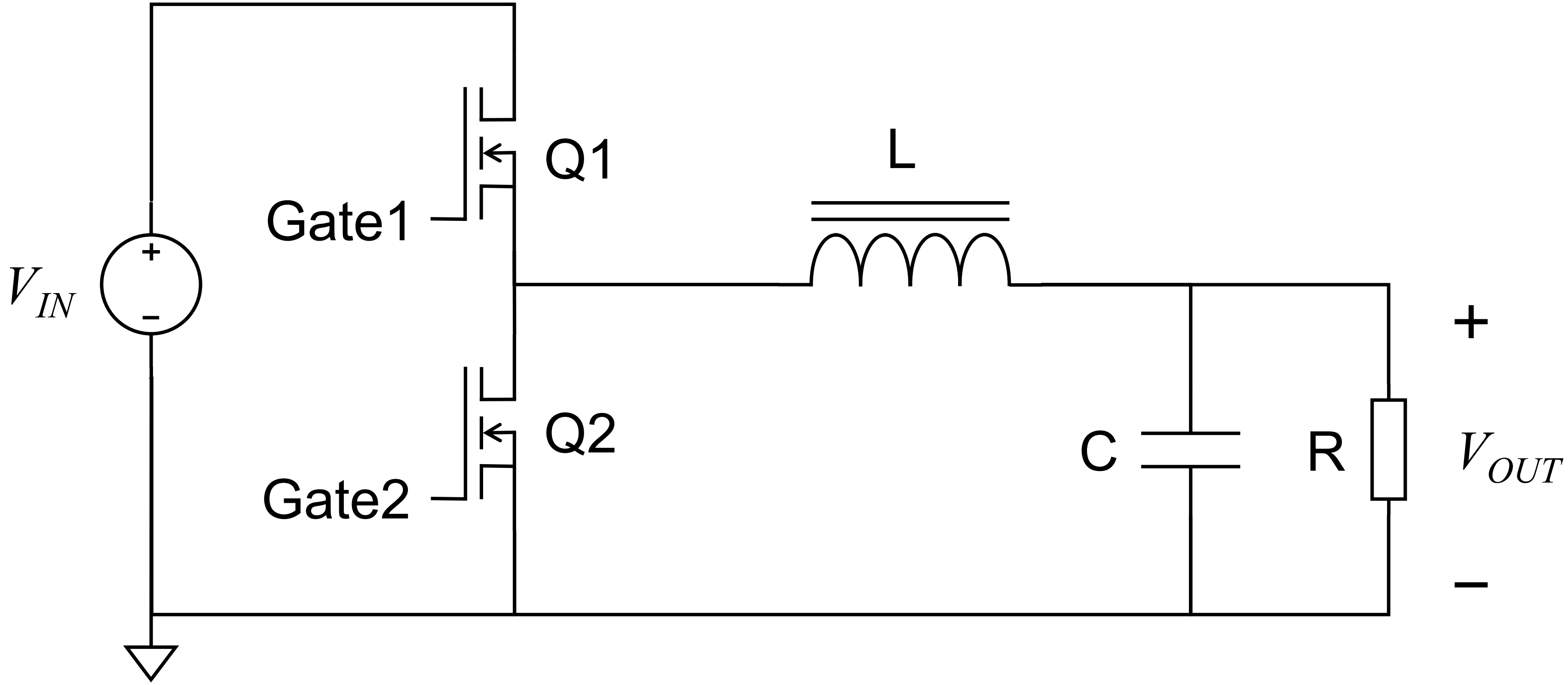


Buck Converter

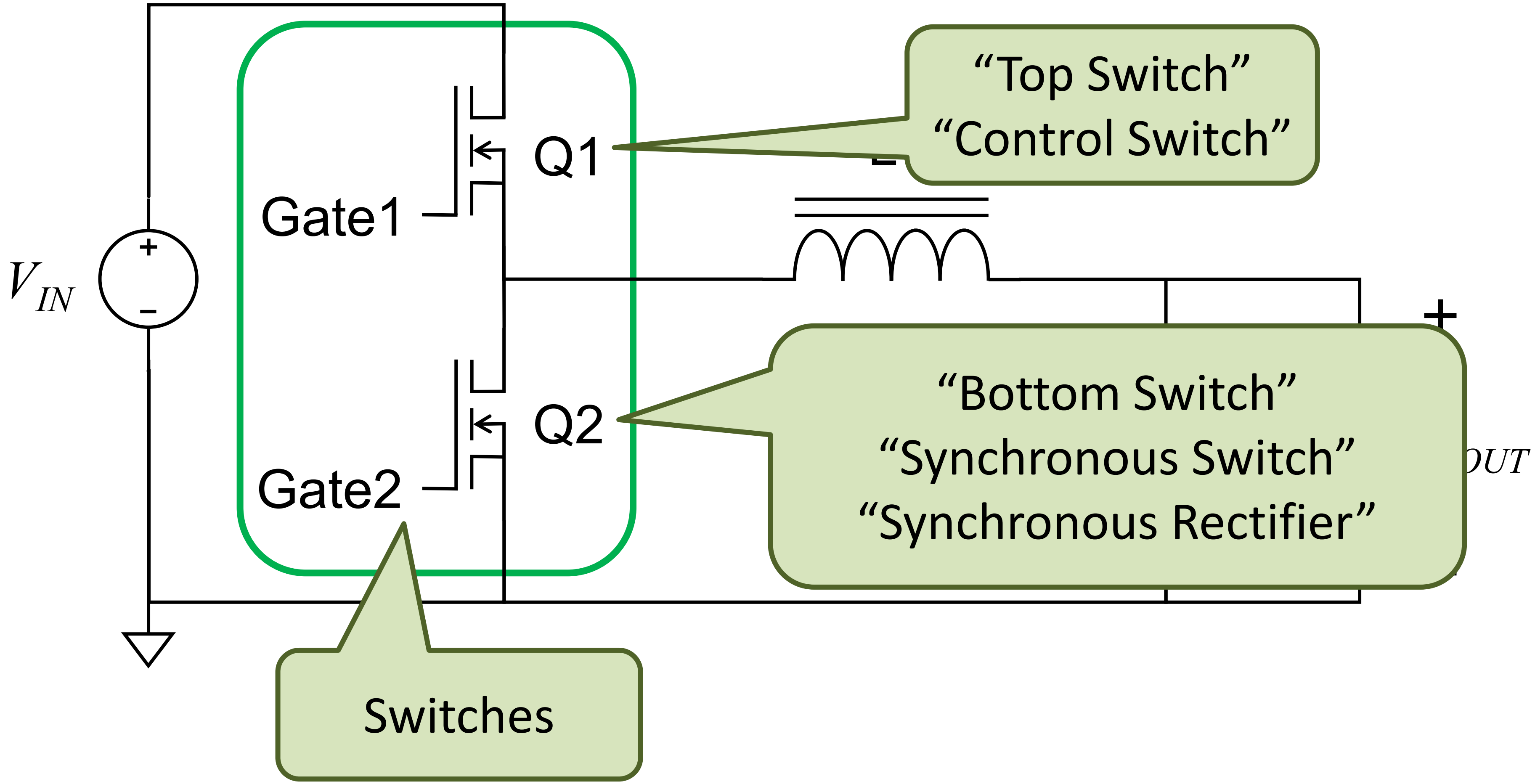
Switch Mode Concept



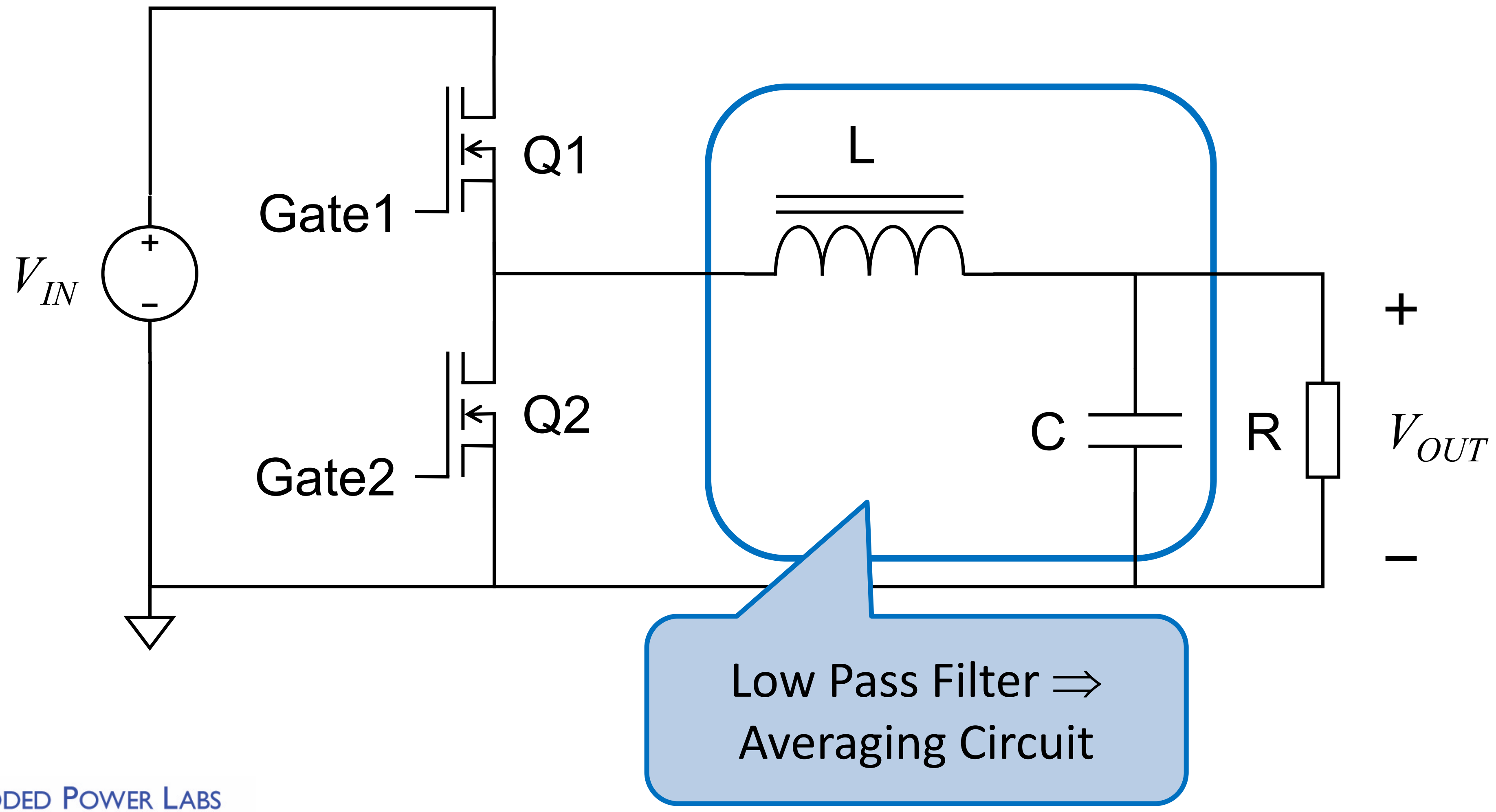
Buck Converter



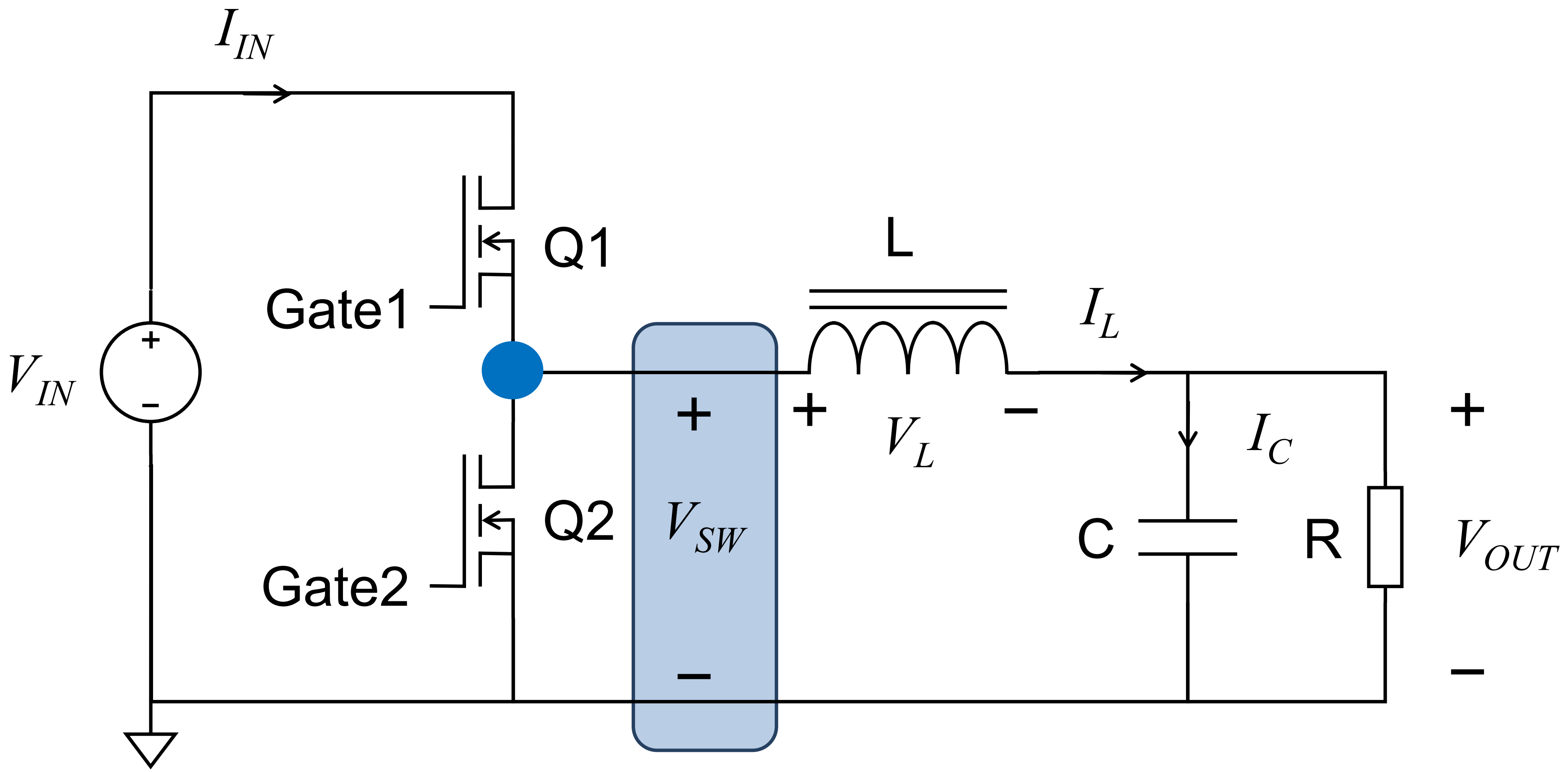
Buck Converter



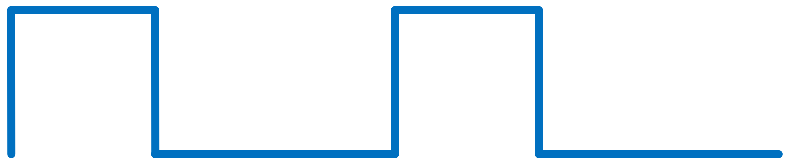
Buck Converter



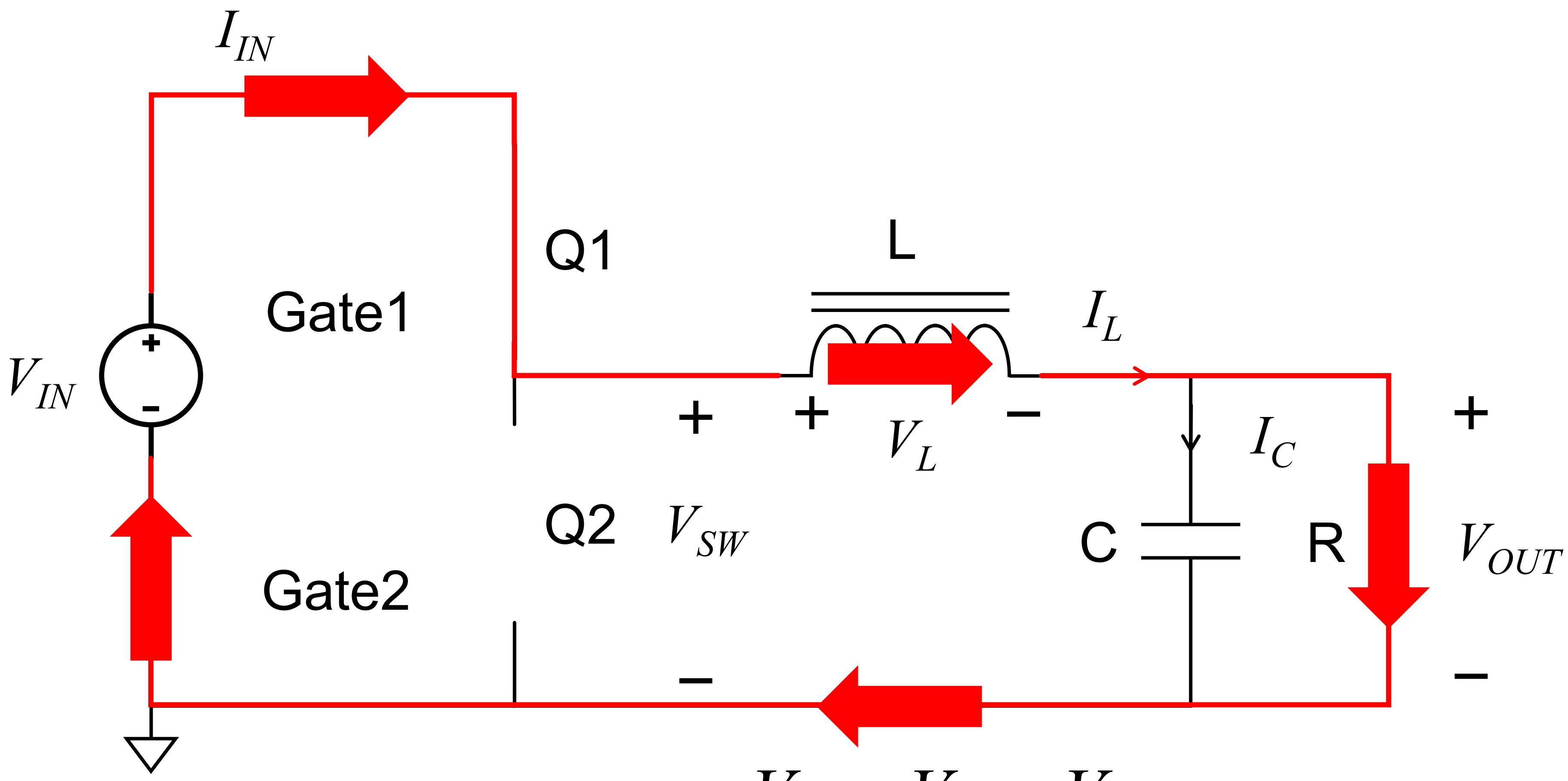
Buck Converter



"Switch Node"

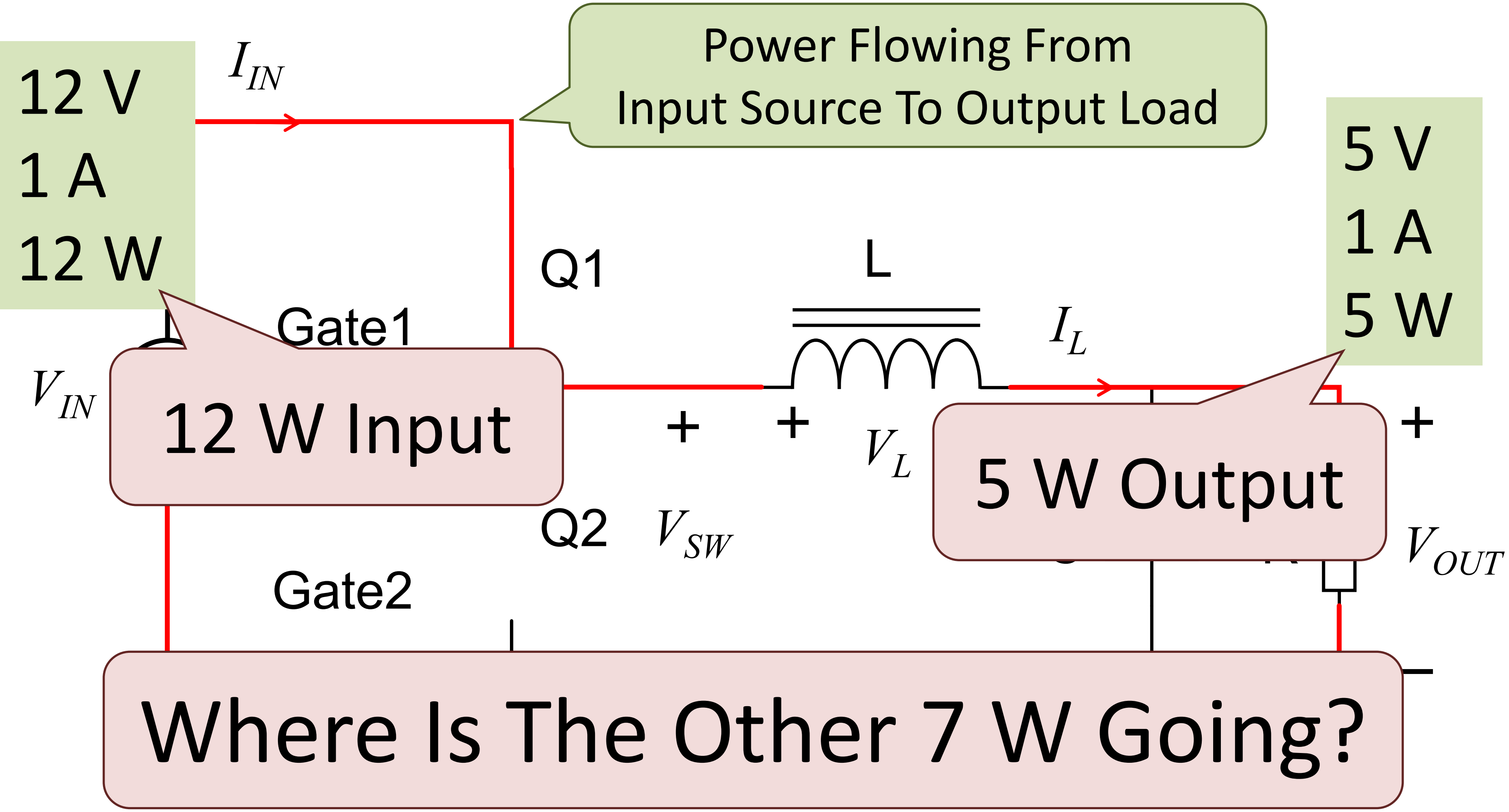


Buck Converter On Time

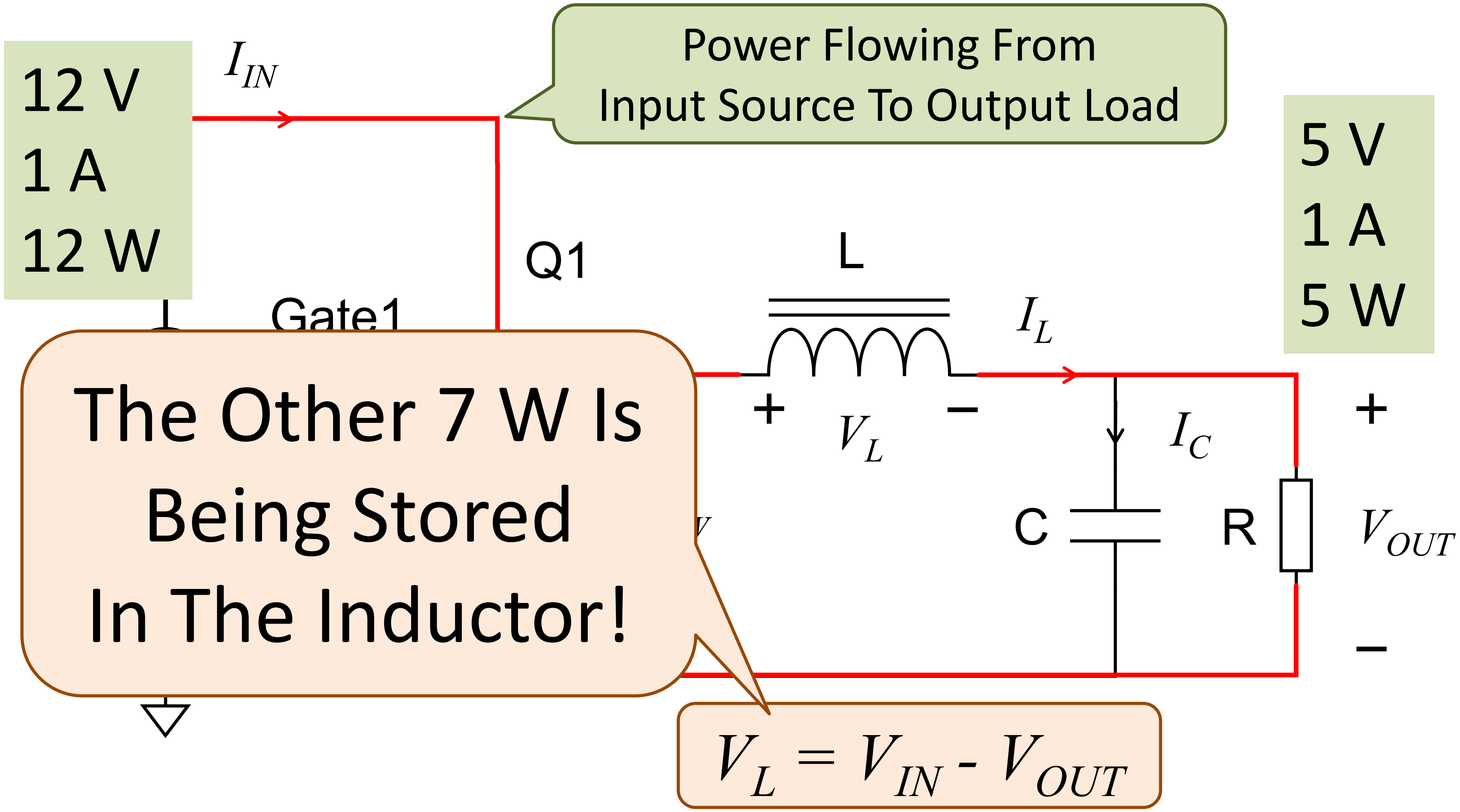


$$V_L = V_{IN} - V_{OUT}$$

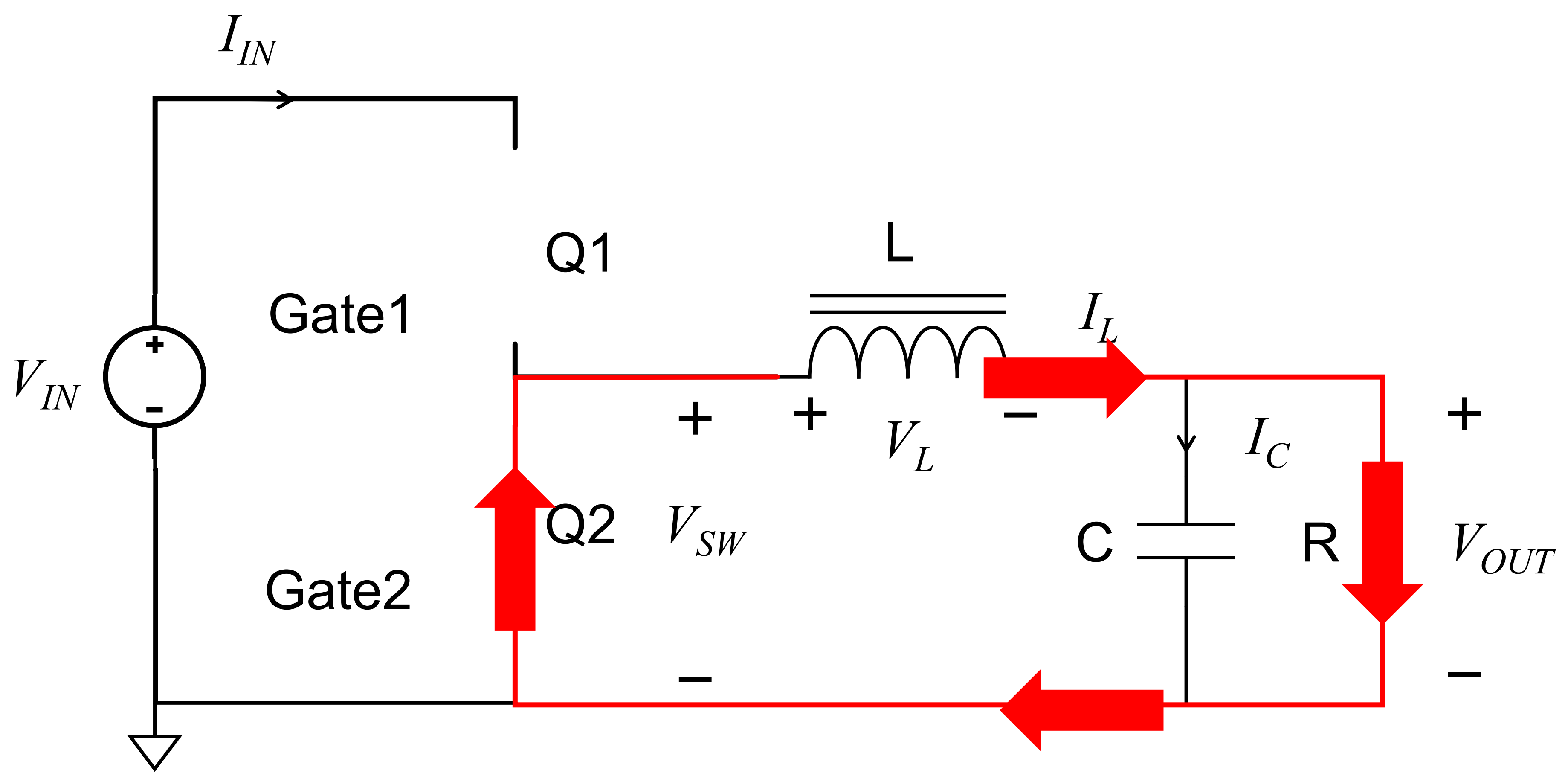
Buck Converter On Time



Buck Converter On Time

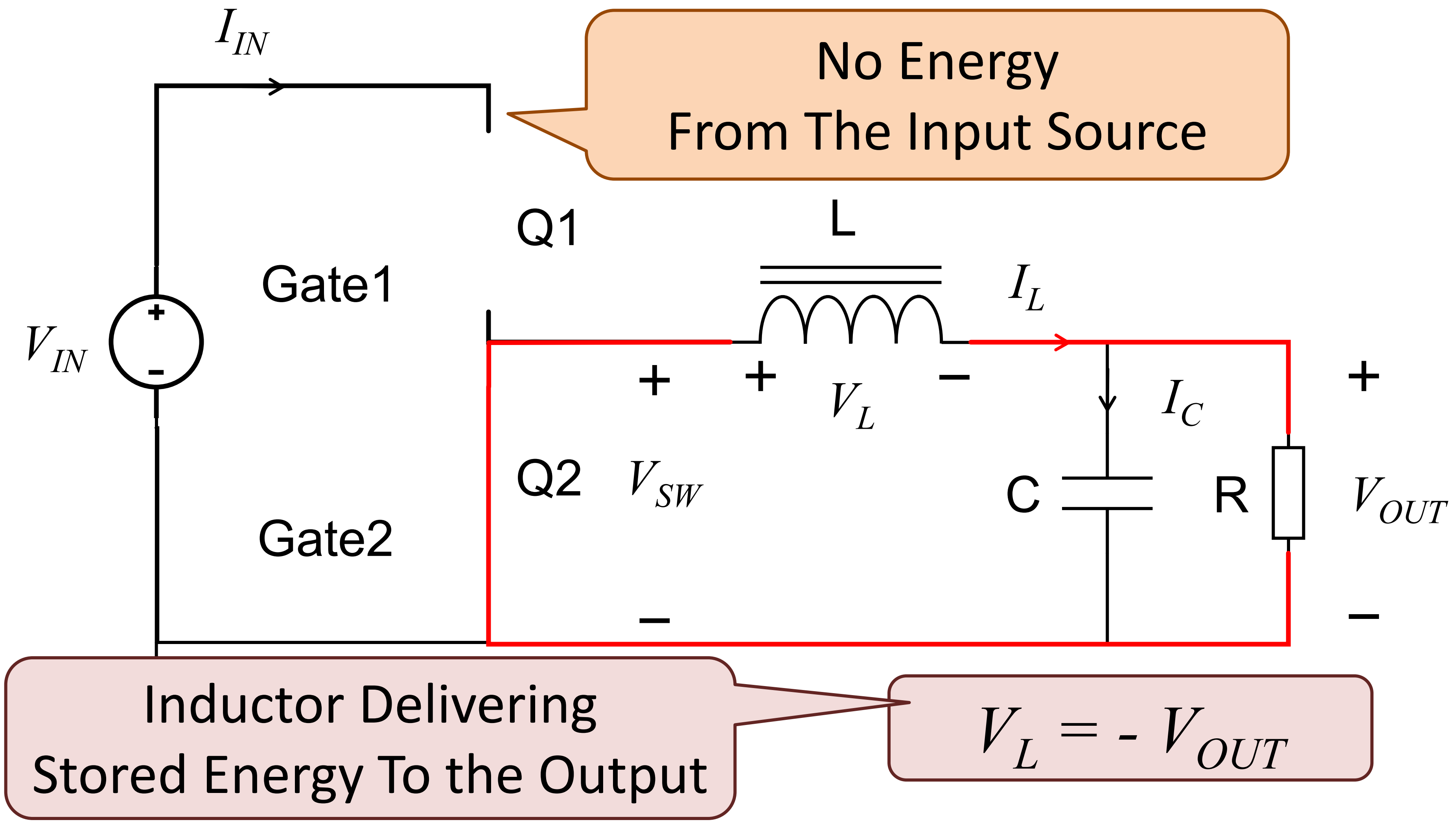


Buck Converter Off Time



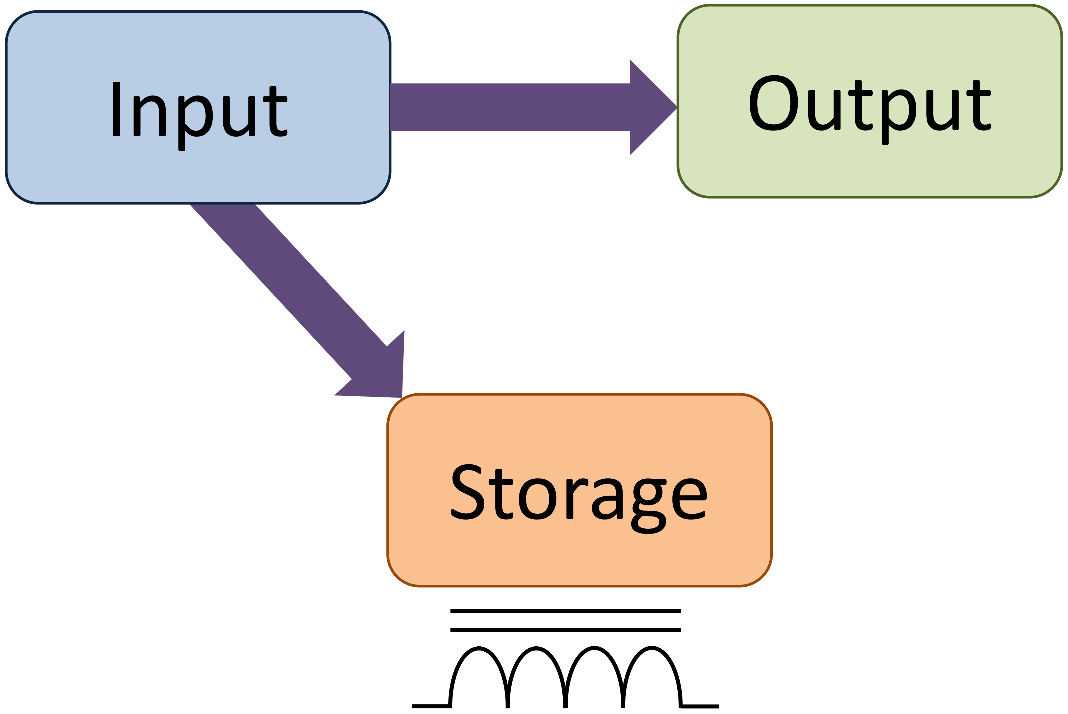
$$V_L = - V_{OUT}$$

Buck Converter Off Time

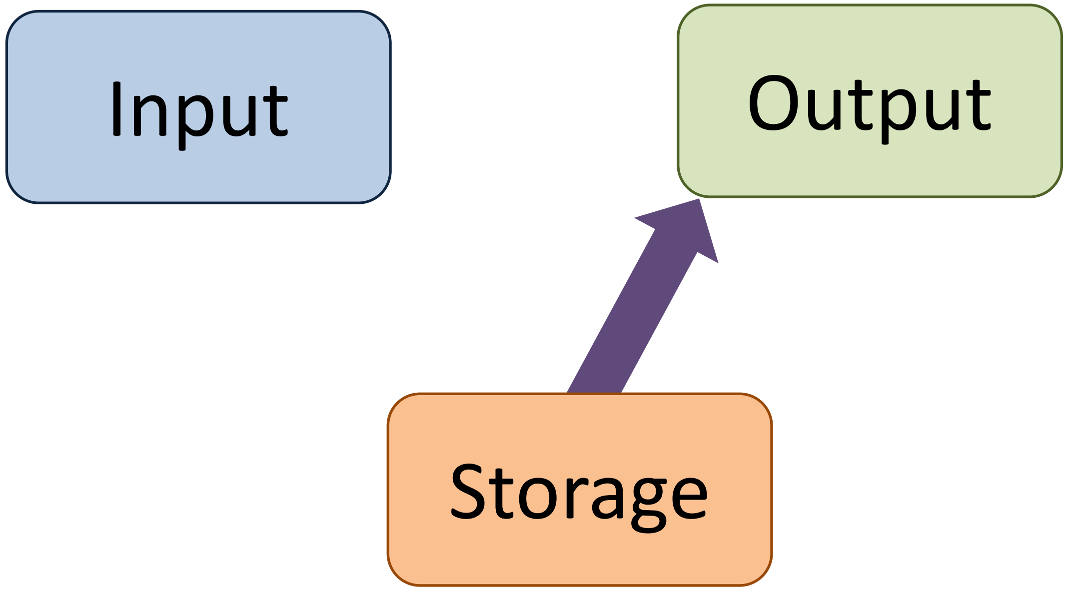


Buck Converter Energy Flow

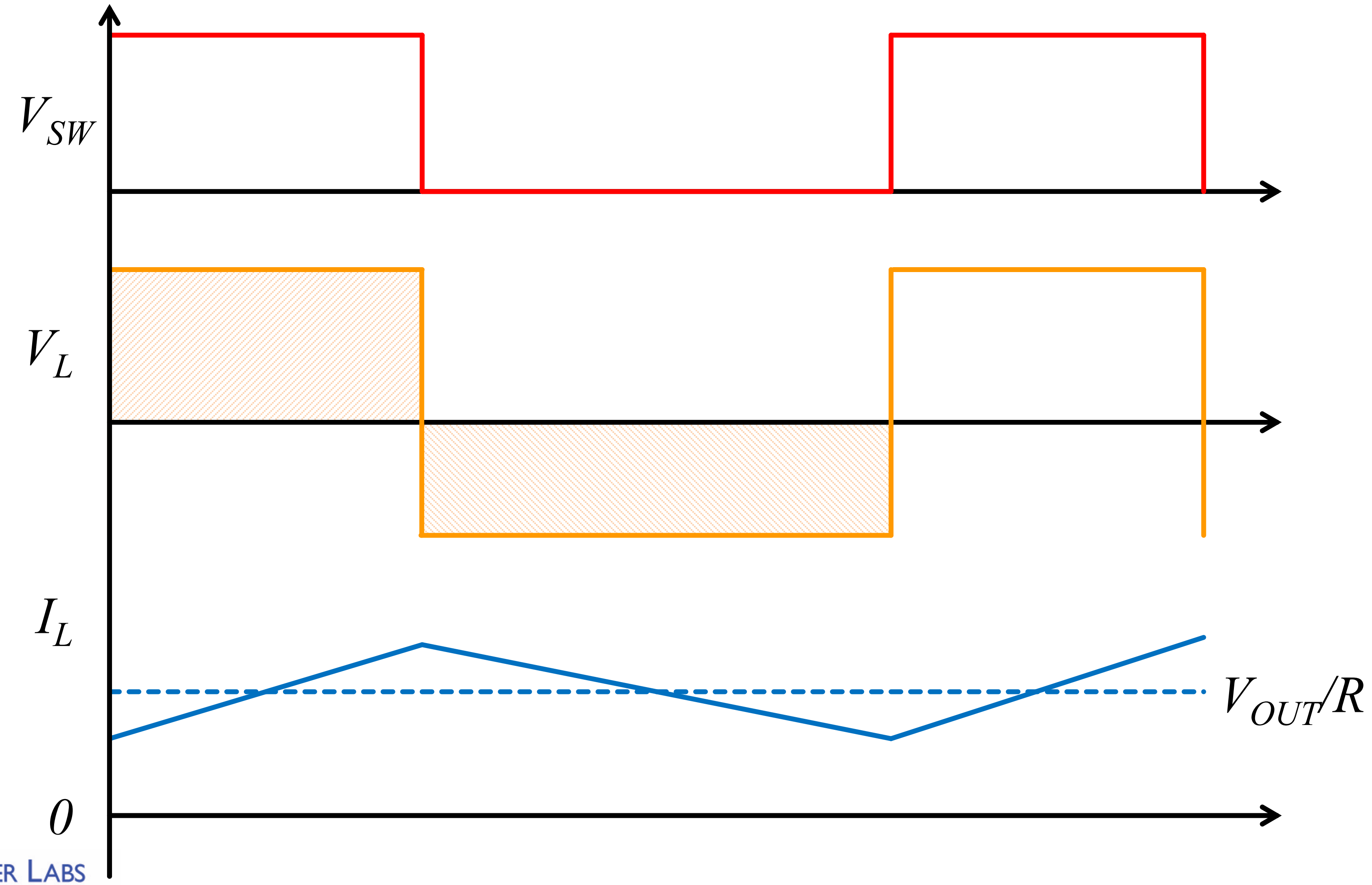
On Time



Off Time



Buck Converter
Inductor Voltage And Current



Inductor Volt-Second Balance

$$V_L(T_{ON})\cdot T_{ON} + V_L(T_{OFF})\cdot T_{OFF} = 0$$

$$(V_{IN} - V_{OUT})\cdot T_{ON} + (-V_{OUT})\cdot T_{OFF} = 0$$

$$(V_{IN} - V_{OUT})\cdot D\cdot T_{SW} + (-V_{OUT})\cdot(1-D)\cdot T_{SW} = 0$$

$$(V_{IN} - V_{OUT})\cdot D + (-V_{OUT})\cdot(1-D) = 0$$

$$D\cdot V_{IN} - D\cdot V_{OUT} - V_{OUT} + D\cdot V_{OUT} = 0$$

$$D\cdot V_{IN} - V_{OUT} = 0$$

$$V_{OUT} = D\cdot V_{IN}$$

Open Loop Buck Converter Waveforms

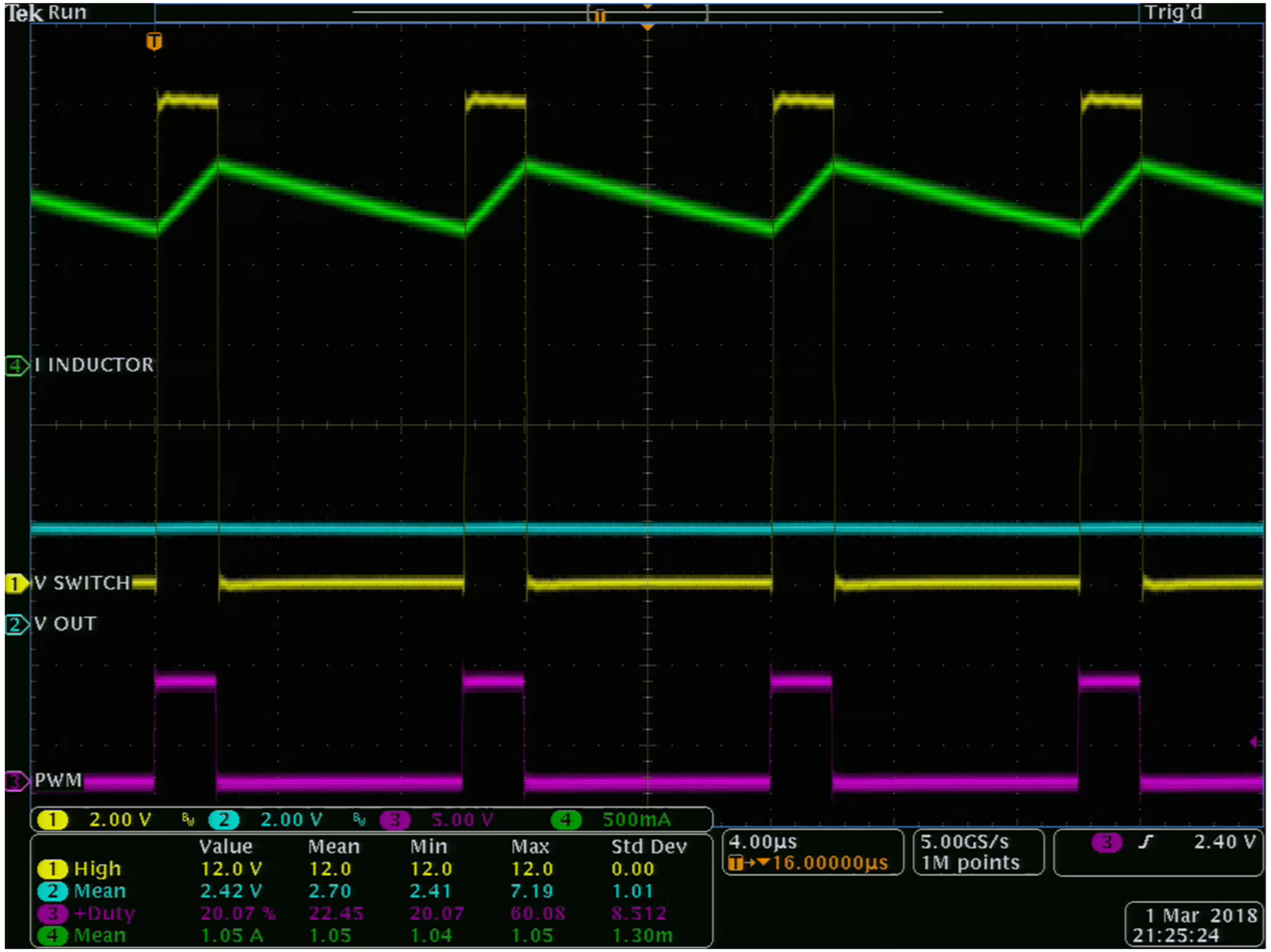
F_{SW} = 100 kHz D = 25% V_{IN} = 12 V V_{OUT} = 3.0 V I_{LOAD} = 1 A (Constant)

V_{SW}

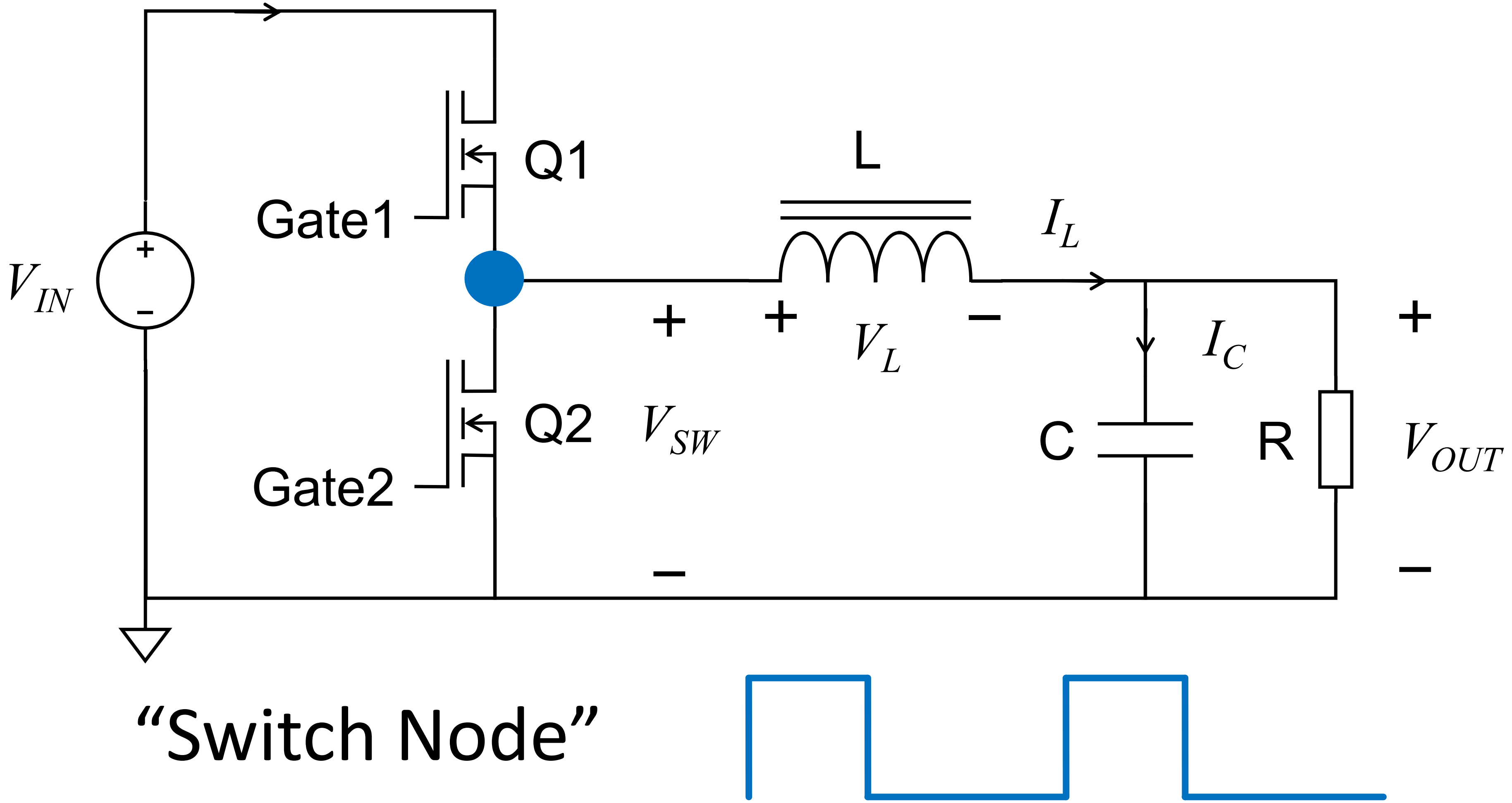
I_L

V_{OUT}

PWM



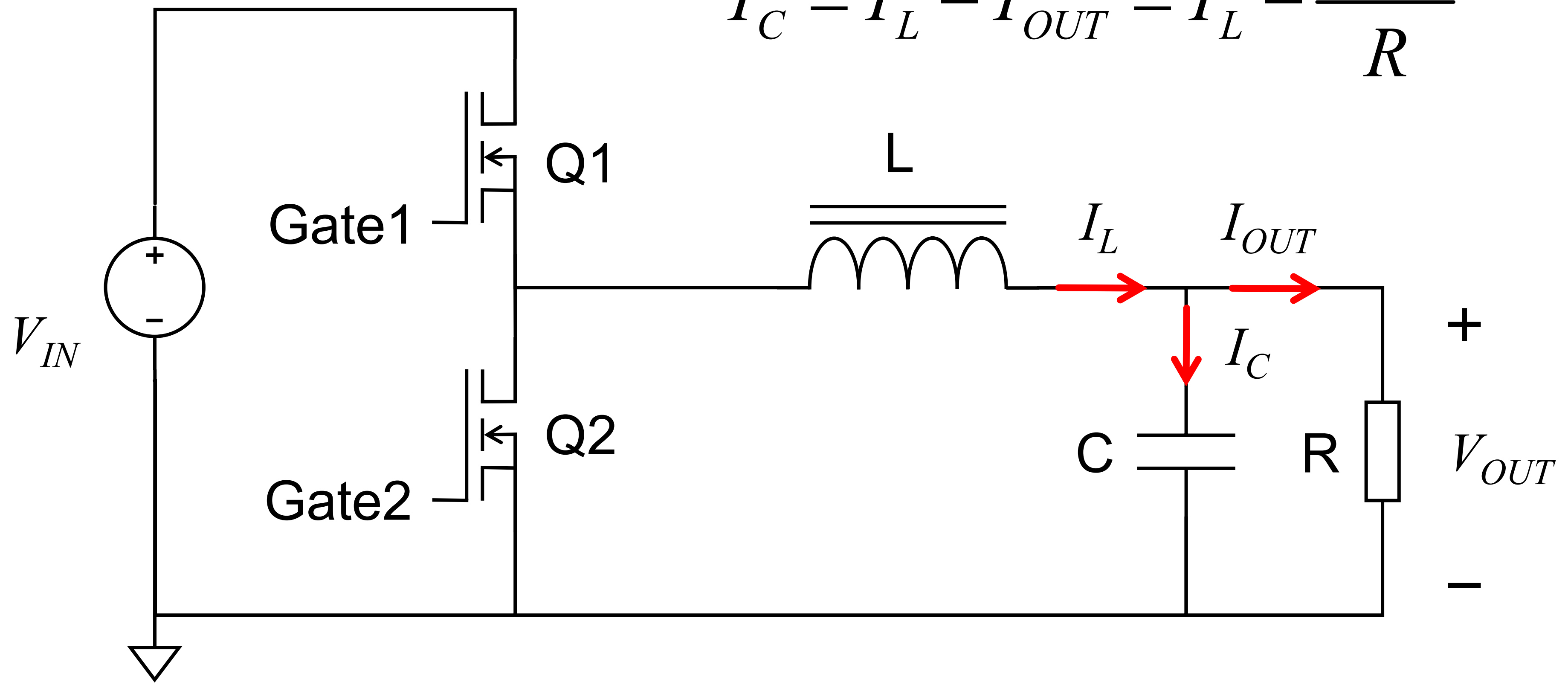
Video Lab 2
Buck Converter Waveforms



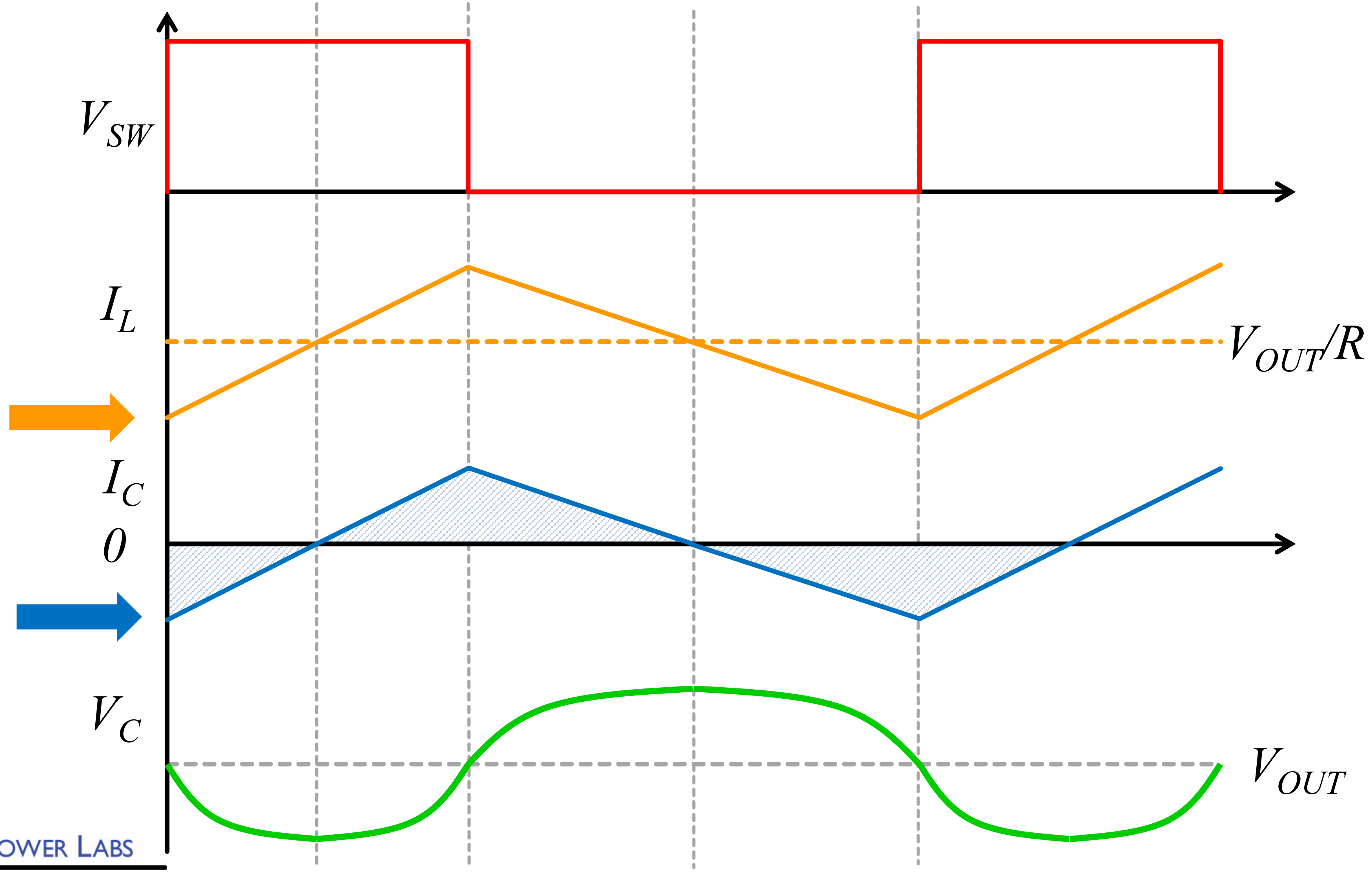
“Switch Node”

Buck Converter
Capacitor Current And Voltage

$$I_C = I_L - I_{OUT} = I_L - \frac{V_{OUT}}{R}$$

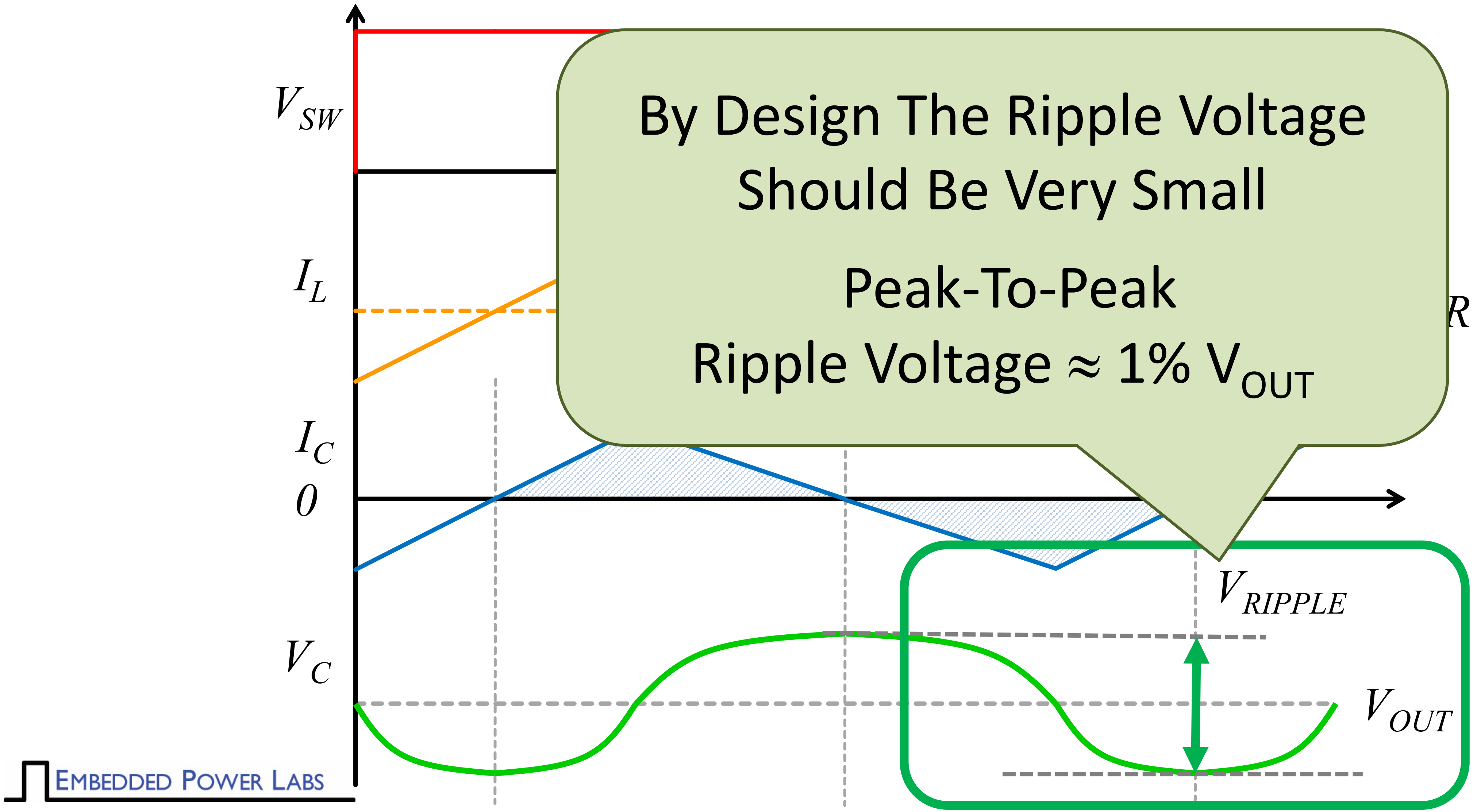


Buck Converter
Capacitor Current And Voltage

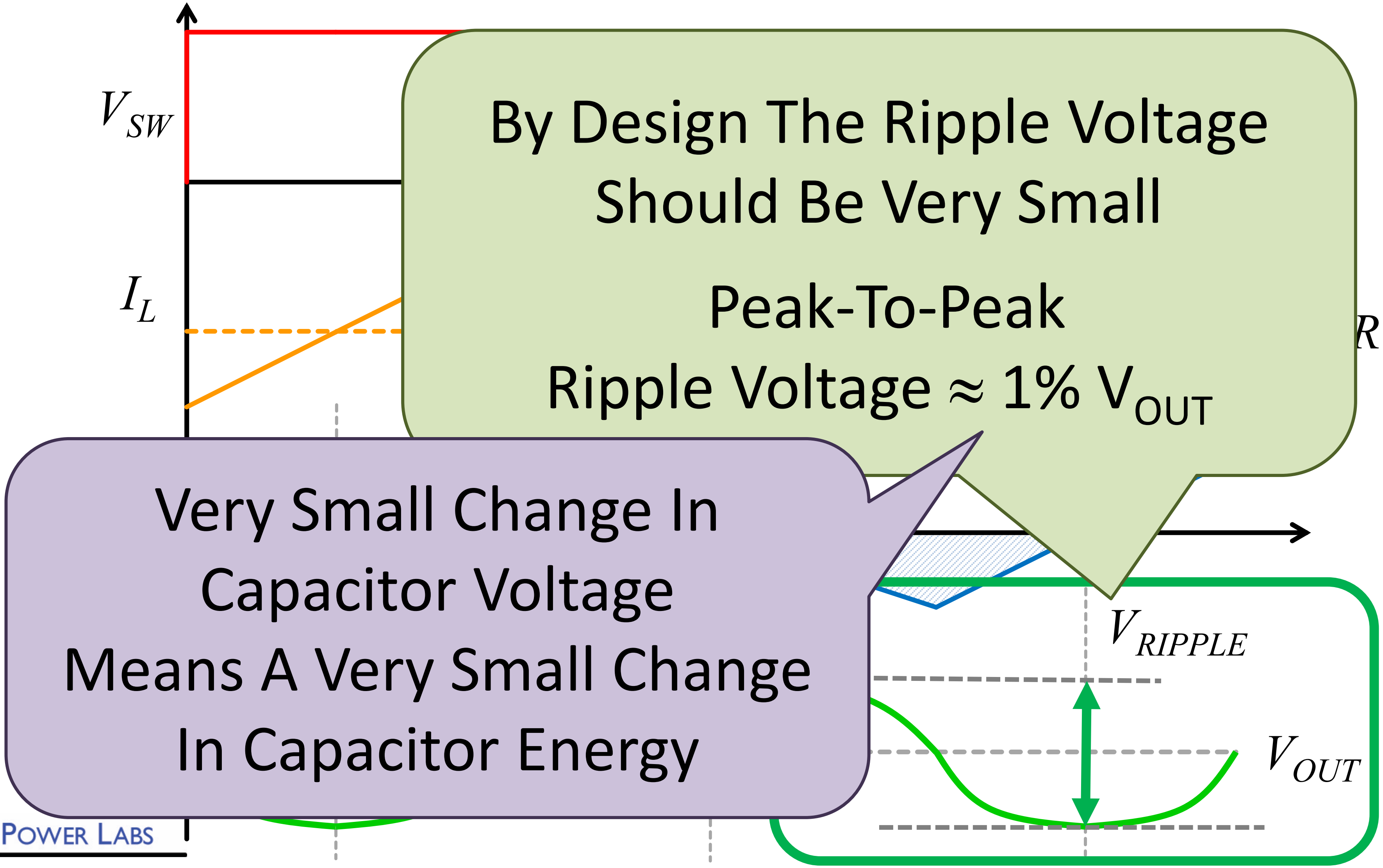


EMBEDDED POWER LABS

Buck Converter
Capacitor Current And Voltage



Buck Converter
Capacitor Current And Voltage



Capacitor Charge Balance

$$I_C(T_{ON})\cdot T_{ON} + I_C(T_{OFF})\cdot T_{OFF} = 0$$

$$I_C(T_{ON}) = I_L - \frac{V_{OUT}}{R}$$

$$I_C(T_{OFF}) = I_L - \frac{V_{OUT}}{R}$$

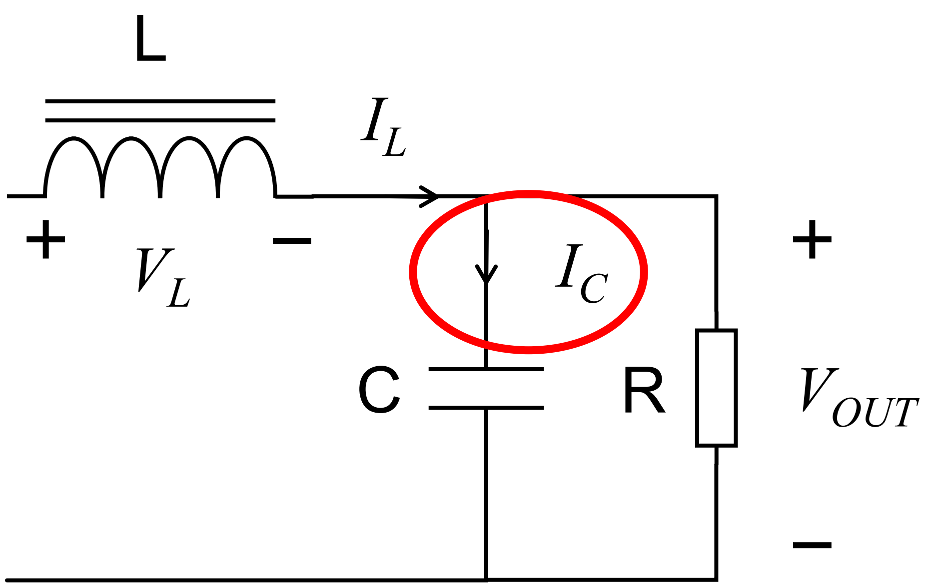
$$\left(I_L - \frac{V_{OUT}}{R}\right)\cdot T_{ON} + \left(I_L - \frac{V_{OUT}}{R}\right)\cdot T_{OFF} = 0$$

$$\left(I_L - \frac{V_{OUT}}{R}\right)\cdot T_{SW} = 0$$

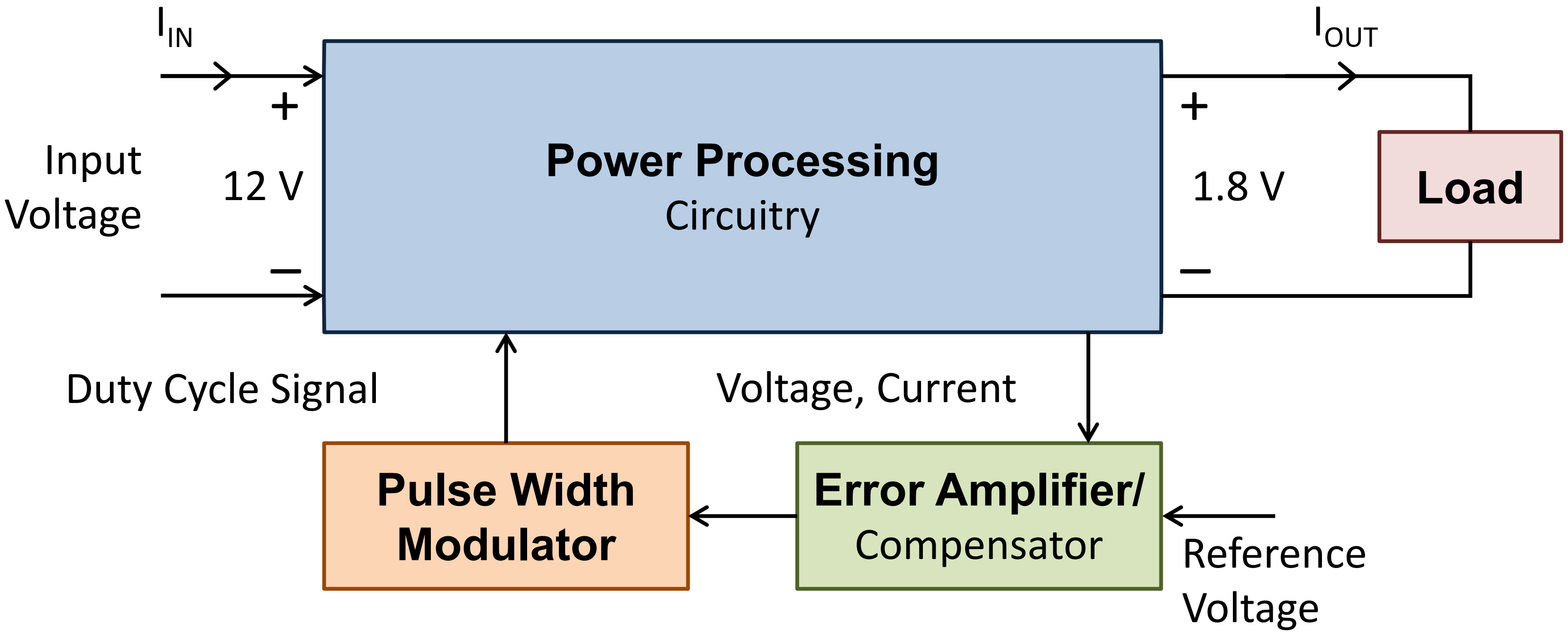
$$I_L - \frac{V_{OUT}}{R} = 0$$

$$I_L = \frac{V_{OUT}}{R}$$

This Sets The Inductor Current Rating



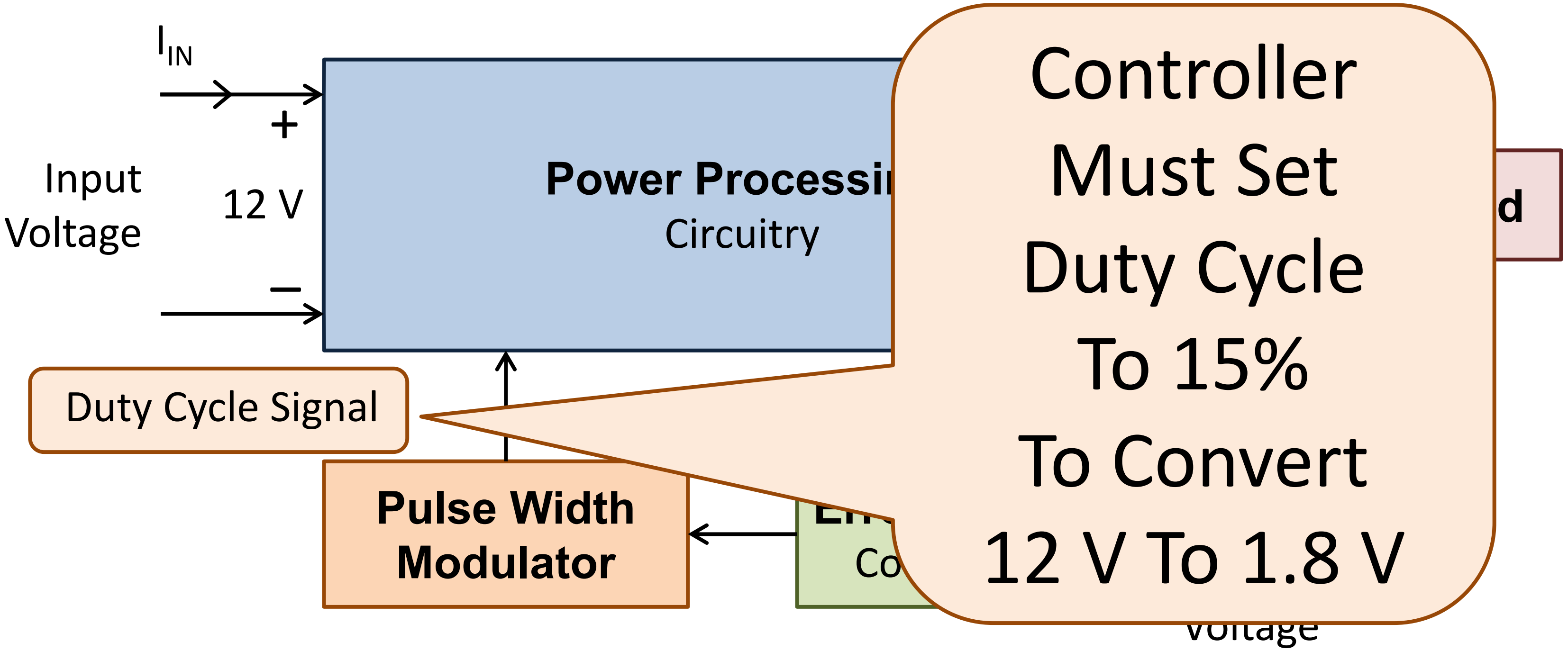
Buck Converter With Feedback



$$V_{OUT} = 1.8\,\text{V}$$
$$V_{IN} = 12\,\text{V}$$
$$I_{OUT} = 5\,\text{A}$$

Closed Loop Control:
Output Voltage Held Constant

Buck Converter With Feedback



$$V_{OUT} = 1.8\,\text{V}$$

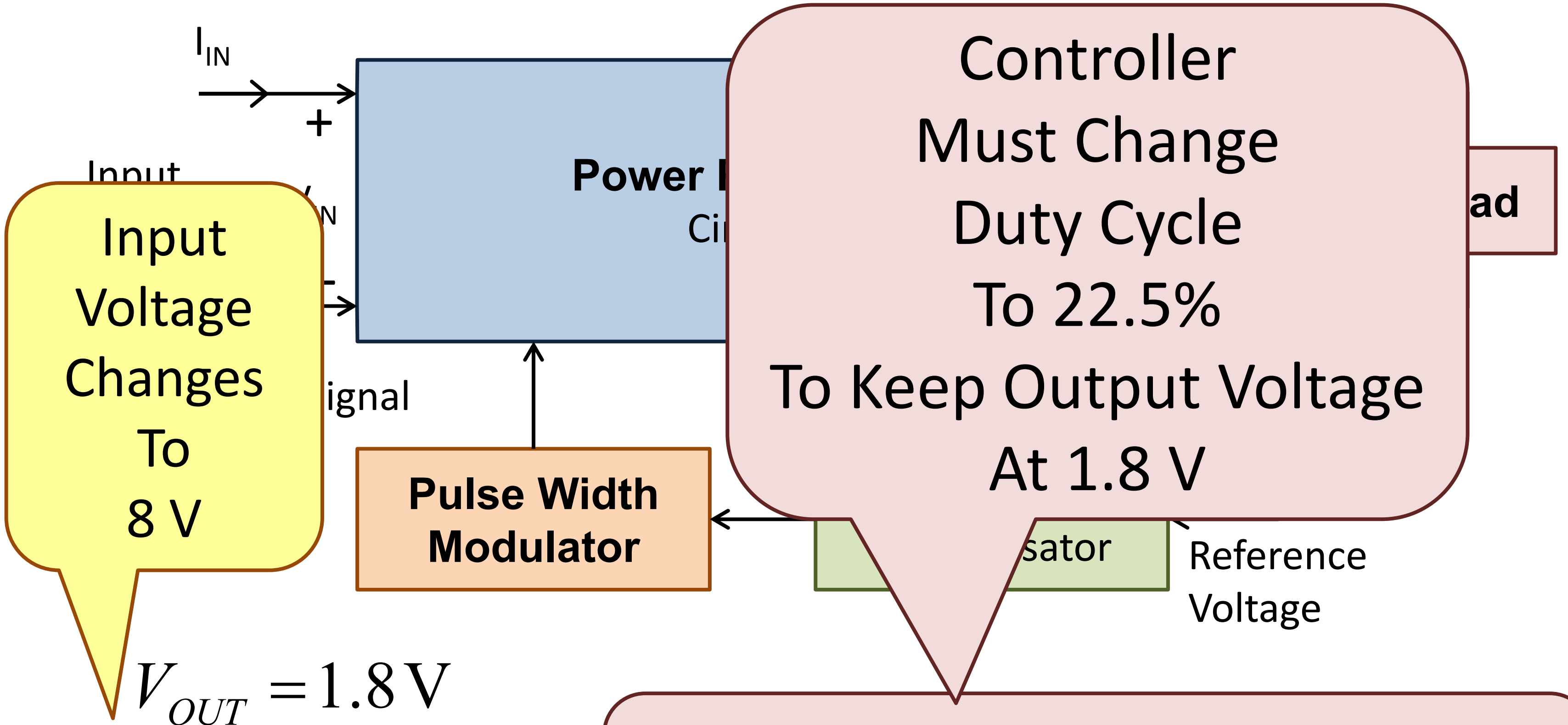
$$V_{IN} = 12\,\text{V}$$

$$I_{OUT} = 5\,\text{A}$$

$$V_{OUT} = D \cdot V_{IN}$$

$$D = \frac{V_{OUT}}{V_{IN}} = \frac{1.8\,\text{V}}{12\,\text{V}} = 0.15 = 15\%$$

Buck Converter With Feedback



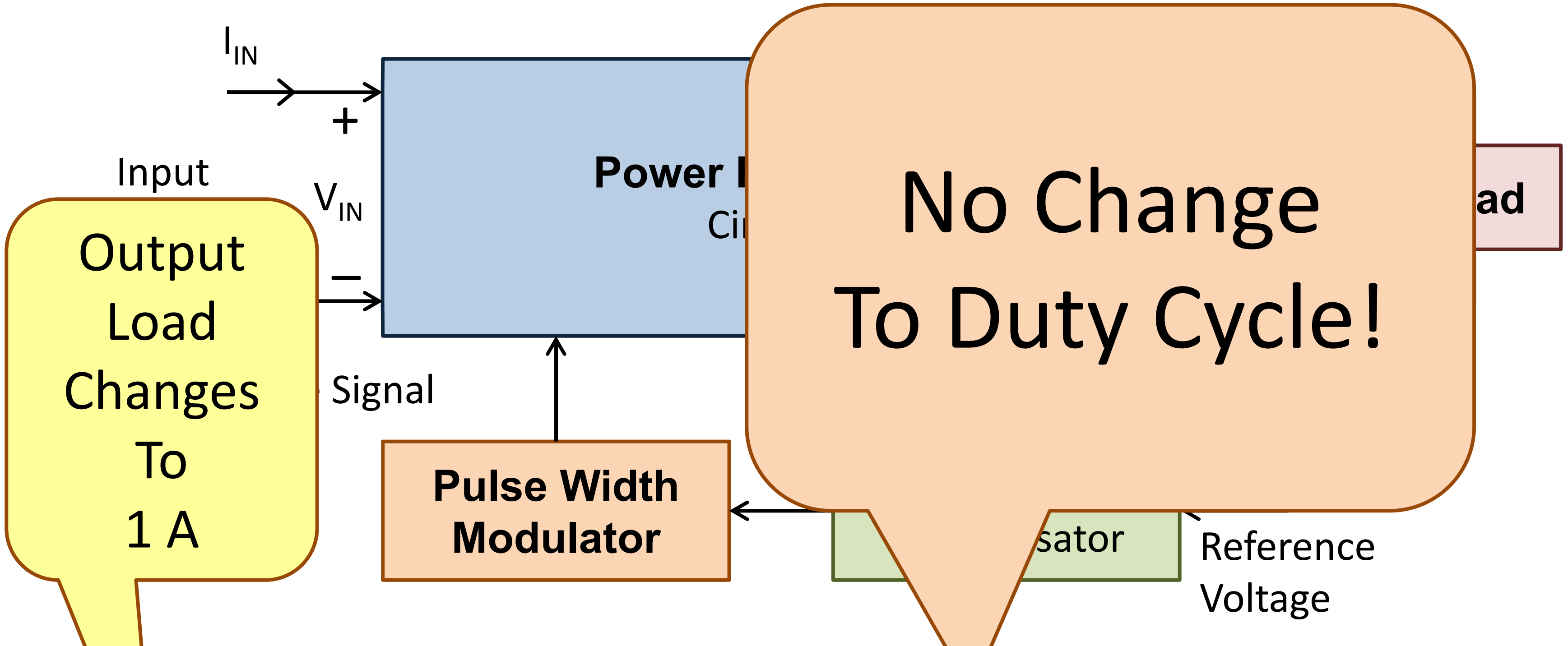
$$V_{OUT} = 1.8\,\text{V}$$

$$V_{IN} = 8\,\text{V}$$

$$I_{OUT} = 5\,\text{A}$$

$$D = \frac{V_{OUT}}{V_{IN}} = \frac{1.8\,\text{V}}{8\,\text{V}} = 0.225 = 22.5\%$$

Buck Converter With Feedback



$V_{OUT} = 1.8\,\text{V}$

$V_{IN} = 8\,\text{V}$

$I_{OUT} = 1\,\text{A}$

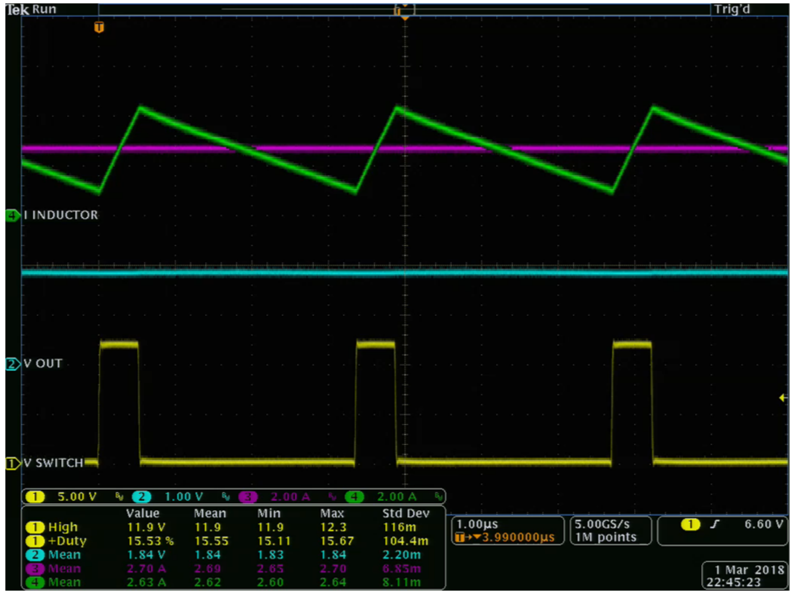
$$D = \frac{V_{OUT}}{V_{IN}} = \frac{1.8\,\text{V}}{8\,\text{V}} = 0.225 = 22.5\%$$

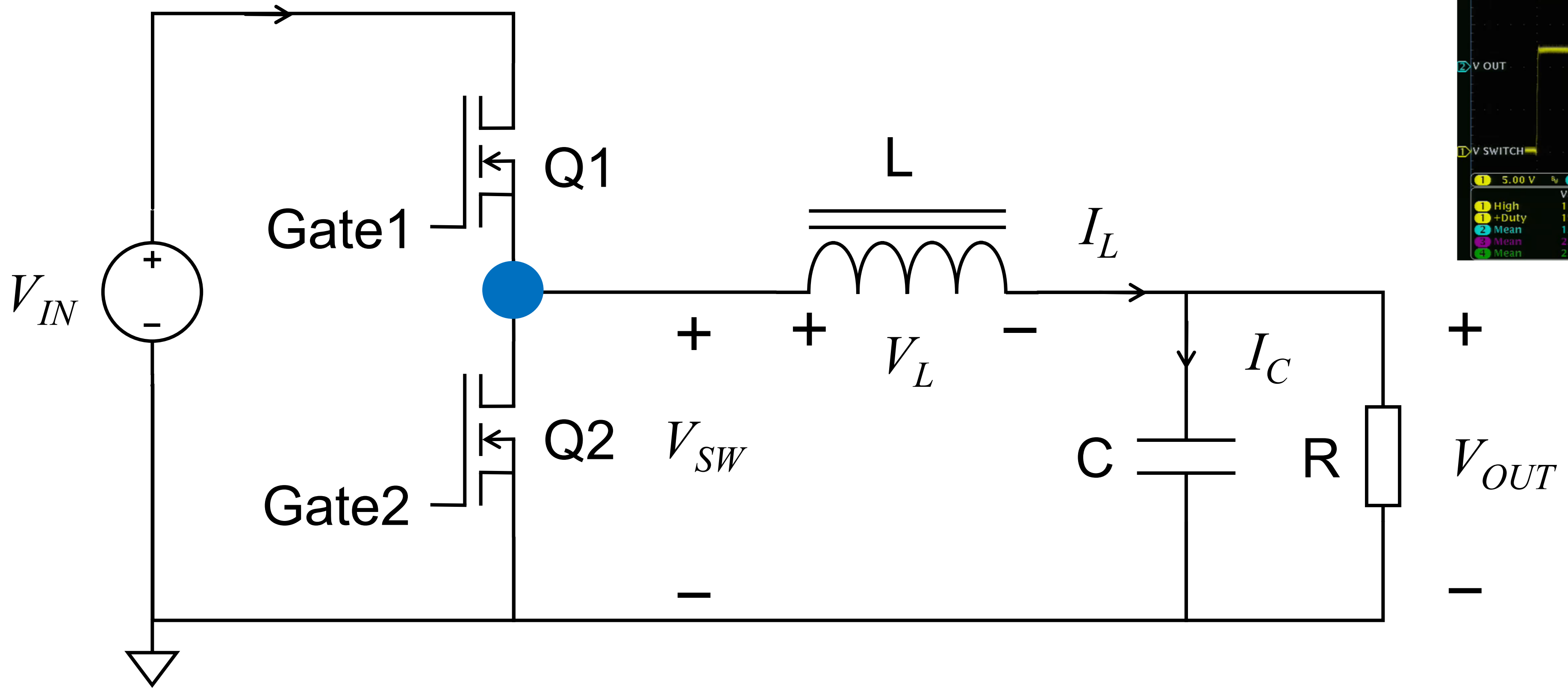
Buck Converter With Feedback

Not Exactly True
Duty Cycle Changes
To Compensate
For Changes In Losses

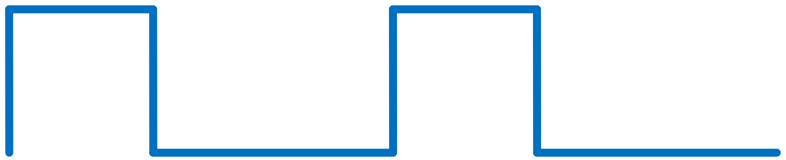
See Example
In Appendix I

Video Lab 3
Buck Converter With Feedback

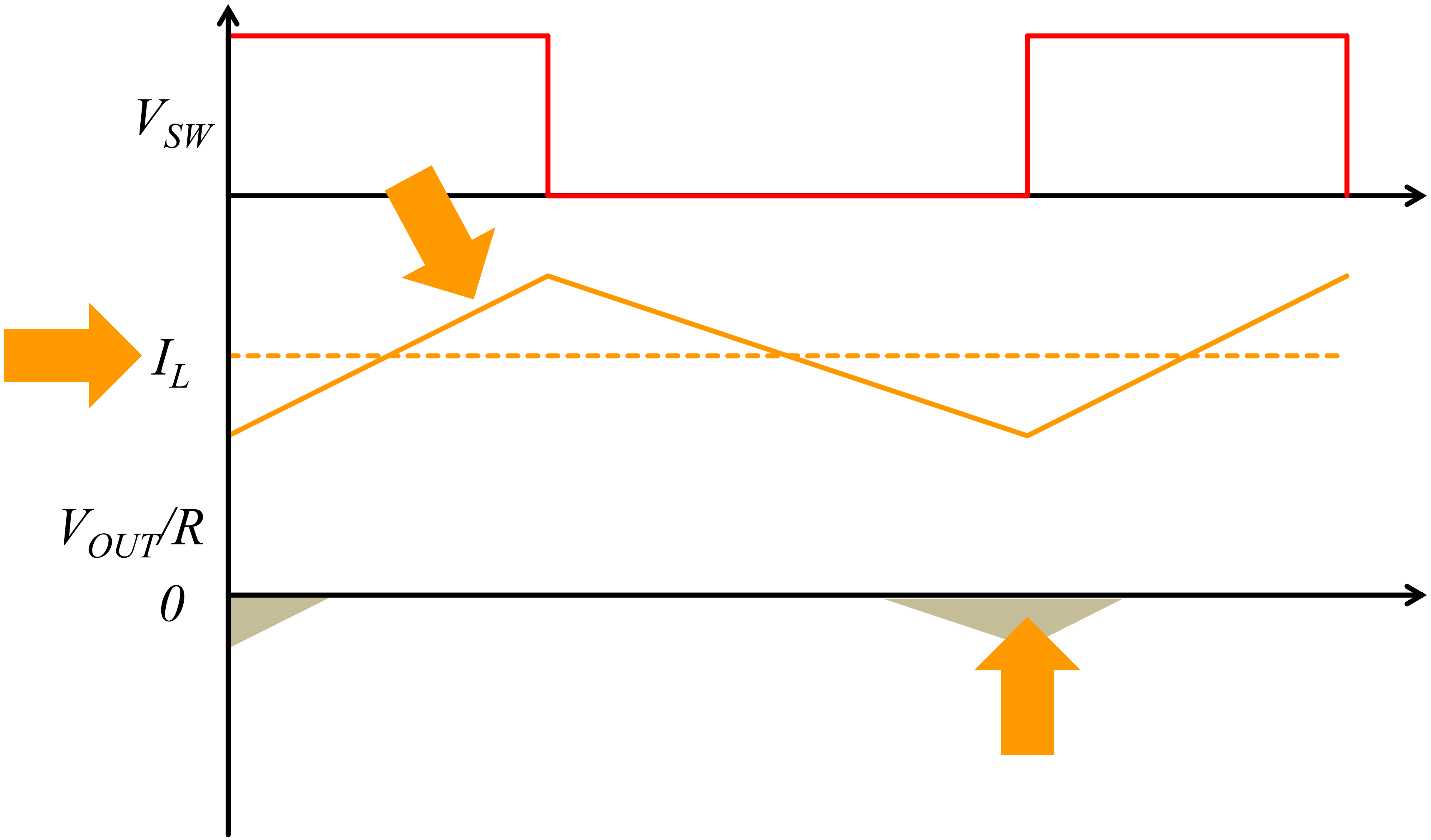




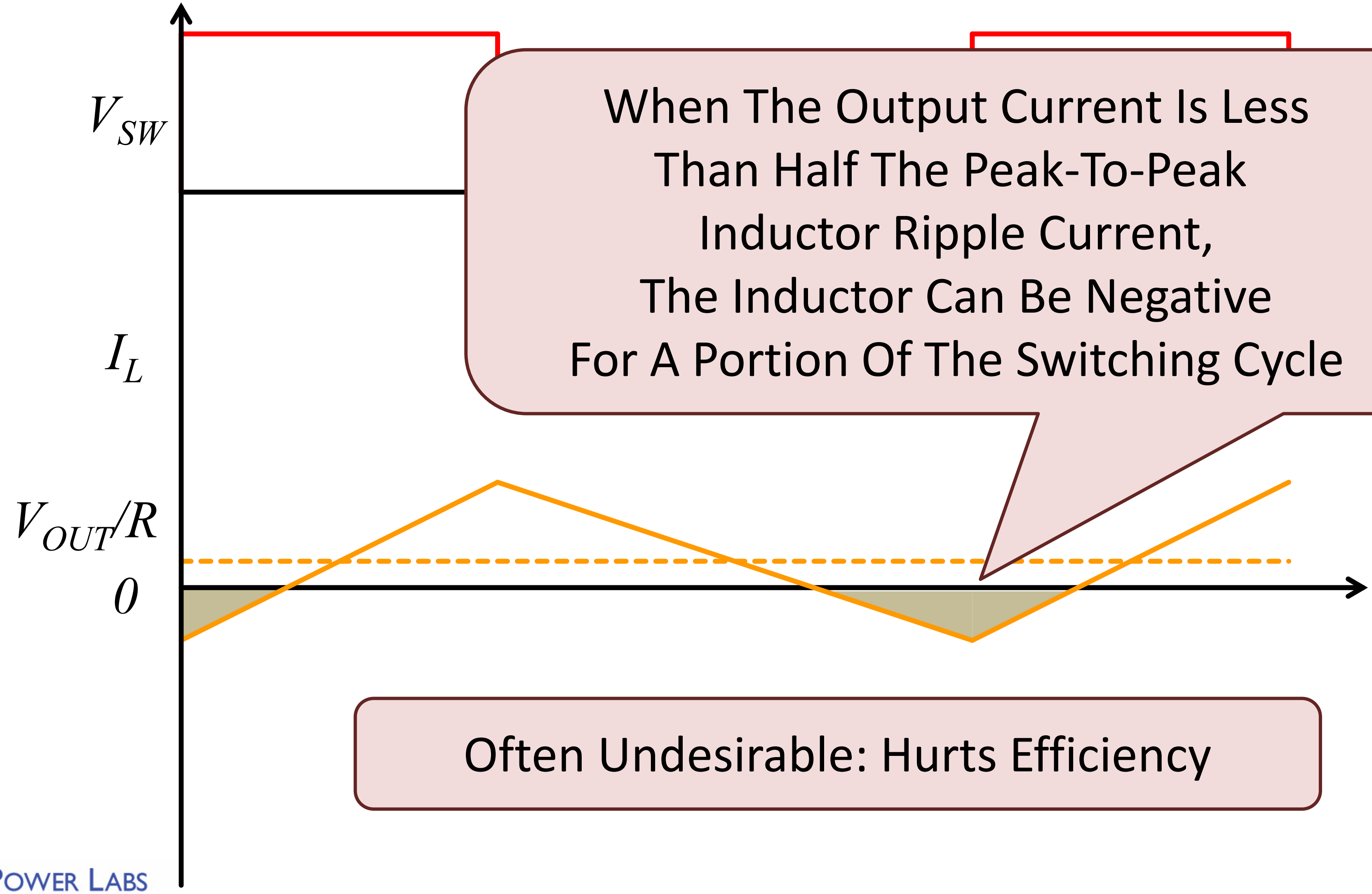
“Switch Node”



Discontinuous Conduction Mode (DCM)



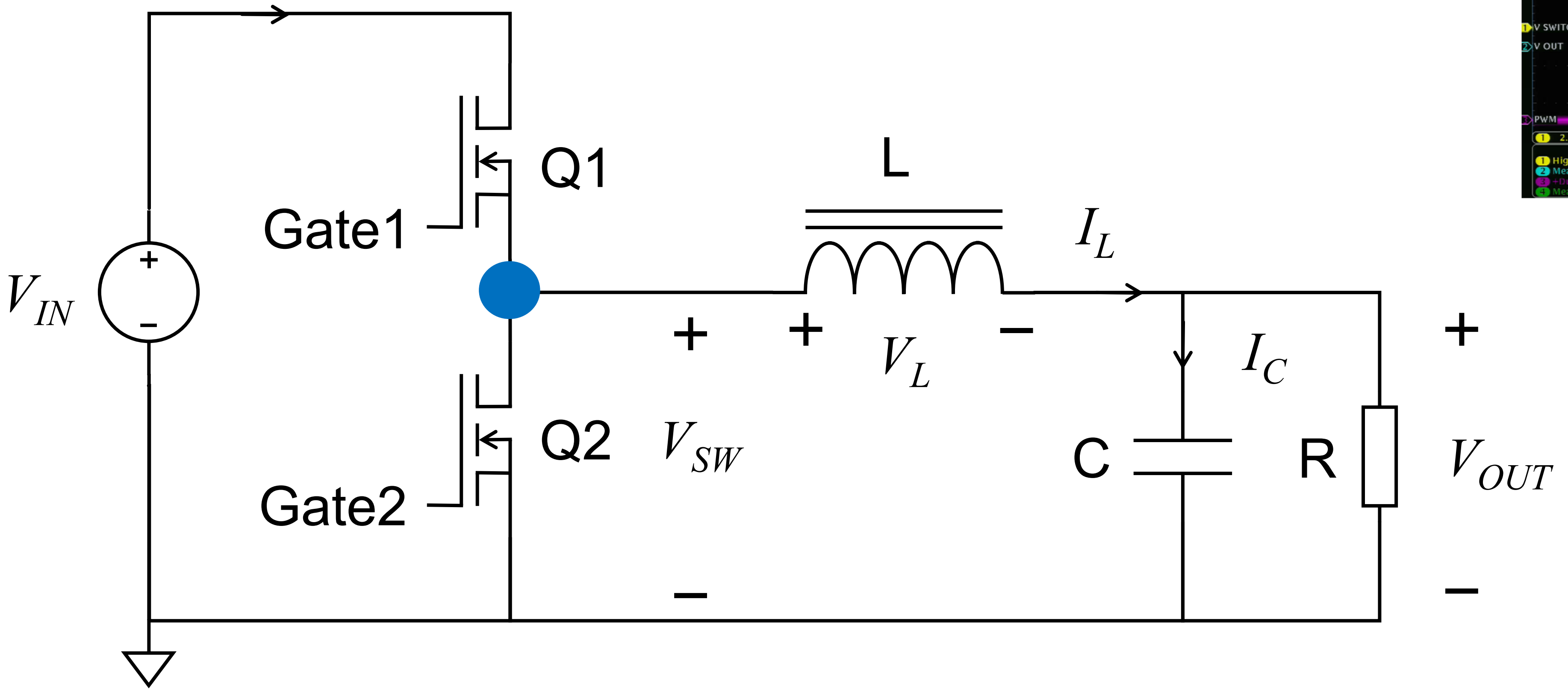
Discontinuous Conduction Mode (DCM)



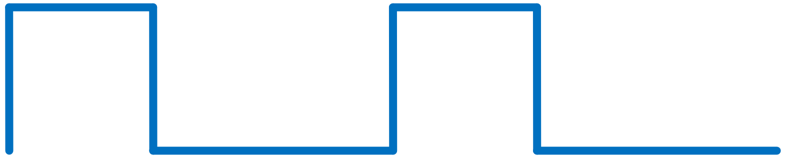
When The Output Current Is Less Than Half The Peak-To-Peak Inductor Ripple Current, The Inductor Can Be Negative For A Portion Of The Switching Cycle

Often Undesirable: Hurts Efficiency

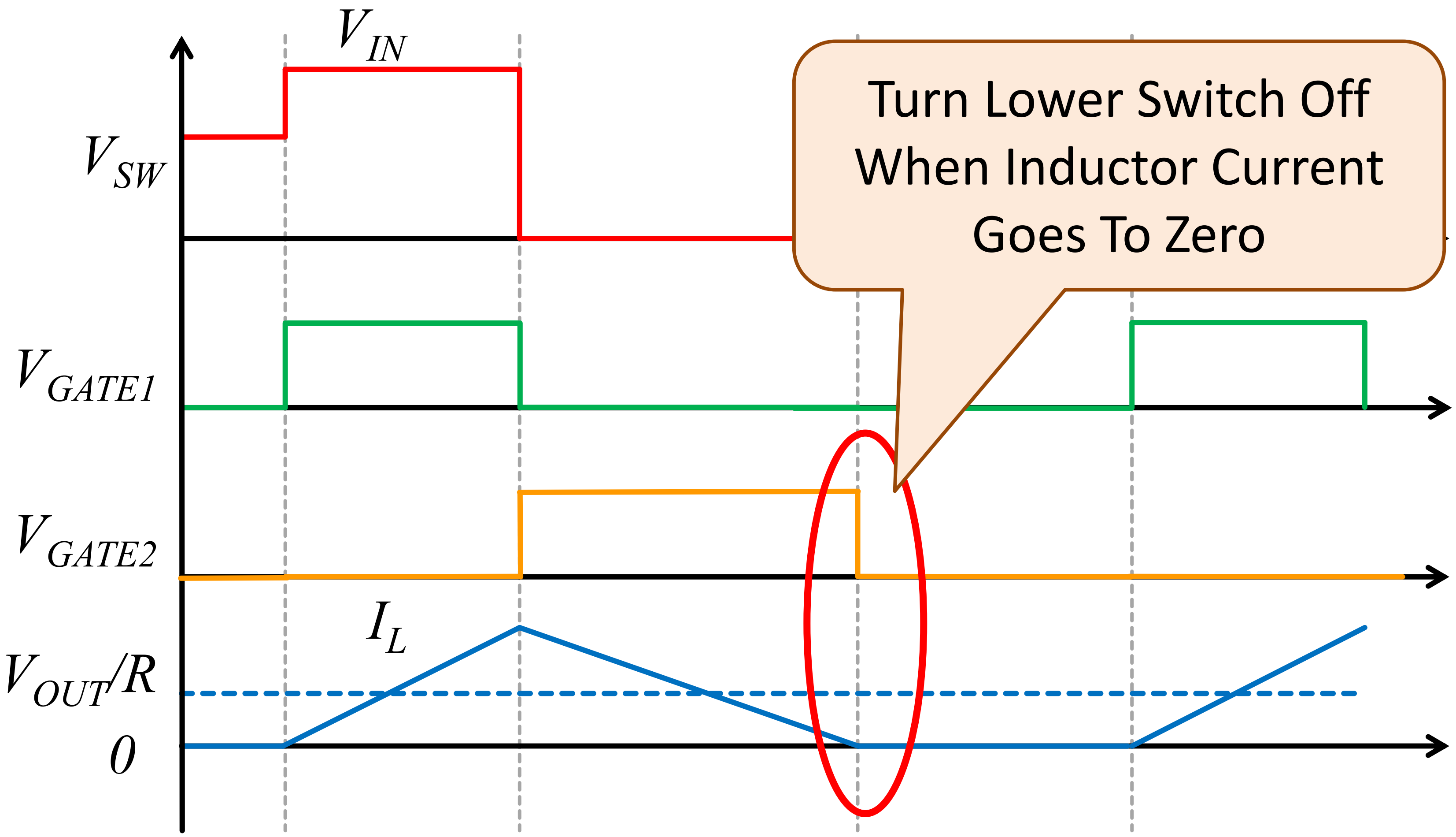
Video Lab 4
Buck Converter: Negative Inductor Current



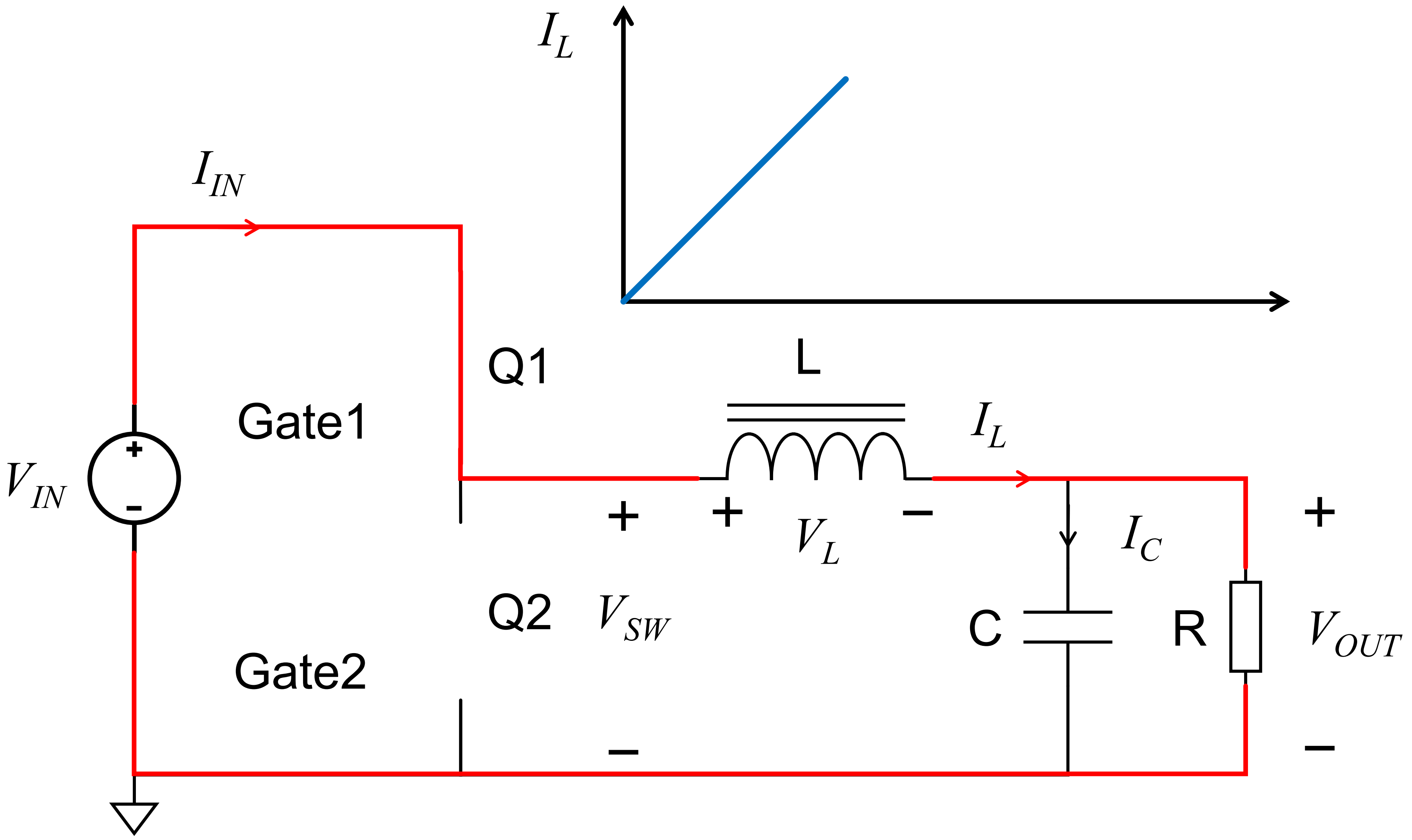
"Switch Node"



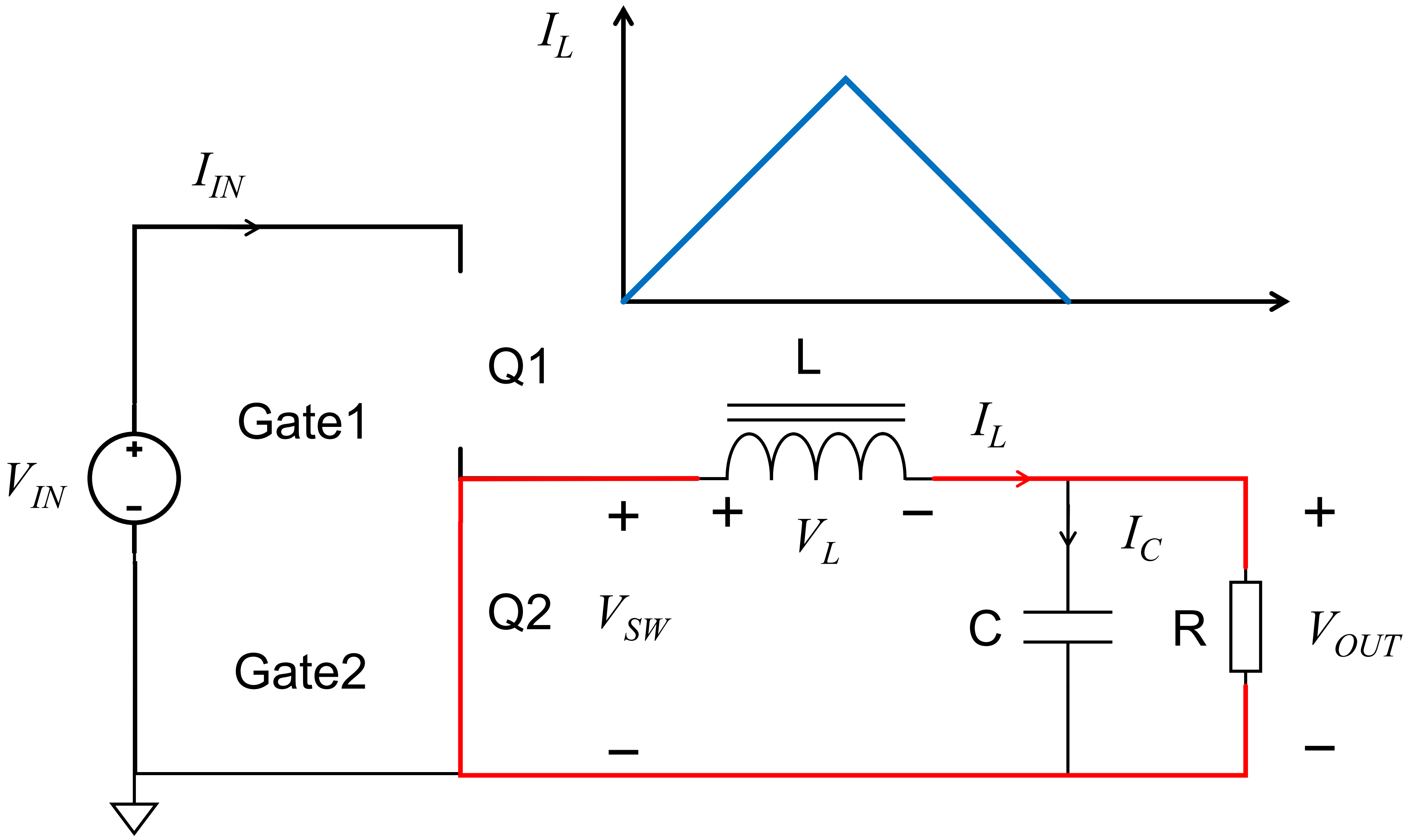
Discontinuous Conduction Mode (DCM)



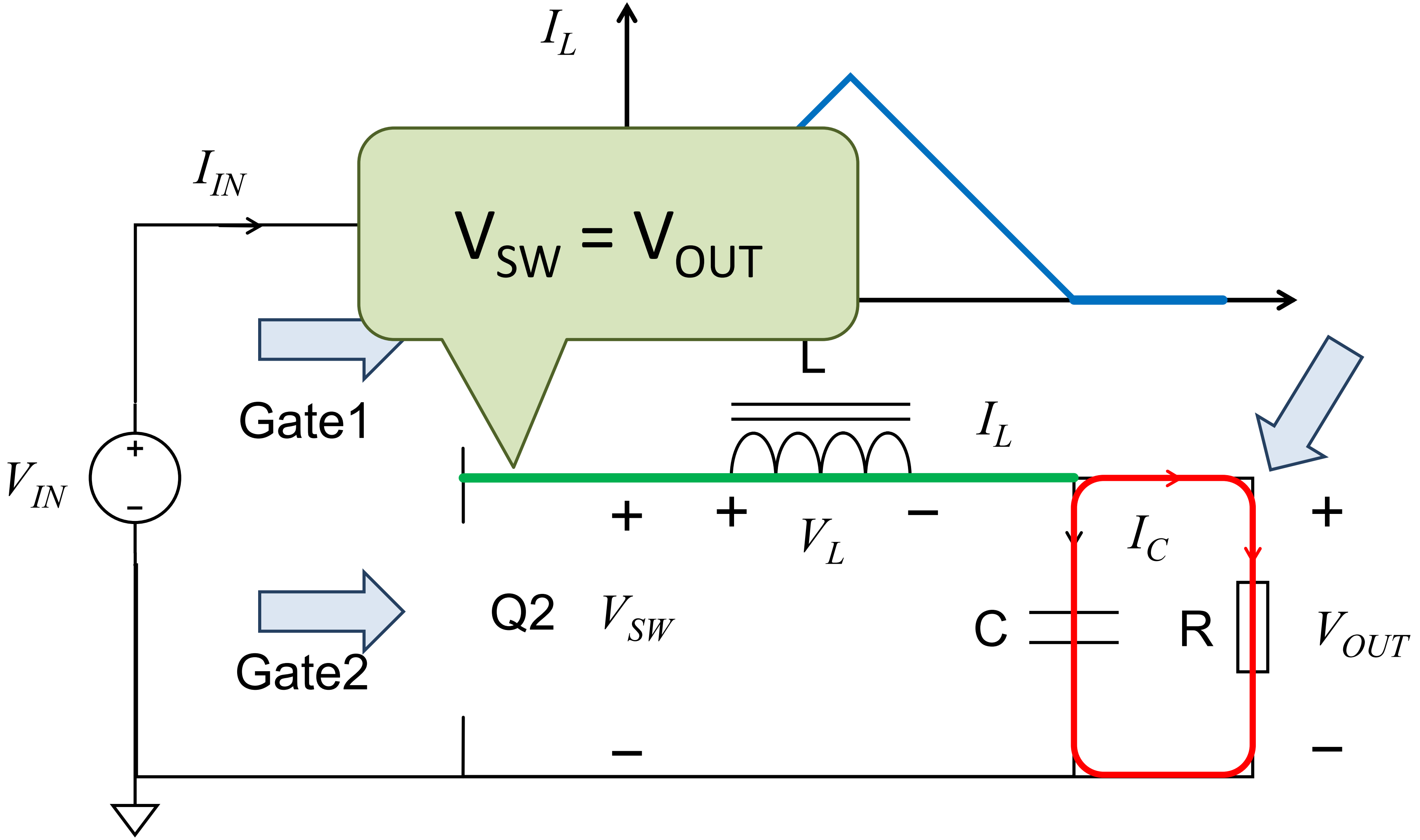
DCM Buck Converter On Time



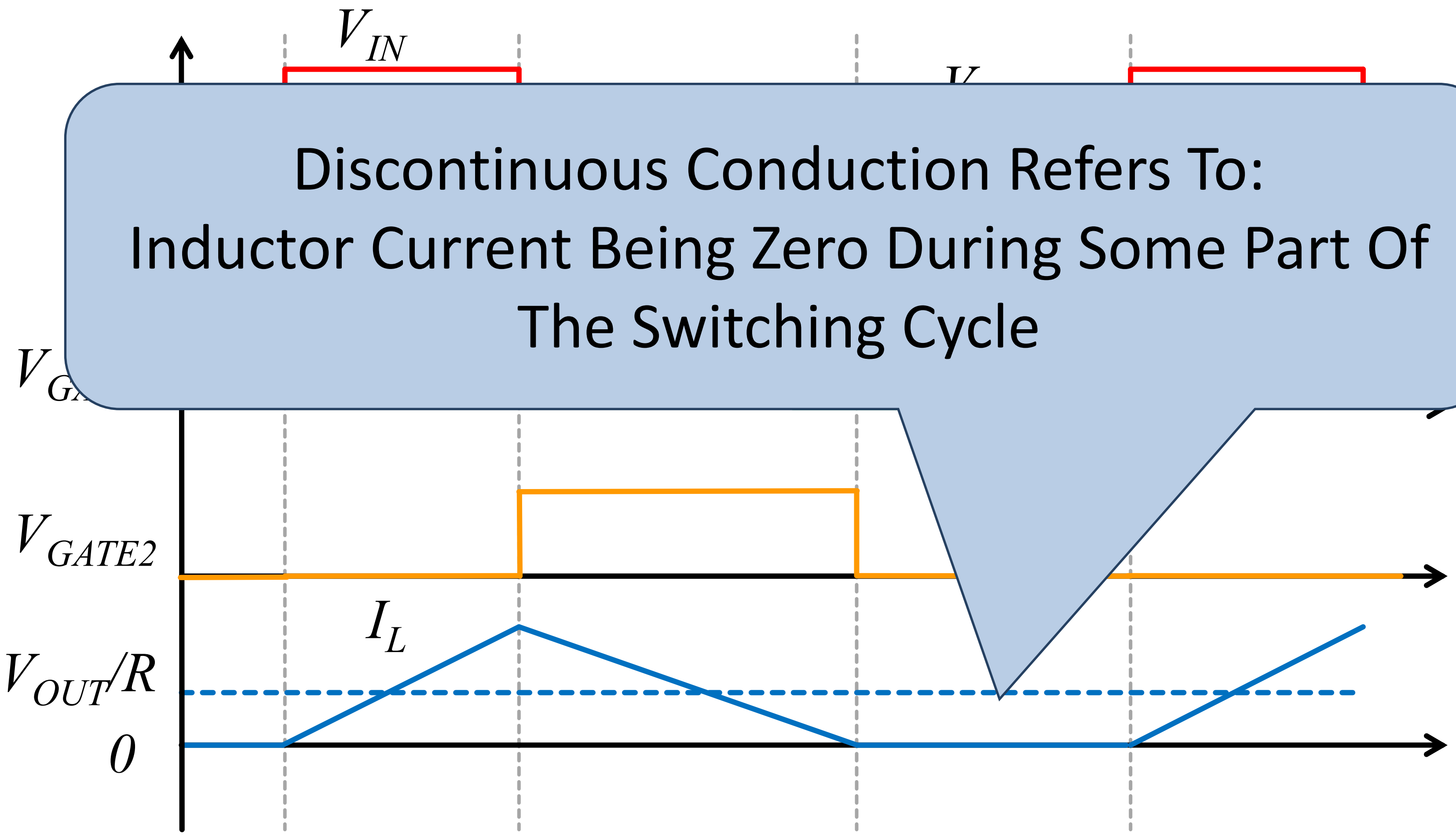
DCM Buck Converter Off Time



Buck Converter Idle Time



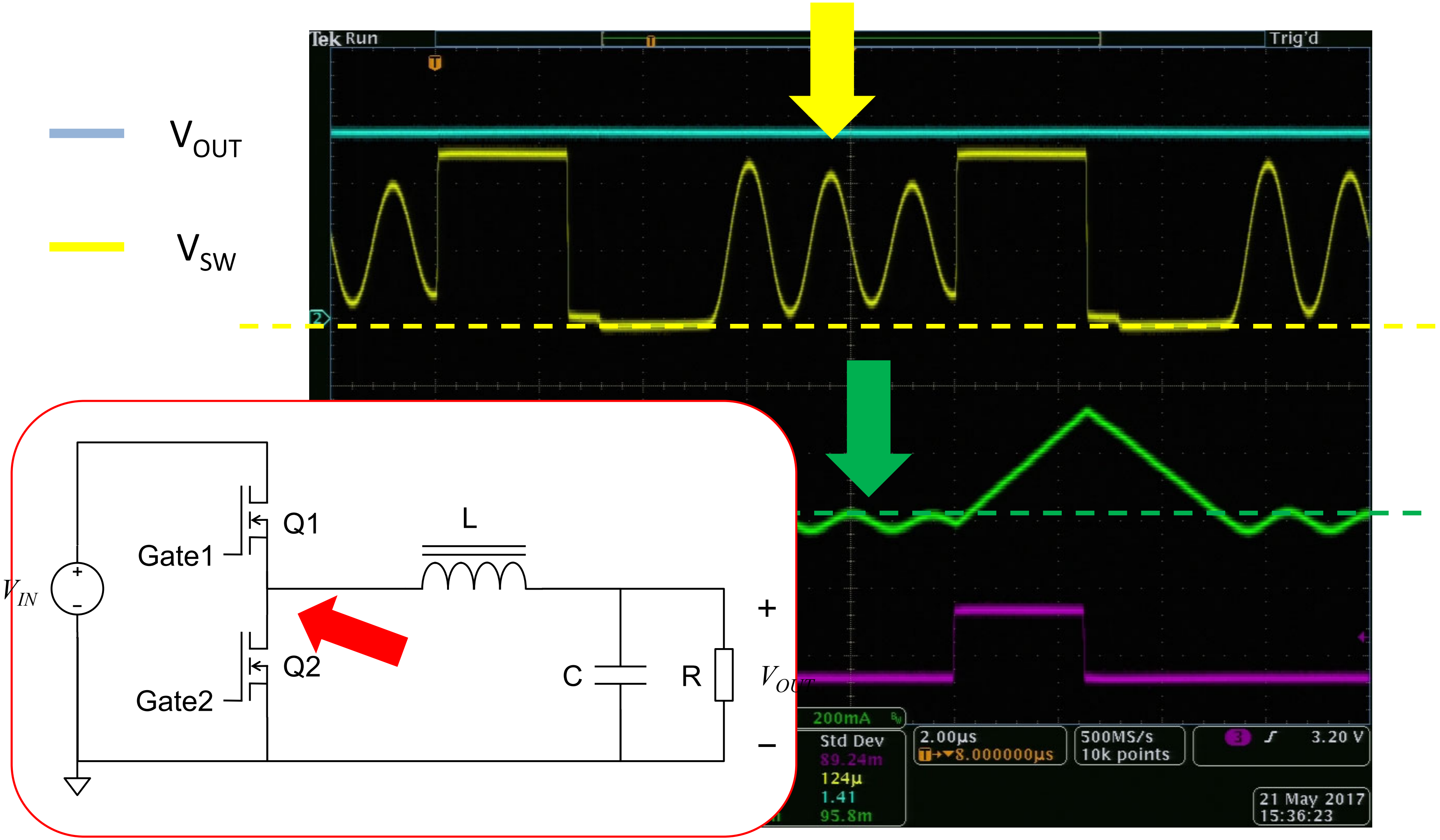
Discontinuous Conduction Mode (DCM)



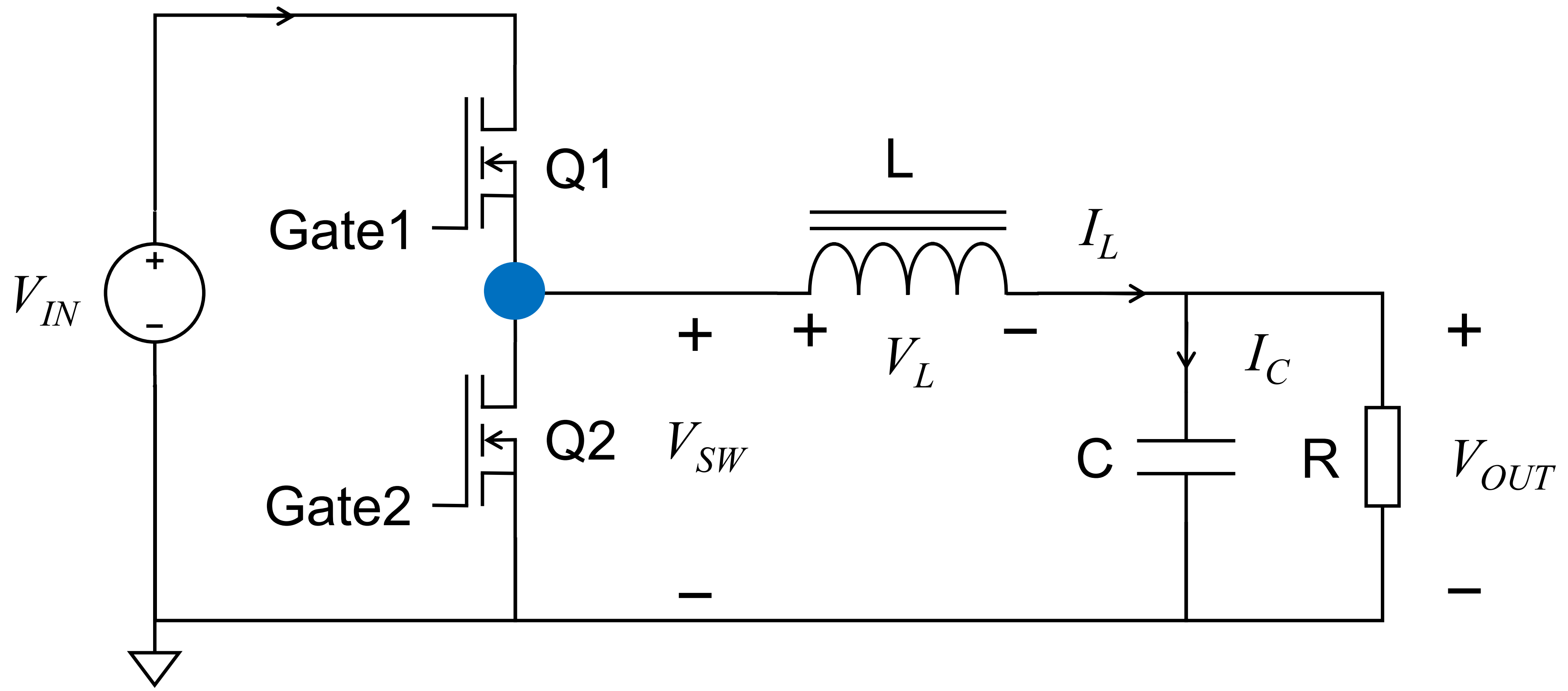
Discontinuous Conduction Refers To:
Inductor Current Being Zero During Some Part Of The Switching Cycle

Buck DCM Waveforms

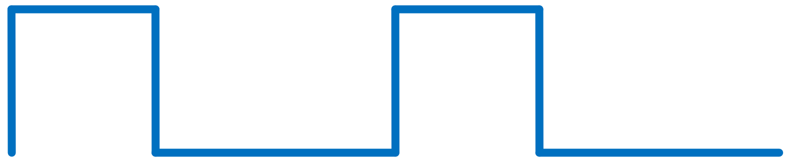
Buck DCM Waveforms



Video Lab 5
Buck Converter Waveforms: DCM



"Switch Node"



Buck DCM Waveforms

Gate Drive IC Detects Negative Sync Switch Current And Turns Off The Sync Switch

Conversion Characteristic

- For Buck Converter Operating In Discontinuous Conduction Mode (DCM):

$$V_{OUT} = \frac{2}{1 + \sqrt{1 + \frac{4}{D^2} \cdot \frac{2 \cdot L}{R \cdot T_{SW}}}} \cdot V_{IN}$$

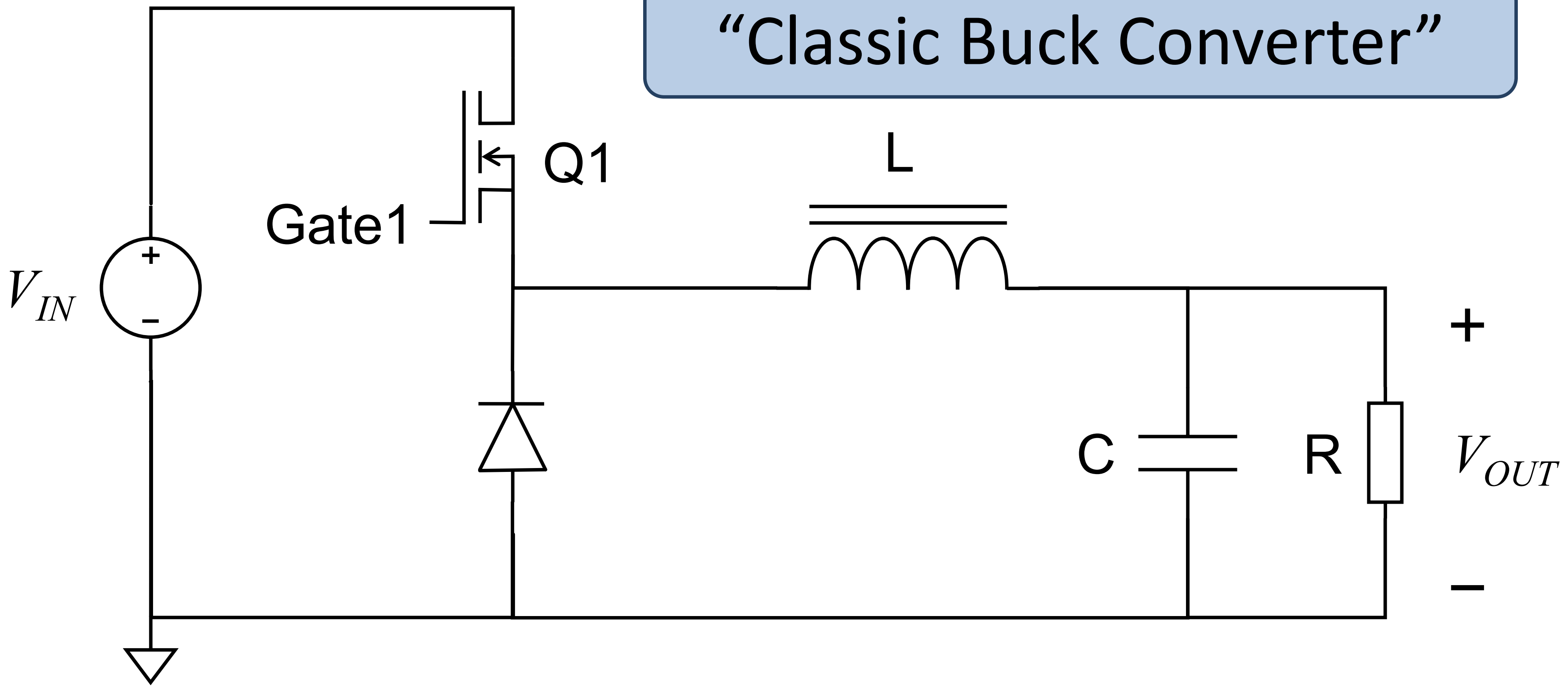
Derivation In Appendix

Nonlinear With Duty Cycle

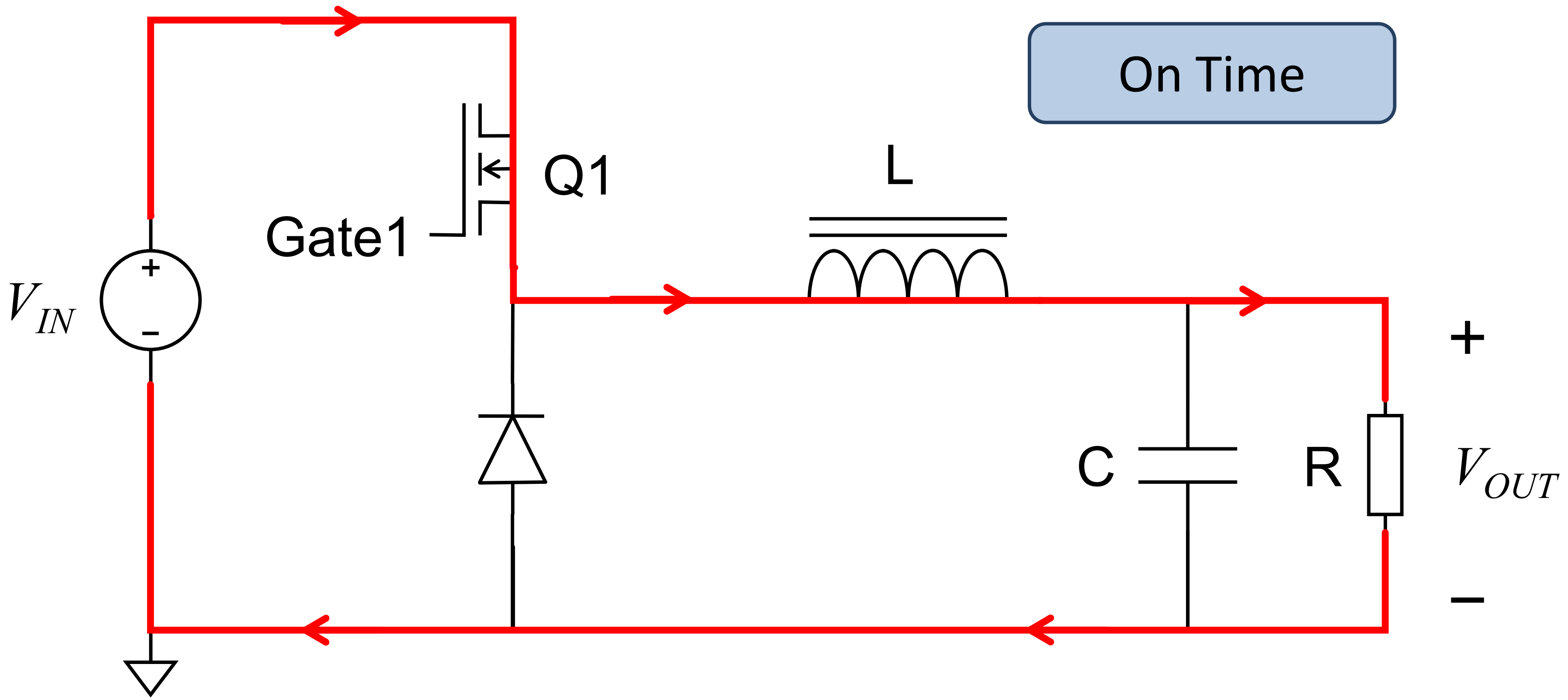
Dependent On Load Resistance

Buck Converter w/Diode

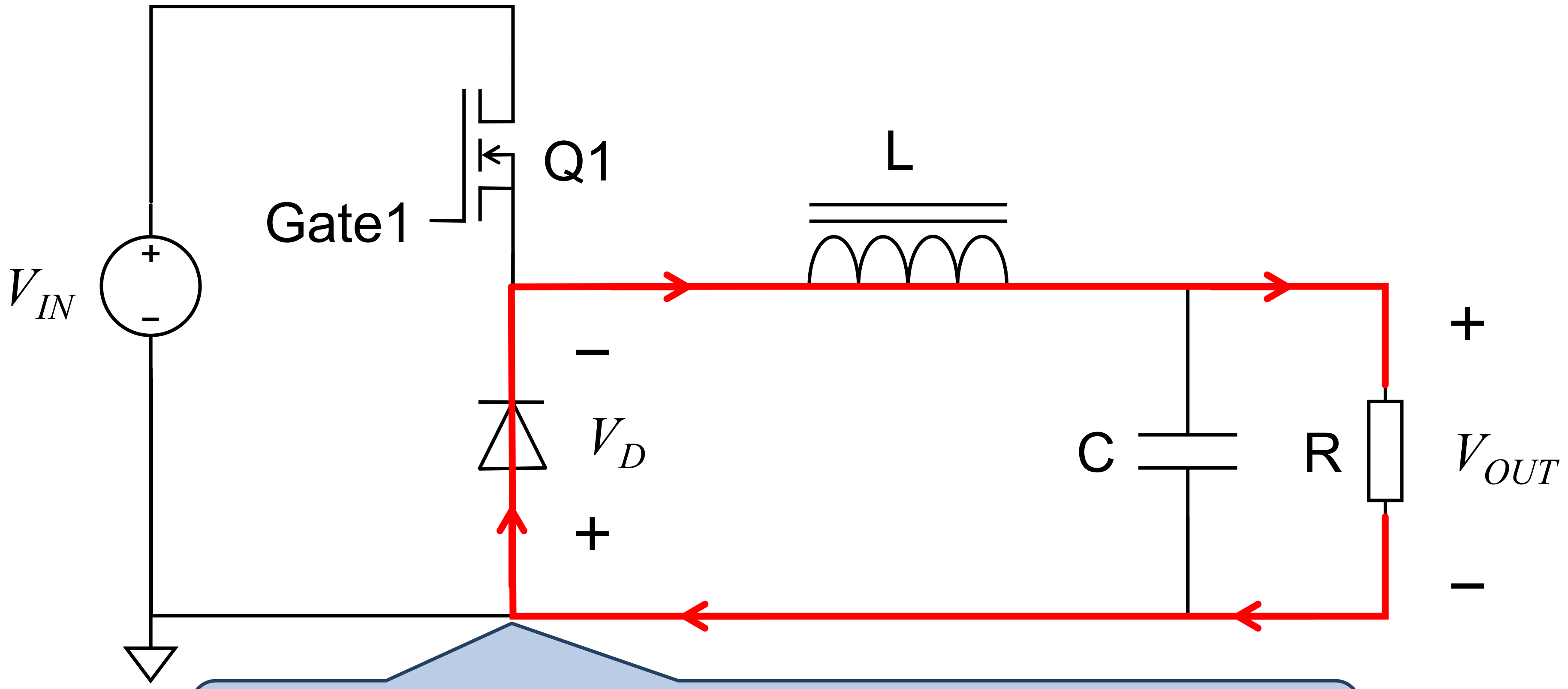
“Classic Buck Converter”



Buck Converter w/Diode: On Time

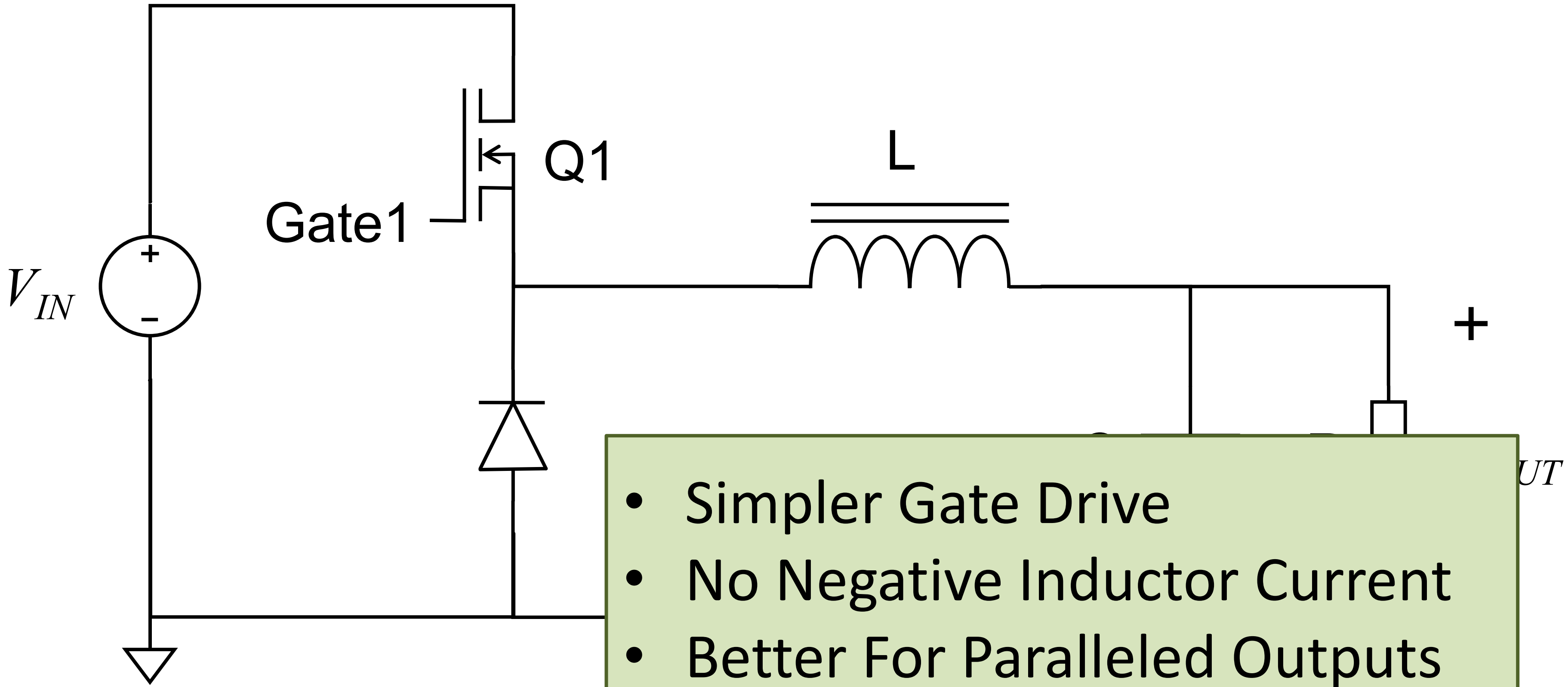


Buck Converter w/Diode: Off Time



For Good Efficiency, Diode Forward Voltage Must Be Much Less Than Output Voltage

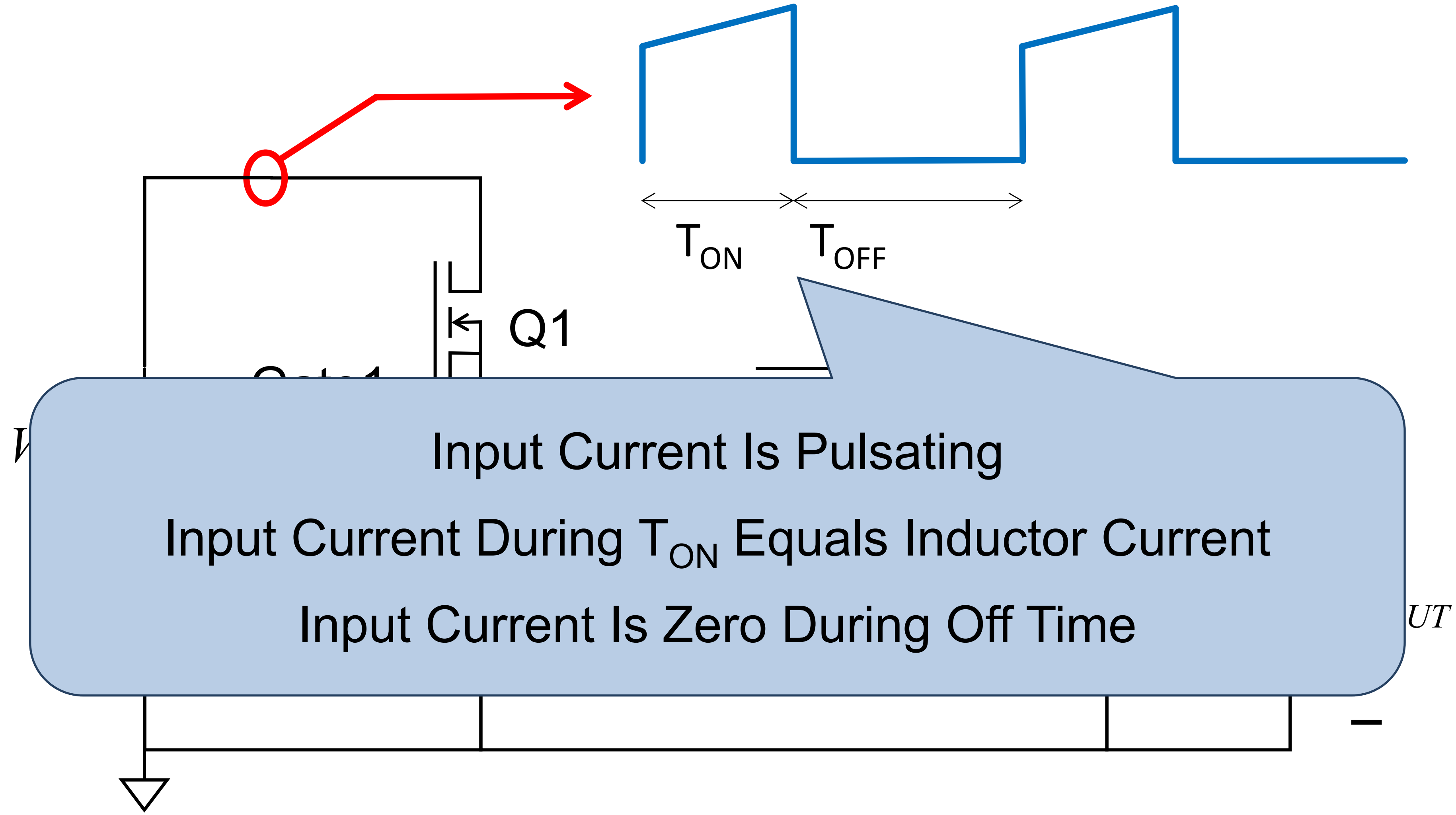
Buck Converter w/Diode



- Simpler Gate Drive
- No Negative Inductor Current
- Better For Paralleled Outputs
- Less Expensive

EMBEDDED POWER LABS

Buck Converter Input Current

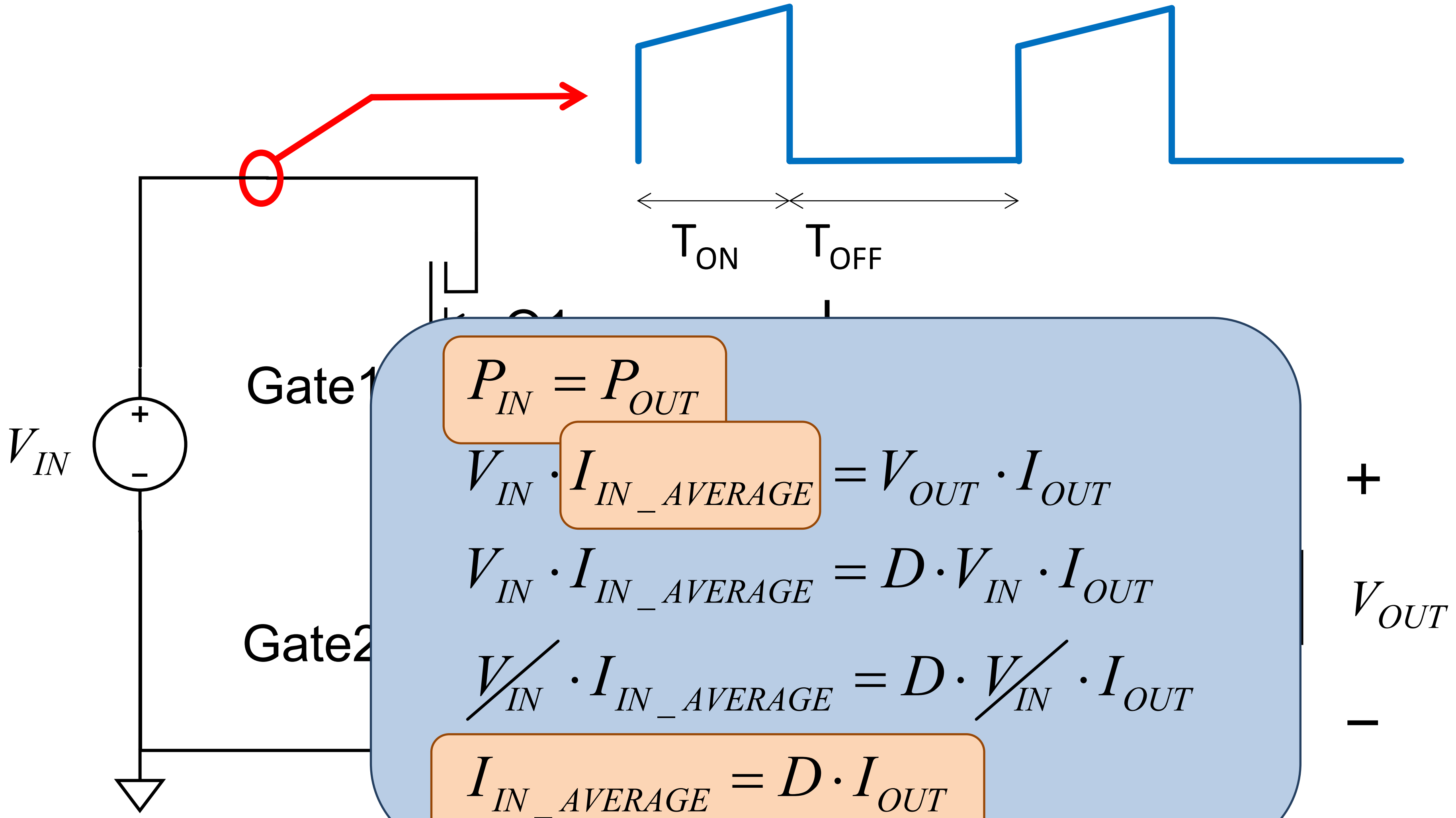


Input Current Is Pulsating

Input Current During T_{ON} Equals Inductor Current

Input Current Is Zero During Off Time

Buck Converter Input Current



$$P_{IN} = P_{OUT}$$

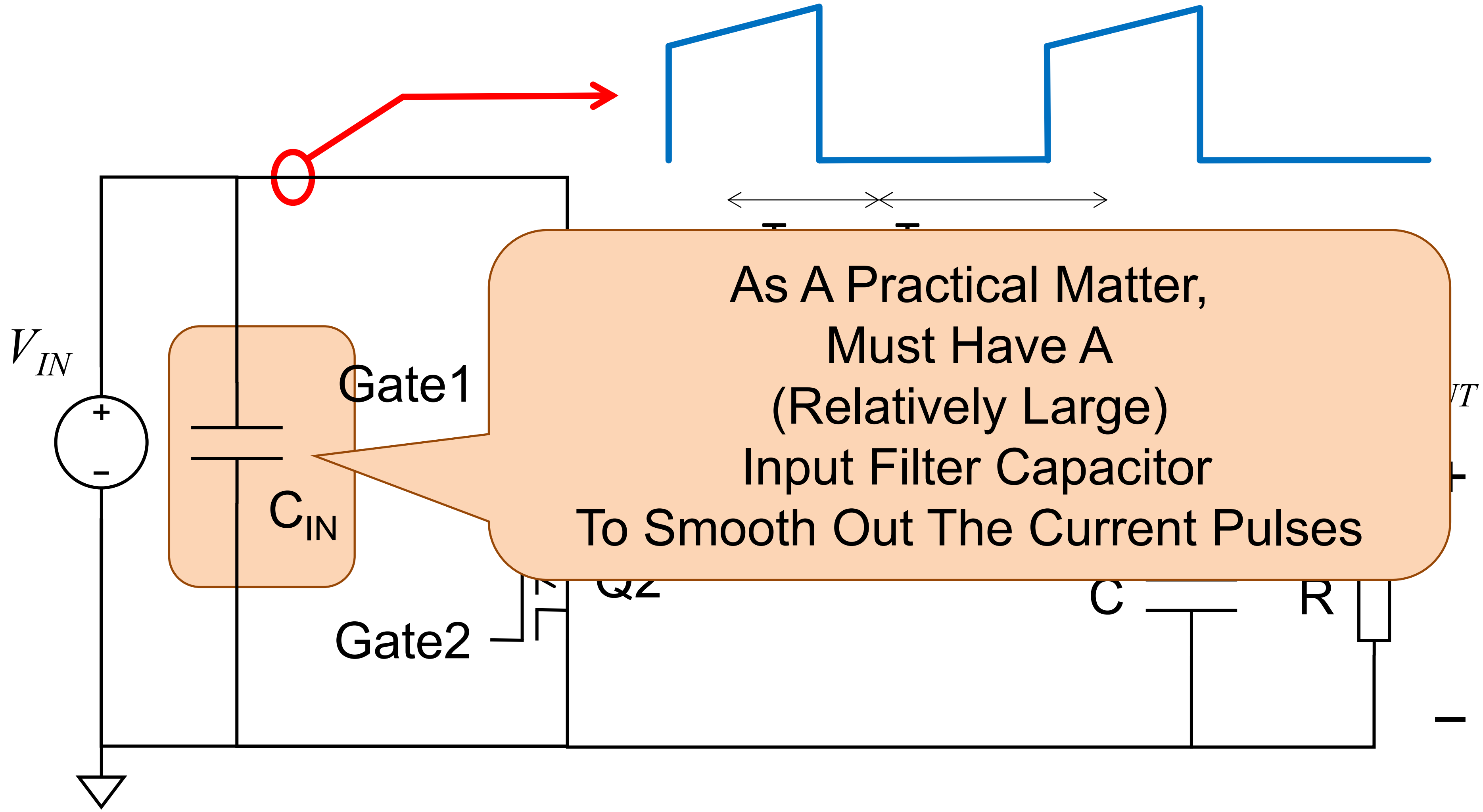
$$V_{IN} \cdot I_{IN\_AVERAGE} = V_{OUT} \cdot I_{OUT}$$

$$V_{IN} \cdot I_{IN\_AVERAGE} = D \cdot V_{IN} \cdot I_{OUT}$$

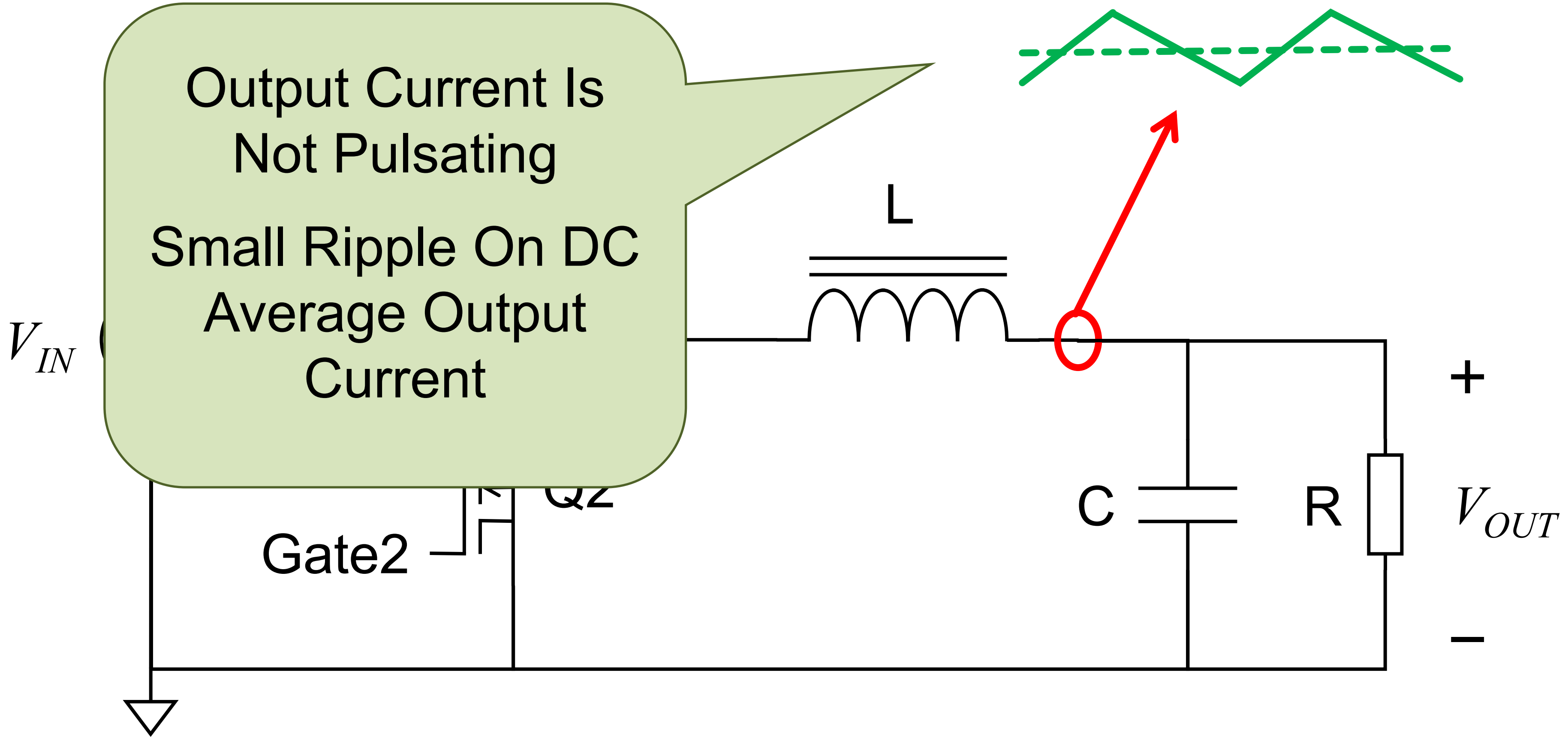
$$\cancel{V_{IN}} \cdot I_{IN\_AVERAGE} = D \cdot \cancel{V_{IN}} \cdot I_{OUT}$$

$$I_{IN\_AVERAGE} = D \cdot I_{OUT}$$

Buck Converter Input Current



Buck Converter Output Current



Buck Converter

Advantages

- Low Cost
- Low Parts Count
- Good Efficiency
- Simple To Control
- Many Controller ICs Available
- Available As Fully Integrated Converters

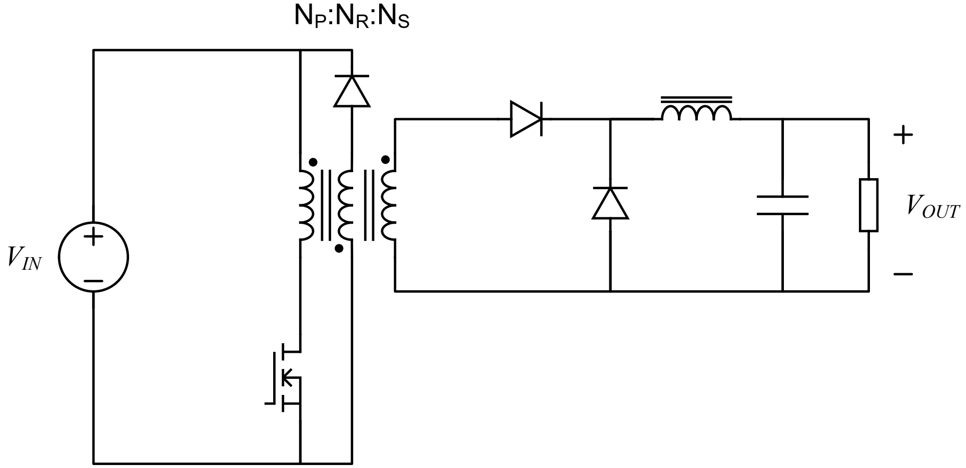
Disadvantages

- Non-Isolated
- Only Voltage Step Down
- Floating Gate Drive For Control MOSFET
- Protecting Against Control Switch Short Failure

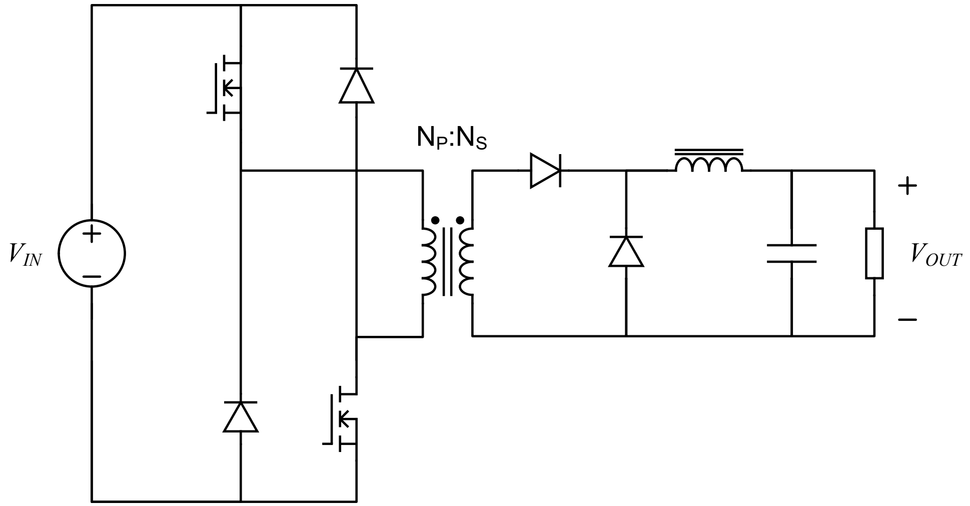
Transformer Isolated Buck Converters

Transformer Isolated Buck Converters
Forward Converters

Single Transistor Forward Converter



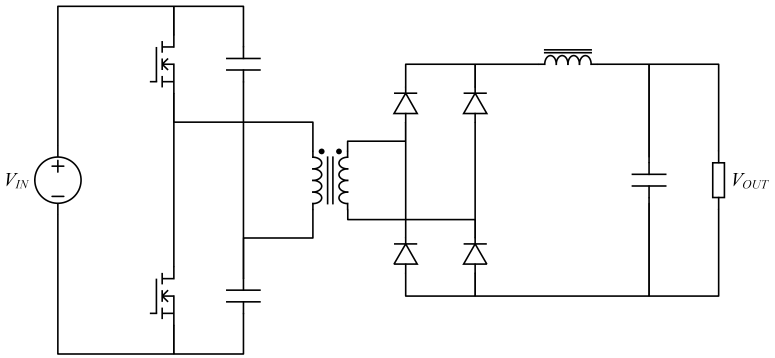
Two Transistor Forward Converter



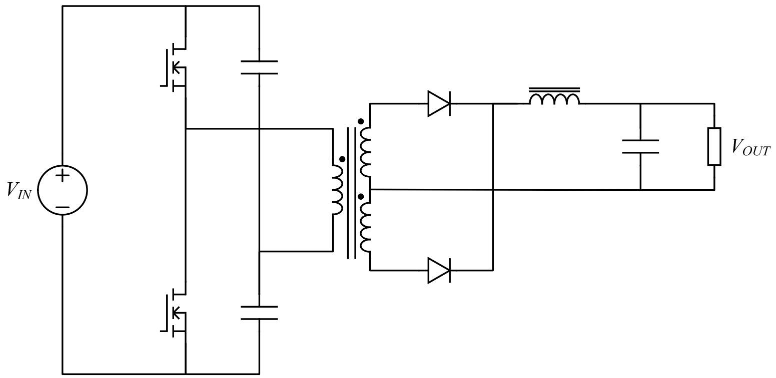
Described In The Appendices

Transformer Isolated Buck Converters
Half And Full Bridge Converters

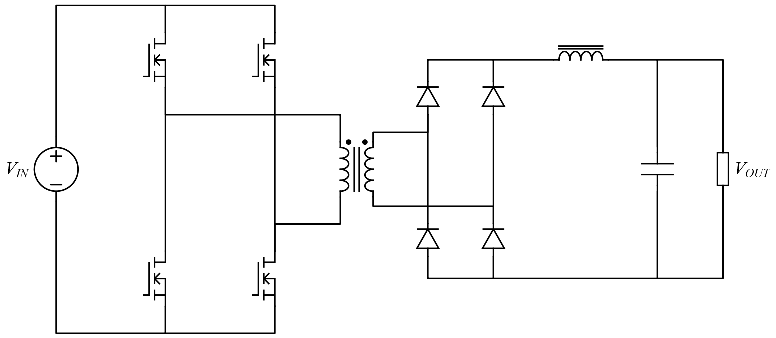
Half Bridge Converter
With Full Wave Rectifier Output



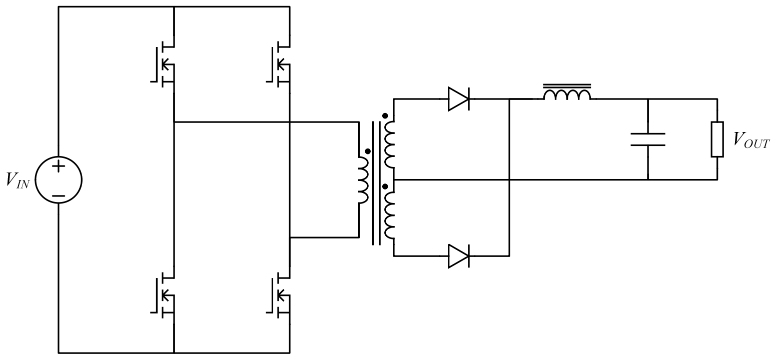
Half Bridge Converter
With Center Tapped Rectifier Output



Full Bridge Converter
With Full Wave Rectifier Output



Full Bridge Converter
With Center Tapped Rectifier Output

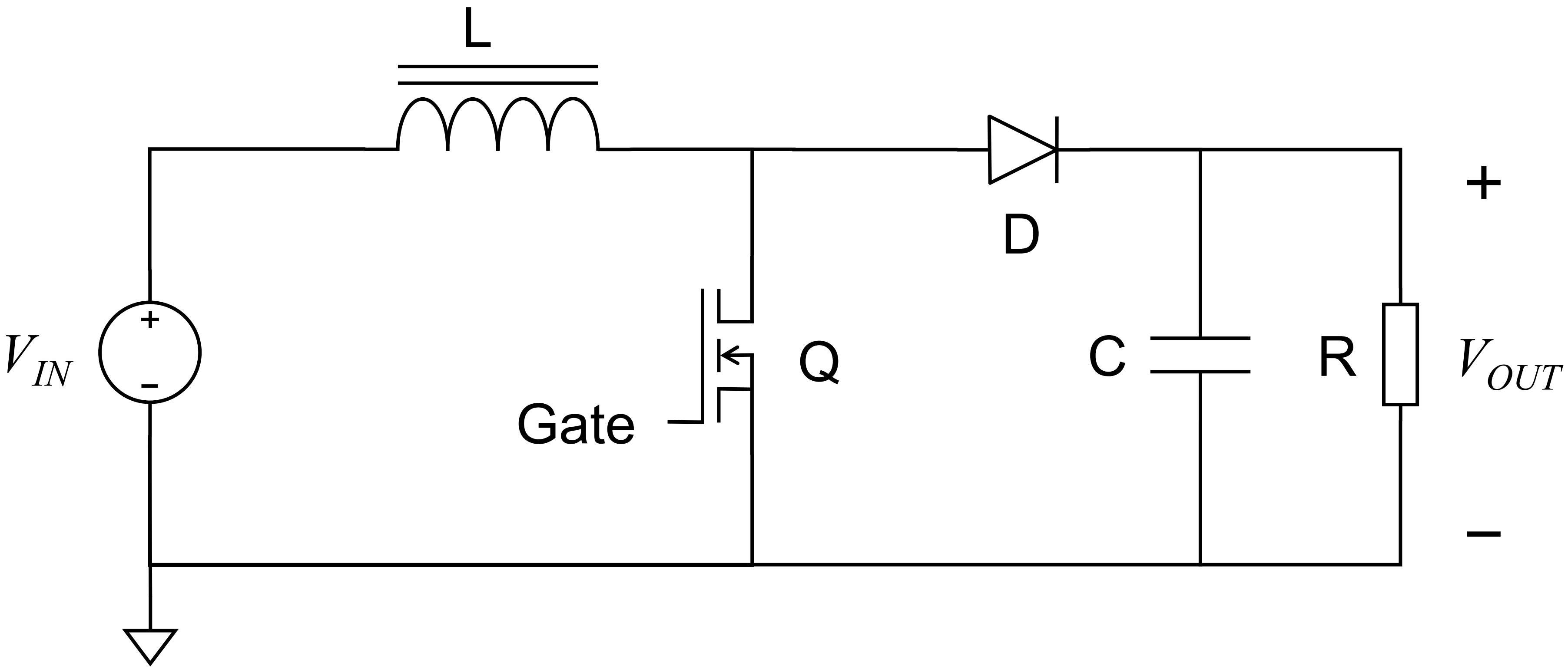


EMBEDDED POWER LABS

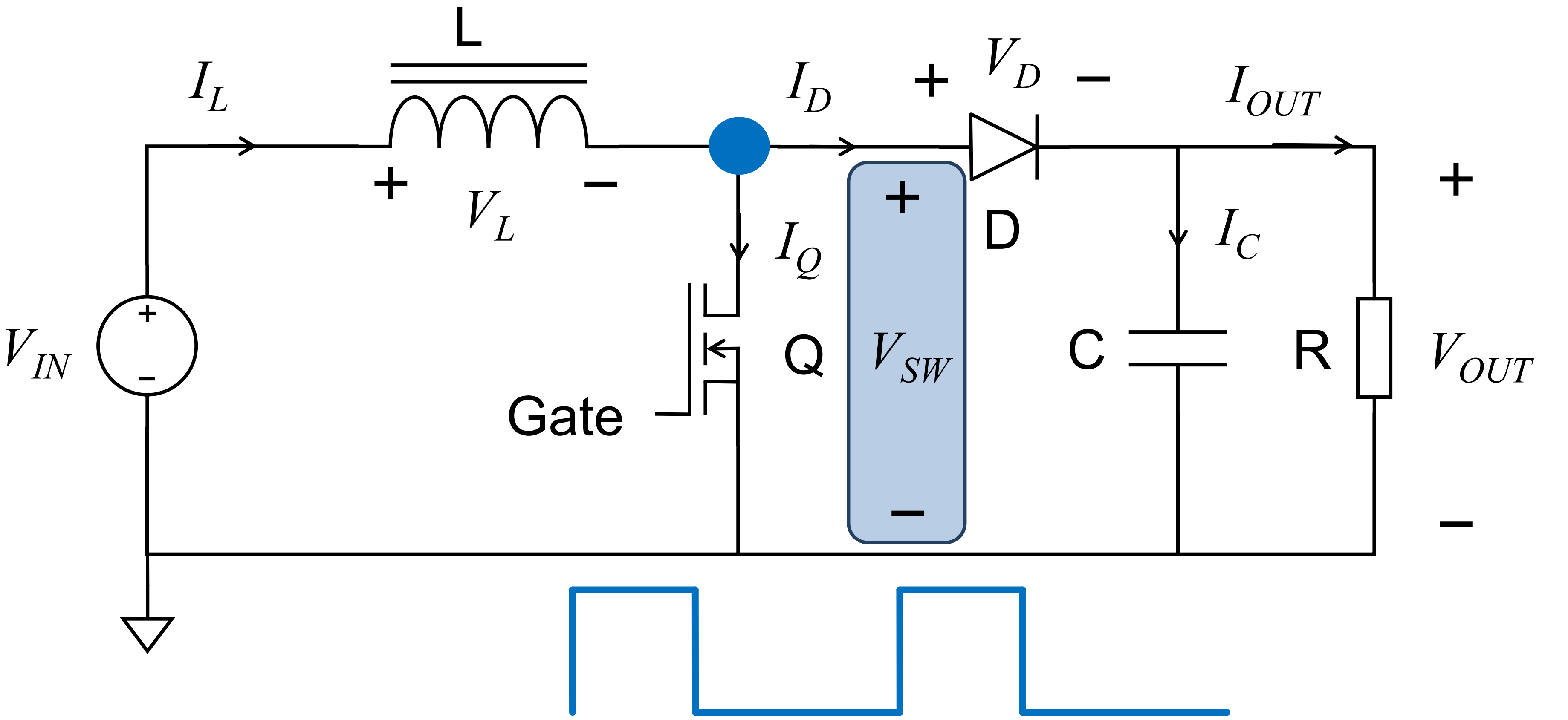
Described In The Appendices

Boost Converter

Boost Converter

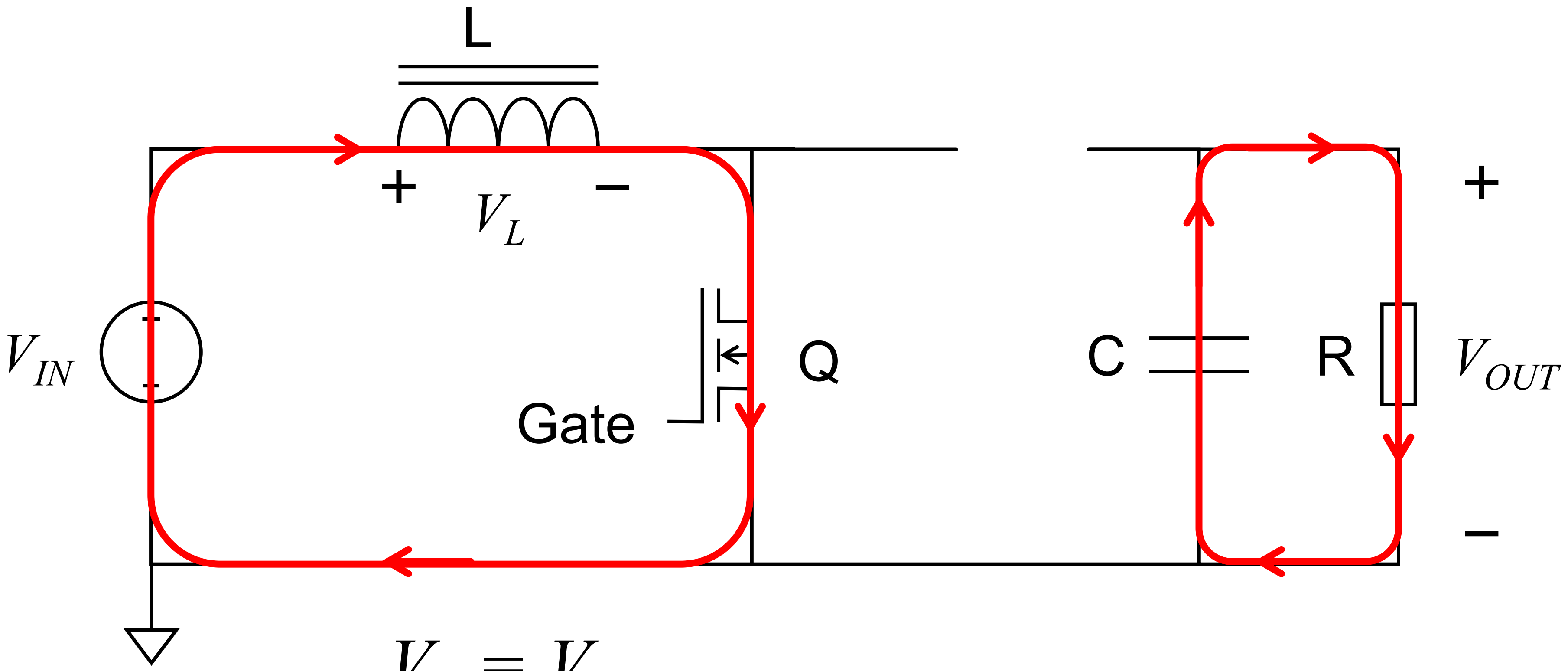


Boost Converter



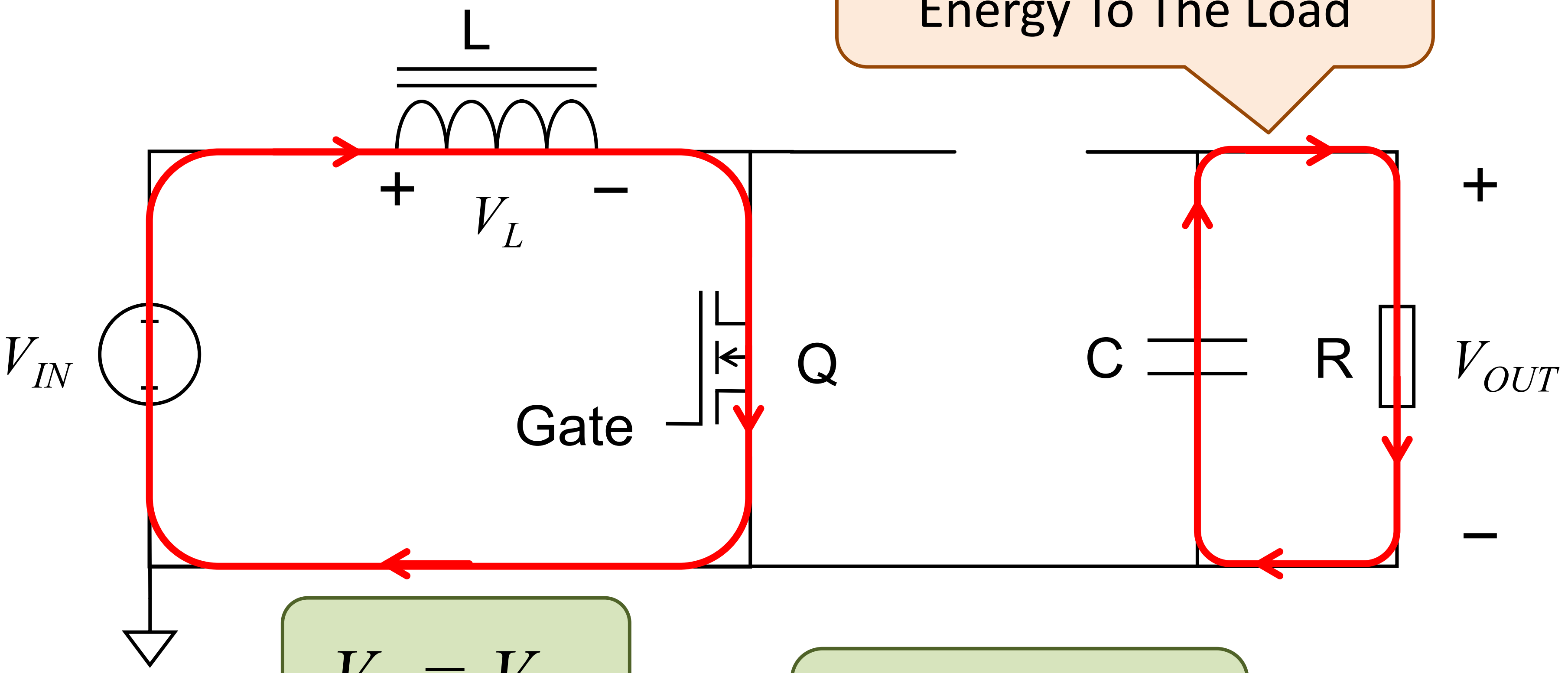
"Switch Node"

Boost Converter: On Time



$$V_L = V_{IN}$$

Boost Converter: On Time

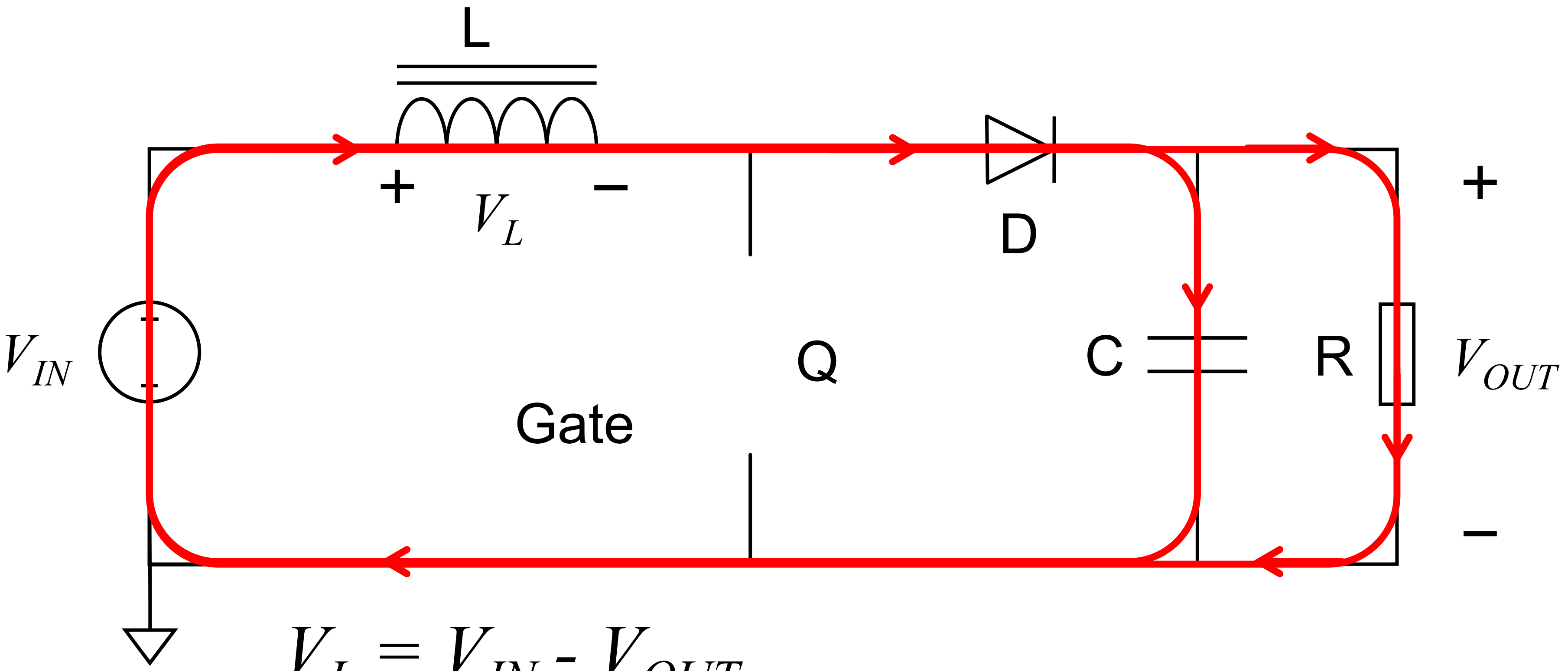


Capacitor Delivering Energy To The Load

$$V_L = V_{IN}$$

Storing Energy In The Inductor

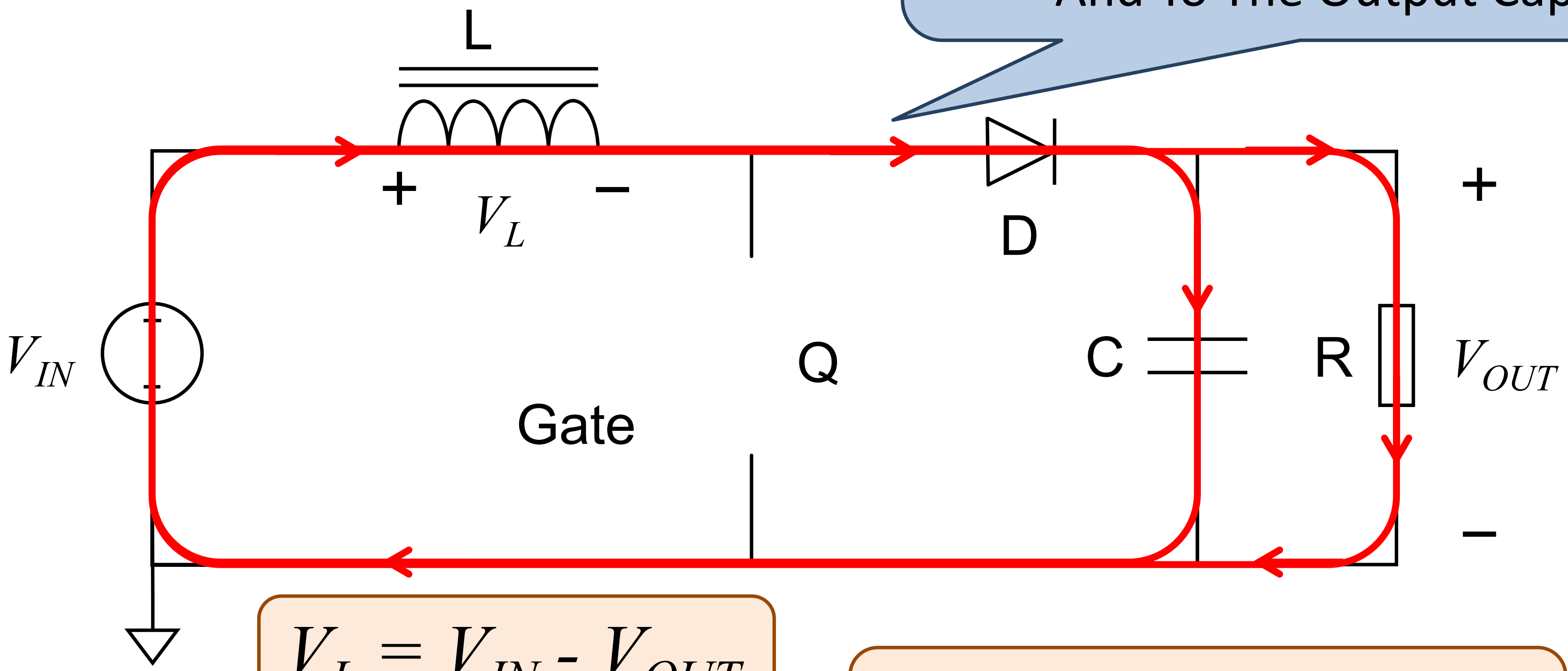
Boost Converter: Off Time



$$V_L = V_{IN} - V_{OUT}$$

Boost Converter: Off Time

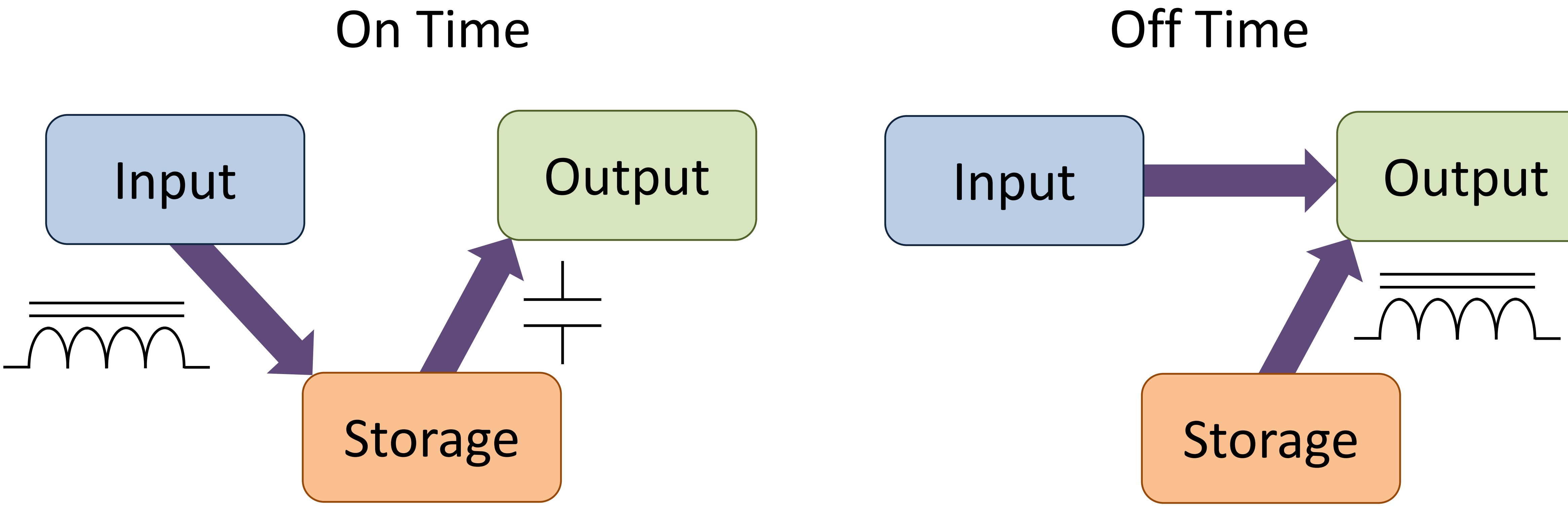
Power Flowing
From Input To The Output Load
And To The Output Capacitor



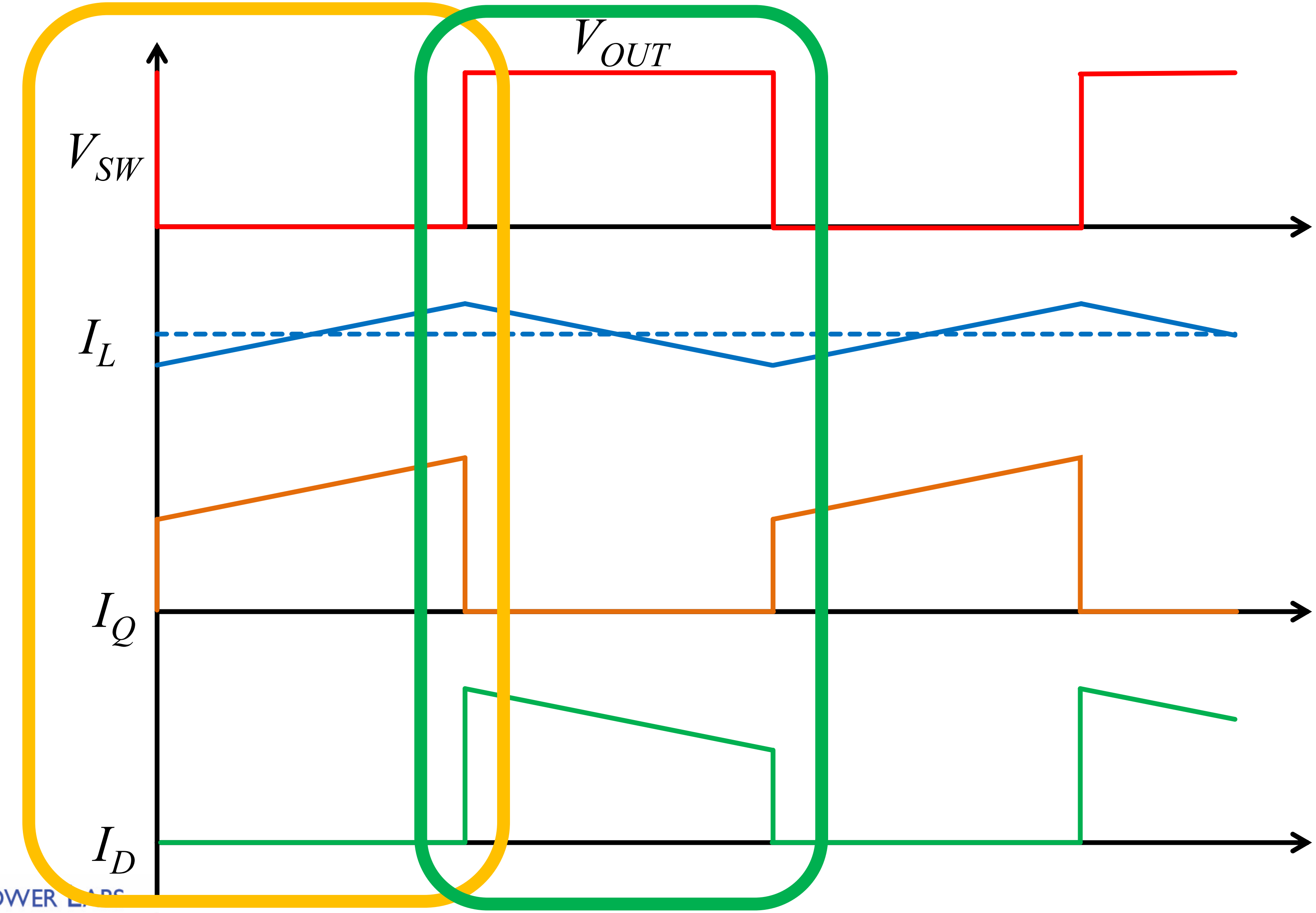
$$V_L = V_{IN} - V_{OUT}$$

Also Releasing Stored Energy
In The Inductor

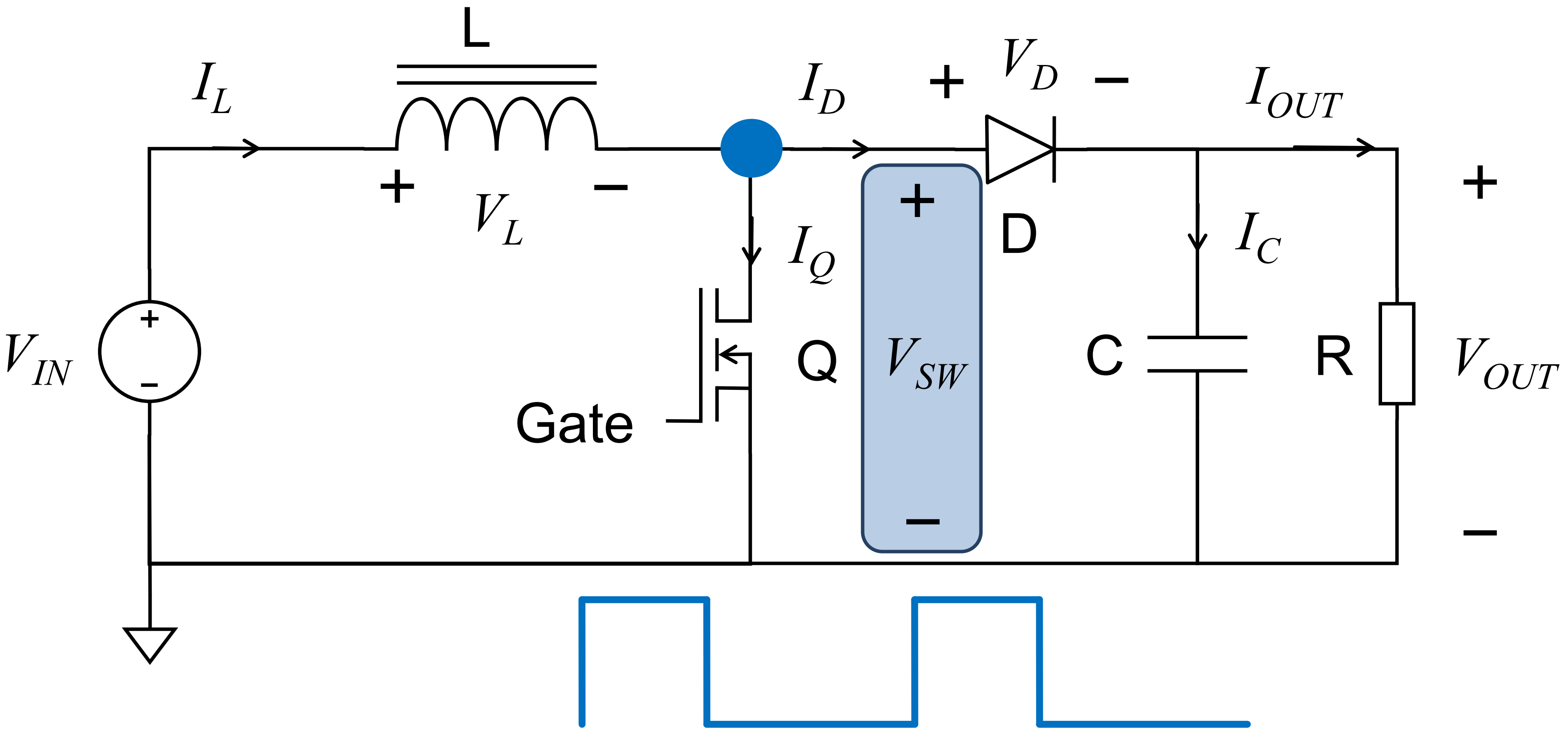
Boost Converter Energy Flow



Boost Converter Waveforms



Video Lab 6
Open Loop Boost Converter Waveforms



"Switch Node"

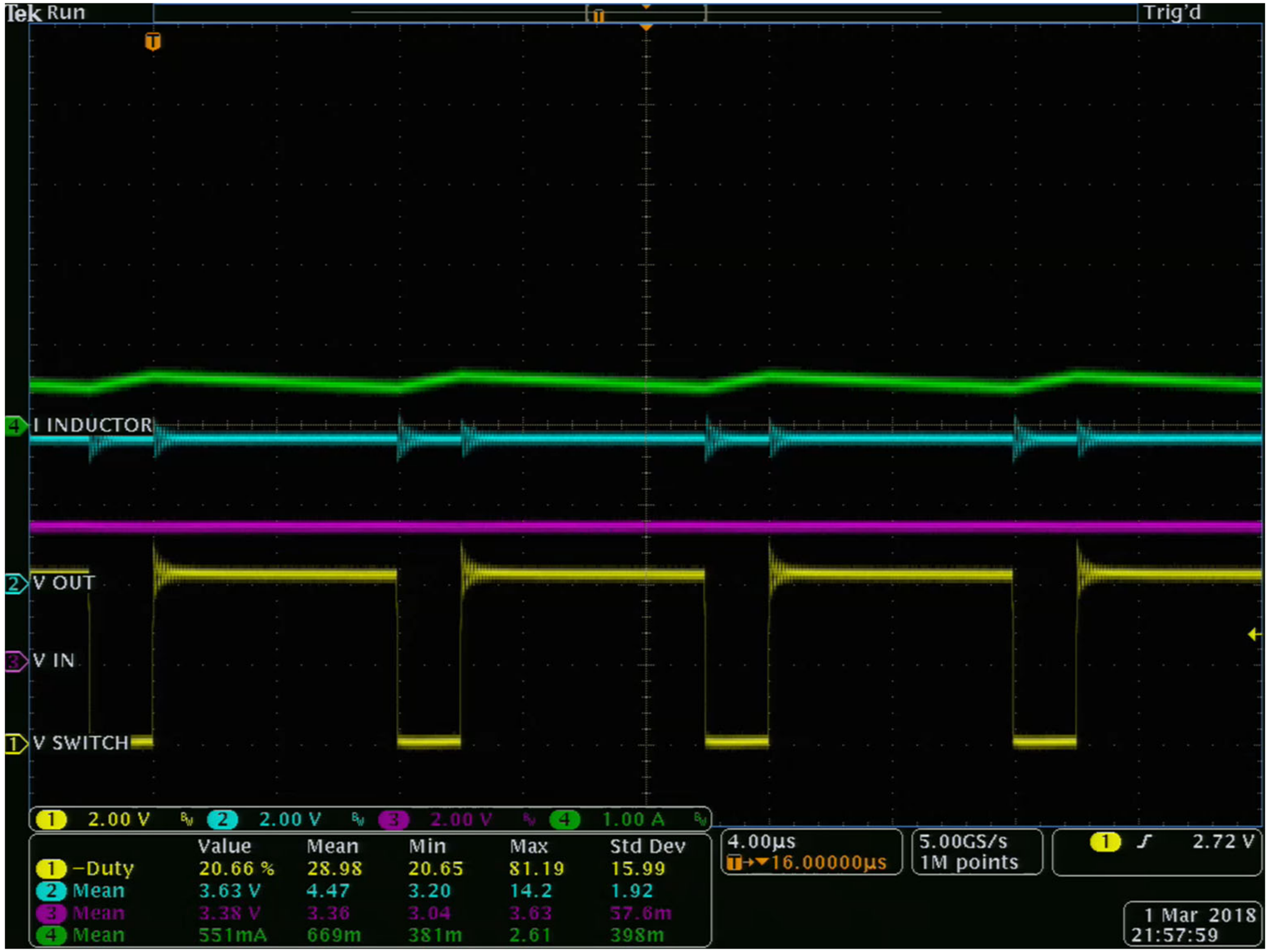
Open Loop Boost Converter Waveforms

I_L

V_{OUT}

V_{IN} (3.3 V)

V_{SW}



Conversion Ratio

$$V_L(T_{ON})\cdot T_{ON}+V_L(T_{OFF})\cdot T_{OFF}=0$$

$$V_{IN}\cdot T_{ON}+(V_{IN}-V_{OUT})\cdot T_{OFF}=0$$

$$V_{IN}\cdot D\cdot T_{SW}+(V_{IN}-V_{OUT})\cdot(1-D)\cdot T_{SW}=0$$

$$V_{IN}\cdot D+(V_{IN}-V_{OUT})\cdot(1-D)=0$$

$$D\cdot V_{IN}+V_{IN}-D\cdot V_{IN}-V_{OUT}+D\cdot V_{OUT}=0$$

$$V_{IN}-V_{OUT}+D\cdot V_{OUT}=0$$

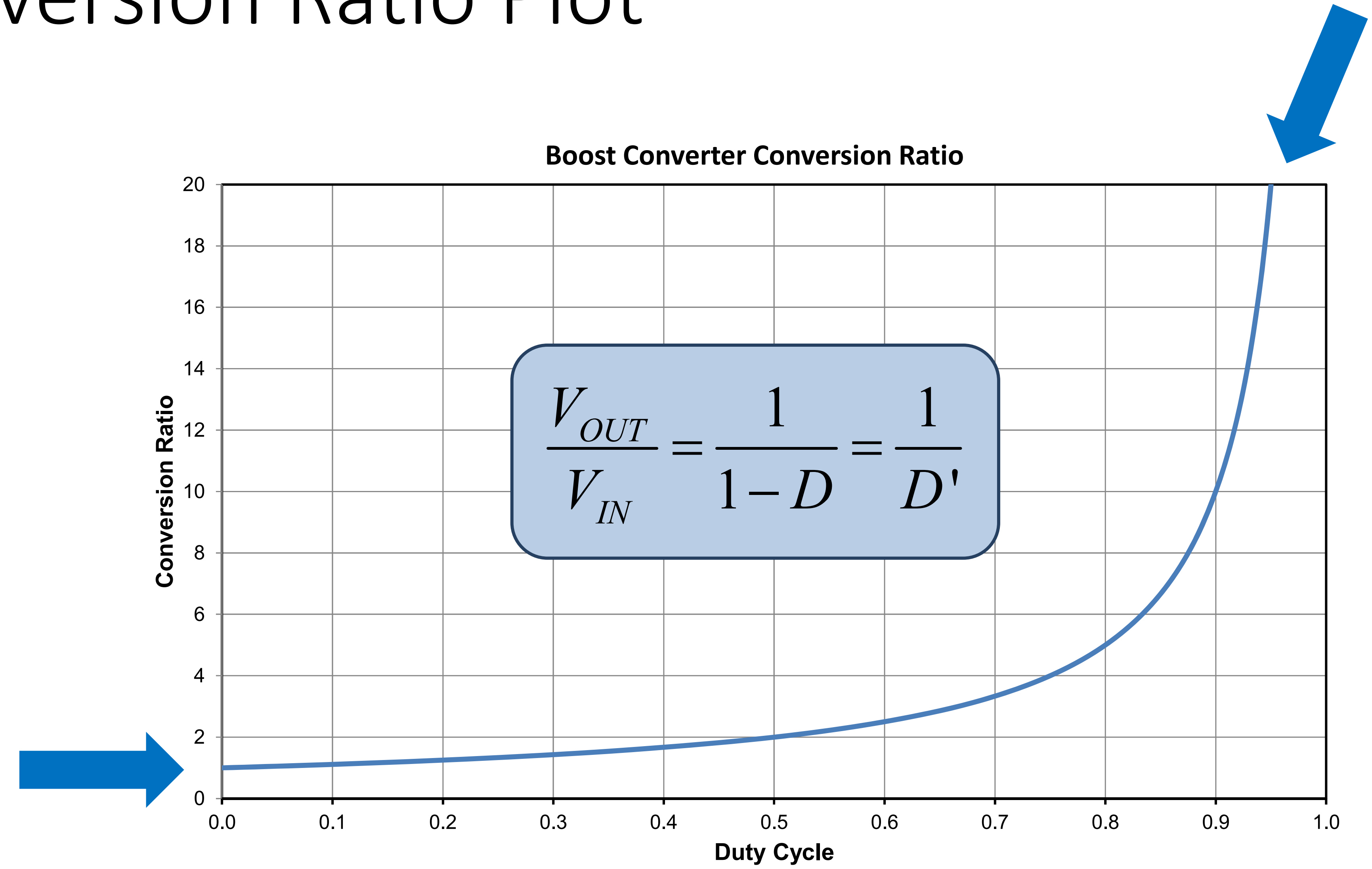
$$V_{OUT}-D\cdot V_{OUT}=V_{IN}$$

$$(1-D)\cdot V_{OUT}=V_{IN}$$

$$V_{OUT}=\frac{1}{1-D}\cdot V_{IN}=\frac{V_{IN}}{D'}$$

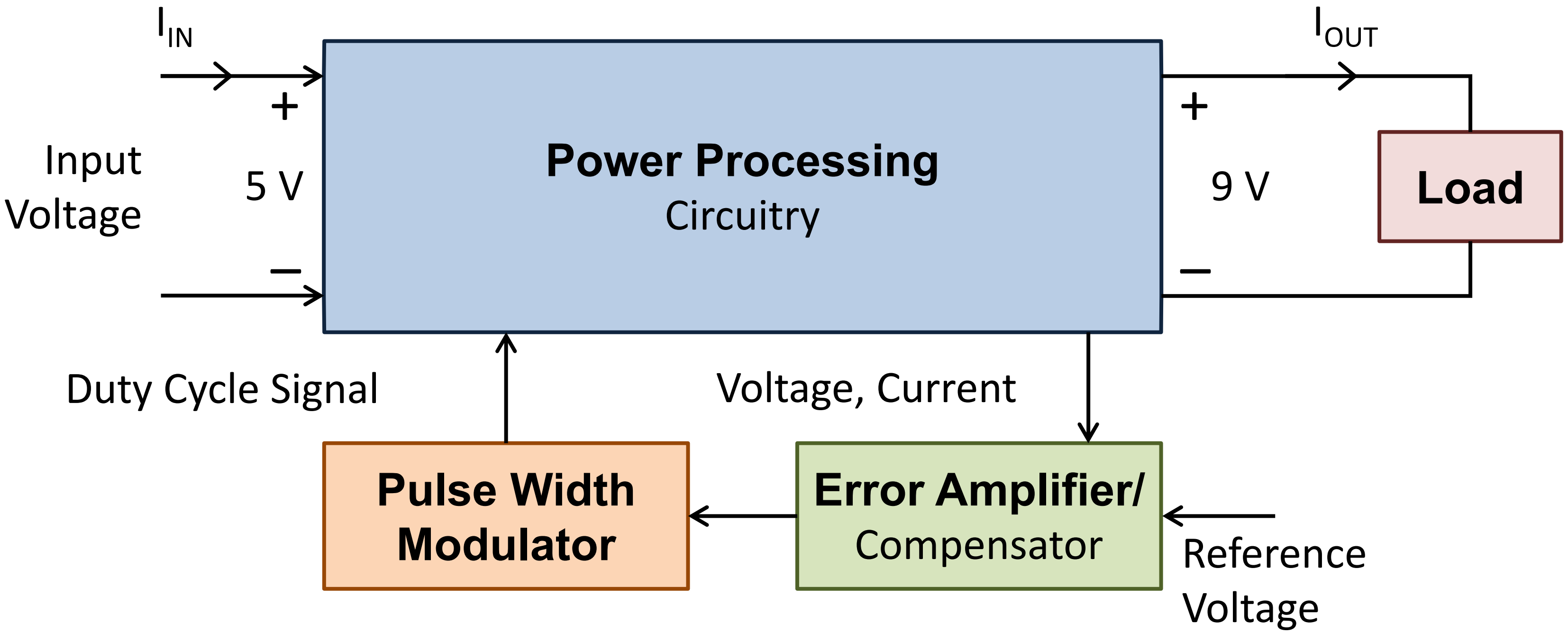
$$D\rightarrow 1\Rightarrow V_{OUT}\rightarrow\infty$$

Conversion Ratio Plot



$$\frac{V_{OUT}}{V_{IN}} = \frac{1}{1-D} = \frac{1}{D'}$$

Example

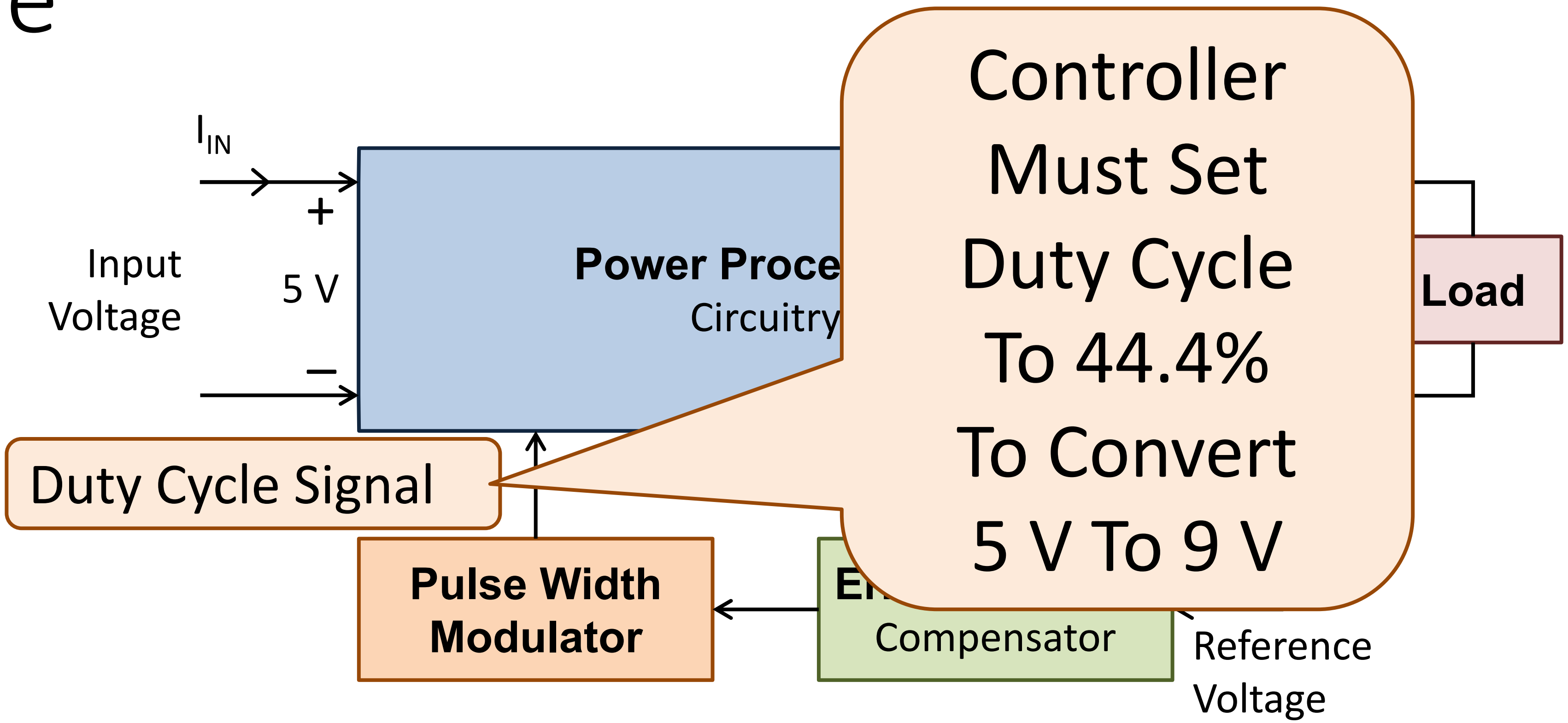


$$V_{OUT} = \frac{1}{D'} V_{IN} = \frac{1}{1-D} \cdot V_{IN}$$

$$1 - D = \frac{V_{IN}}{V_{OUT}}$$

$$D = 1 - \frac{V_{IN}}{V_{OUT}}$$

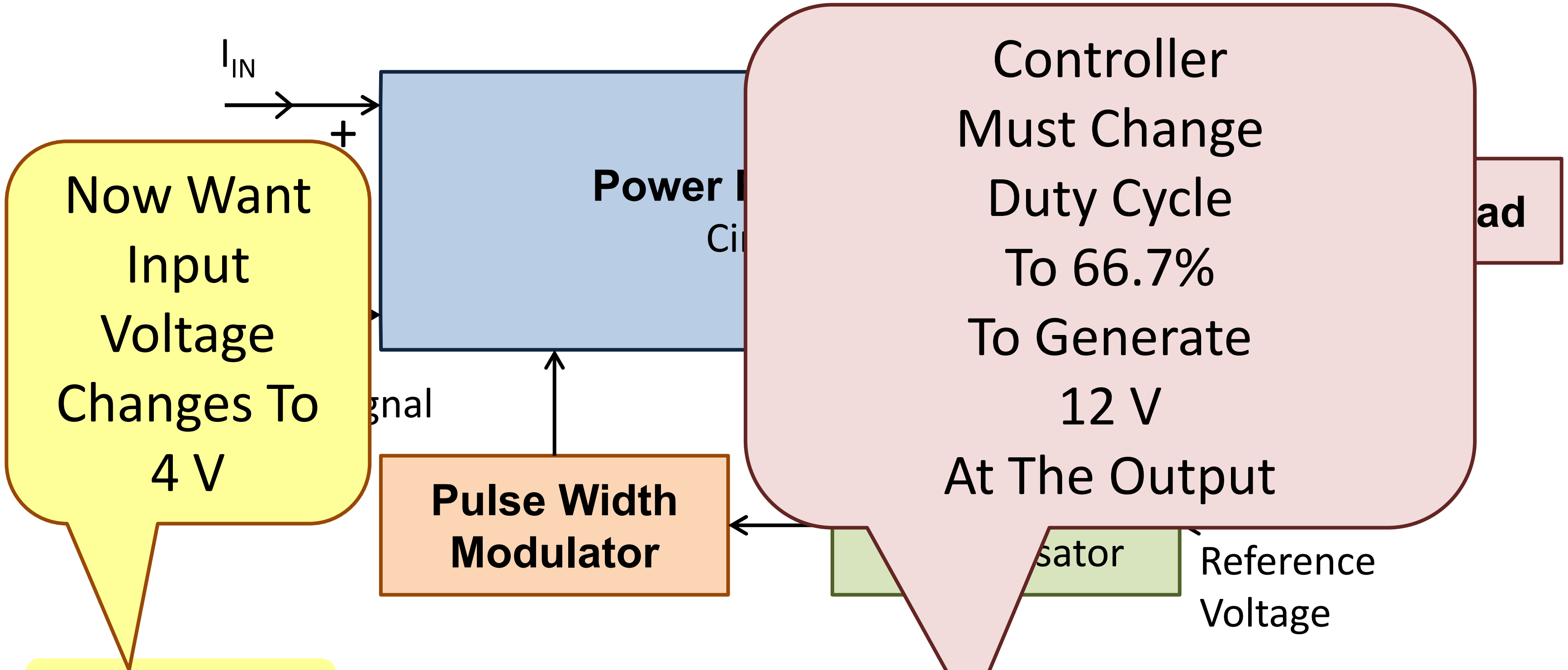
Example



$$V_{IN} = 5\,\text{V}$$
$$V_{OUT} = 9\,\text{V}$$
$$I_{OUT} = 5\,\text{A}$$

$$D = 1 - \frac{V_{IN}}{V_{OUT}} = 1 - \frac{5\,\text{V}}{9\,\text{V}}$$
$$= 0.444 = 44.4\%$$

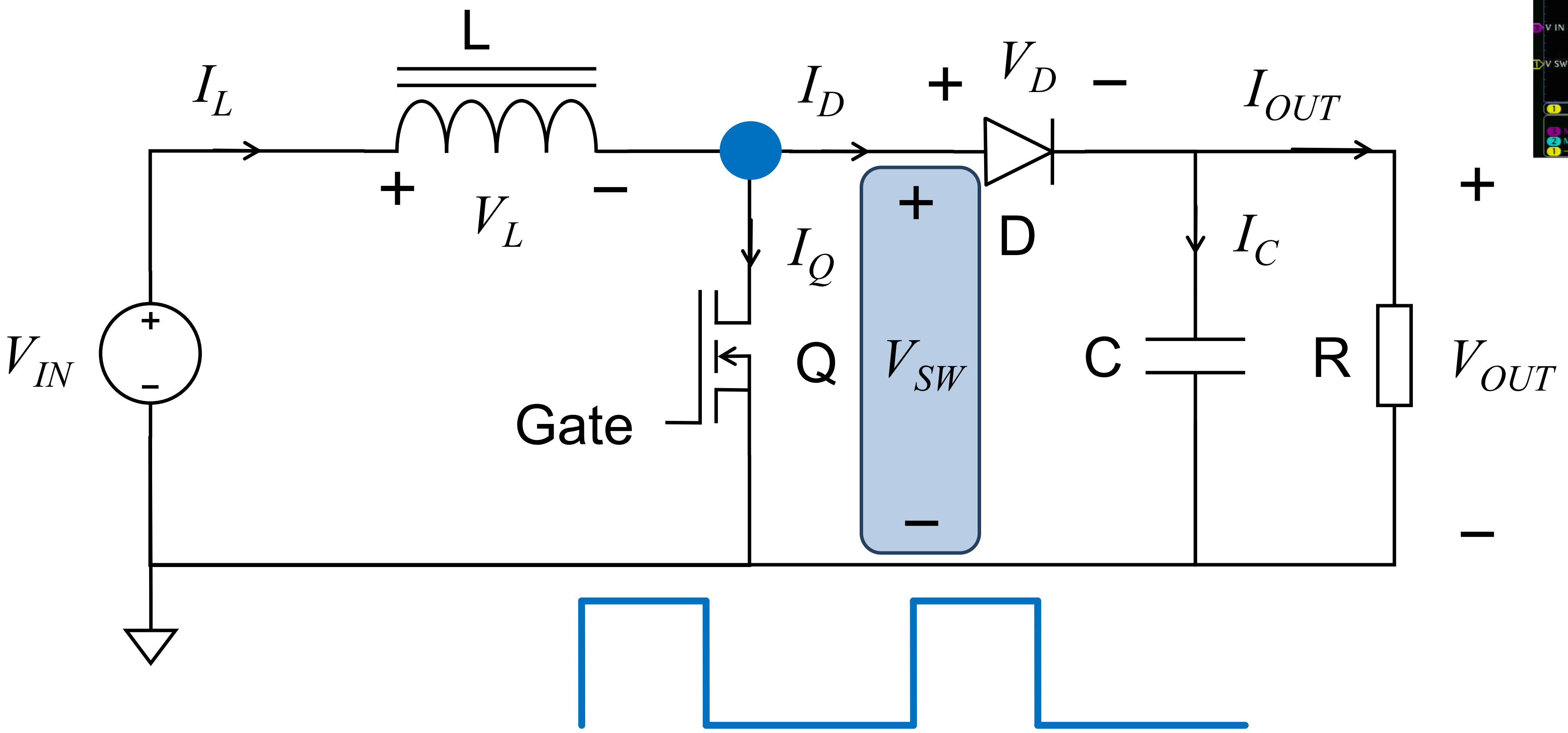
Example



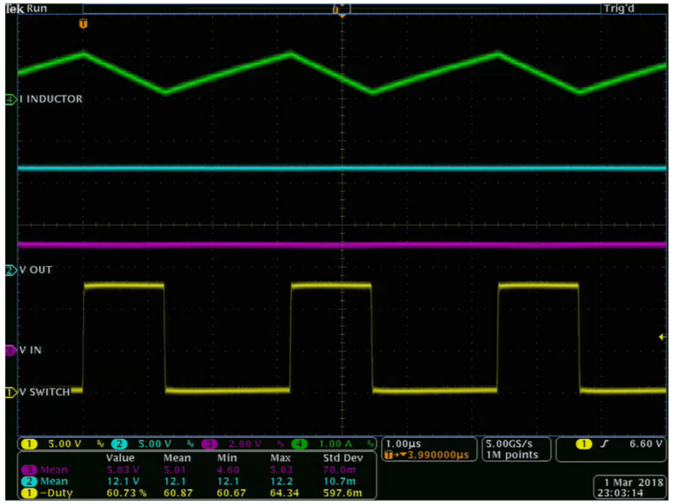
$$V_{IN} = 4\,\text{V}$$
$$V_{OUT} = 12\,\text{V}$$
$$I_{OUT} = 5\,\text{A}$$

$$D = 1 - \frac{V_{IN}}{V_{OUT}} = 1 - \frac{4\,\text{V}}{12\,\text{V}}$$
$$= 0.667 = 66.7\%$$

Video Lab 7
Boost Converter With Feedback



“Switch Node”



Inductor Current

$$I_C(T_{ON}) \cdot T_{ON} + I_C(T_{OFF}) \cdot T_{OFF} = 0$$

Capacitor Charge Balance

$$I_{OUT} = \frac{1}{R_{OUT}} \cdot V_{OUT}$$

$$I_{OUT} \cdot T_{ON} + (I_L - I_{OUT}) \cdot T_{OFF} = 0$$

$$I_{OUT} \cdot D \cdot T_{SW} + (I_L - I_{OUT}) \cdot (1-D) \cdot T_{SW} = 0$$

$$I_{OUT} \cdot D + (I_L - I_{OUT}) \cdot (1-D) = 0$$

$$D \cdot I_{OUT} + I_{OUT} - D \cdot I_{OUT} - I_L + D \cdot I_L = 0$$

$$I_{OUT} - I_L + D \cdot I_L = 0$$

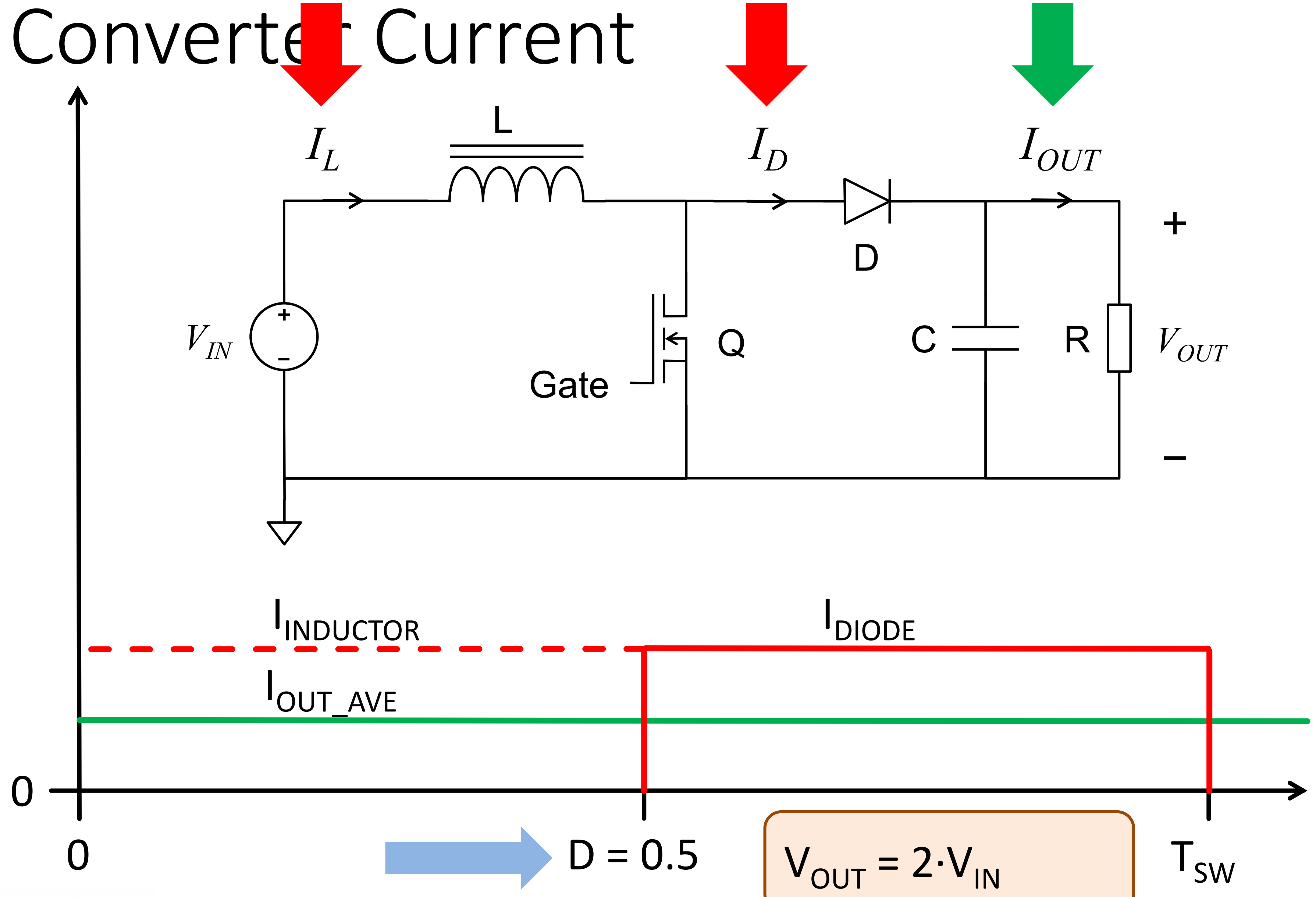
$$I_L - D \cdot I_L = I_{OUT}$$

$$(1-D) \cdot I_L = I_{OUT}$$

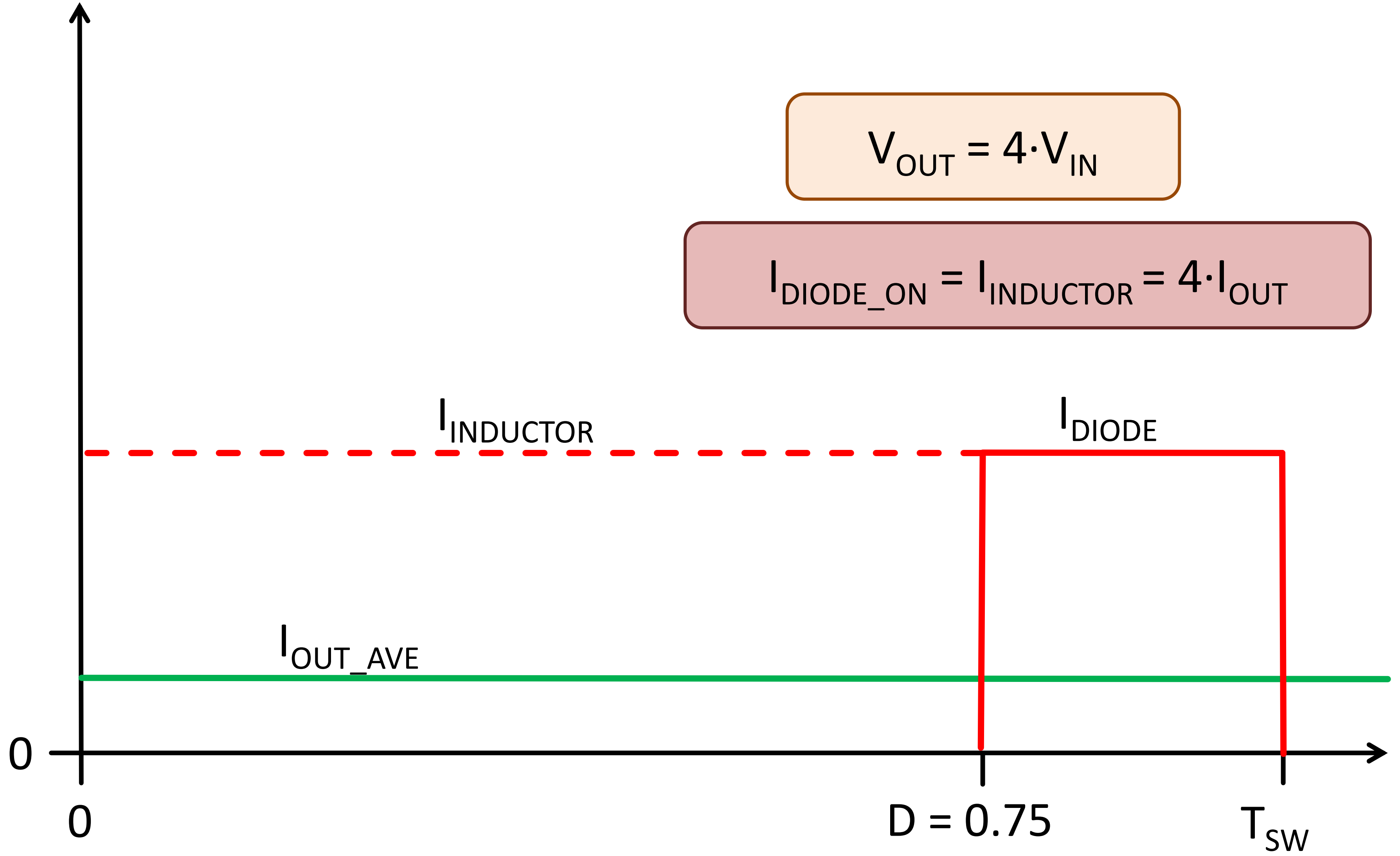
$$I_L = \frac{1}{1-D} \cdot I_{OUT} = \frac{1}{D'} \cdot I_{OUT}$$

$$D \to 1 \Rightarrow I_L \to \infty$$

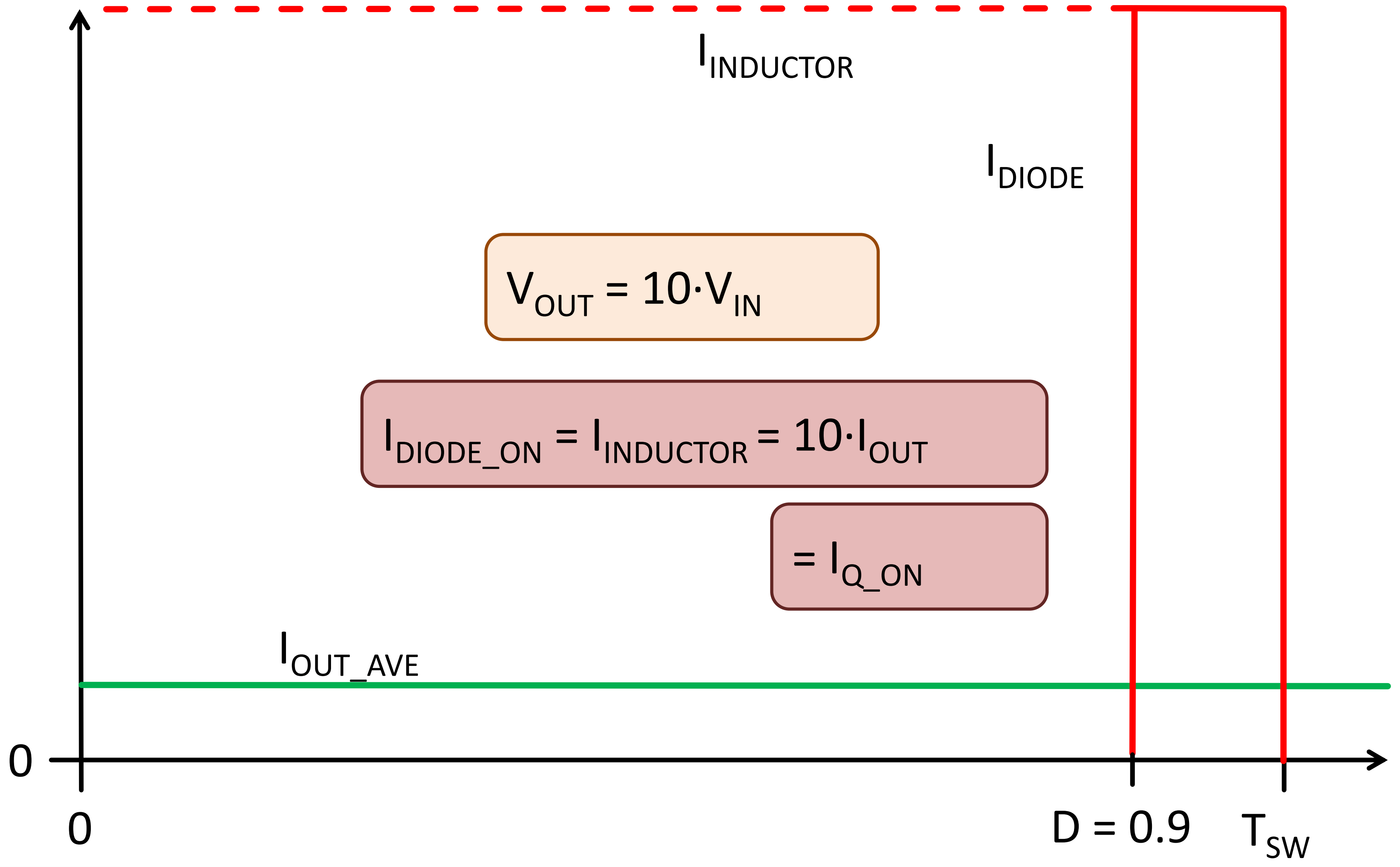
Boost Converter Current



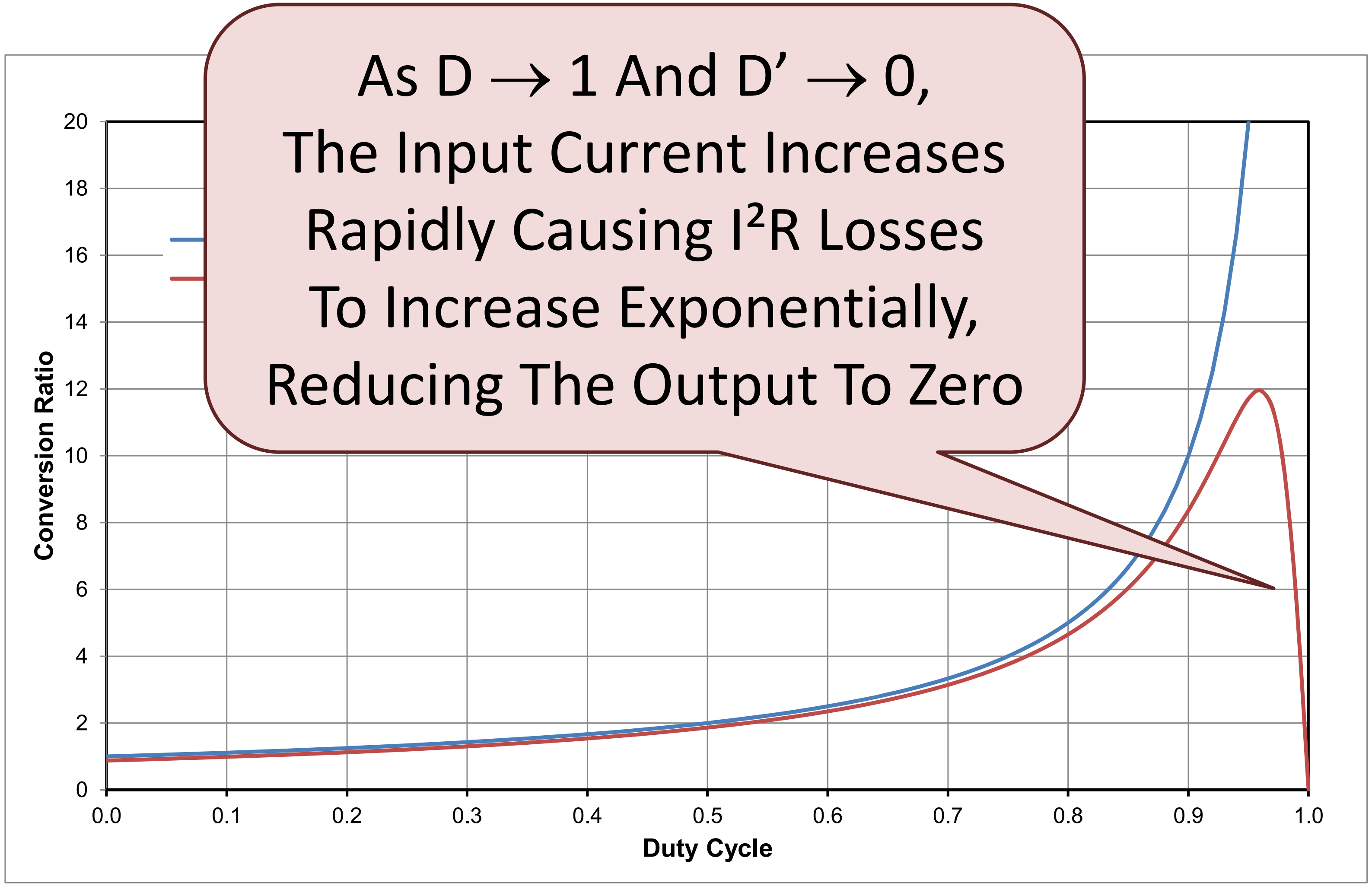
Boost Converter Current



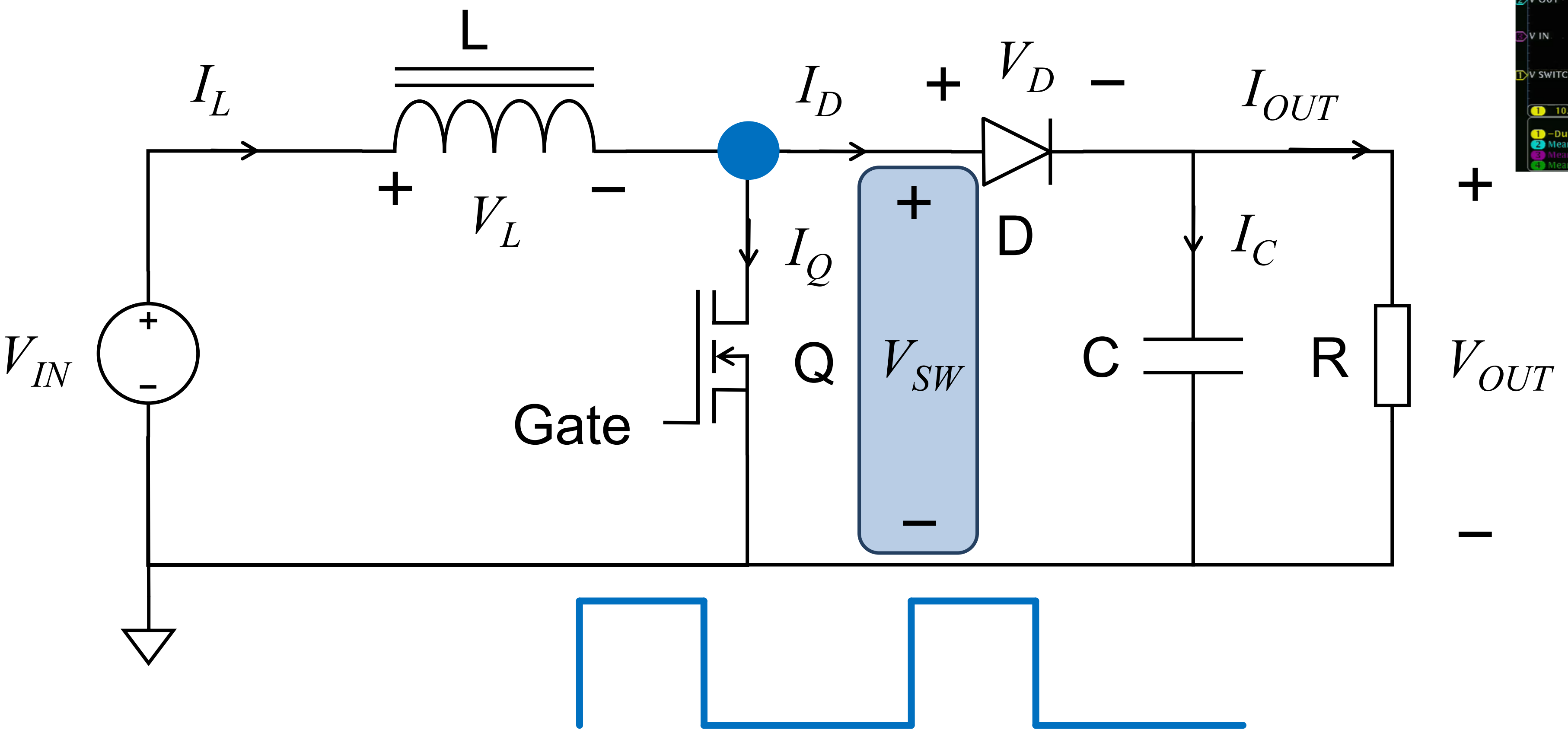
Boost Converter Current



Conversion Ratio Plot

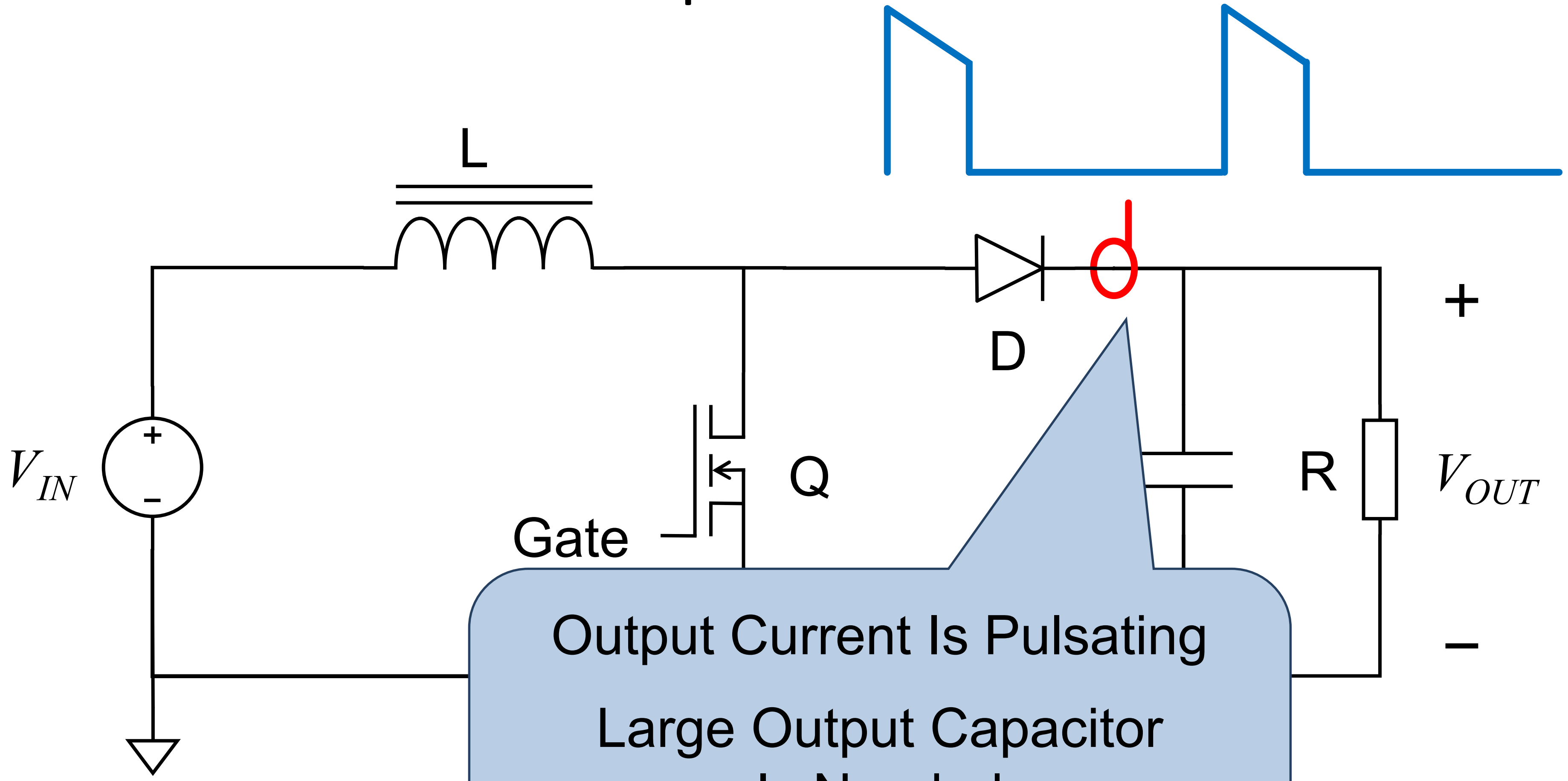


Video Lab 8
Boost Converter At High Duty Cycle



“Switch Node”

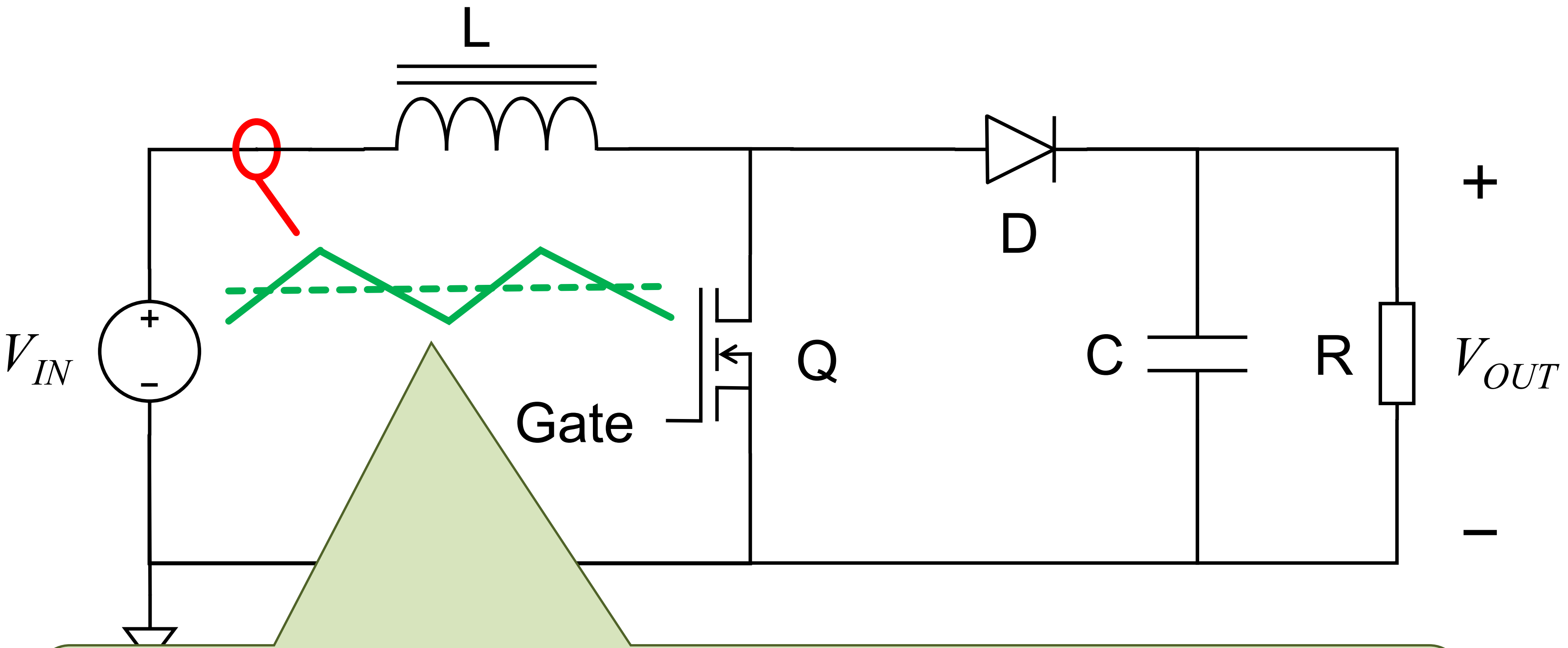
Boost Converter Output Current



Output Current Is Pulsating

Large Output Capacitor
Is Needed
To Smooth To DC

Boost Converter Input Current



Input Current Is Not Pulsating

Small Ripple On DC Average Output Current

Boost Converter

Advantages

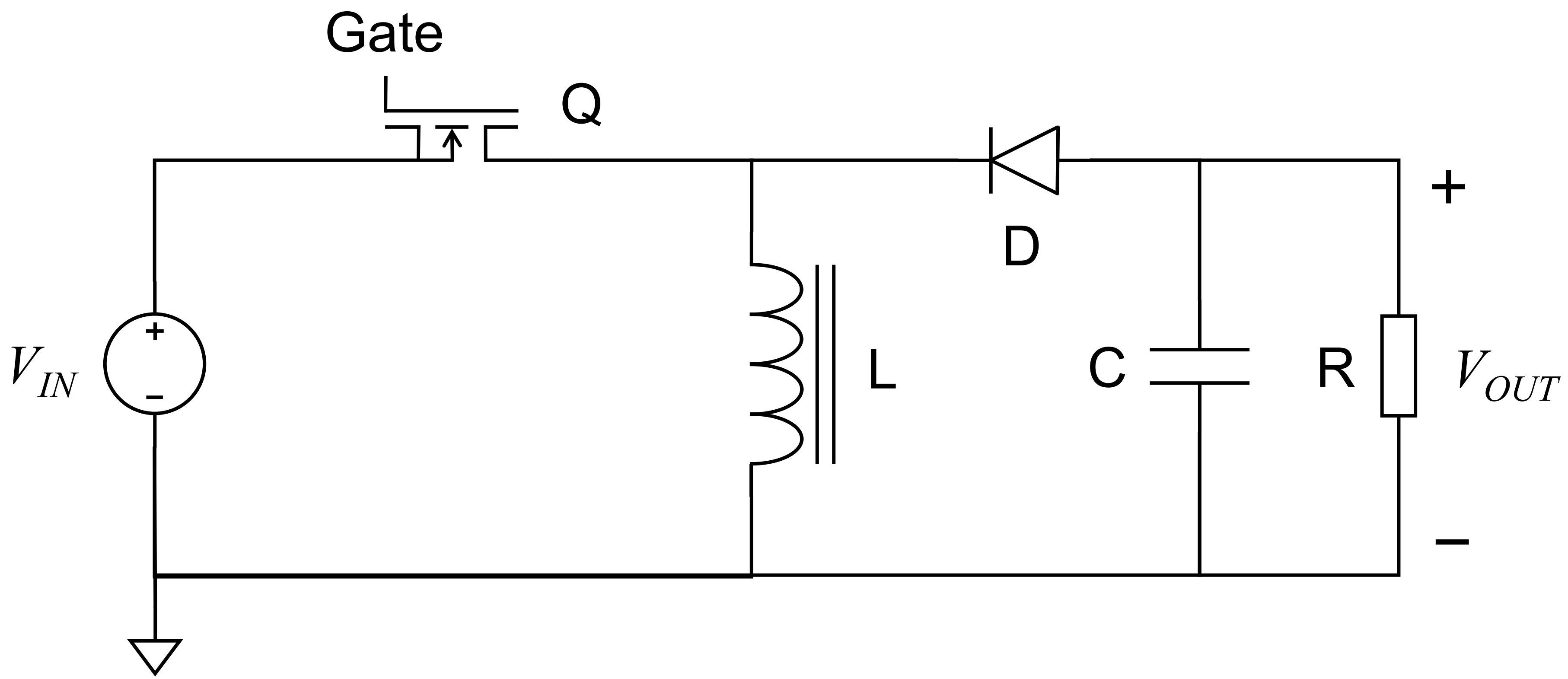
- Low Cost
- Low Parts Count

Disadvantages

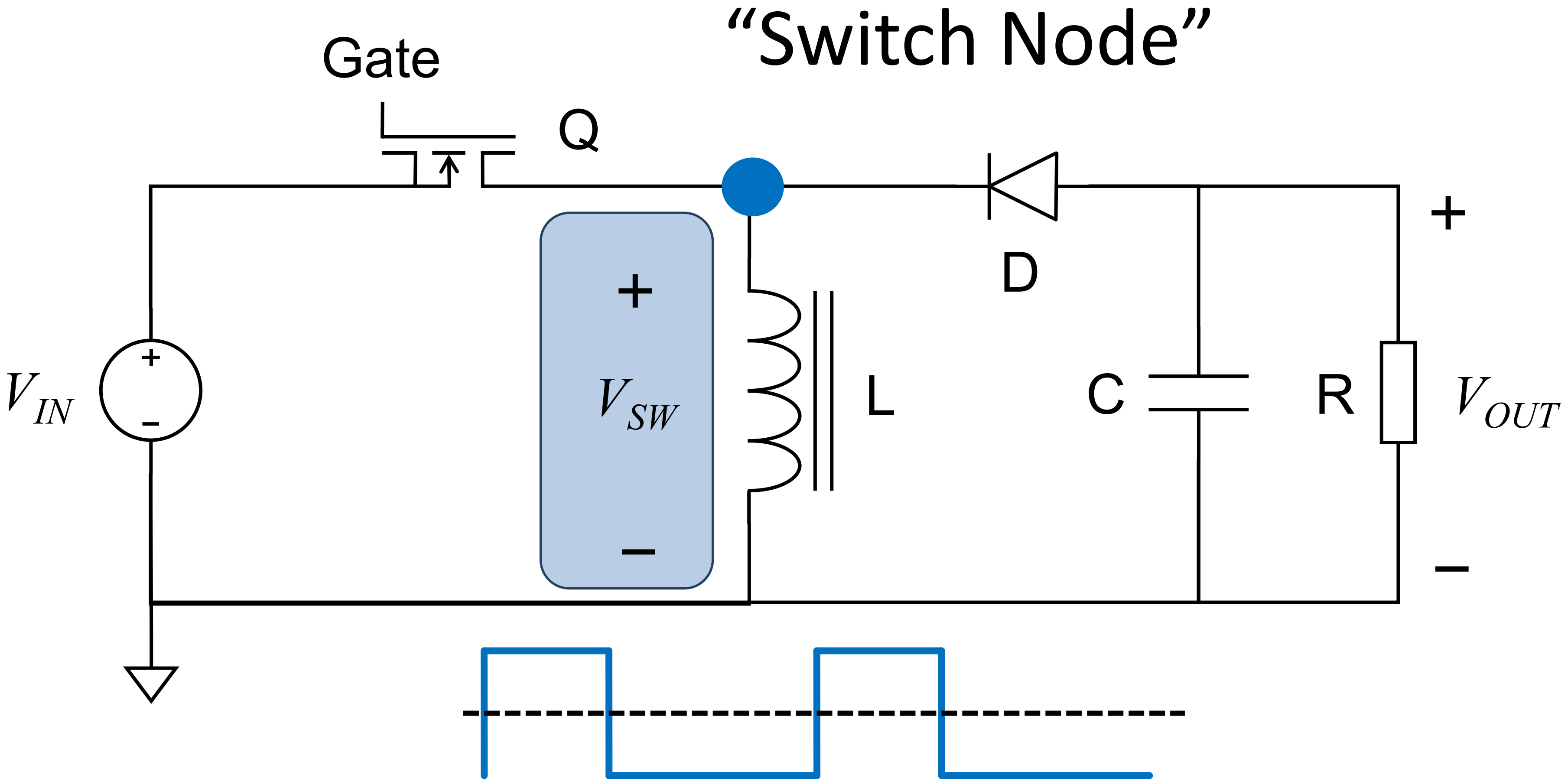
- Non-Isolated
- Only Voltage Step Up
- High Efficiency Is Difficult
- Continuous Conduction Mode Harder To Control

Buck-Boost Converter

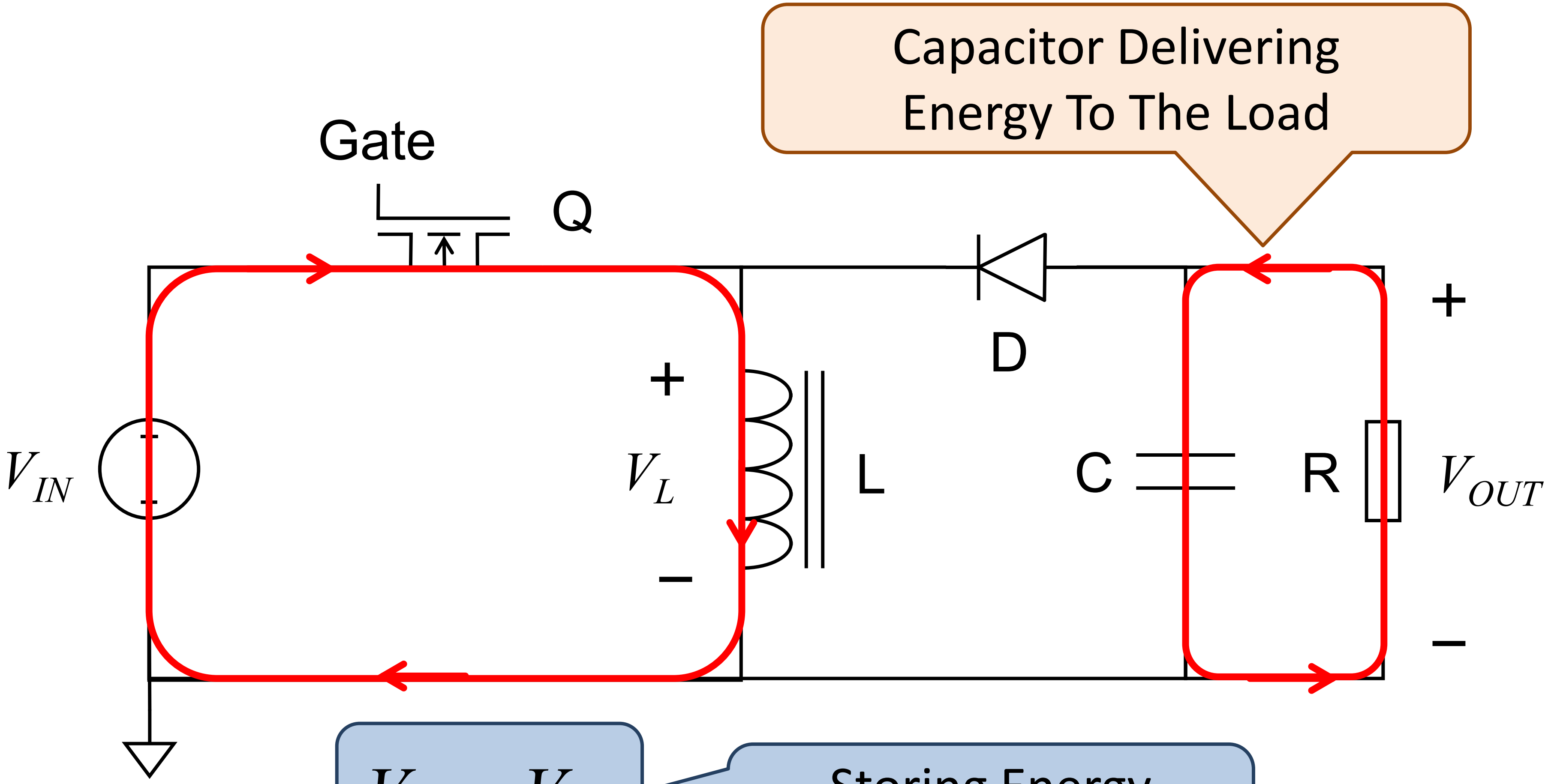
Buck-Boost Converter



Buck-Boost Converter



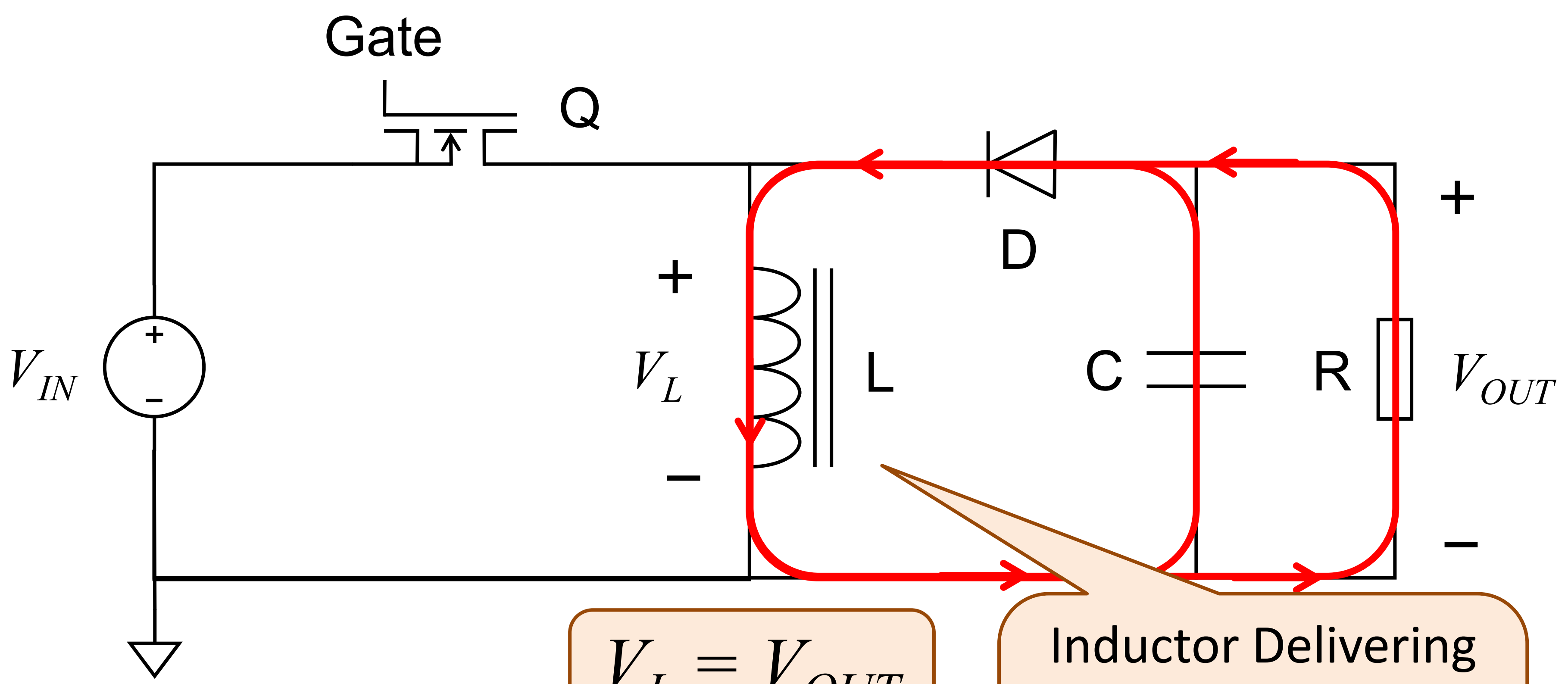
Buck-Boost Converter On Time



$$V_L = V_{IN}$$

Storing Energy
In The Inductor

Buck-Boost Converter Off Time



$$V_L = V_{OUT}$$

Inductor Delivering Stored Energy To The Load And Output Capacitor

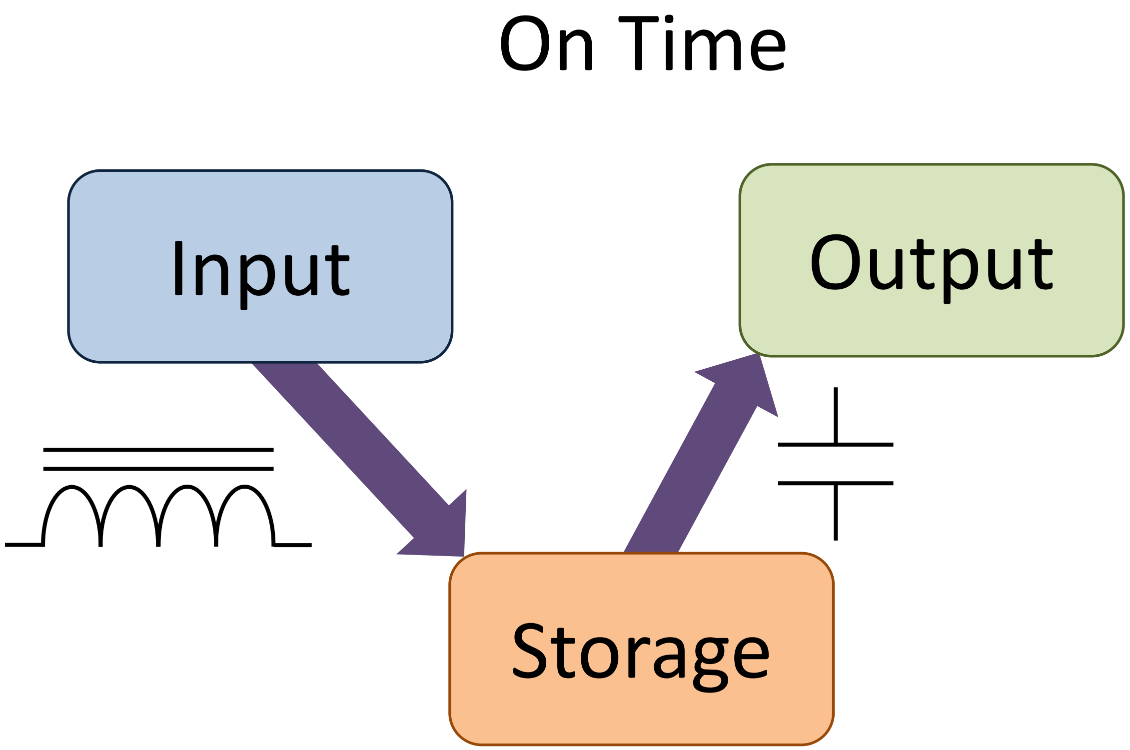
Inductor Volt-Second Balance

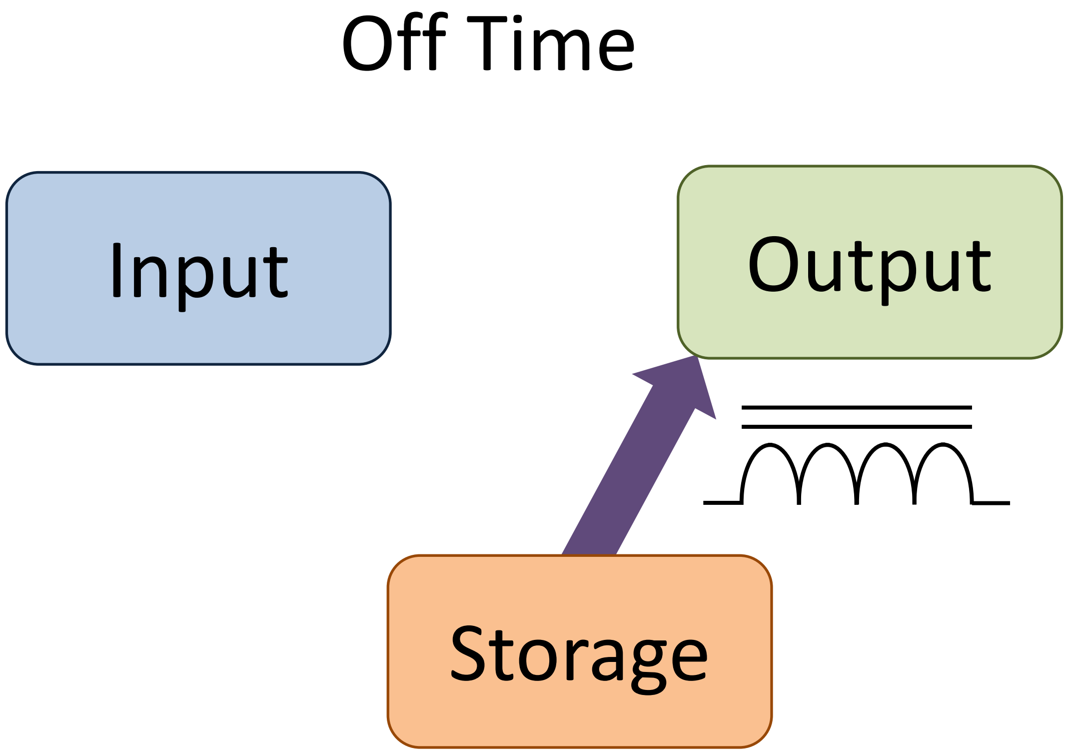
If V_{IN} Is Positive

$$V_{IN} \cdot T_{ON} + V_{OUT} \cdot T_{OFF} = 0$$

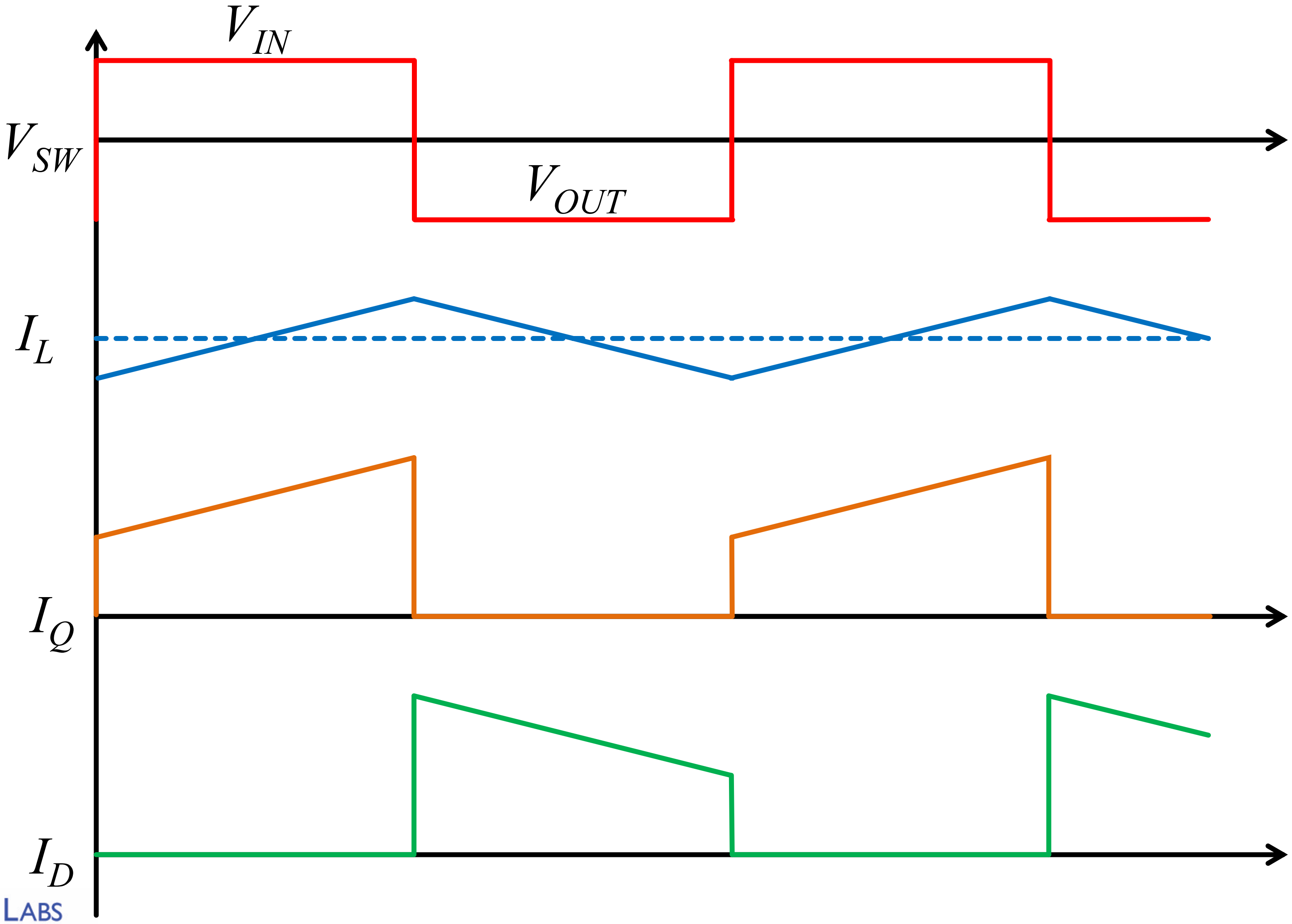
If V_{OUT} Must Be Negative

Buck-Boost Converter Energy Flow





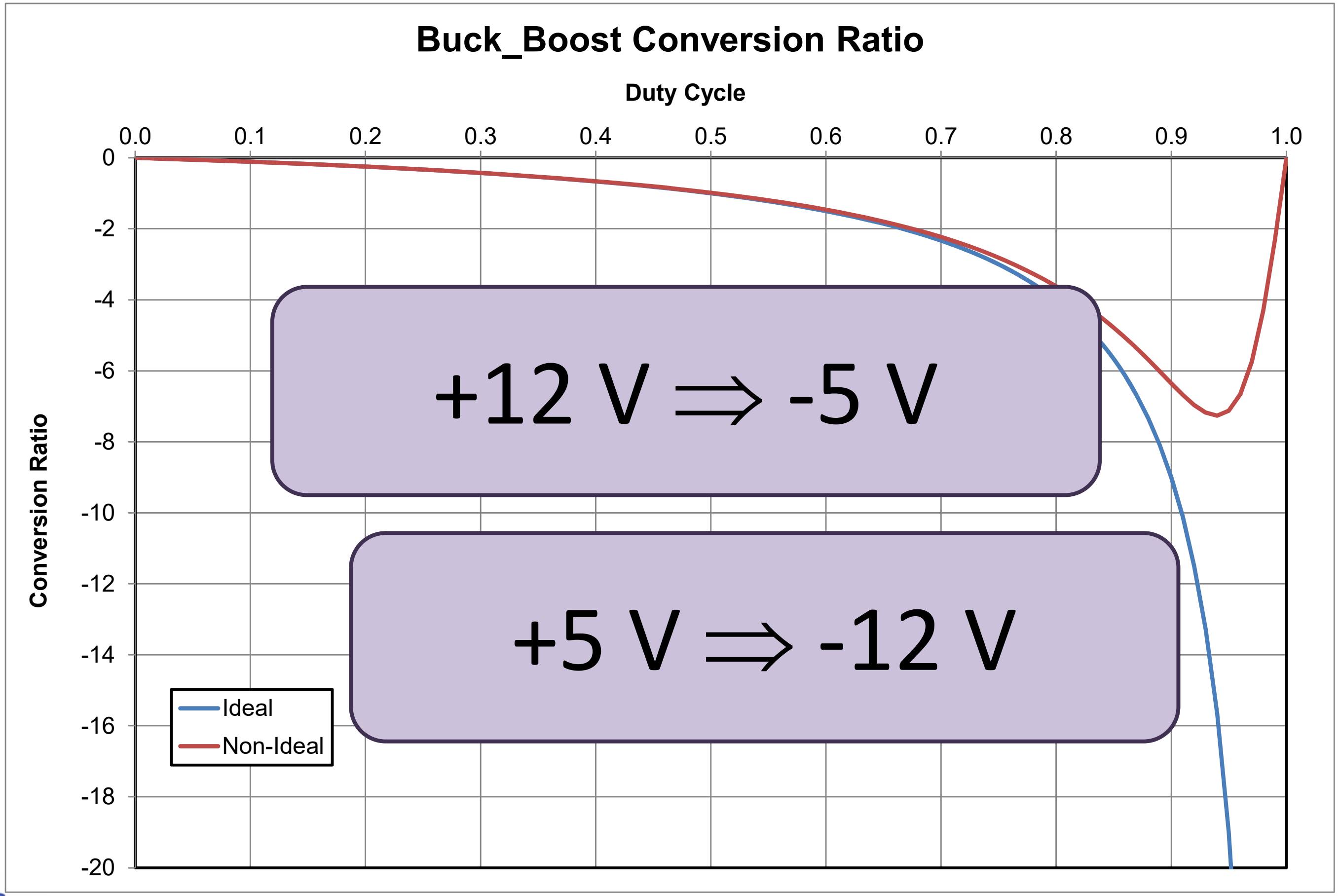
Buck-Boost Converter Waveforms



Embedded Power Labs

Buck-Boost Conversion Ratio

$$V_{OUT} = -\frac{D}{1-D} \cdot V_{IN} = -\frac{D}{D'} \cdot V_{IN}$$



Buck-Boost Converter

Advantages

- Negative Voltage From Positive Voltage
- Low Cost
- Low Parts Count

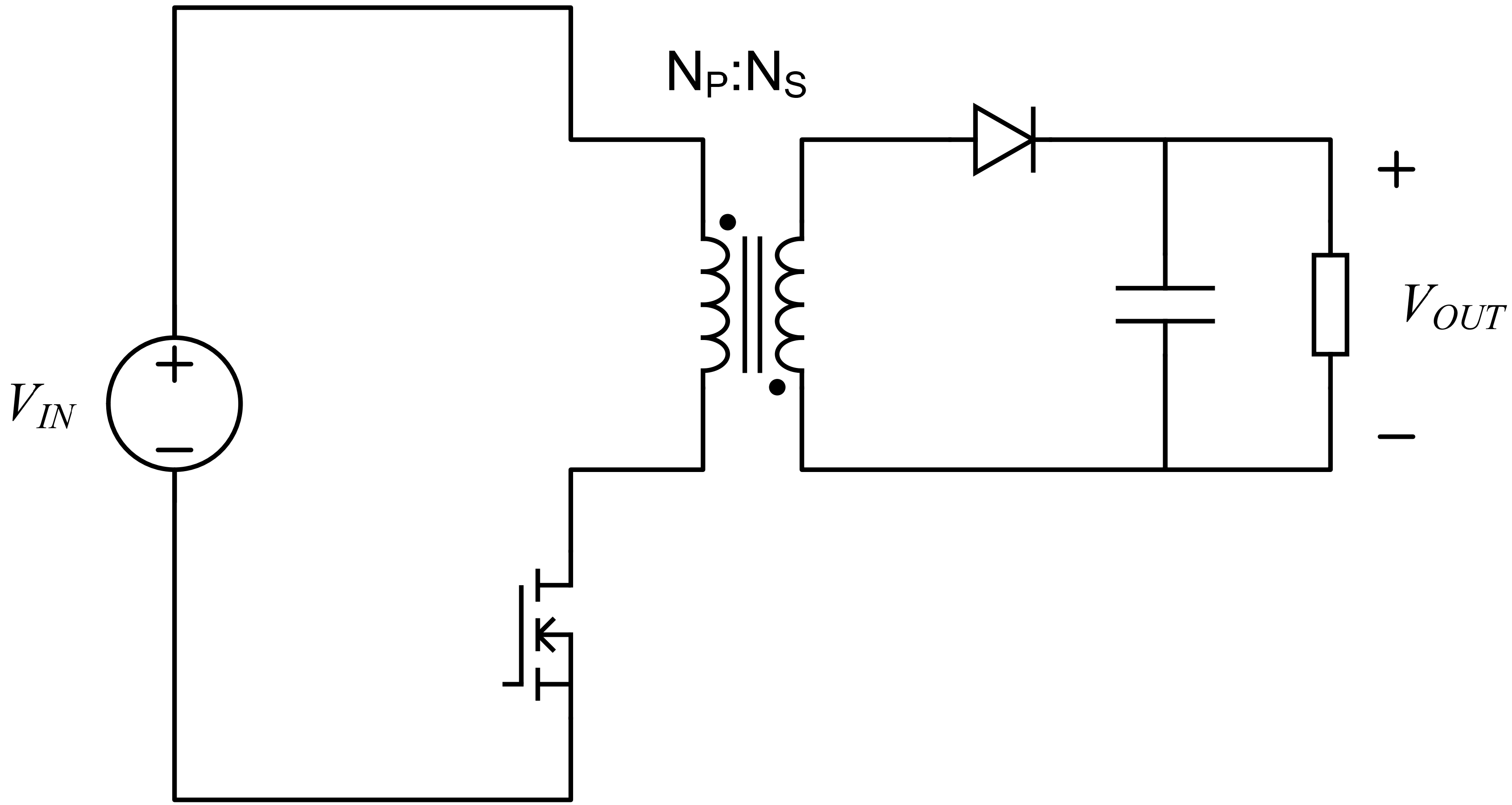
No Direct Flow Of Energy From Input To Output

Disadvantages

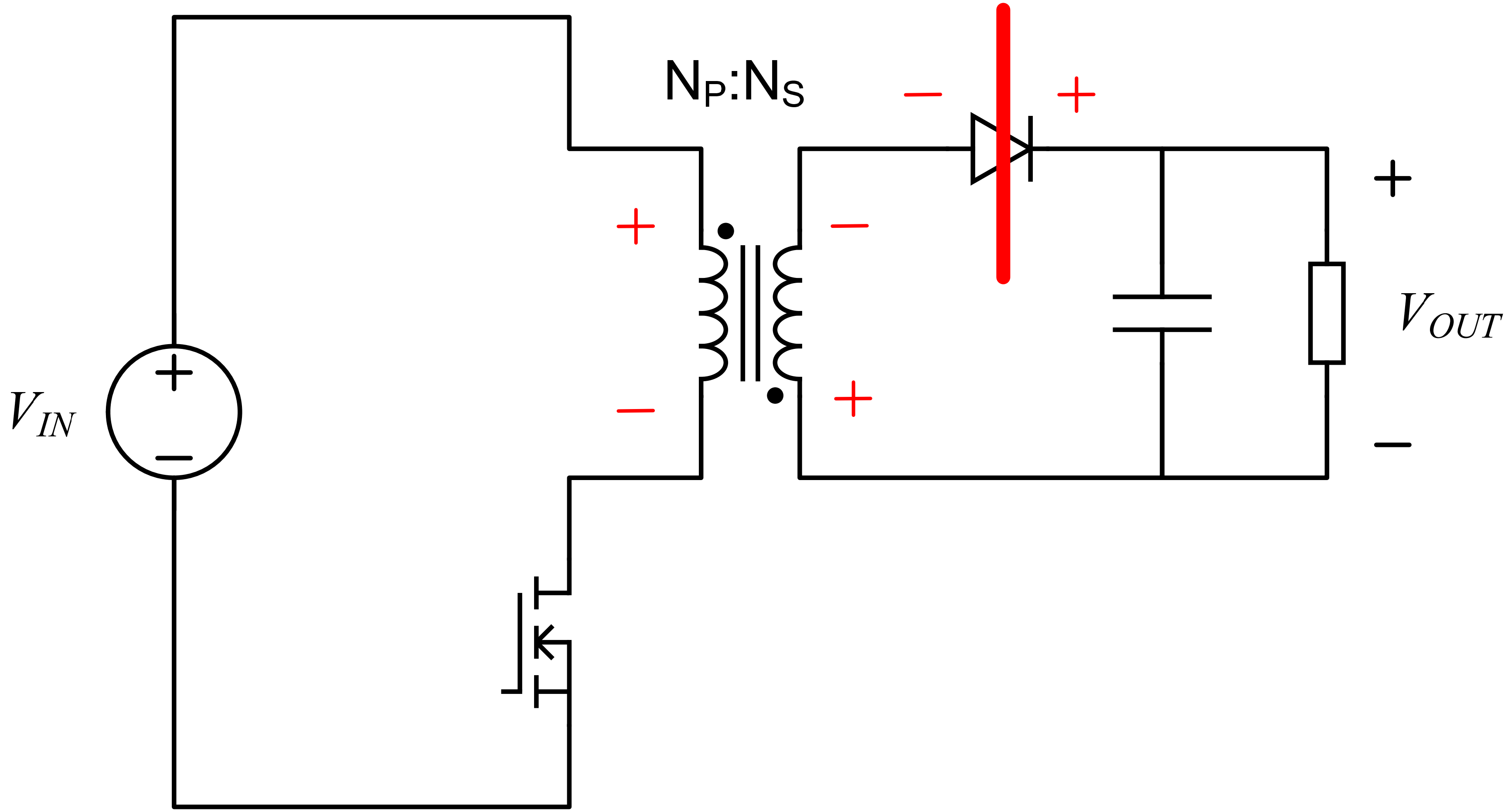
- Non-Isolated
- High Voltage Stress On Transistor And Diode
- Continuous Conduction Mode Harder To Control
- High Efficiency Is Difficult

Flyback Converter (Transformer Isolated Buck-Boost Converter)

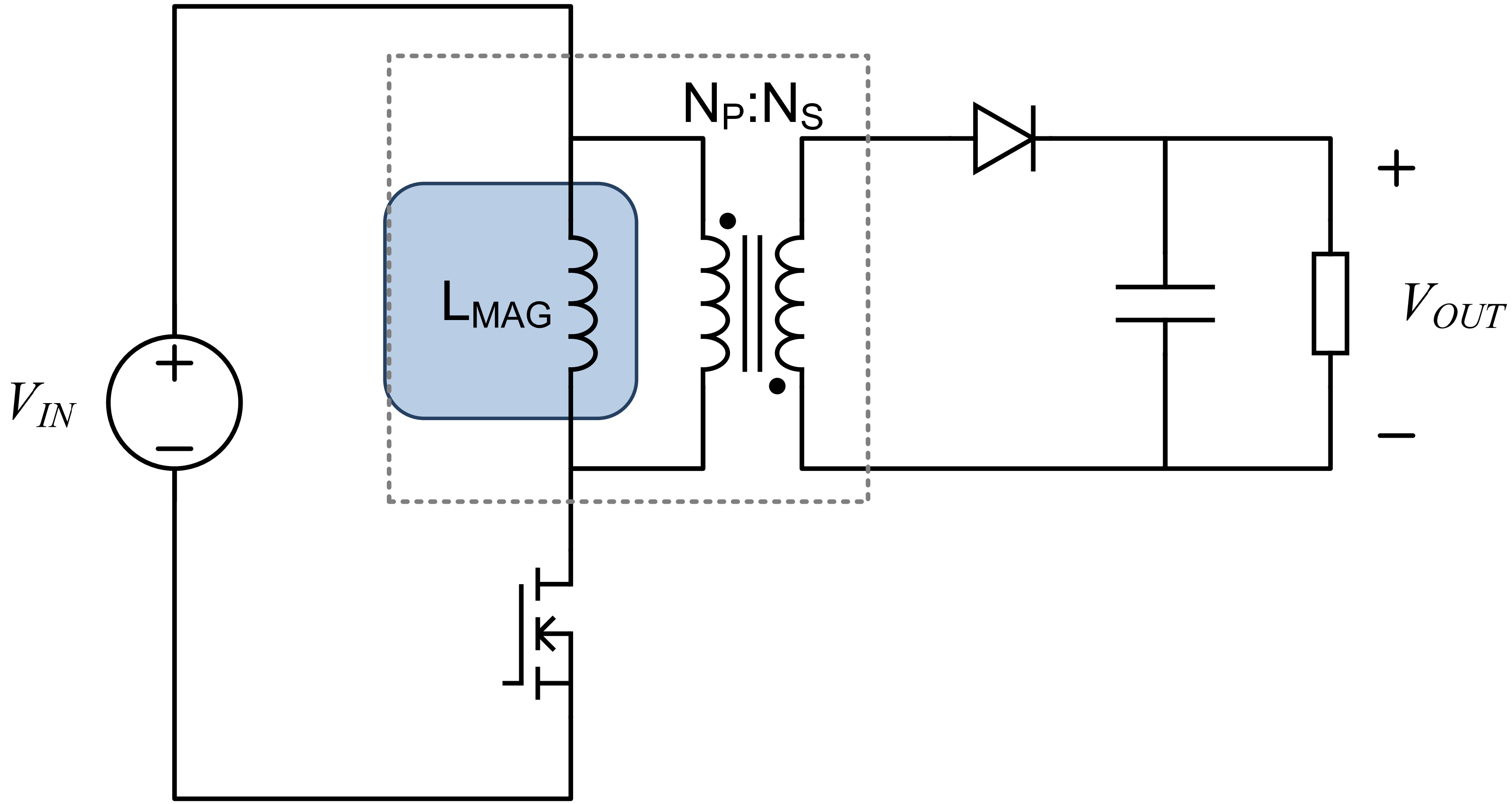
Flyback Converter



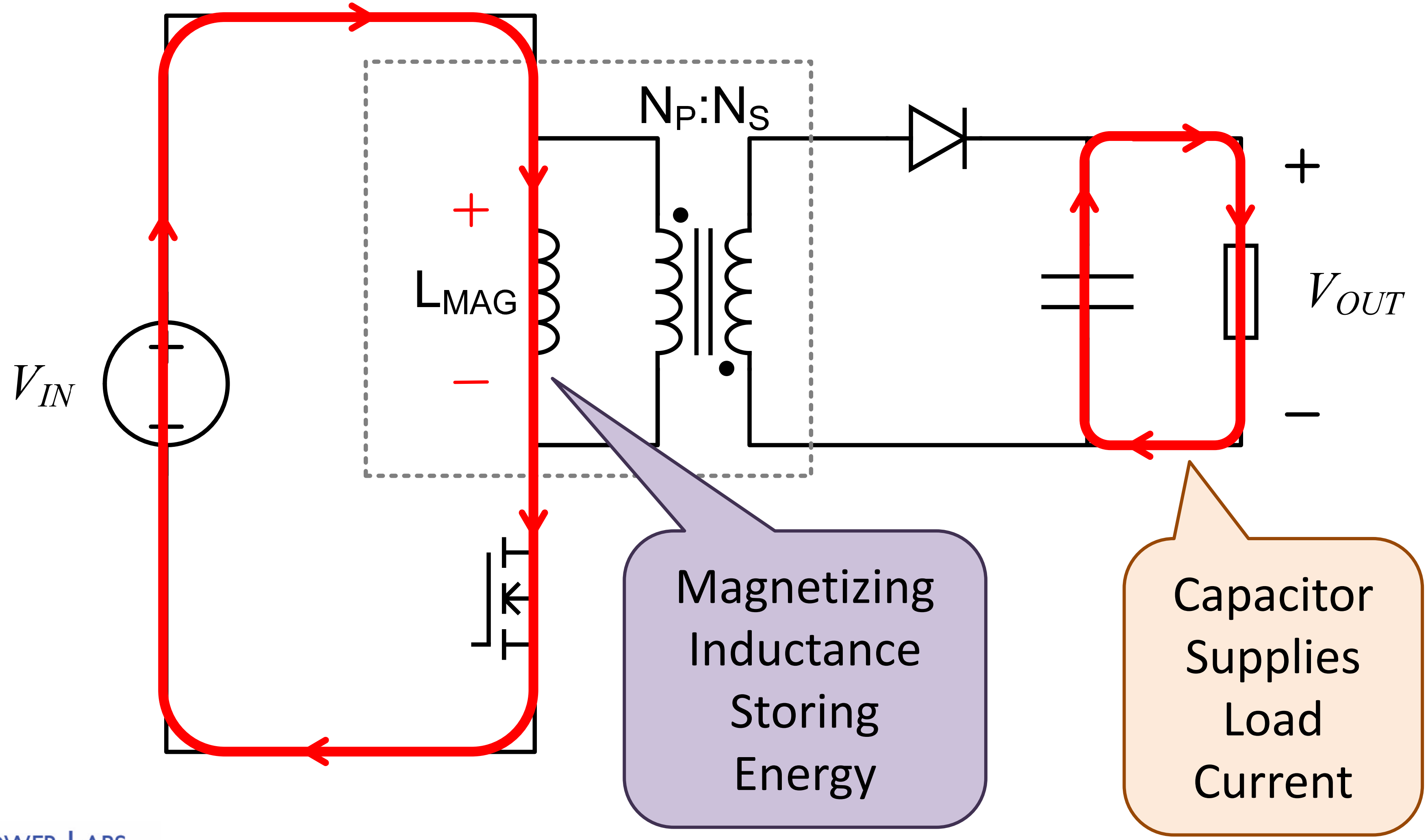
Flyback Converter



Flyback Converter

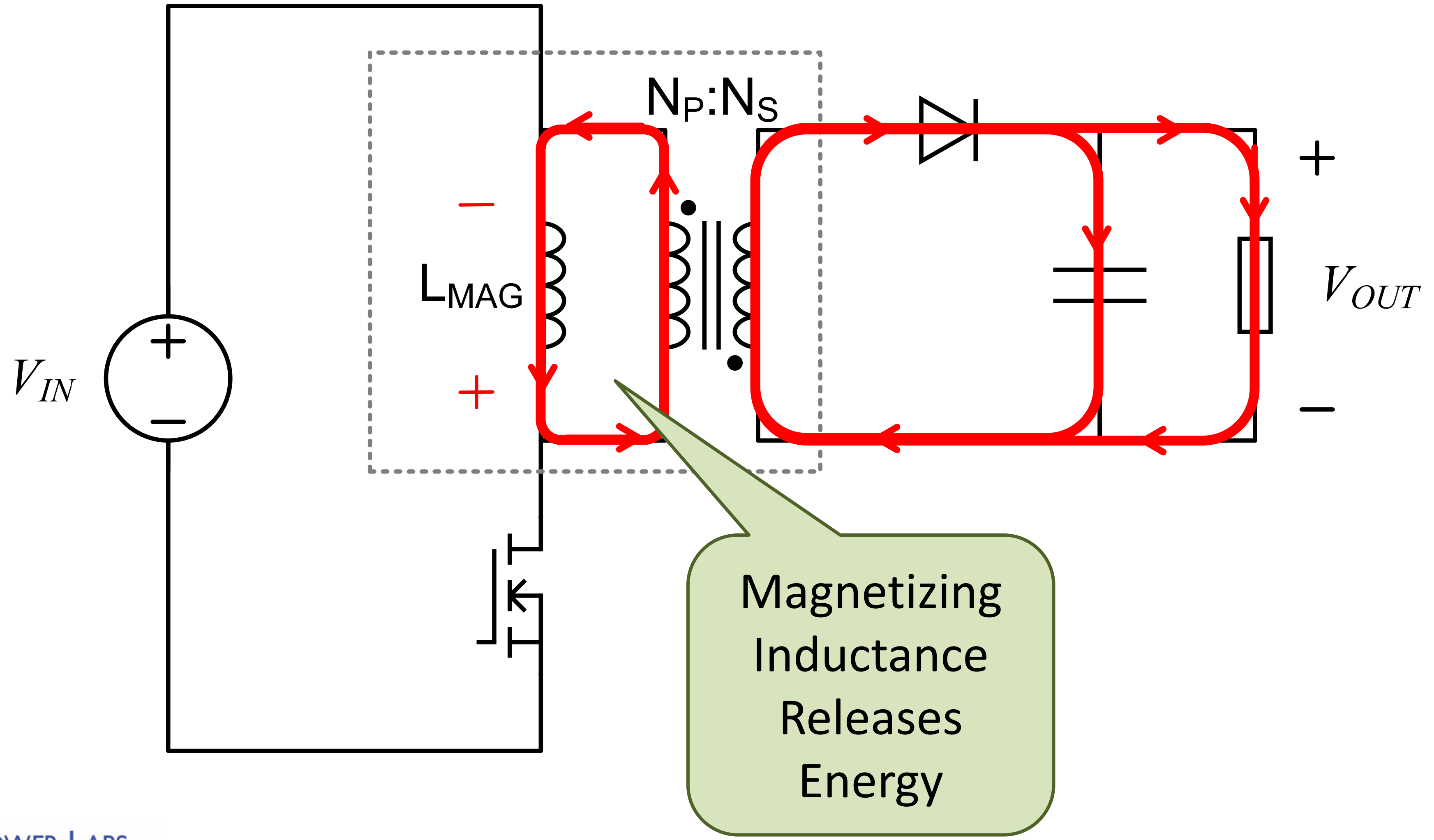


Flyback Converter On Time

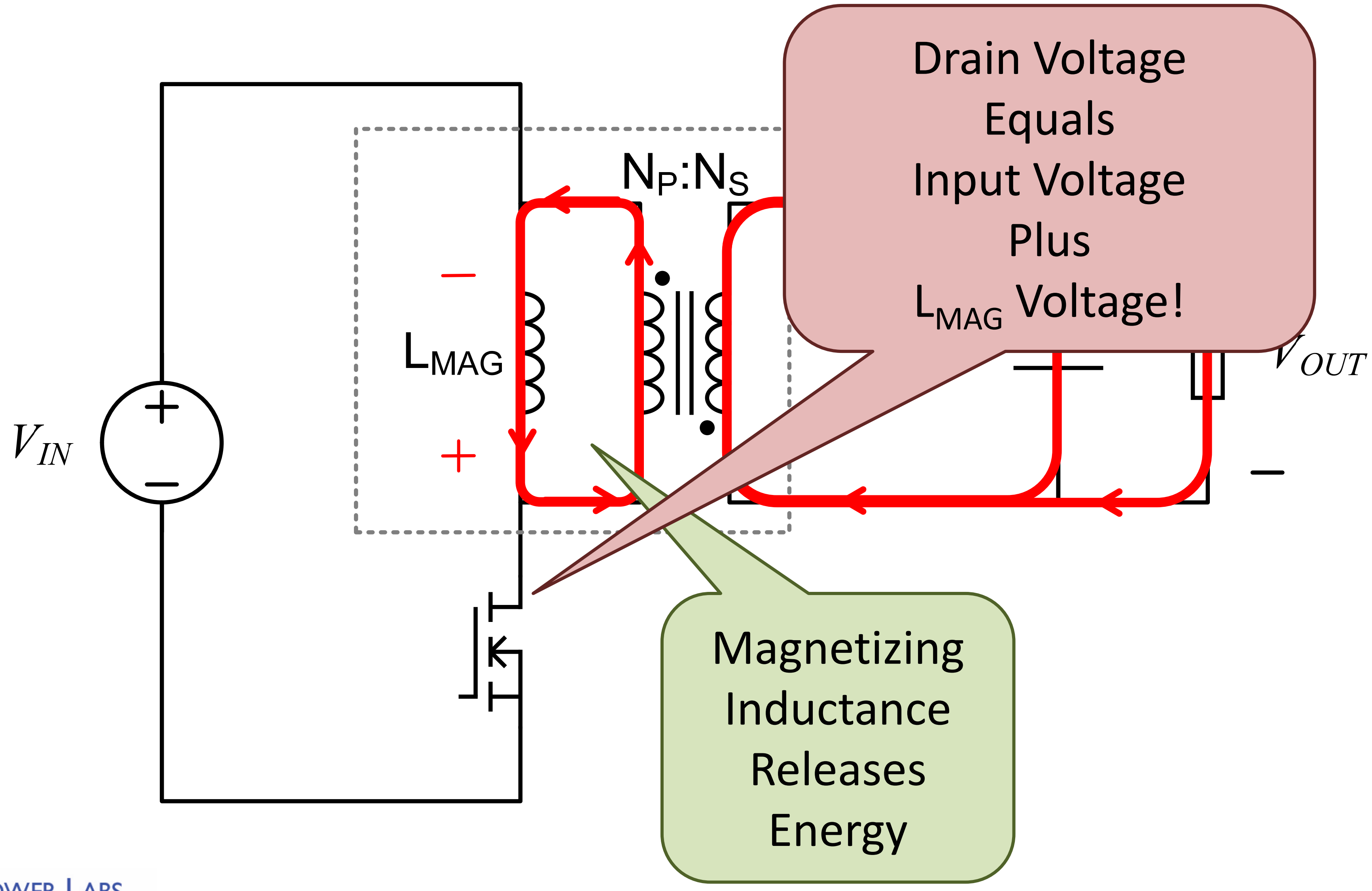


EMBEDDED POWER LABS

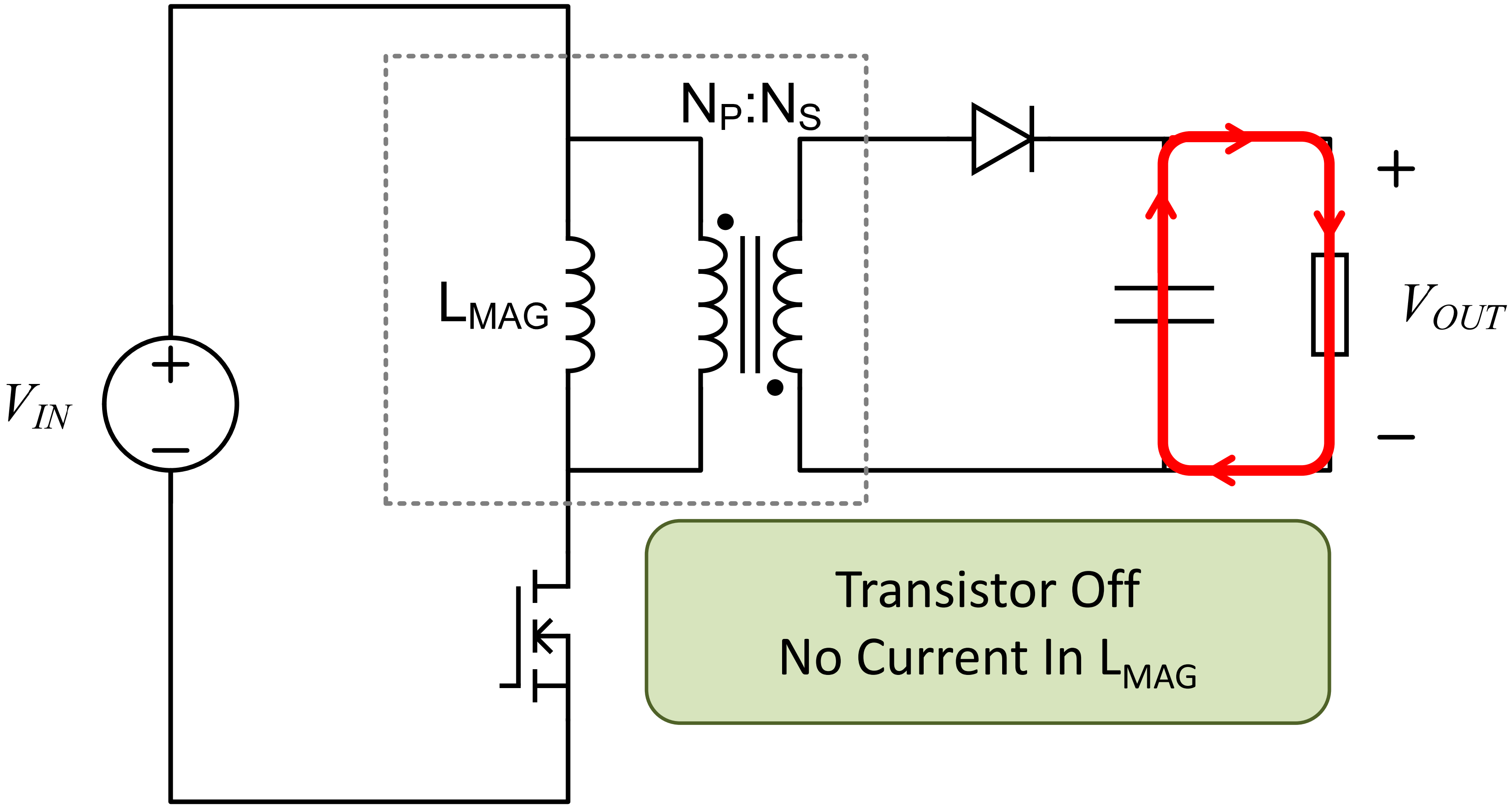
Flyback Converter Reset Time



Flyback Converter Reset Time

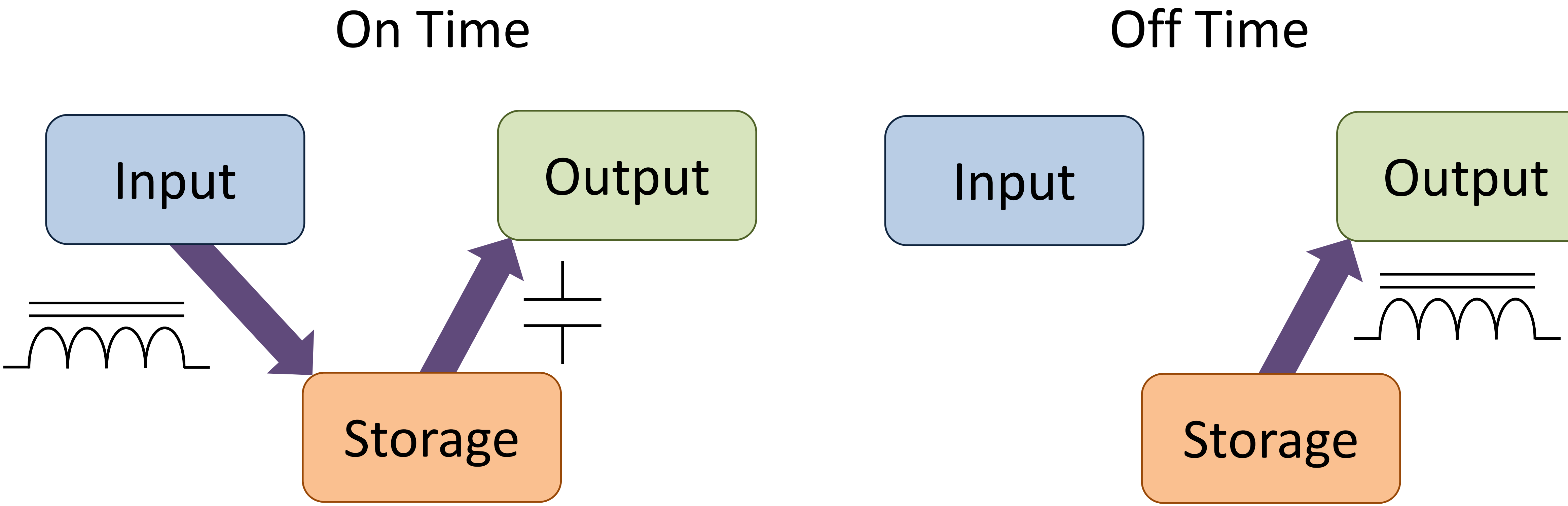


Flyback Converter Idle Time



Transistor Off
No Current In L_{MAG}

Flyback Converter Energy Flow



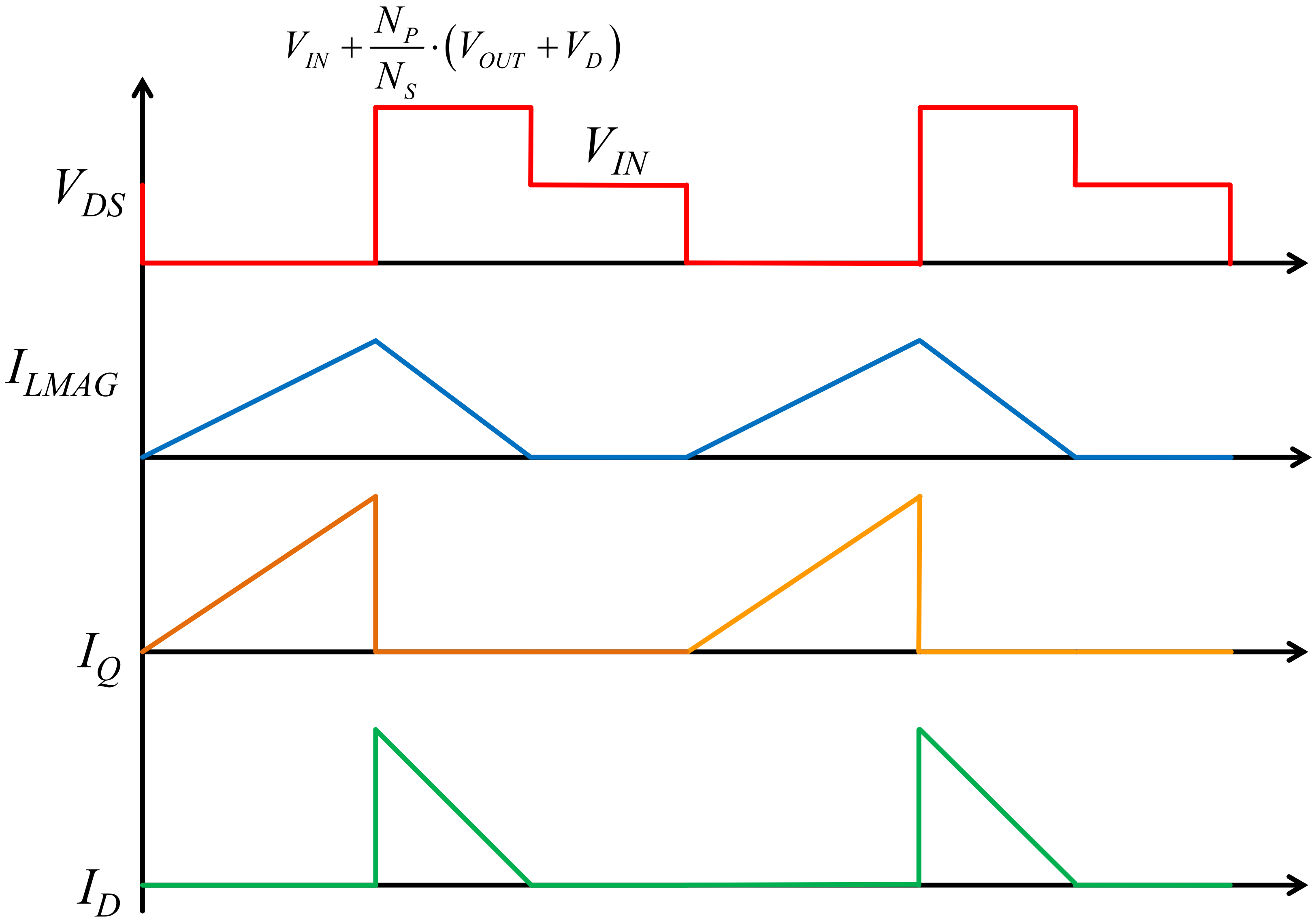
Flyback Energy Flow

On Time:
Store Energy In Flyback
Inductor Core
“Fill The Bucket”

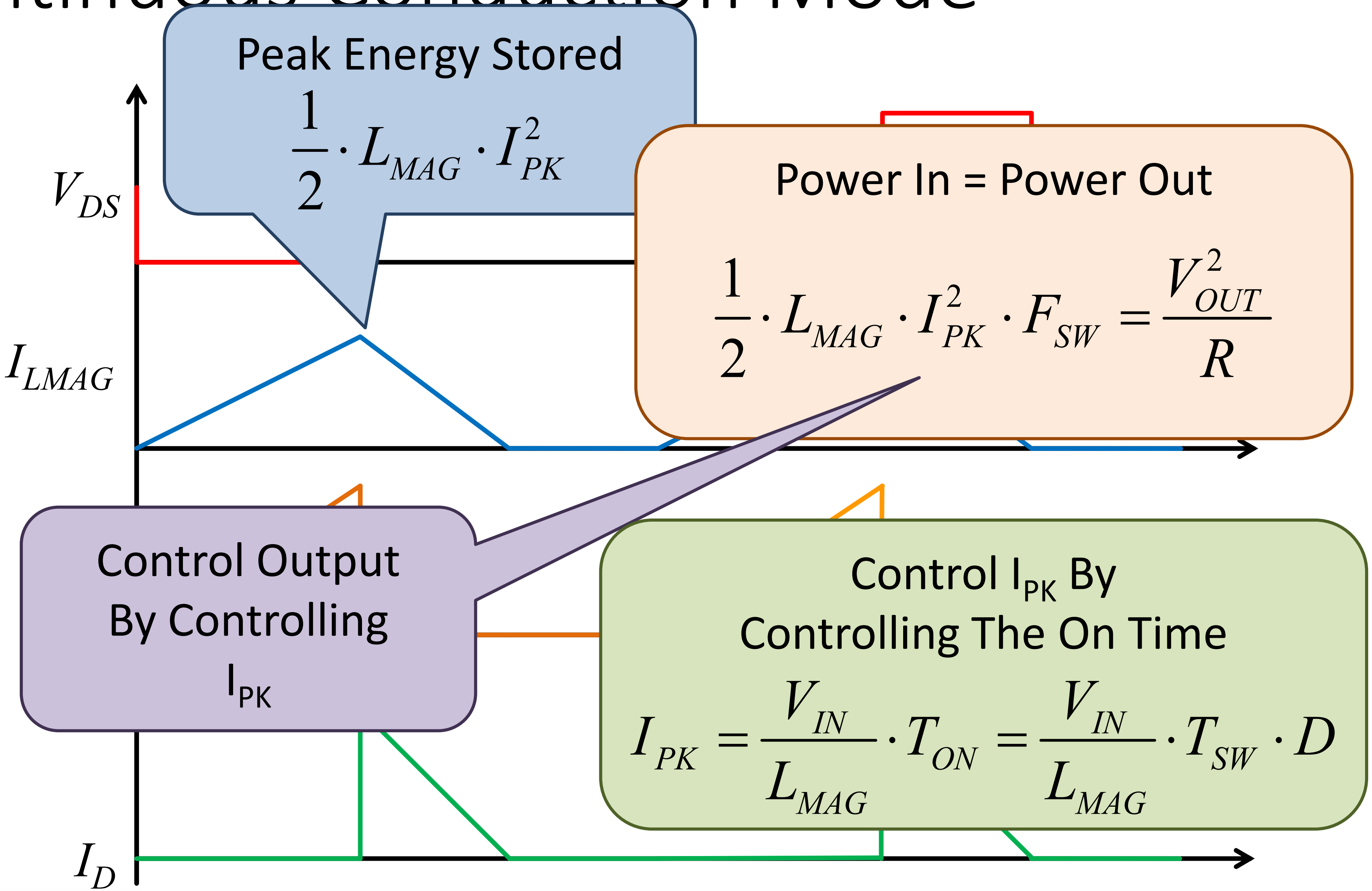
Reset Time:
Deliver Stored Energy
To The Output
“Empty The Bucket”

Idle Time:
“Take A Break”

Discontinuous Conduction Mode



Discontinuous Conduction Mode



Discontinuous Conduction Mode

$$\frac{V_{OUT}^2}{R} = \frac{1}{2} \cdot L_{MAG} \cdot I_{PK}^2 \cdot F_{SW}$$

Start With
Power In = Power Out

$$= \frac{1}{2} \cdot L_{MAG} \cdot \left(\frac{V_{IN} \cdot T_{ON}}{L_{MAG}} \right)^2 \cdot F_{SW}$$

$$V_{OUT}^2 = \frac{R \cdot L_{MAG}}{2} \cdot \frac{V_{IN}^2 \cdot T_{ON}^2}{L_{MAG}^2} \cdot \frac{1}{T_{SW}} \cdot \frac{T_{SW}}{T_{SW}}$$

$$= \frac{R \cdot T_{SW}}{2 \cdot L_{MAG}} \cdot \frac{V_{IN}^2 \cdot T_{ON}^2}{T_{SW}^2} = \frac{R \cdot T_{SW}}{2 \cdot L_{MAG}} \cdot V_{IN}^2 \cdot D^2$$

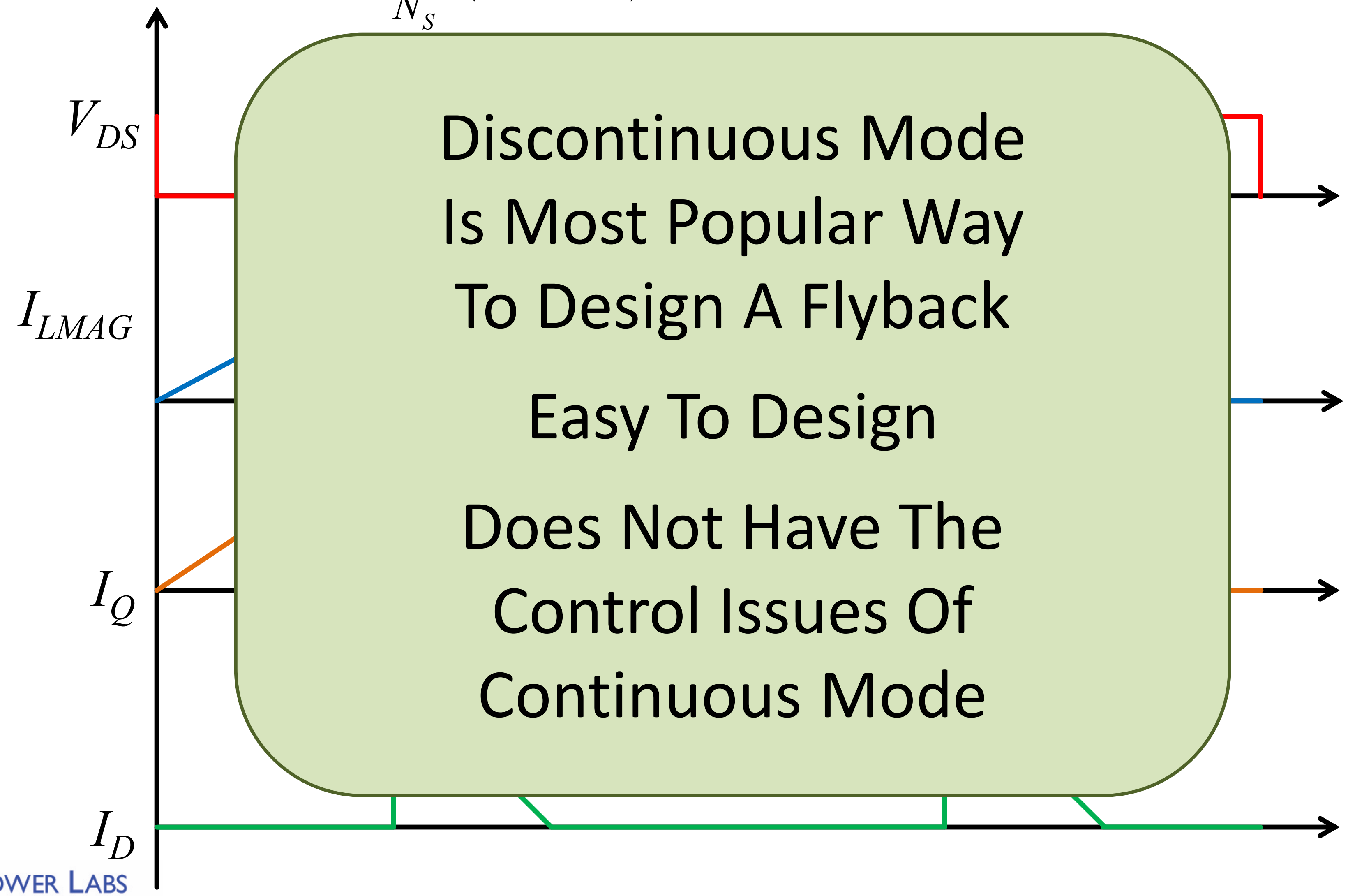
$$V_{OUT} = \sqrt{\frac{R \cdot T_{SW}}{2 \cdot L_{MAG}} \cdot V_{IN}^2 \cdot D^2}$$

Get A
“Buck Like”
Conversion
Ratio

$$V_{OUT} = D \cdot V_{IN} \cdot \sqrt{\frac{R \cdot T_{SW}}{2 \cdot L_{MAG}}}$$

Discontinuous Conduction Mode

$$V_{IN} + \frac{N_P}{N_S} \cdot \left(V_{OUT} + V_D\right)$$

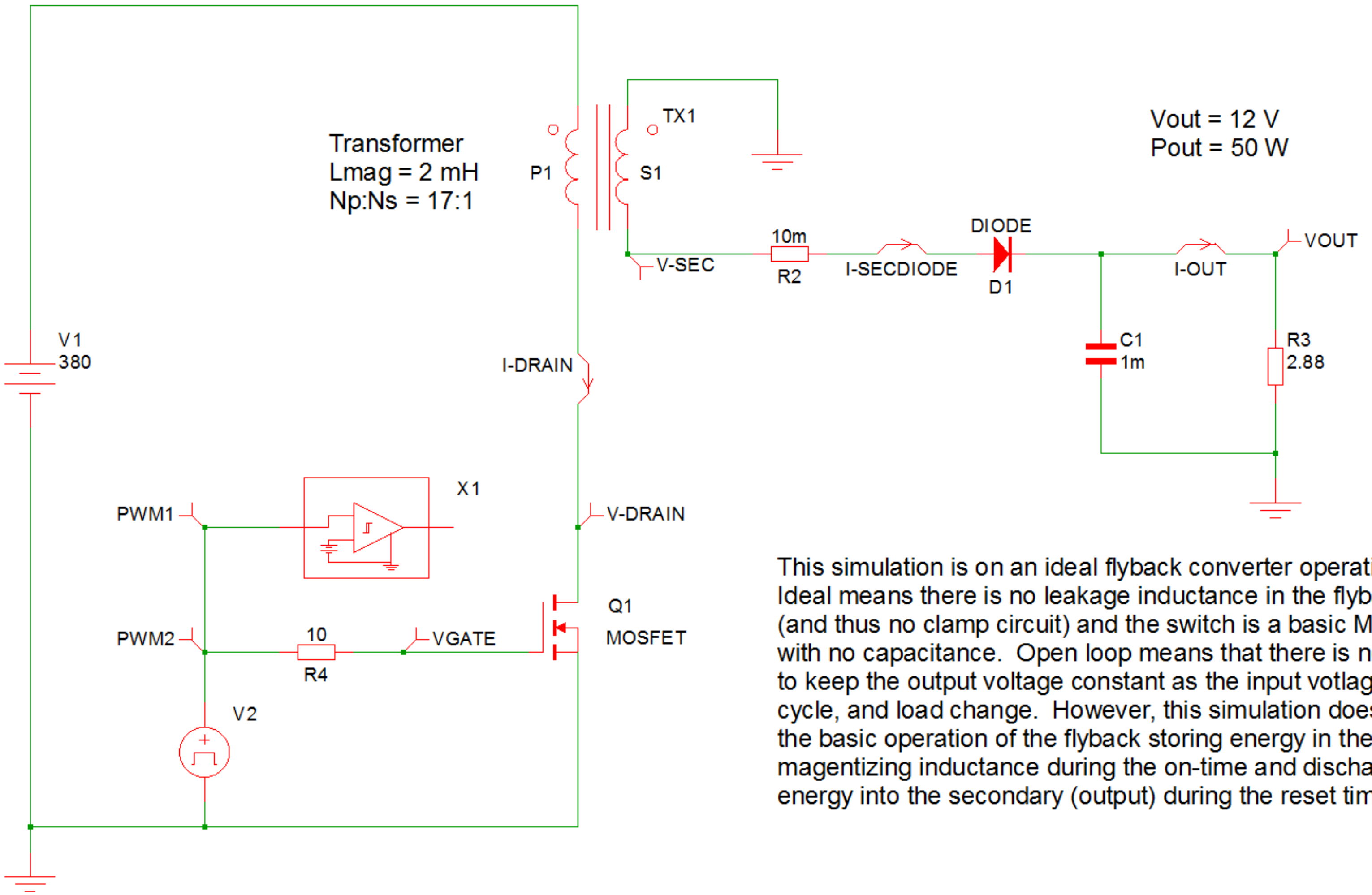


Discontinuous Mode Is Most Popular Way To Design A Flyback

Easy To Design

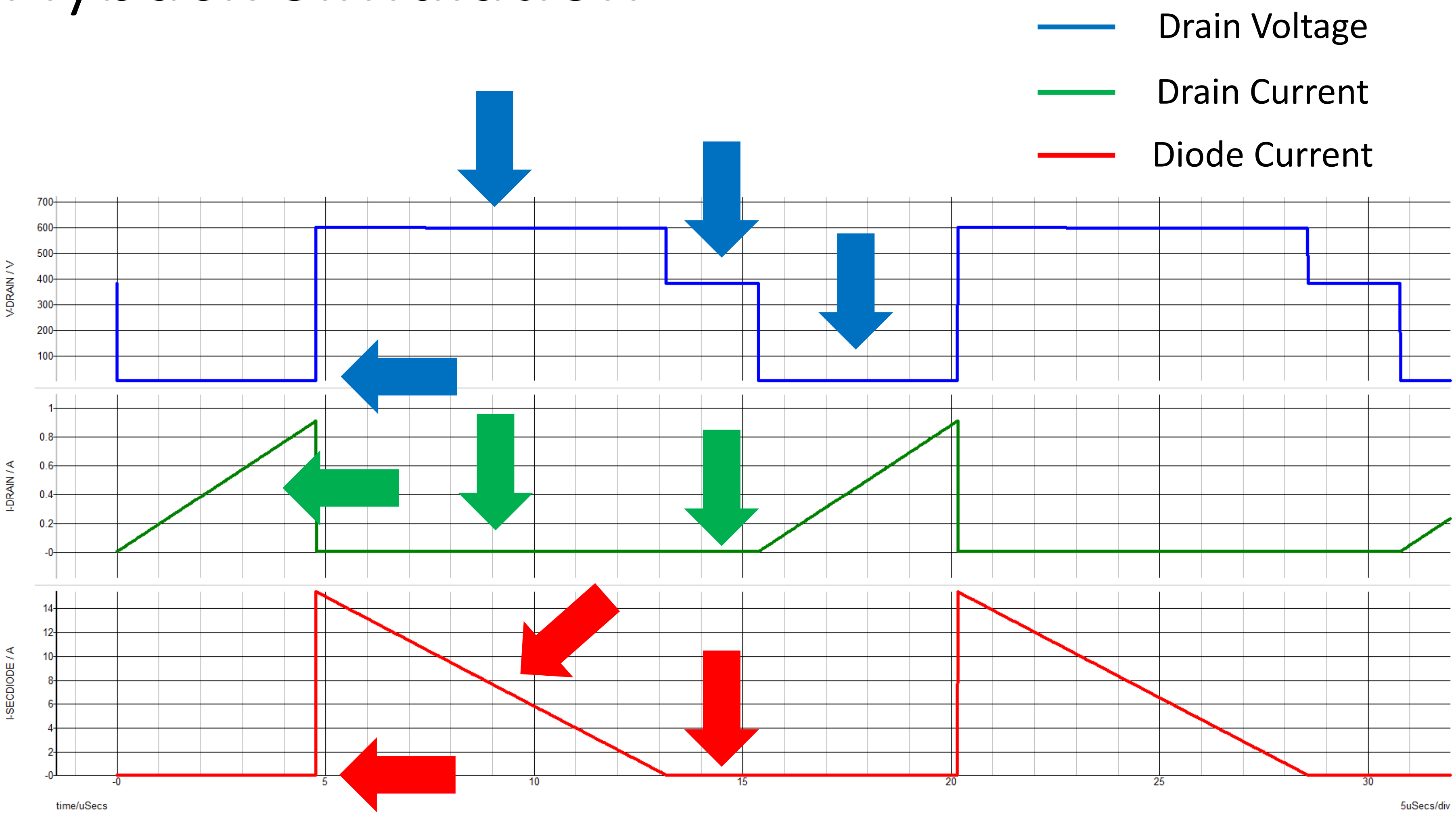
Does Not Have The Control Issues Of Continuous Mode

Discontinuous Conduction Mode
Ideal Flyback Simulation

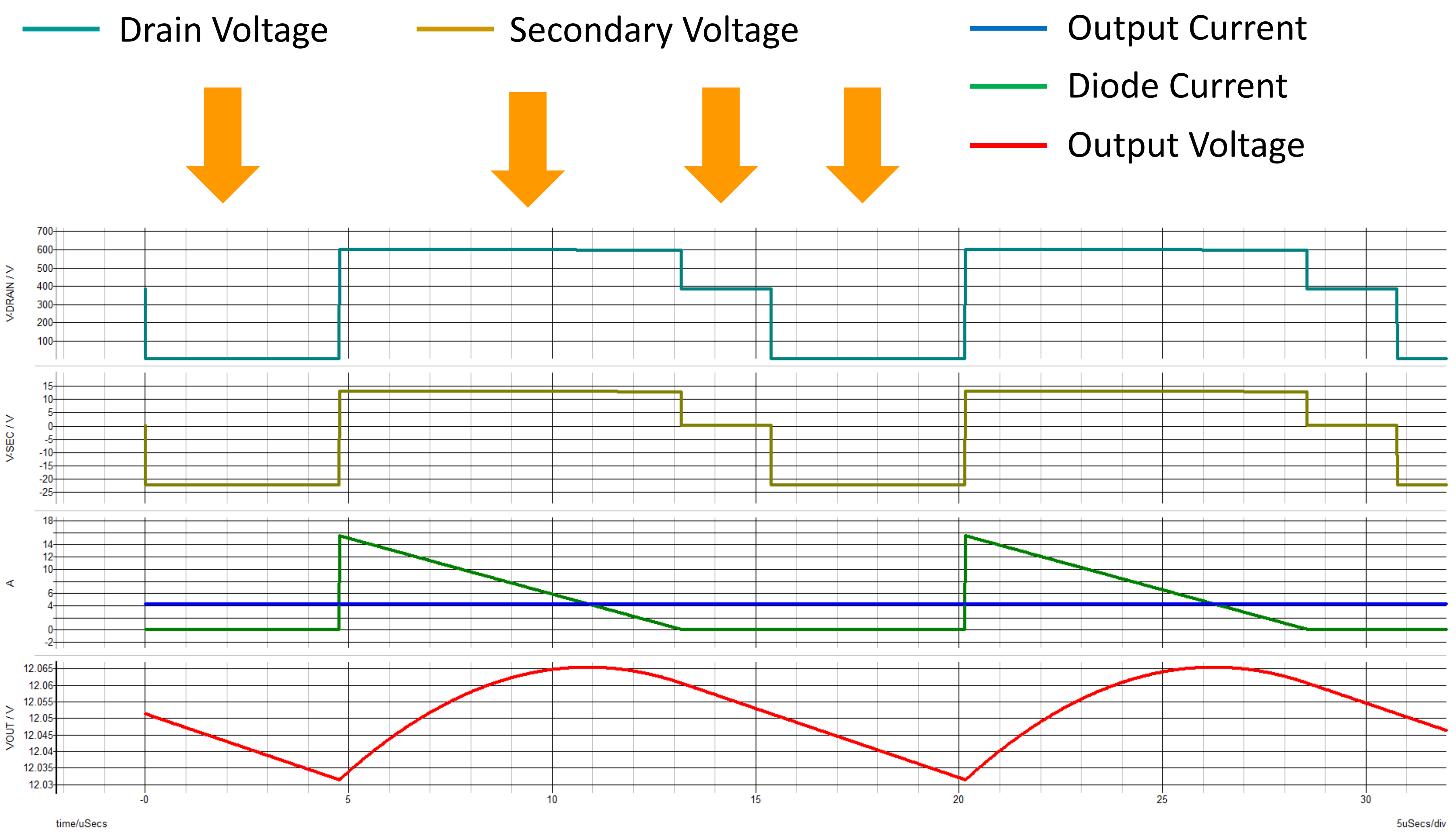


This simulation is on an ideal flyback converter operating open loop. Ideal means there is no leakage inductance in the flyback inductor (and thus no clamp circuit) and the switch is a basic MOSFET with no capacitance. Open loop means that there is no feedback to keep the output voltage constant as the input votlage, duty cycle, and load change. However, this simulation does illustrate the basic operation of the flyback storing energy in the magentizing inductance during the on-time and discharging that energy into the secondary (output) during the reset time.

Discontinuous Conduction Mode
Ideal Flyback Simulation



Discontinuous Conduction Mode
Ideal Flyback Simulation



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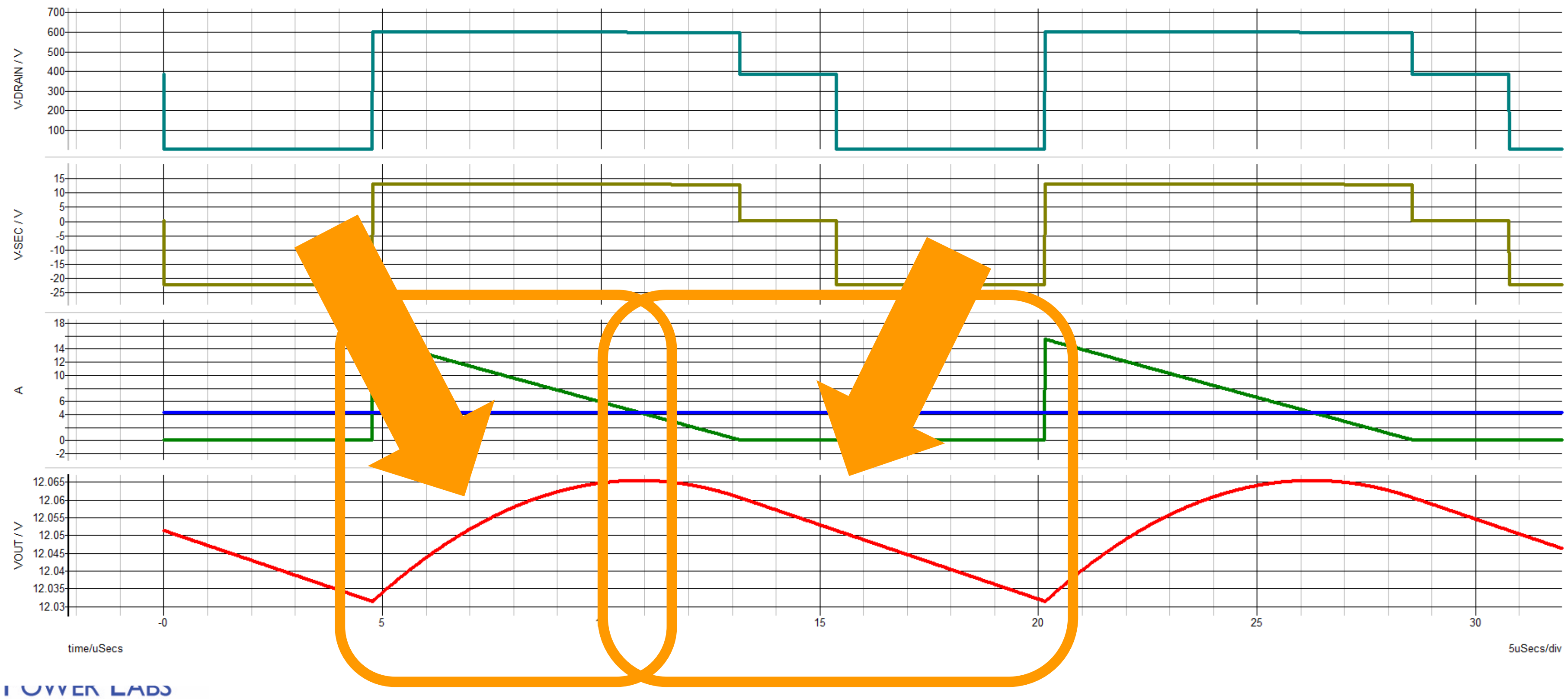
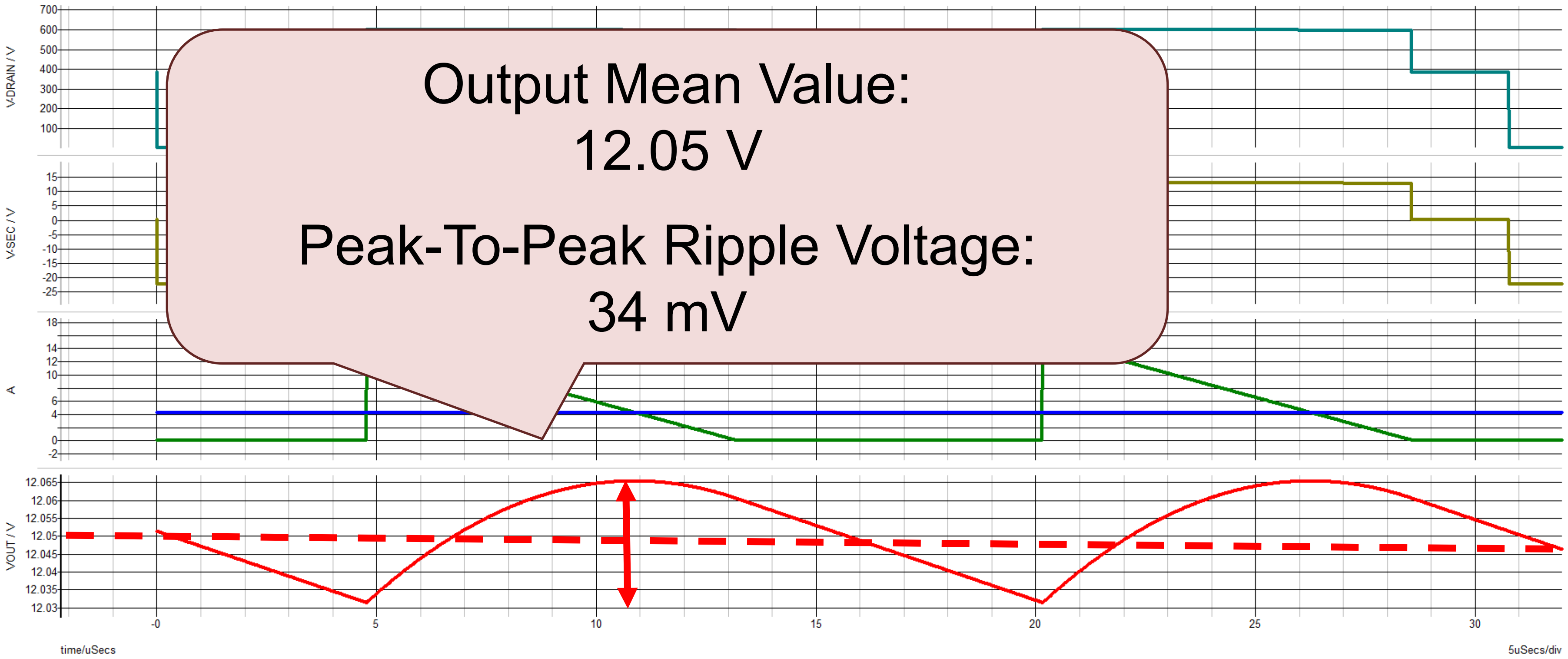
Discontinuous Conduction Mode
Ideal Flyback Simulation

Drain Voltage
Secondary Voltage
Output Current
Diode Current
Output Voltage

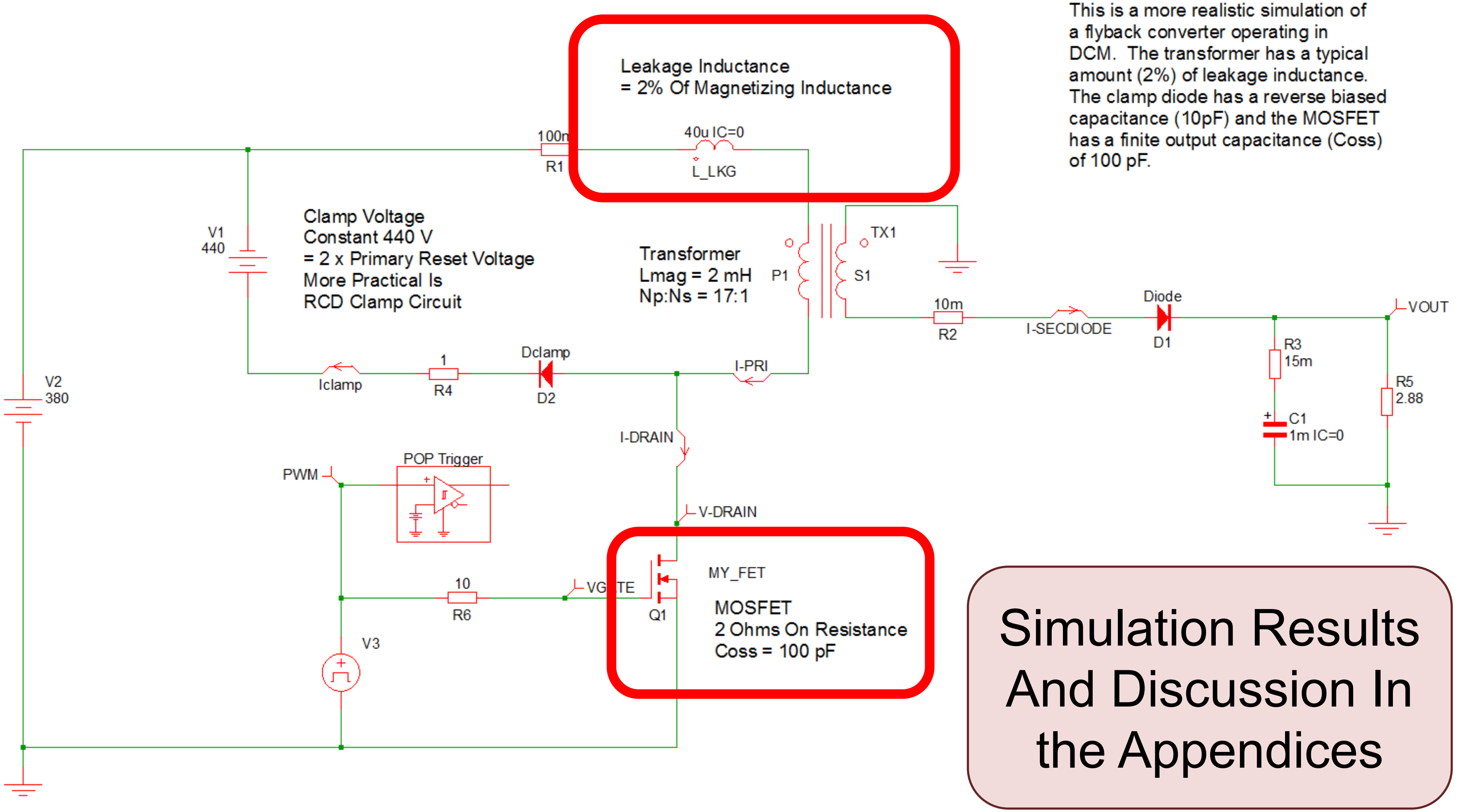


Discontinuous Conduction Mode
Ideal Flyback Simulation

Drain Voltage
Secondary Voltage
Output Current
Diode Current
Output Voltage



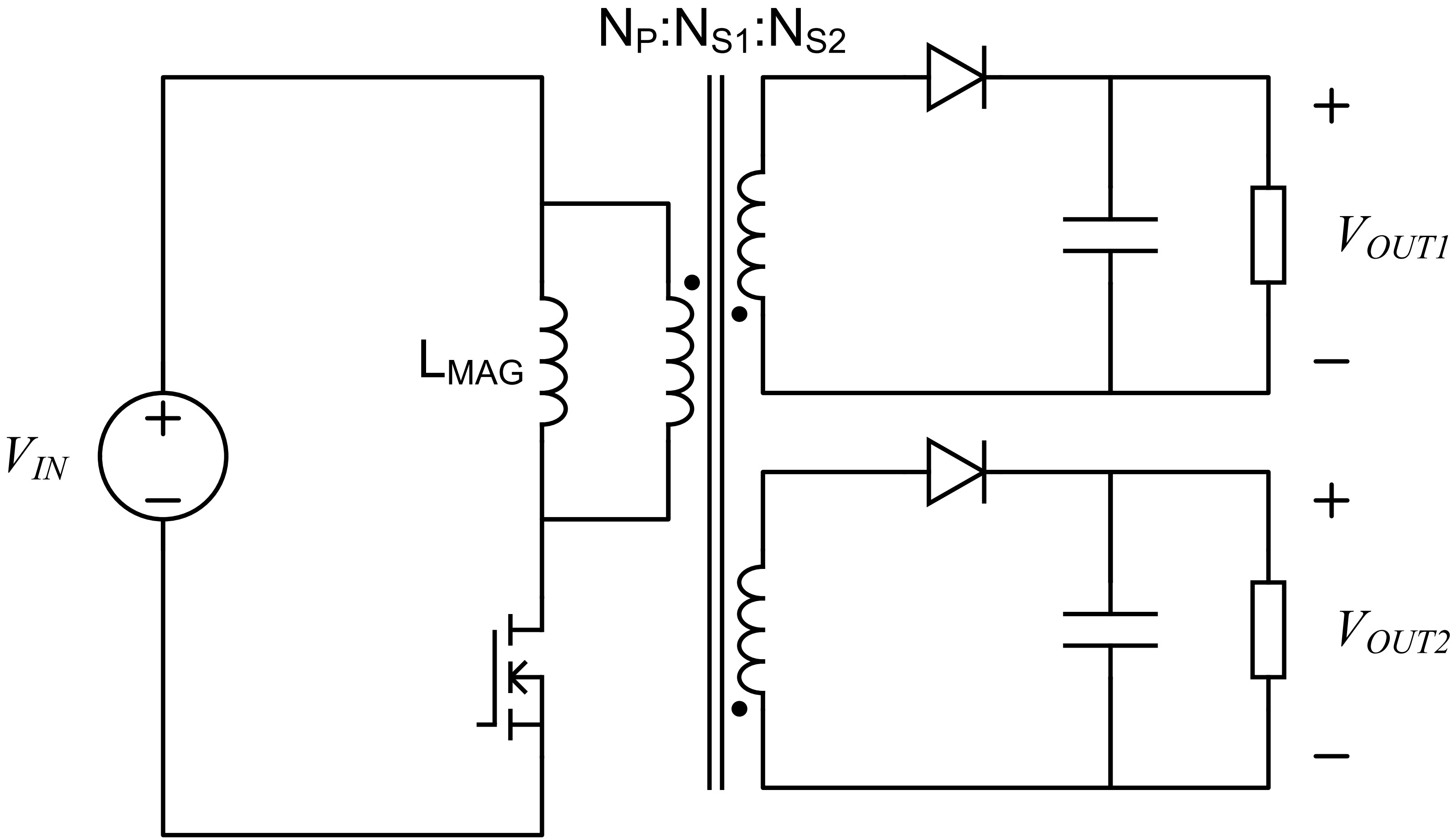
Discontinuous Conduction Mode
More Realistic Flyback Simulation



This is a more realistic simulation of a flyback converter operating in DCM. The transformer has a typical amount (2%) of leakage inductance. The clamp diode has a reverse biased capacitance (10pF) and the MOSFET has a finite output capacitance (Coss) of 100 pF.

Simulation Results And Discussion In the Appendices

Multiple Output Flyback



Flyback Converter

Advantages

- Isolated
- Good Up To 150 W
- Multiple Isolated Outputs Possible
- Easy To Control (Discontinuous Mode)

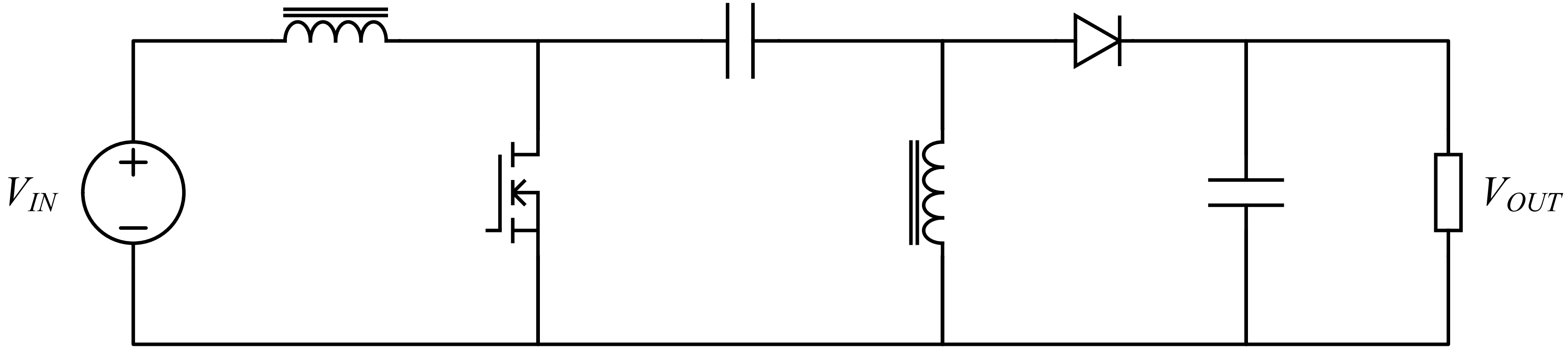
Disadvantages

- Transformer Needed
- Single Transistor: High Voltage Stress On Transistor And Diode
- Large Output Capacitor

SEPIC Topology

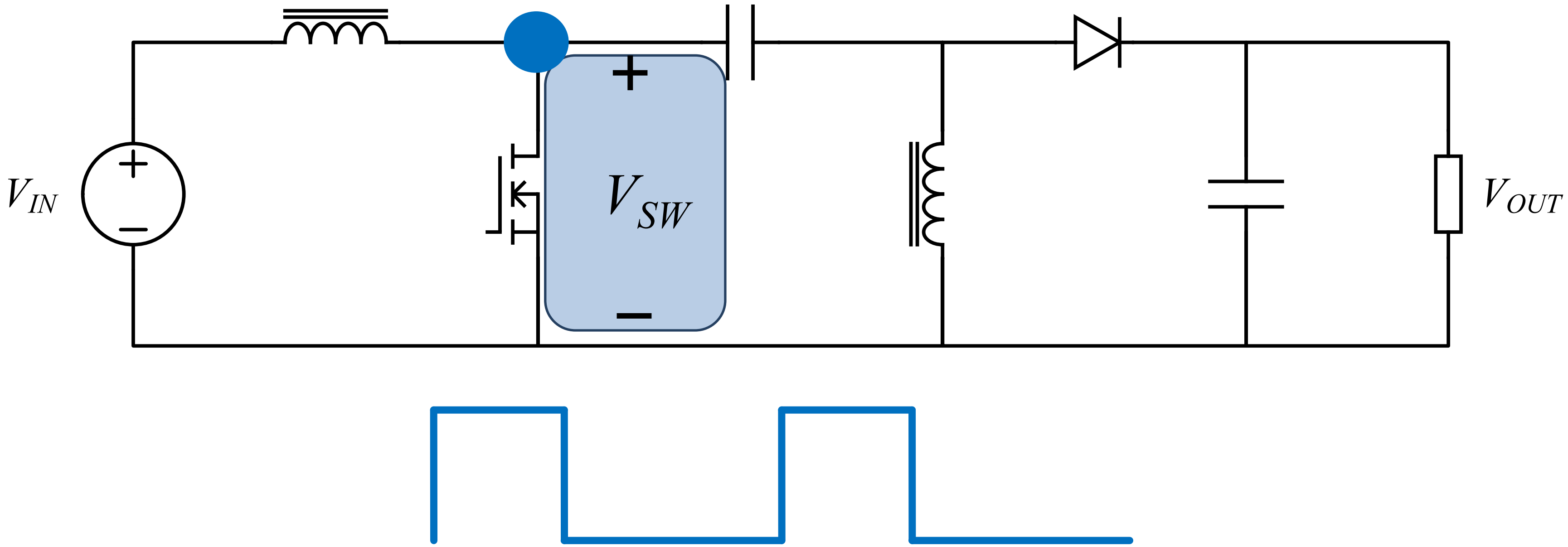
SEPIC

Single-Ended, Primary Inductor Converter



SEPIC

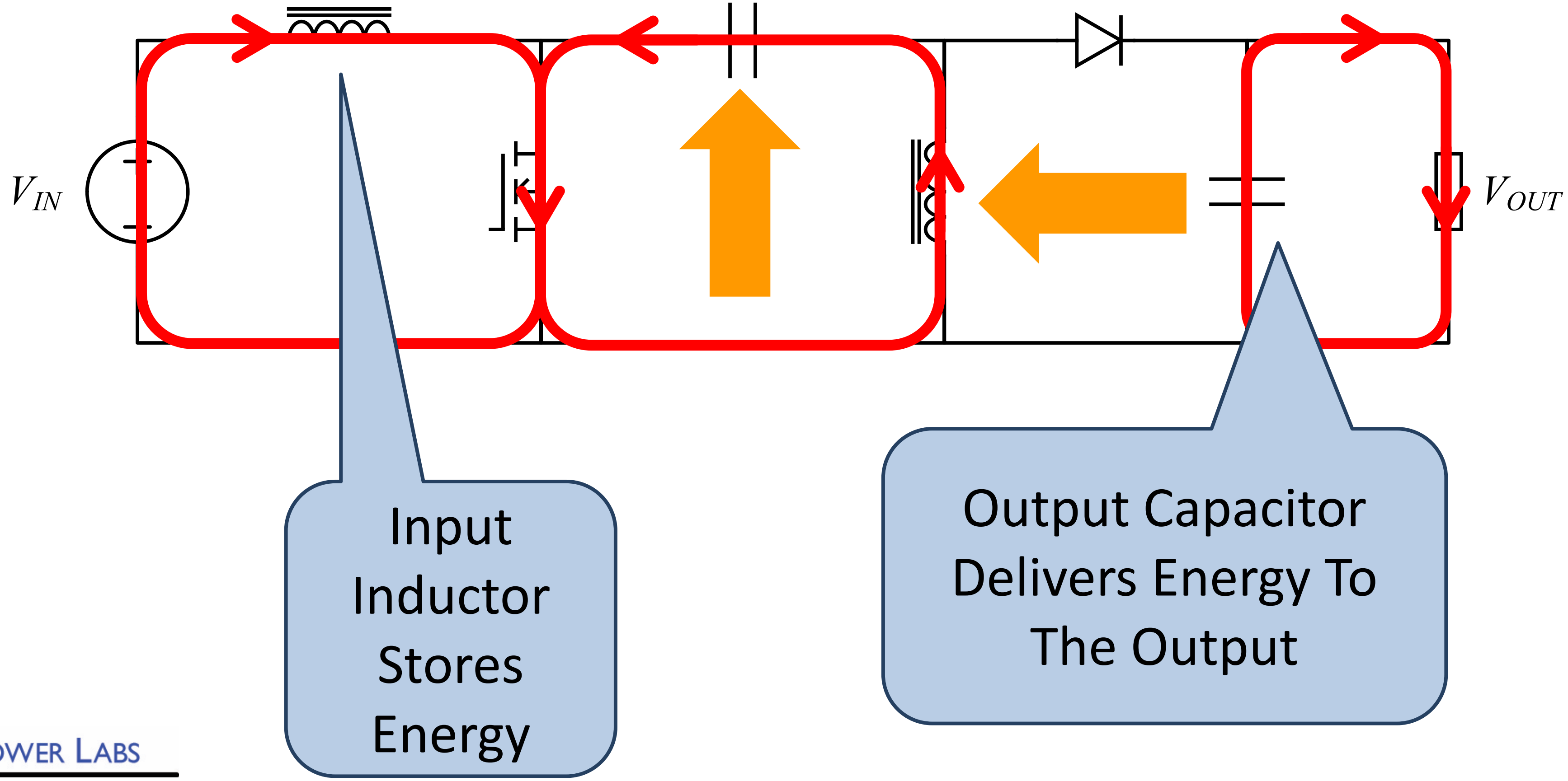
Single-Ended, Primary Inductor Converter



“Switch Node”

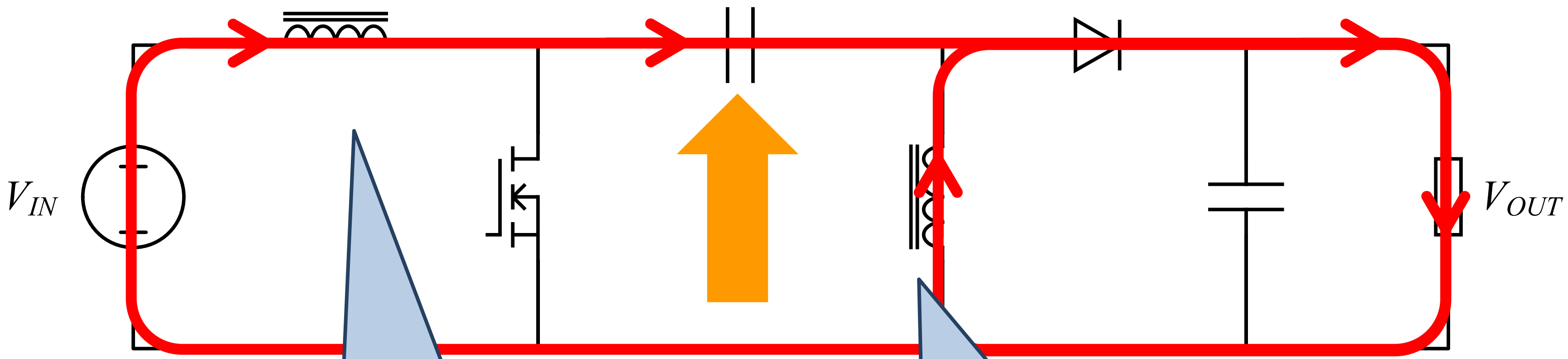
SEPIC On Time

Single-Ended, Primary Inductor Converter



SEPIC Off Time

Single-Ended, Primary Inductor Converter

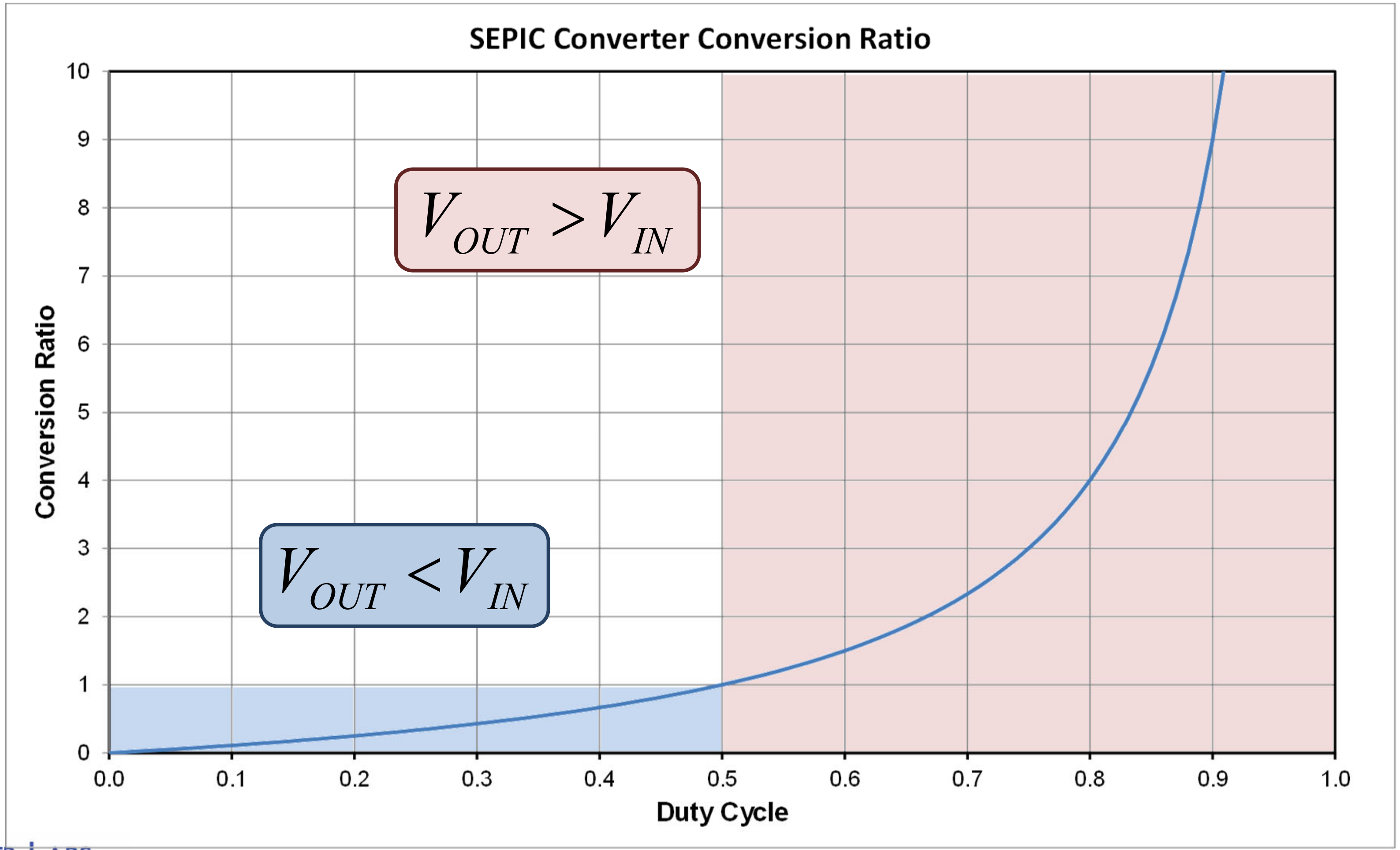


Input Inductor
Deliver Energy
To The Output

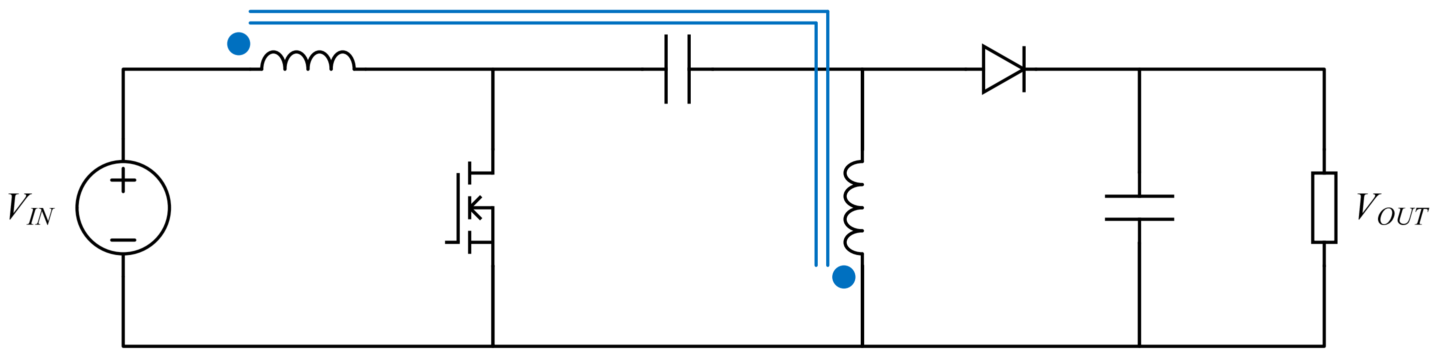
Second Inductor
Also Delivers Energy To The
Output

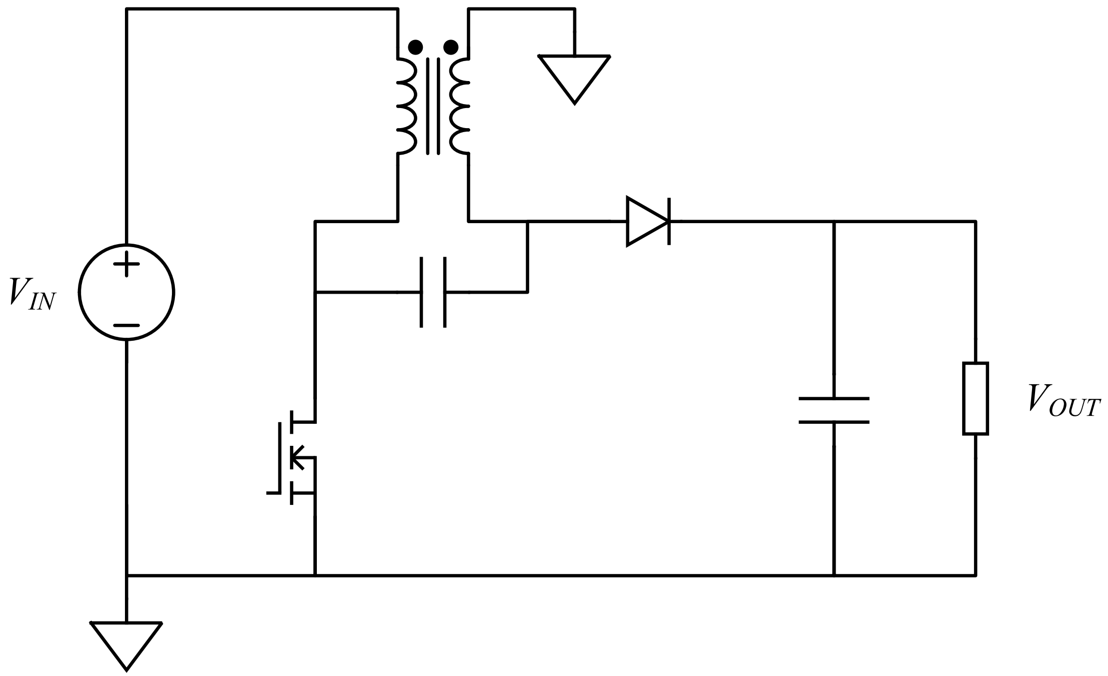
SEPIC Conversion Ratio

$$V_{OUT} = \frac{D}{1-D} \cdot V_{IN} = \frac{D}{D'} \cdot V_{IN}$$



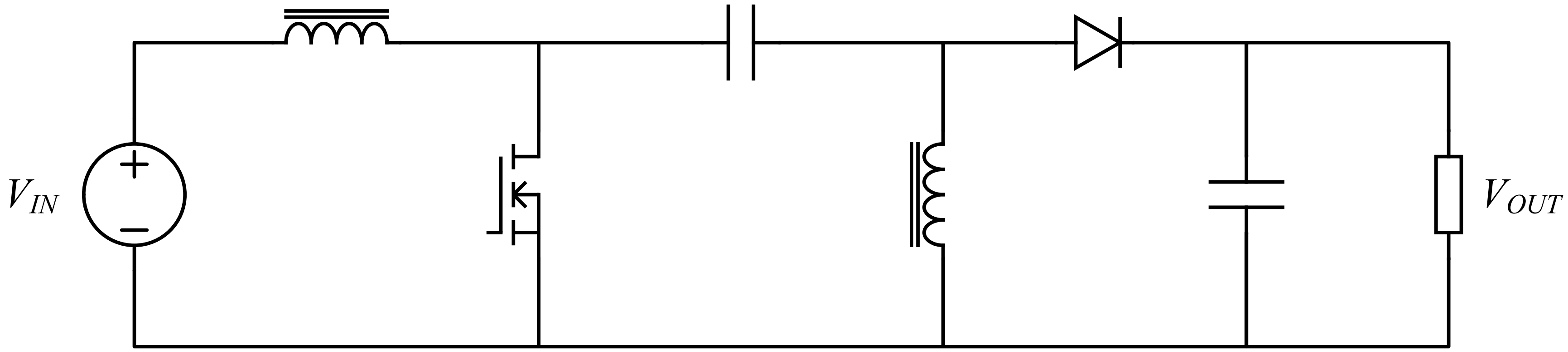
SEPIC With Coupled Inductor





SEPIC

Single-Ended, Primary Inductor Converter



<u>Advantages</u>

- Output Voltage Higher Or Lower Than Input Voltage
- Single Grounded Switch
- Capacitor Blocks Output Short From Input Source

<u>Disadvantages</u>

- All Power Passes Through The "Flying" Capacitor
- Very Difficult Controller Design

Resonant Converters

- More Complex, Typically Specialized Application
  - Very Common: CFL Ballast
- LLC Resonant Converter
  Is An Important Topology Today
  - Complex Operation
  - Complex To Design
  - Offers High Efficiency

Summary

In Part 1 We Covered

- Switch Mode Conversion Principle
  - Lossless Switches Plus Energy Storage
- Inductors And Capacitors As Energy Storage Devices
- Common Switching Converter Topologies
  - Buck Converter
  - Boost Converter
  - Buck-Boost Converter
  - Flyback Converter
  - SEPIC Converter

Questions?

Fundamentals of Switch-Mode Power Conversion Part 2: Control

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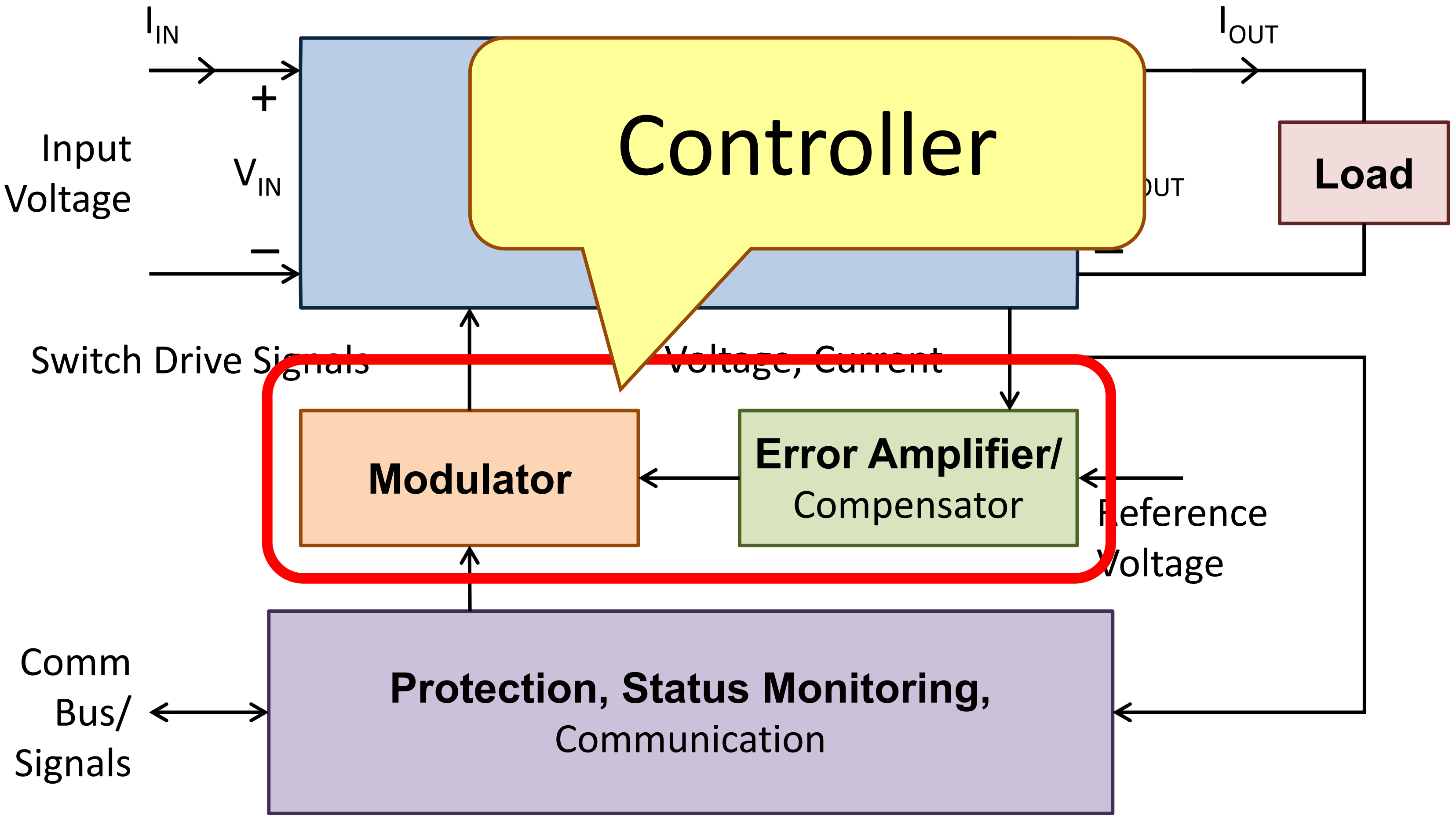
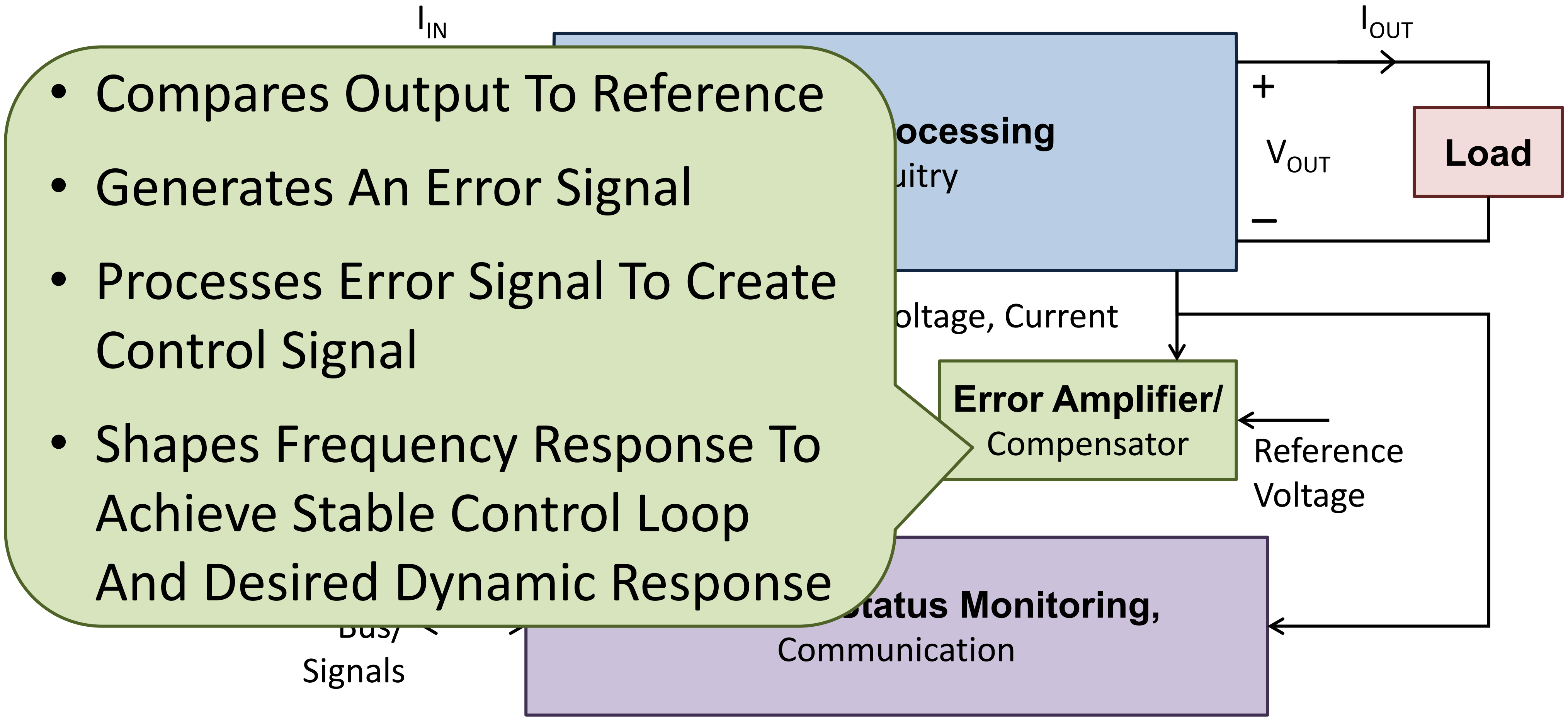
Switch Mode Converter

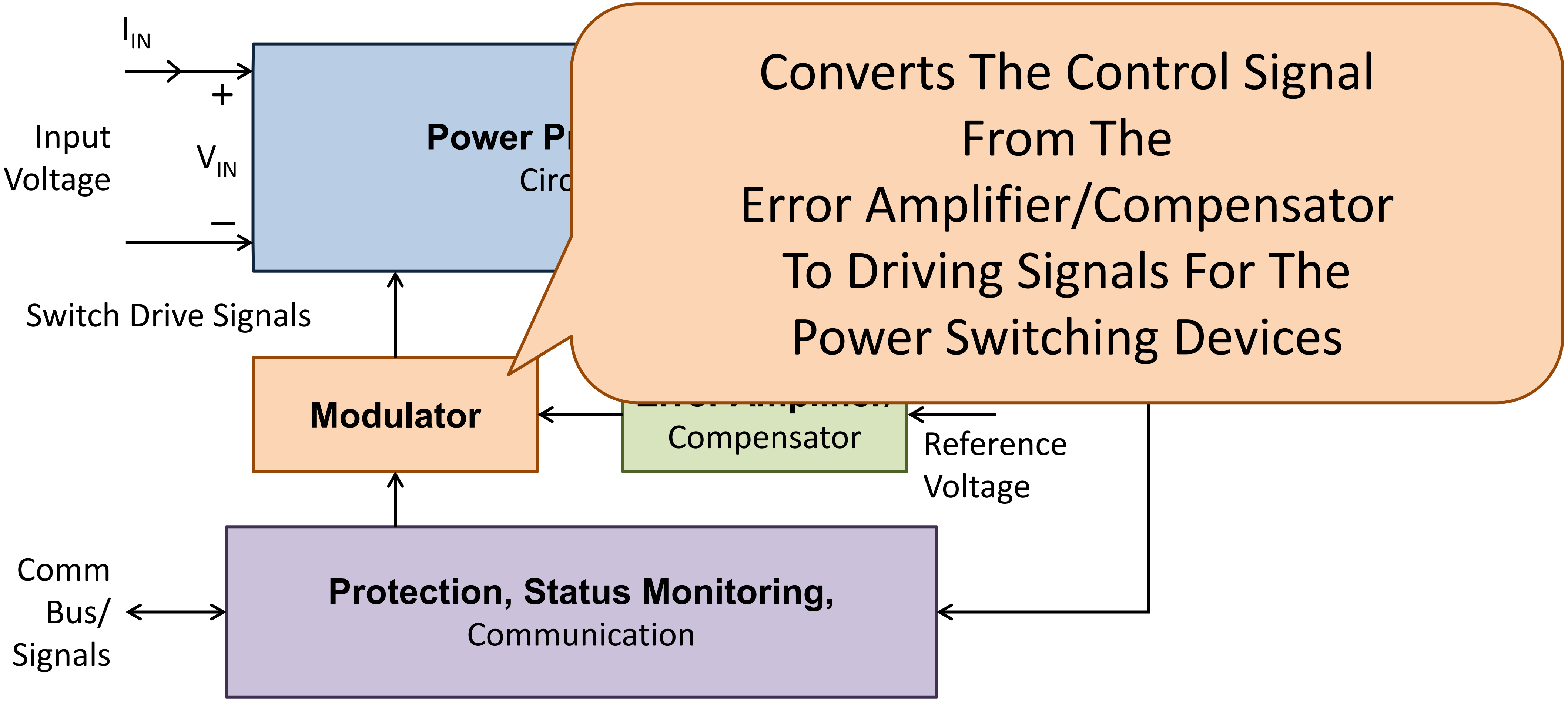


Switch Mode Converter

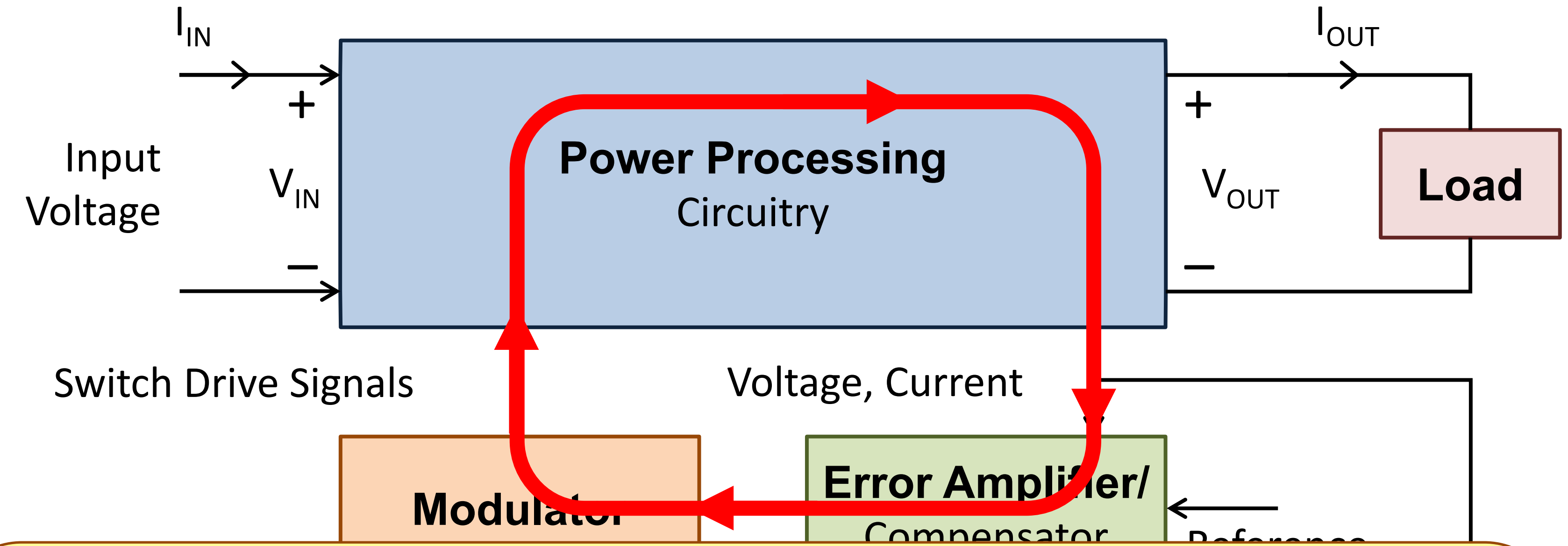
- Compares Output To Reference
- Generates An Error Signal
- Processes Error Signal To Create Control Signal
- Shapes Frequency Response To Achieve Stable Control Loop And Desired Dynamic Response



Switch Mode Converter



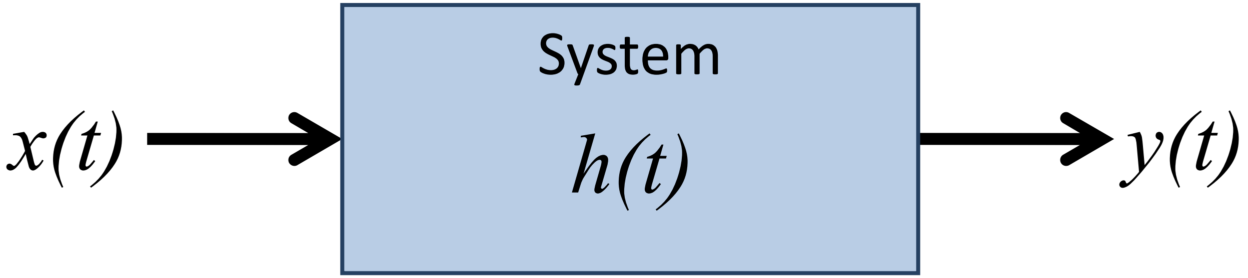
Control Loop



Important:
We Will Be VERY Concerned
With The Frequency Response
Of Signals Flowing Through This Loop

Transfer Functions And Bode Plots

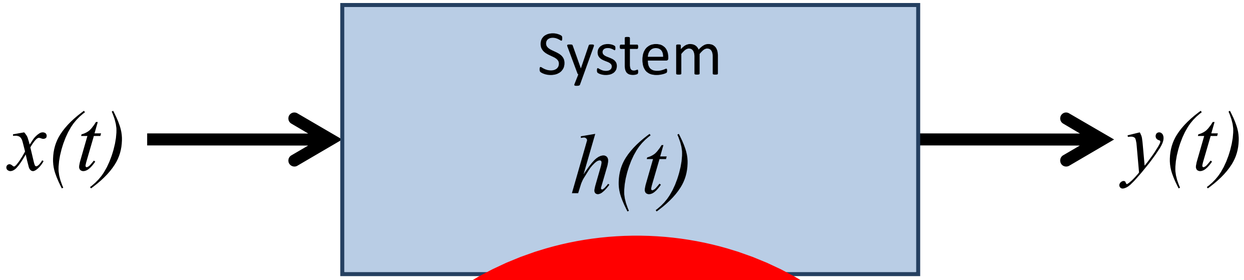
Transfer Function

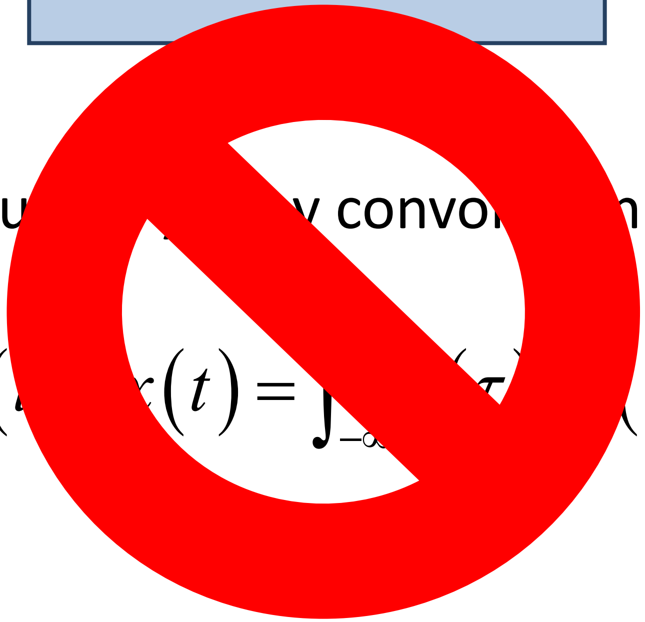


Find output y(t) by convolution

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau) \cdot x(t-\tau)\, d\tau$$

Transfer Function



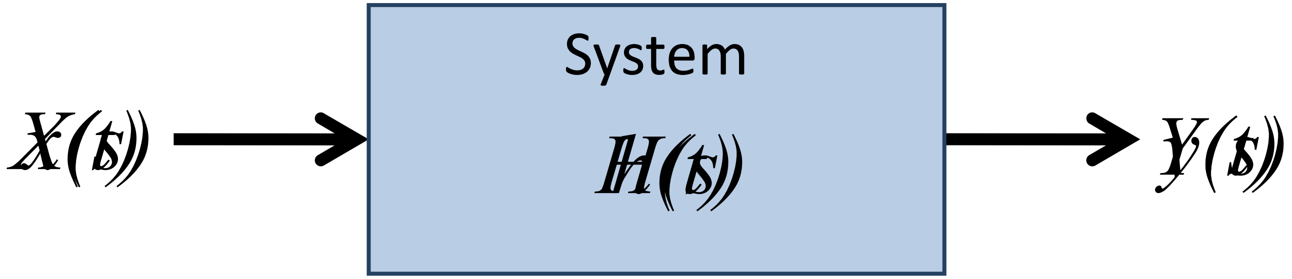


Find ou … y convo … n

$$y(t) = h(\ldots x(t) = \int_{-\infty} \ldots (\tau) \ldots (t-\tau)\,d\tau$$

Transfer Function

Use LaPlace transform to convert to frequency domain



$$Y(s) = H(s) \cdot X(s)$$

H(s)
Is The
"Transfer
Function"

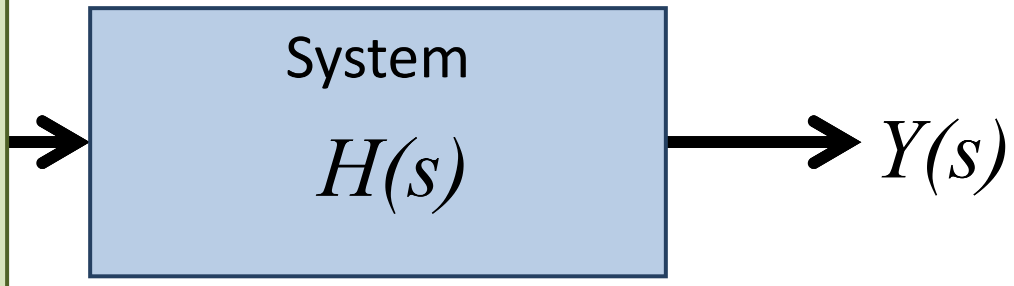
$$H(s) = \frac{Y(s)}{X(s)}$$

Describes The
Frequency Response
Of The System:
Gain
Phase

Transfer Function

If There Is Some Frequency s_Z Which Makes $Y(s_Z) = 0$ Then $H(s_Z) = 0$ And s_Z Is A "Zero" Of The Transfer Function

orm to convert to frequency domain

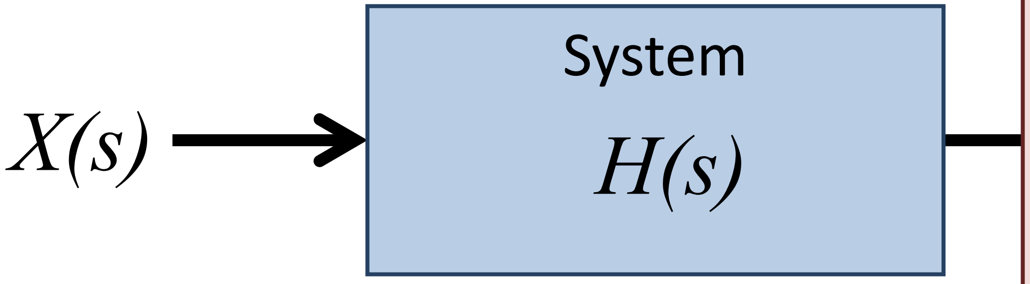


$$Y(s) = H(s) \cdot X(s)$$

$$H(s) = \frac{0}{X(s)}$$

Transfer Function

Use LaPlace transform to convert to frequency

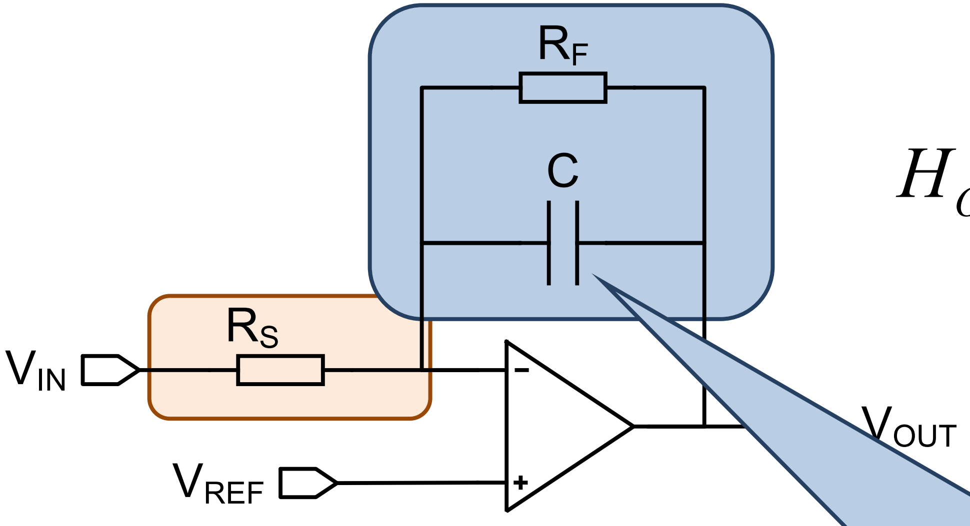


$$Y(s) = H(s) \cdot X(s)$$

$$H(s) = \frac{Y(s)}{0}$$

If There Is Some Frequency s_P Which Makes $X(s_P) = 0$ Then $H(s_P) \rightarrow \infty$ And s_P Is A "Pole" Of The Transfer Function

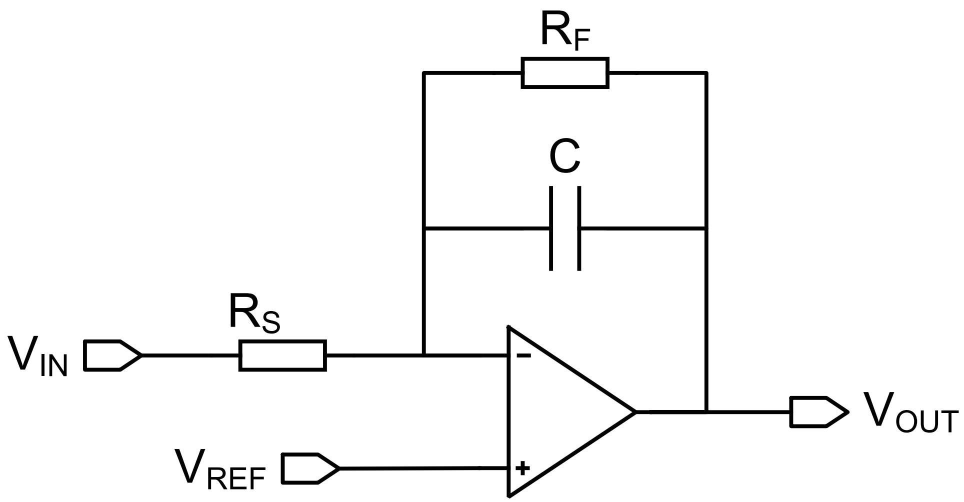
Simple Op-Amp Pole



$$H_{OP-AMP}(s) = -\frac{Z_{FEEDBACK}(s)}{Z_{SOURCE}(s)}$$

As Frequency ↑
Capacitor Impedance ↓
$Z_{FEEDBACK}\downarrow \Rightarrow$ Gain ↓

Simple Op-Amp Pole



$$H_P(s) = -\frac{R_F}{R_S} \cdot \frac{1}{1 + s \cdot R_F \cdot C}$$

$$H_P(s) = G_0 \cdot \frac{1}{1 + \frac{s}{\omega_P}}$$

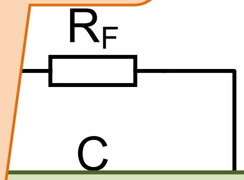
$$G_0 = -\frac{R_F}{R_S}$$

$$\omega_P = 2 \cdot \pi \cdot f_P = \frac{1}{R_F \cdot C}$$

$$f_P = \frac{1}{2 \cdot \pi \cdot R_F \cdot C}$$

Simple Op-Amp Pole

Negative Frequency Valid In s-Domain



$$H_P\left(-\frac{1}{R_F \cdot C}\right) = H_P(-\omega_P)$$

$$= G_0 \cdot \frac{1}{1+\frac{-\omega_P}{\omega_P}} = G_0 \cdot \frac{1}{1+(-1)}$$

$$= G_0 \cdot \left(\frac{1}{0}\right)$$

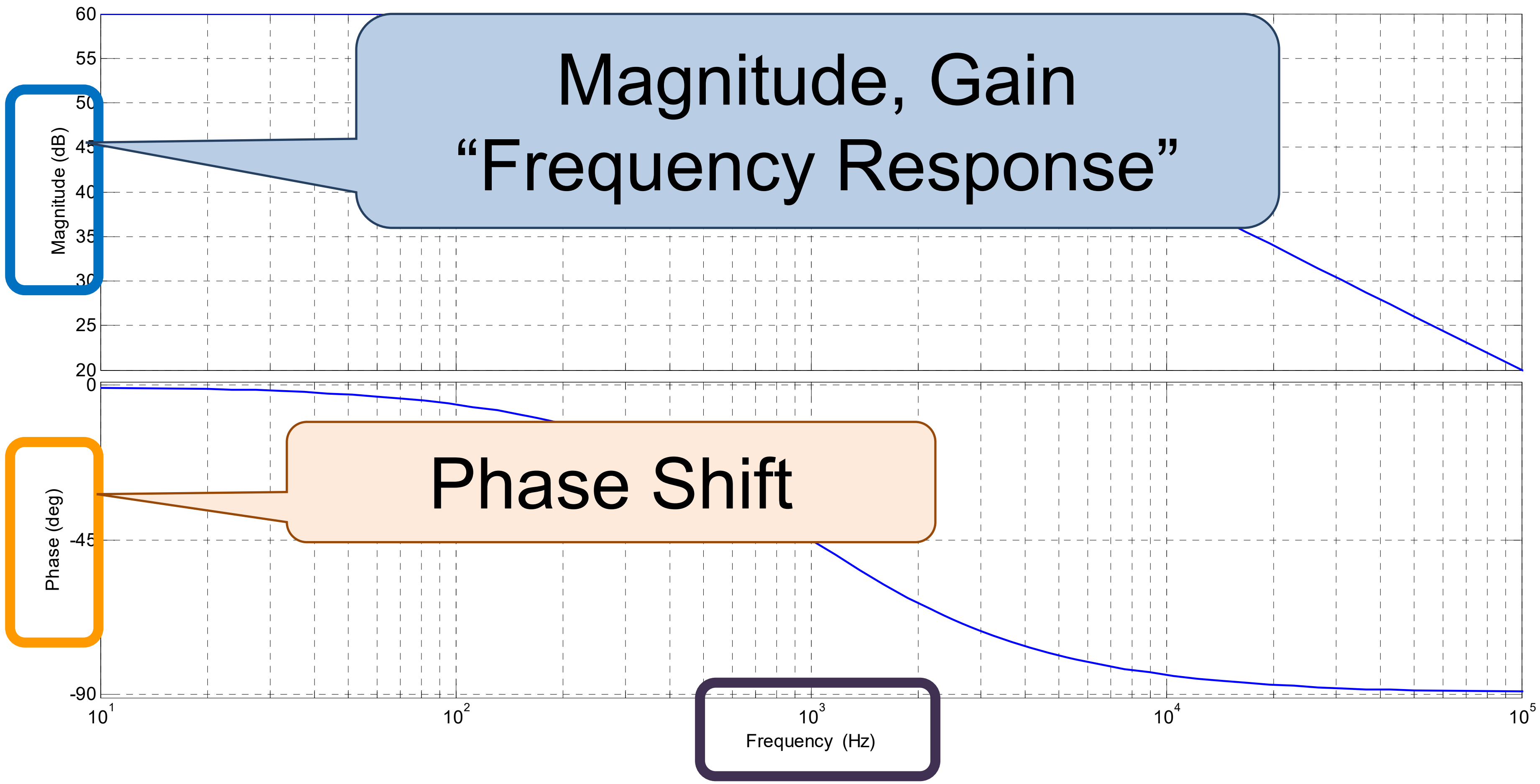
$$H_P(s) = -\frac{R_F}{R_S} \cdot \frac{1}{1 + s \cdot R_F \cdot C}$$

$$H_P(s) = G_0 \cdot \frac{1}{1+\frac{s}{\omega_P}}$$

$$G_0 = -\frac{R_F}{R_S}$$

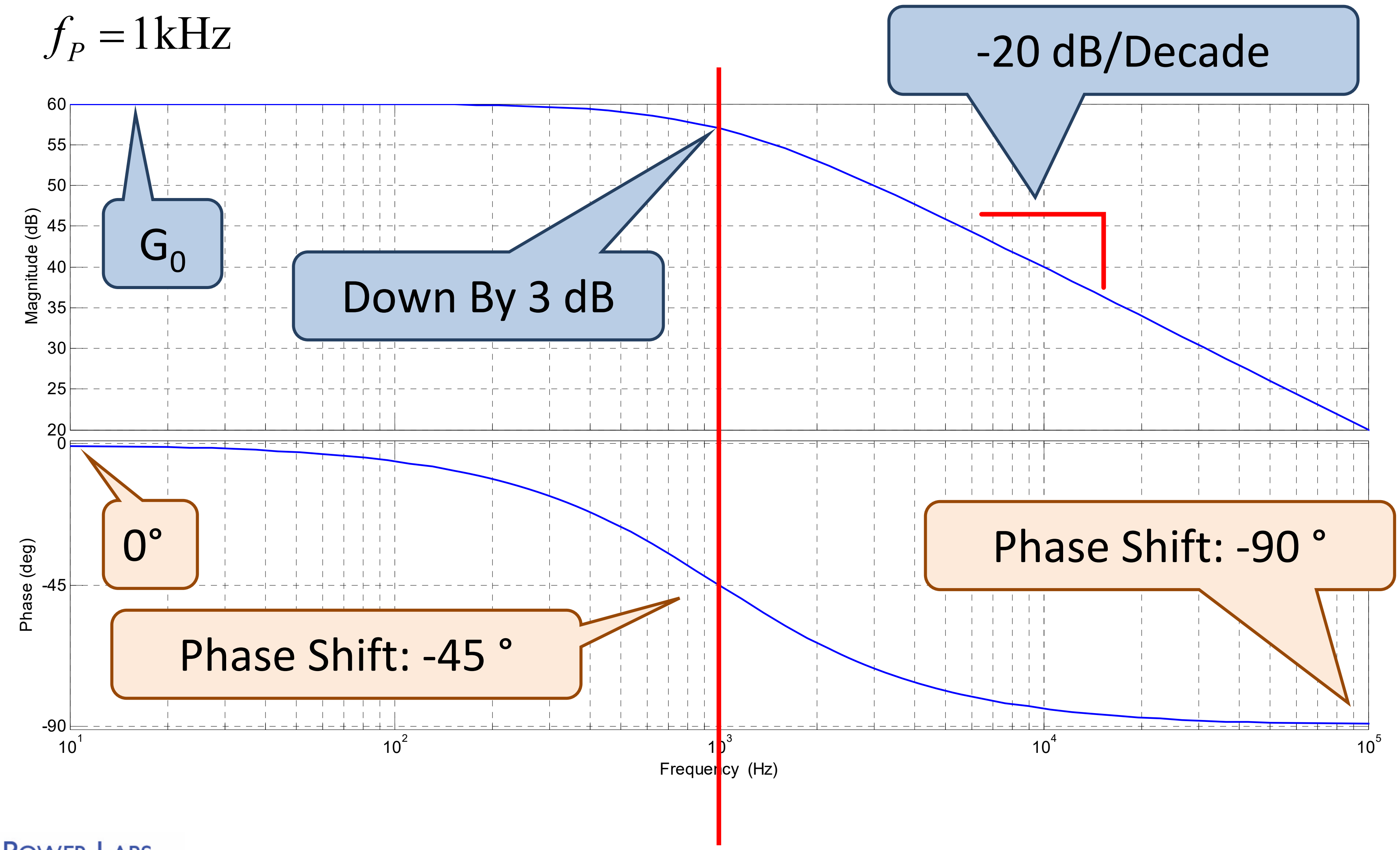
Denominator Is Zero So $-\omega_P$ Is A Pole Of This Transfer Function

Simple Pole Bode Plot



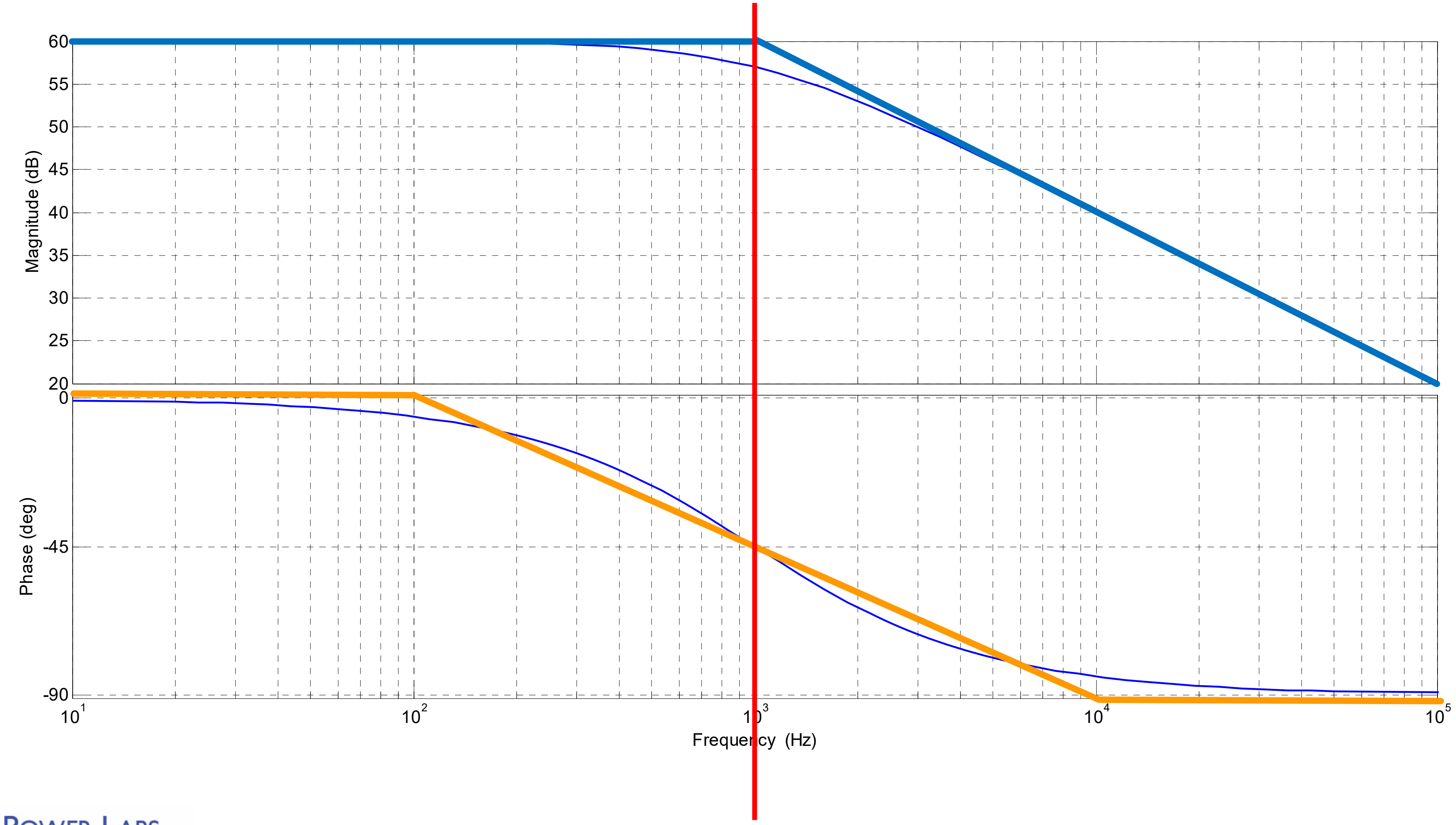
Simple Pole Bode Plot

$$f_P = 1\text{kHz}$$

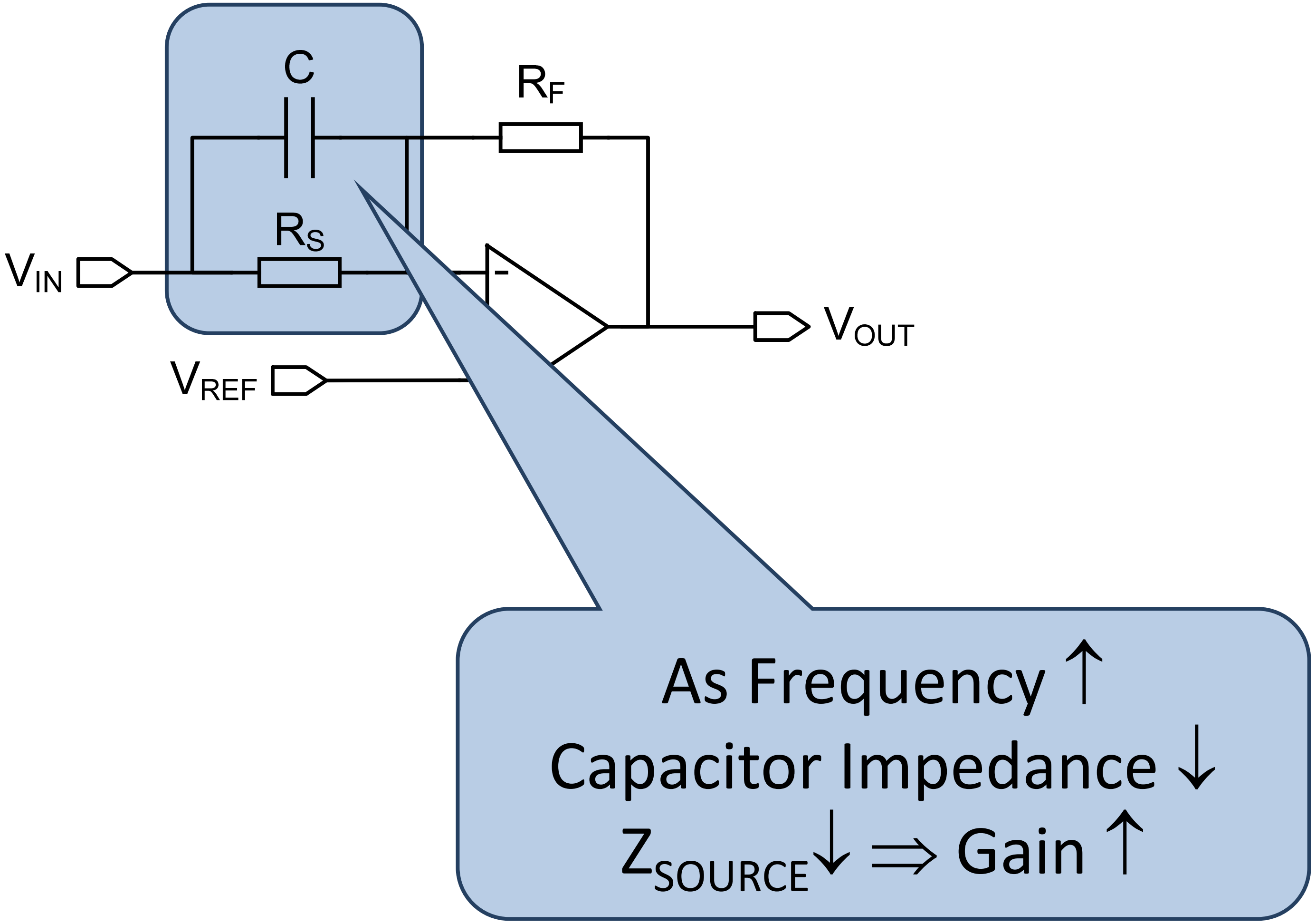


Simple Pole Bode Plot

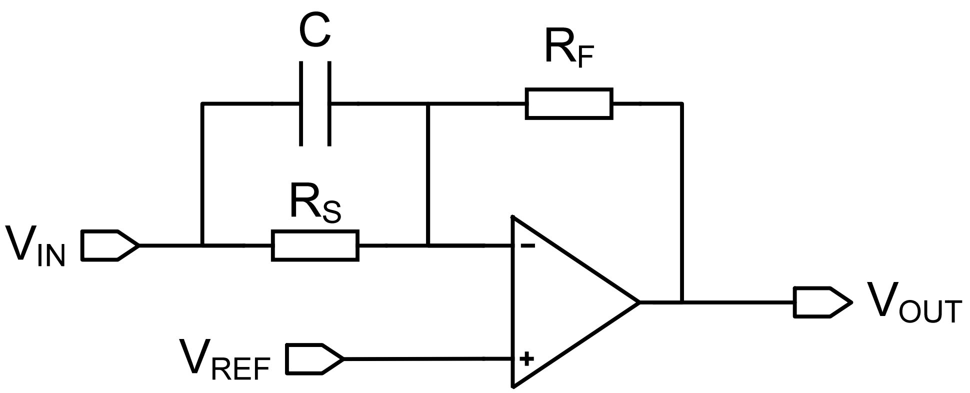
$f_P = 1\text{kHz}$



Simple Zero


C
RF
RS
VIN
VREF
VOUT
As Frequency ↑
Capacitor Impedance ↓
ZSOURCE↓ ⇒ Gain ↑

Simple Zero



$$H_Z(s) = -\frac{R_F}{R_S} \cdot (1 + s \cdot R_S \cdot C)$$

$$H_Z(s) = G_0 \cdot \left(1 + \frac{s}{\omega_Z}\right)$$

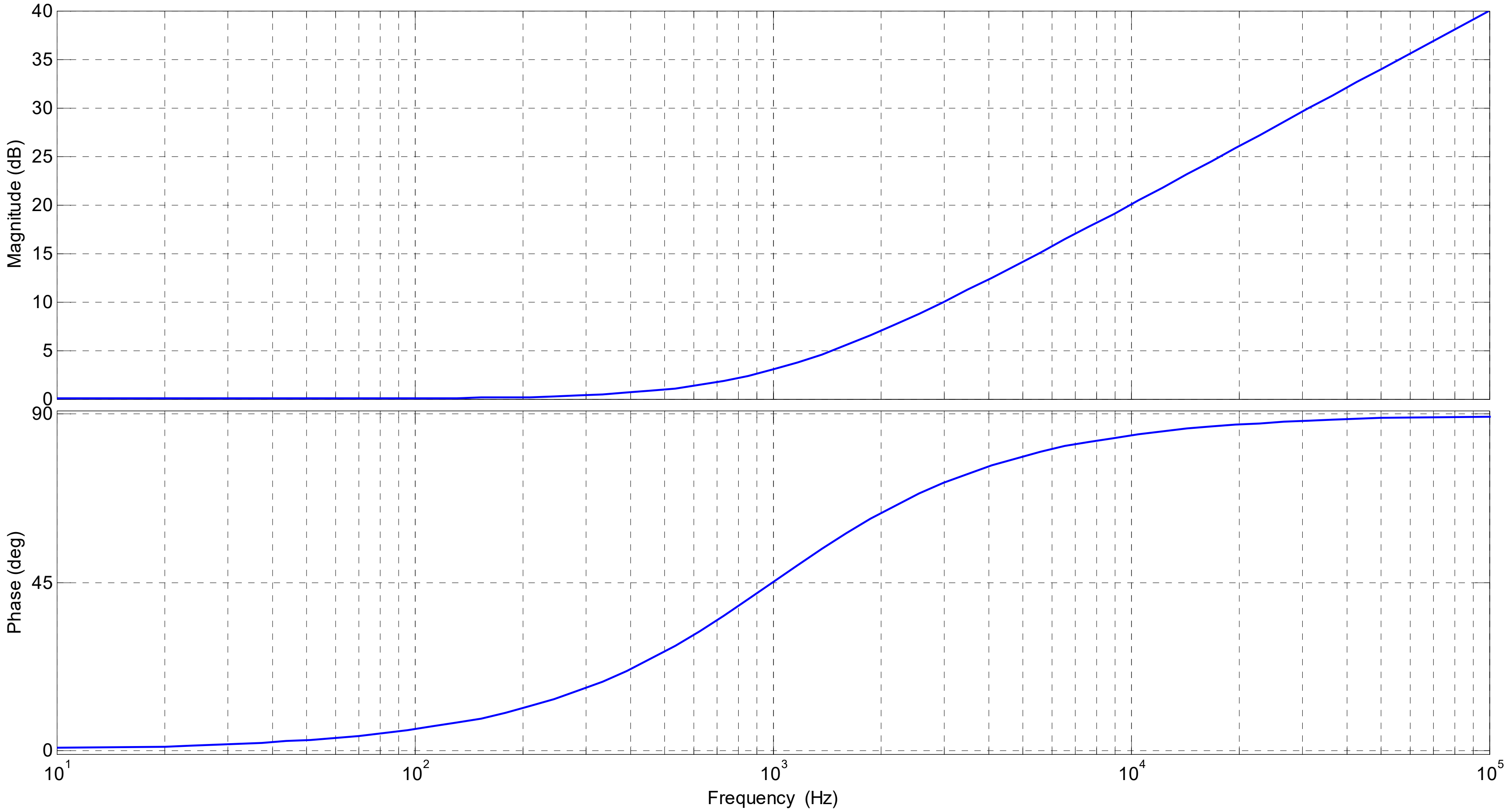
$$G_0 = -\frac{R_F}{R_S}$$

$$\omega_Z = 2 \cdot \pi \cdot f_Z = \frac{1}{R_S \cdot C}$$

$$f_z = \frac{1}{2 \cdot \pi \cdot R_S \cdot C}$$

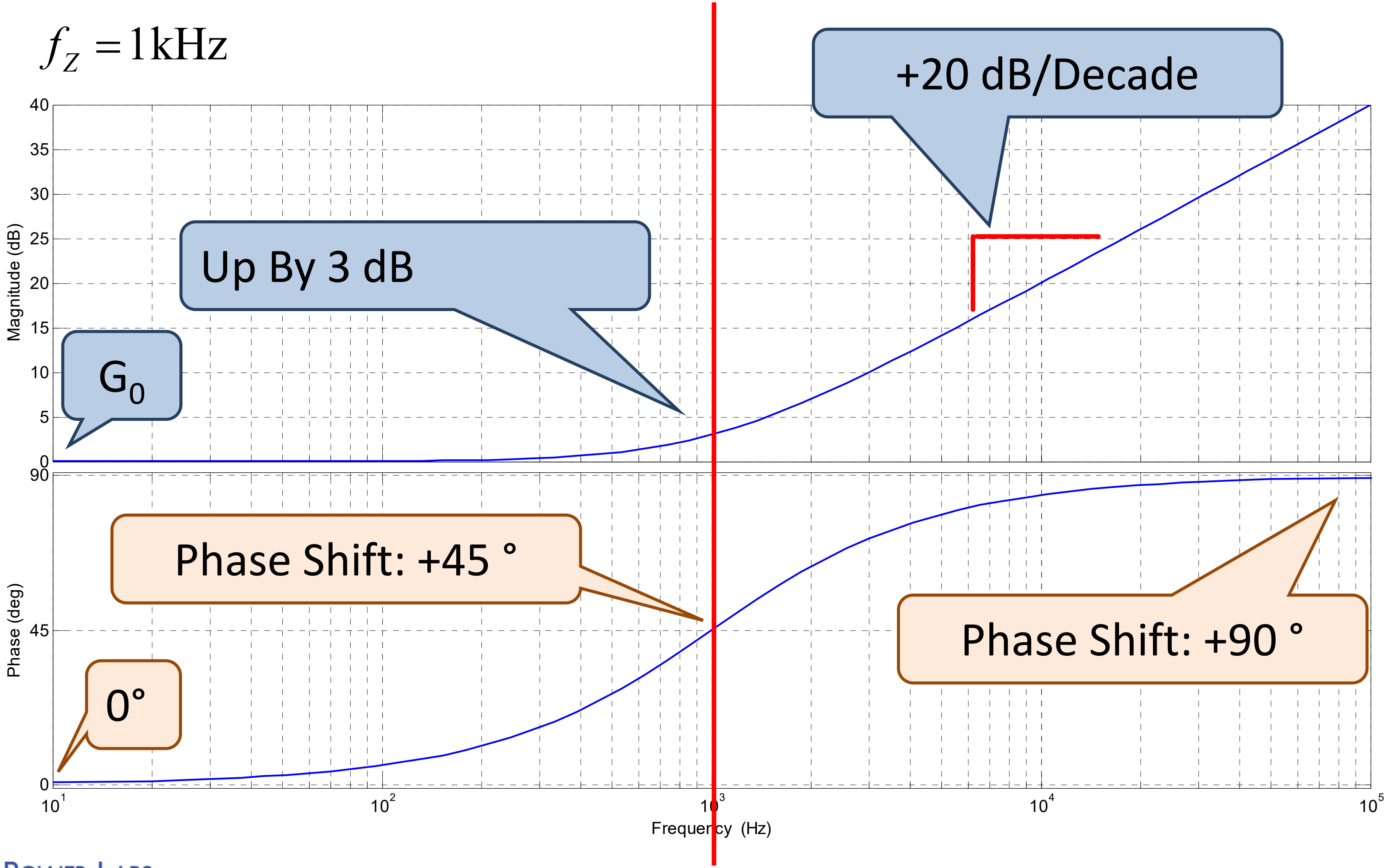
Simple Zero Bode Plot

$$f_Z = 1\text{kHz}$$



Simple Zero Bode Plot

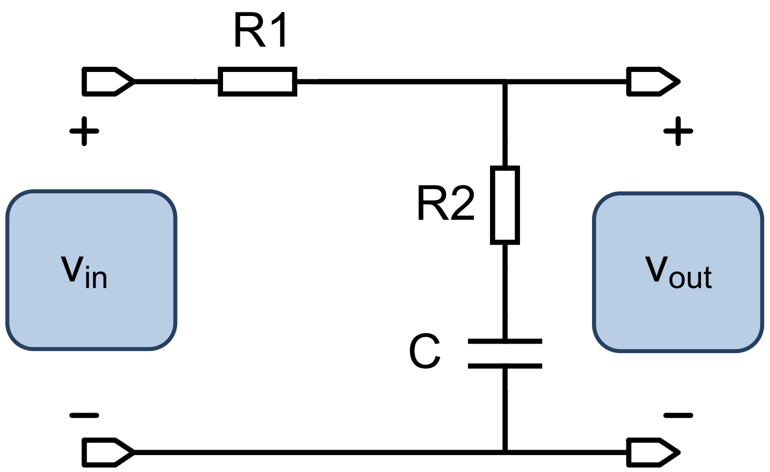
$f_Z = 1\text{kHz}$



EMBEDDED POWER LABS

Example: Simple Transfer Function

$$H(s)=\frac{v_{out}(s)}{v_{in}(s)}$$



$$v_{OUT}(s)=\frac{\left(R2+\frac{1}{s\cdot C}\right)}{R1+\left(R2+\frac{1}{s\cdot C}\right)}\cdot v_{IN}(s)$$

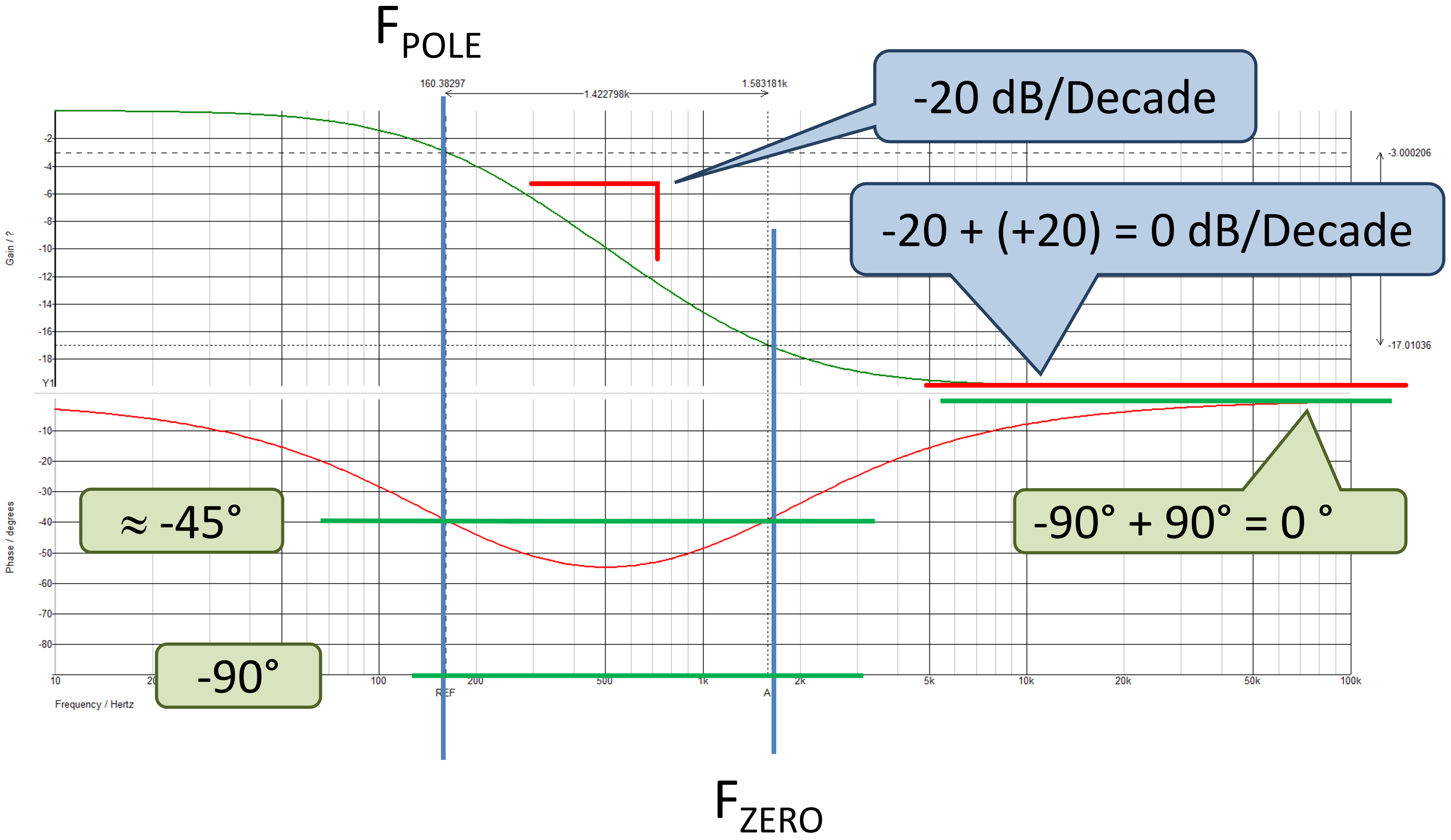
$$H(s)=\frac{v_{out}(s)}{v_{in}(s)}=\frac{1+s\cdot R2\cdot C}{1+s\cdot(R1+R2)\cdot C}$$

$$H(s)=\frac{1+\frac{s}{\omega_Z}}{1+\frac{s}{\omega_P}}$$

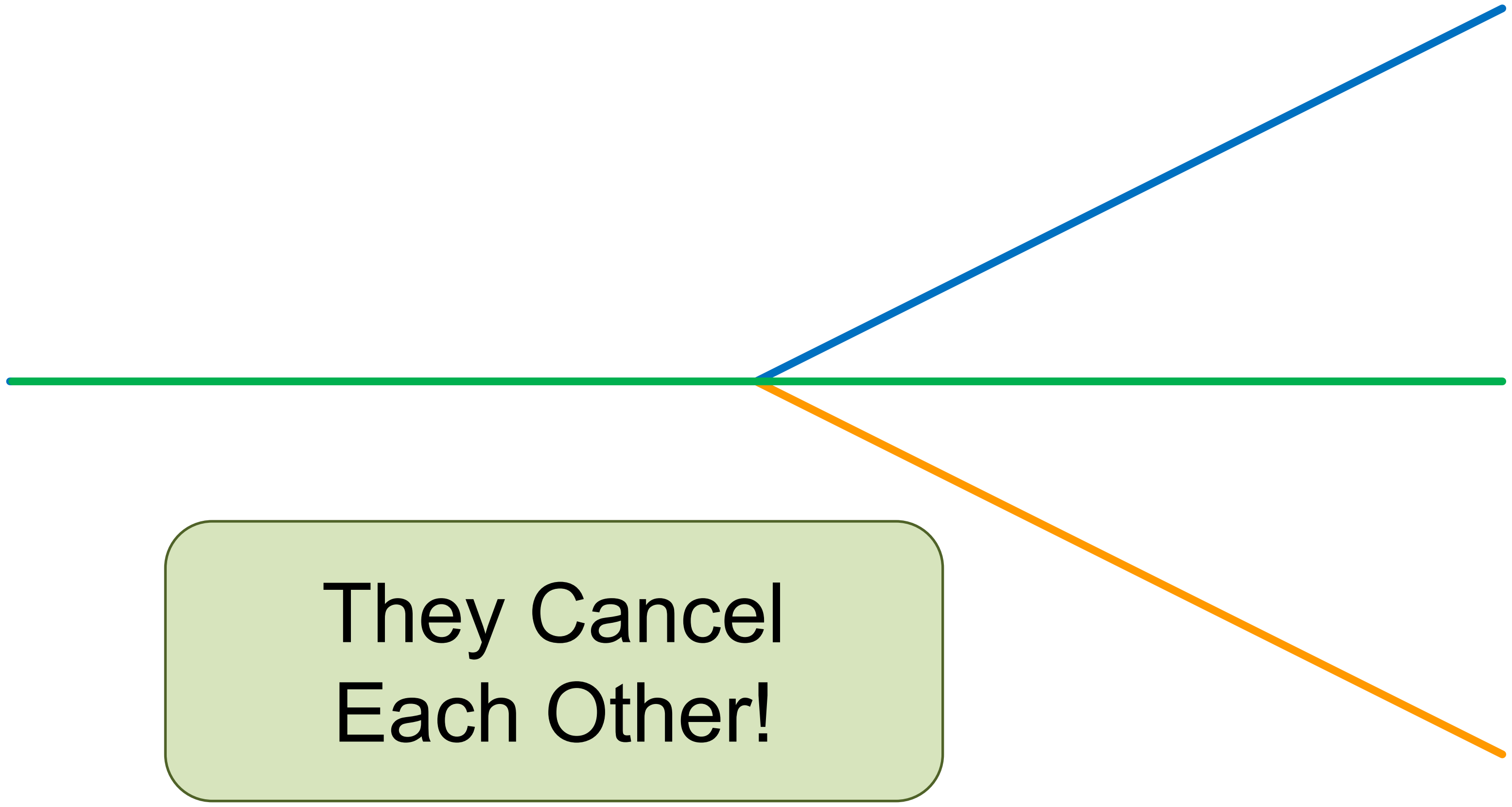
$$\omega_Z=\frac{1}{R2\cdot C}$$

$$\omega_P=\frac{1}{(R1+R2)\cdot C}$$

Simple Transfer Function Bode Plot



What If System Has
Pole And Zero At Same Frequency?



Basic Feedback Loop Review

Basic Feedback Review

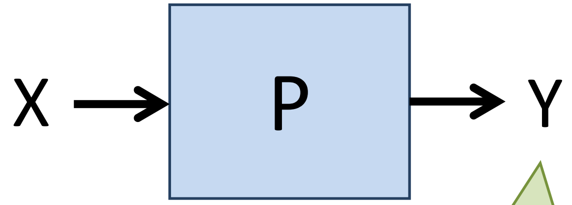
Note

Next 16 Slides Not In Your USB PDF
Or Printed Workbook
They Are In The Version You Can
Download From
Embedded Power Labs Website

Basic Feedback Review

Plant P

In Response To Input X

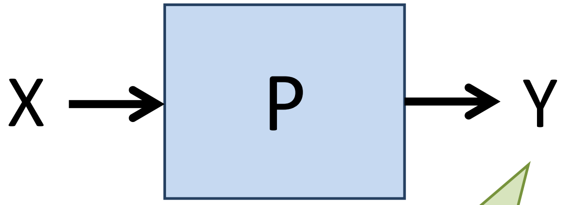


Output Can Be Anything: Voltage, Current, Temperature, Flow Rate, Speed, RPM, Position…

Produces Output Y

Basic Feedback Review

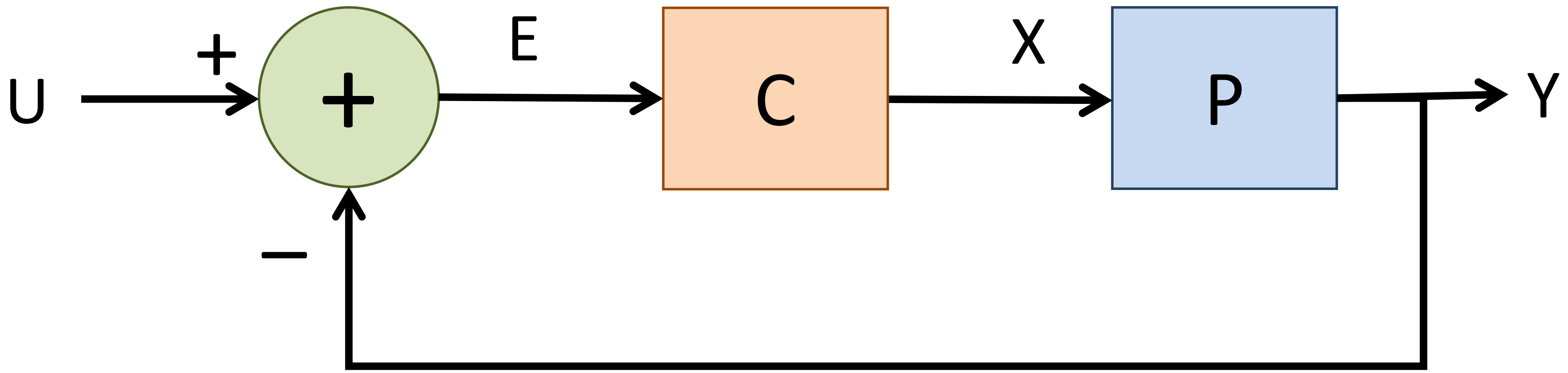
So How Do We Set Input X To Get The Desired Output Y?



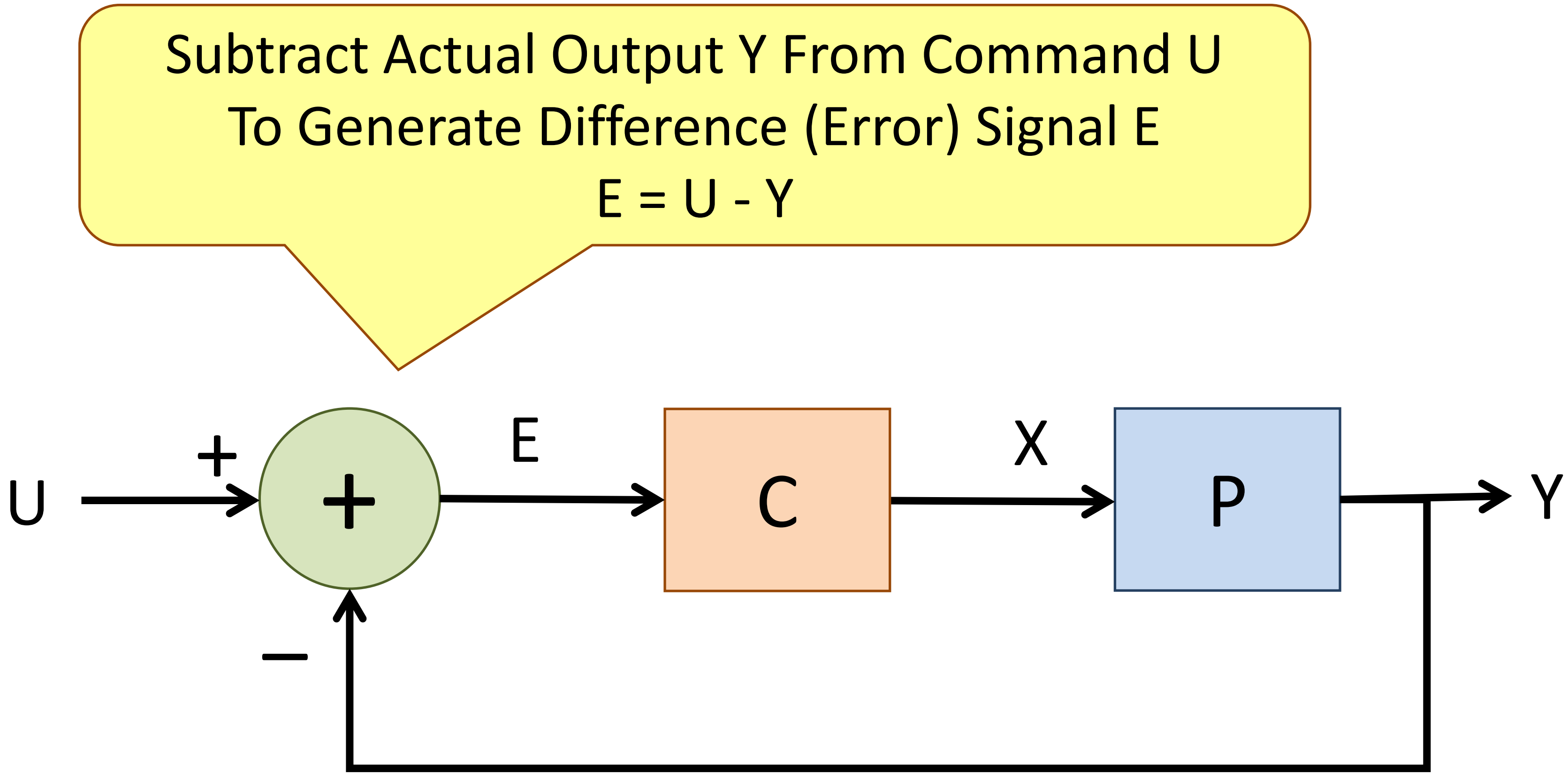
What We Want Is To Have Y Be Some Desired Value

Basic Feedback Review

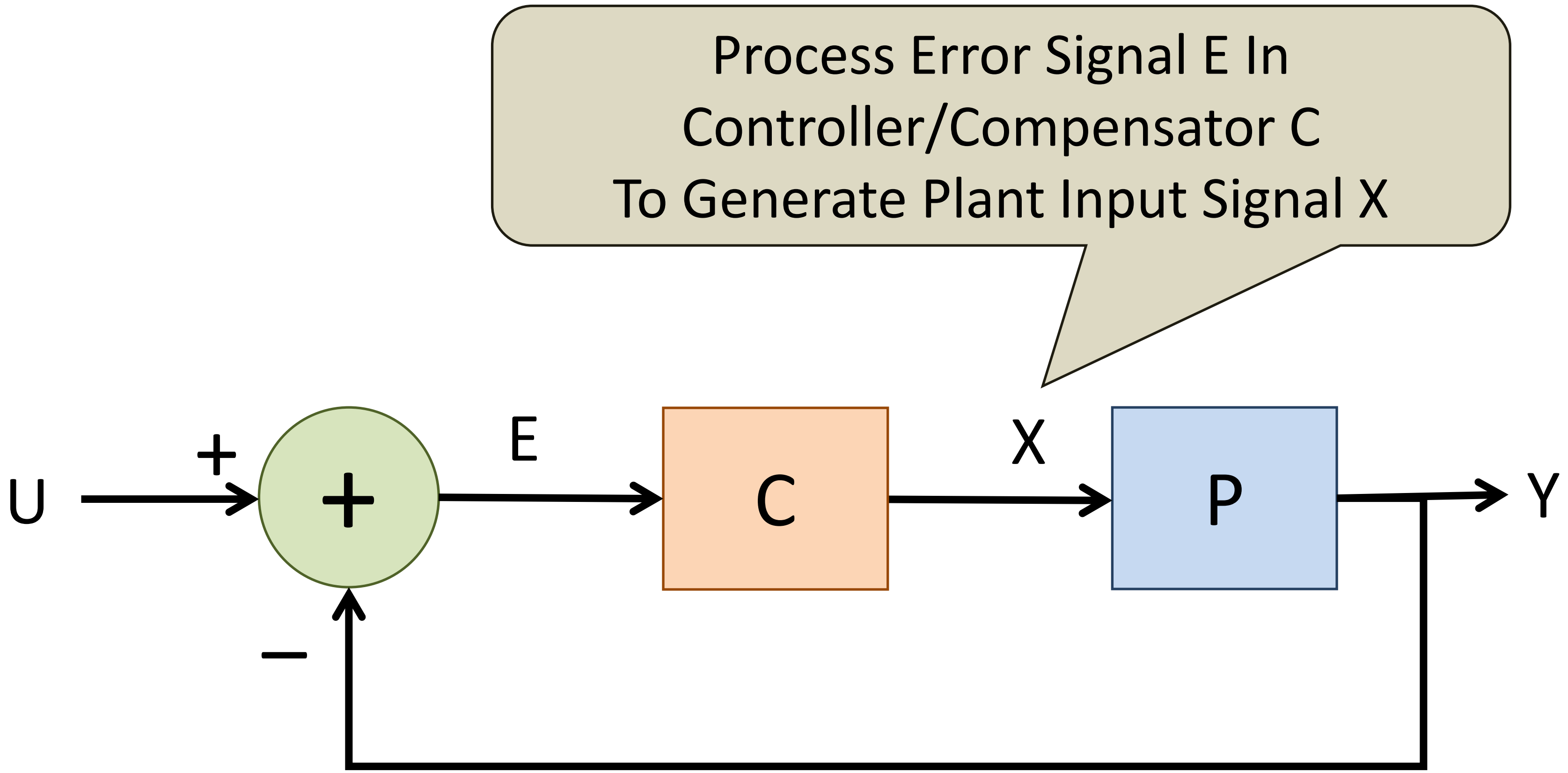
Use Feedback To Get Desired Output Y
In Response To Command U



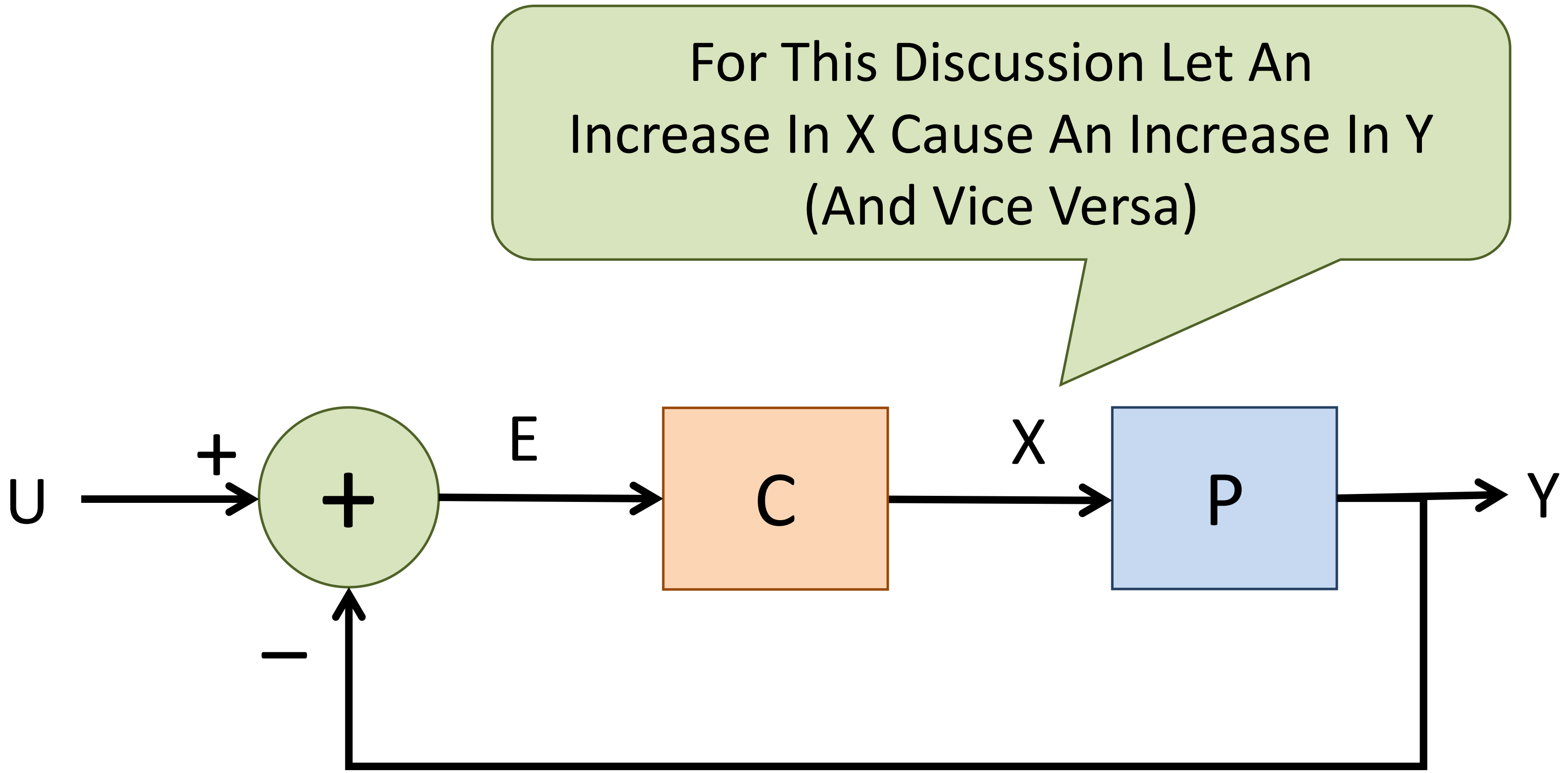
Basic Feedback Review



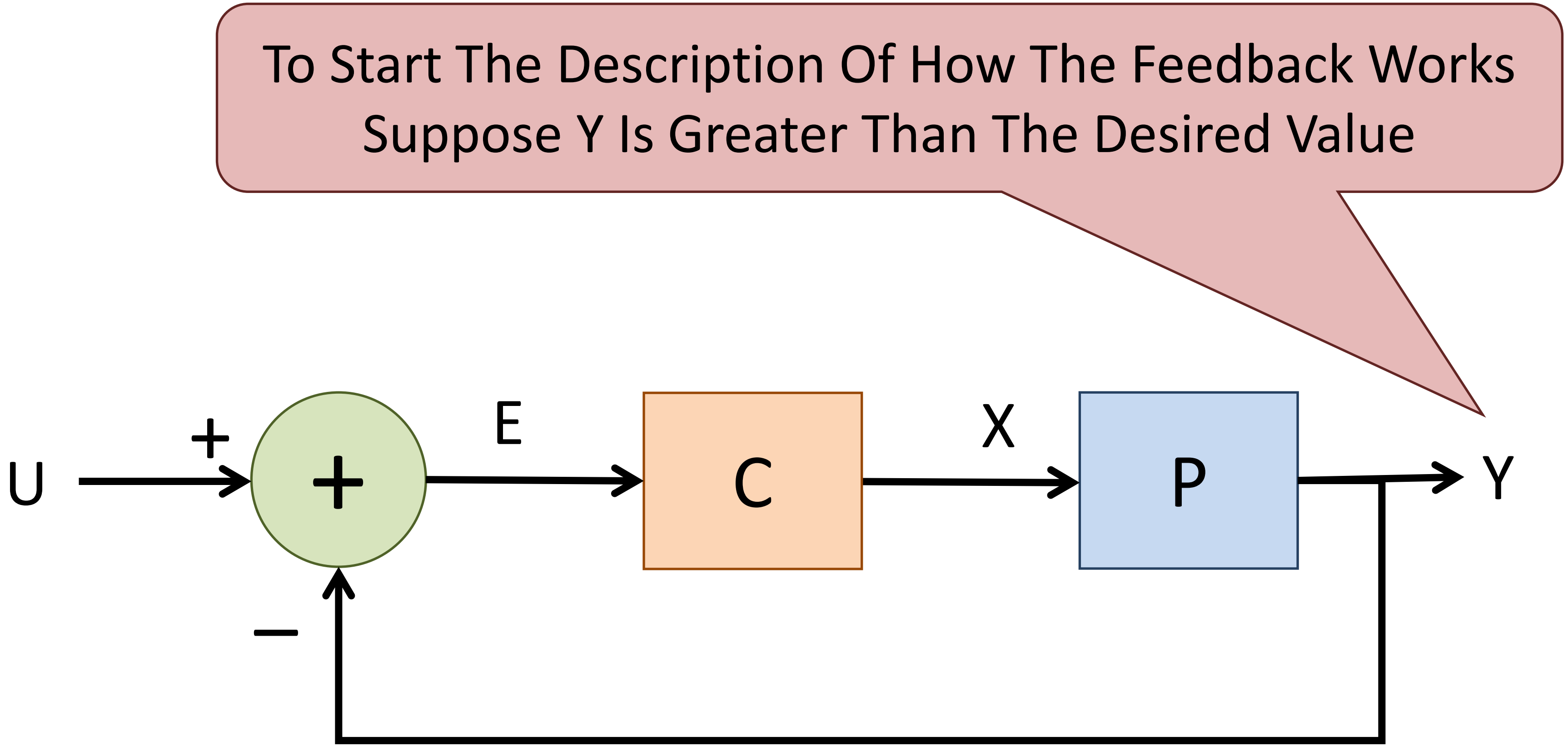
Basic Feedback Review



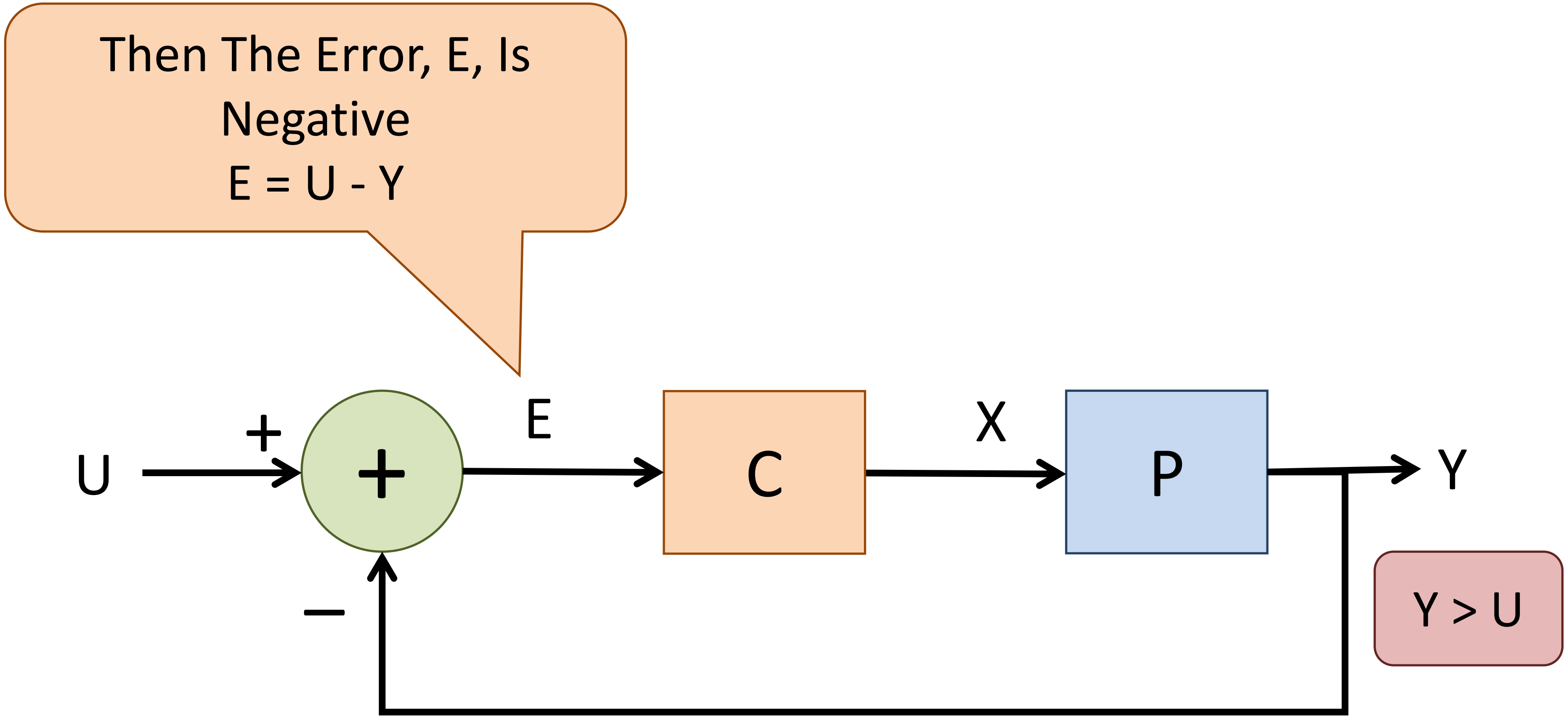
Basic Feedback Review



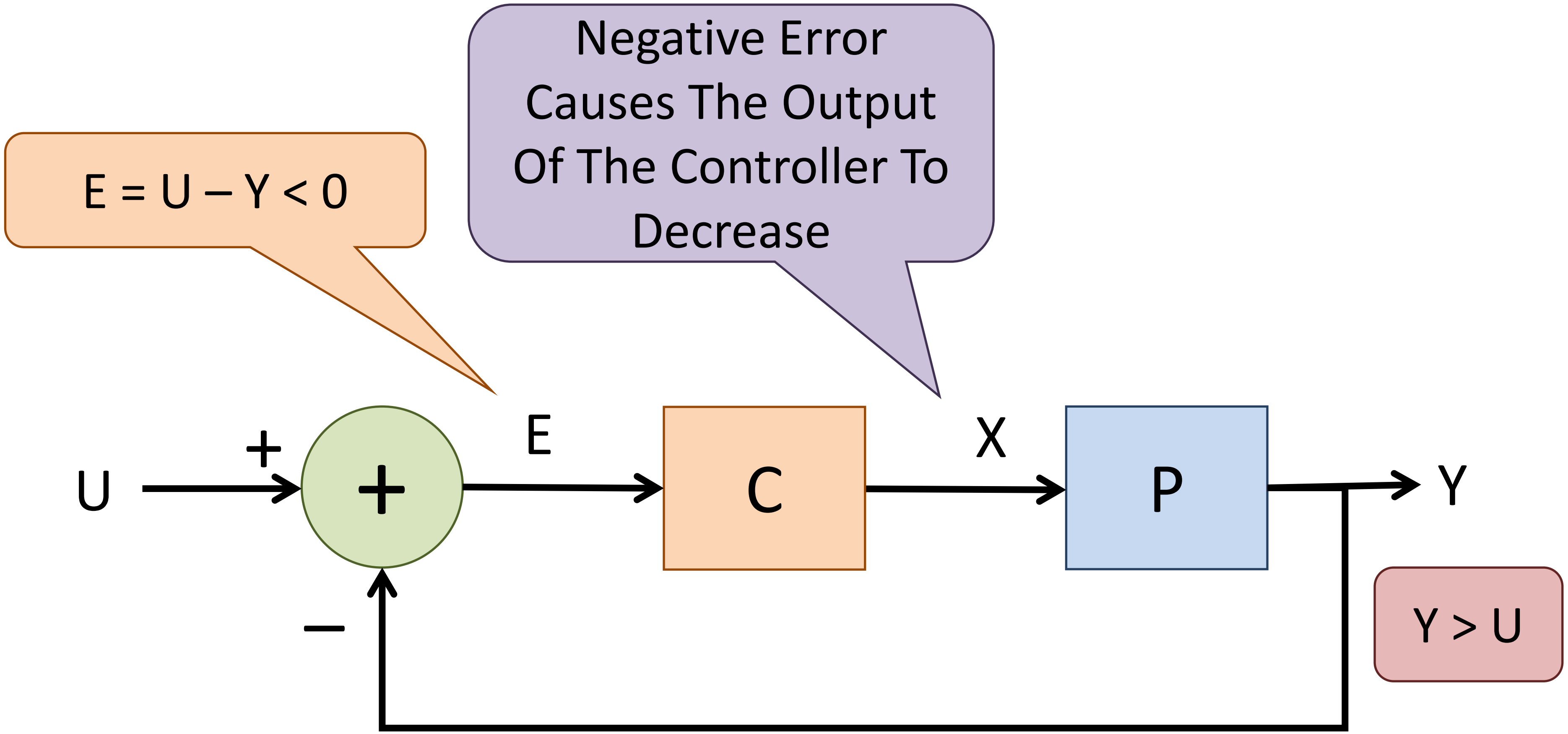
Basic Feedback Review



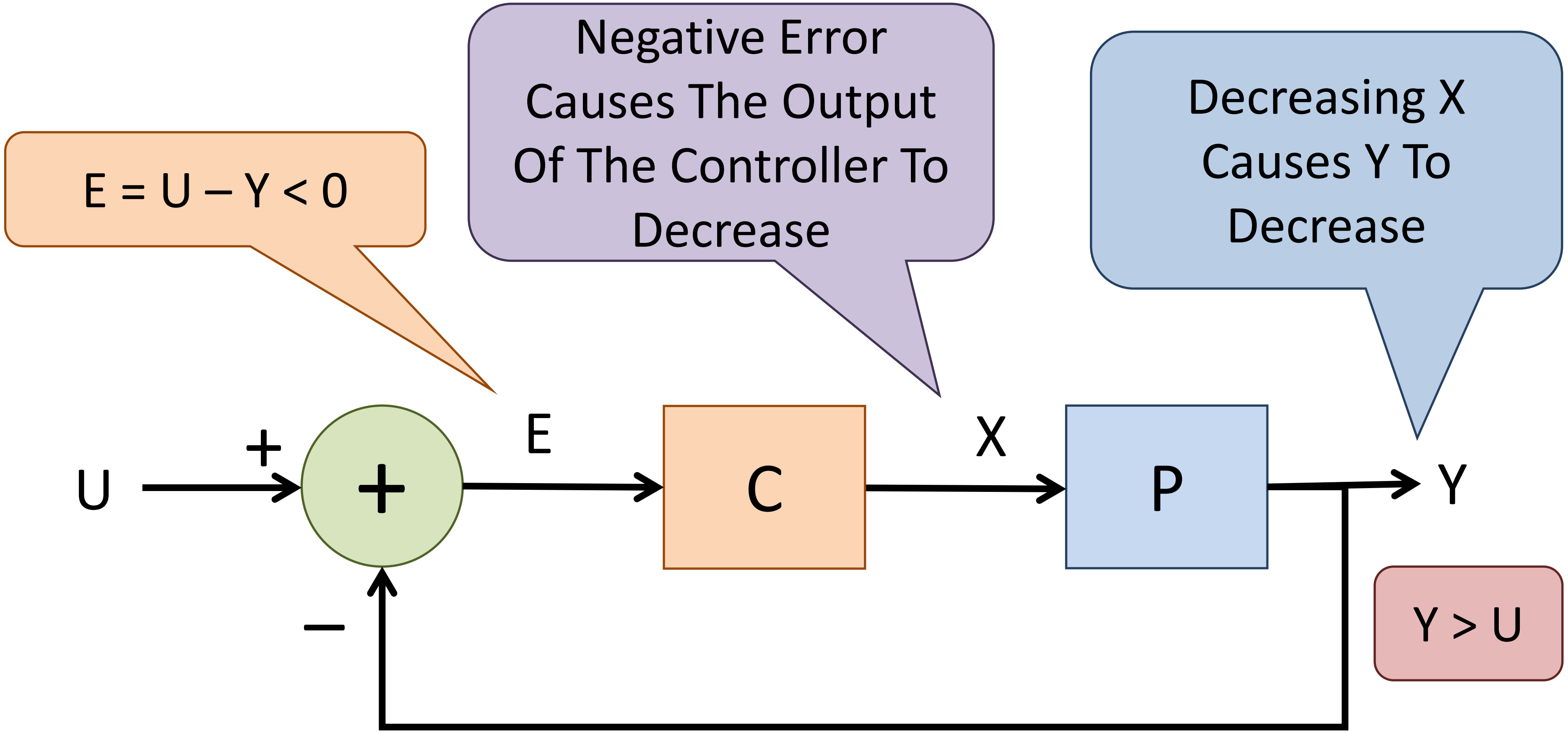
Basic Feedback Review



Basic Feedback Review

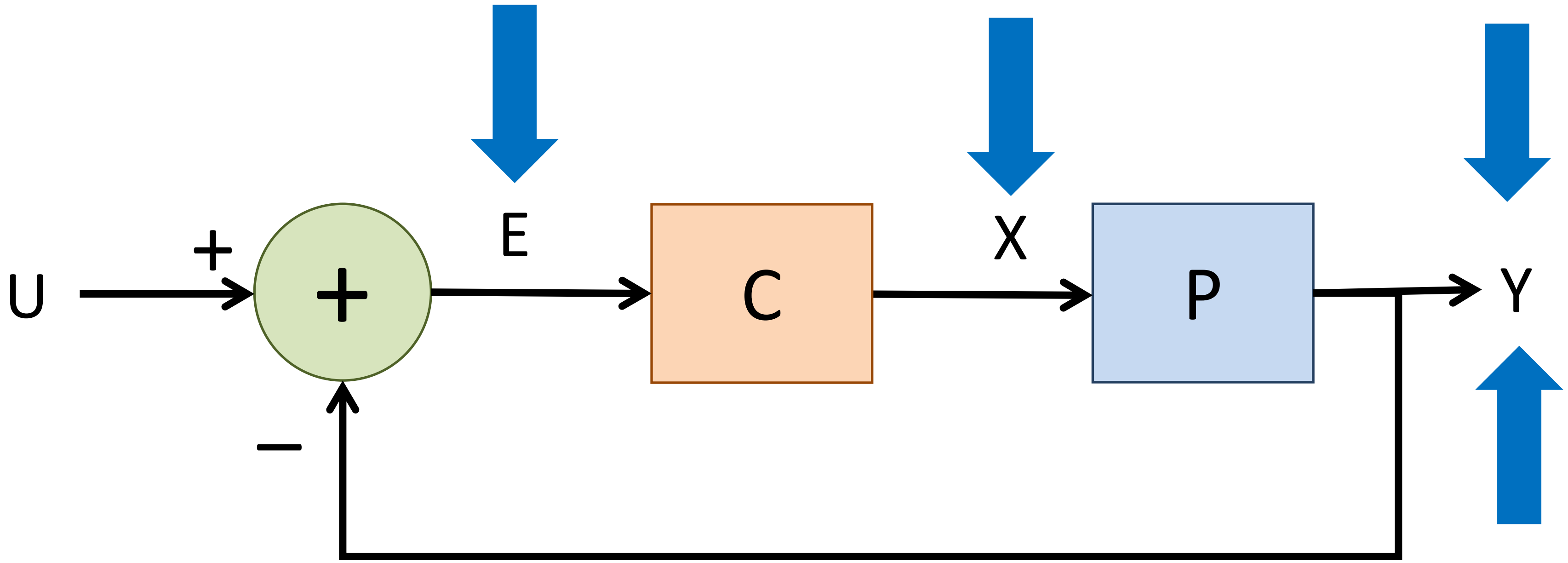


Basic Feedback Review



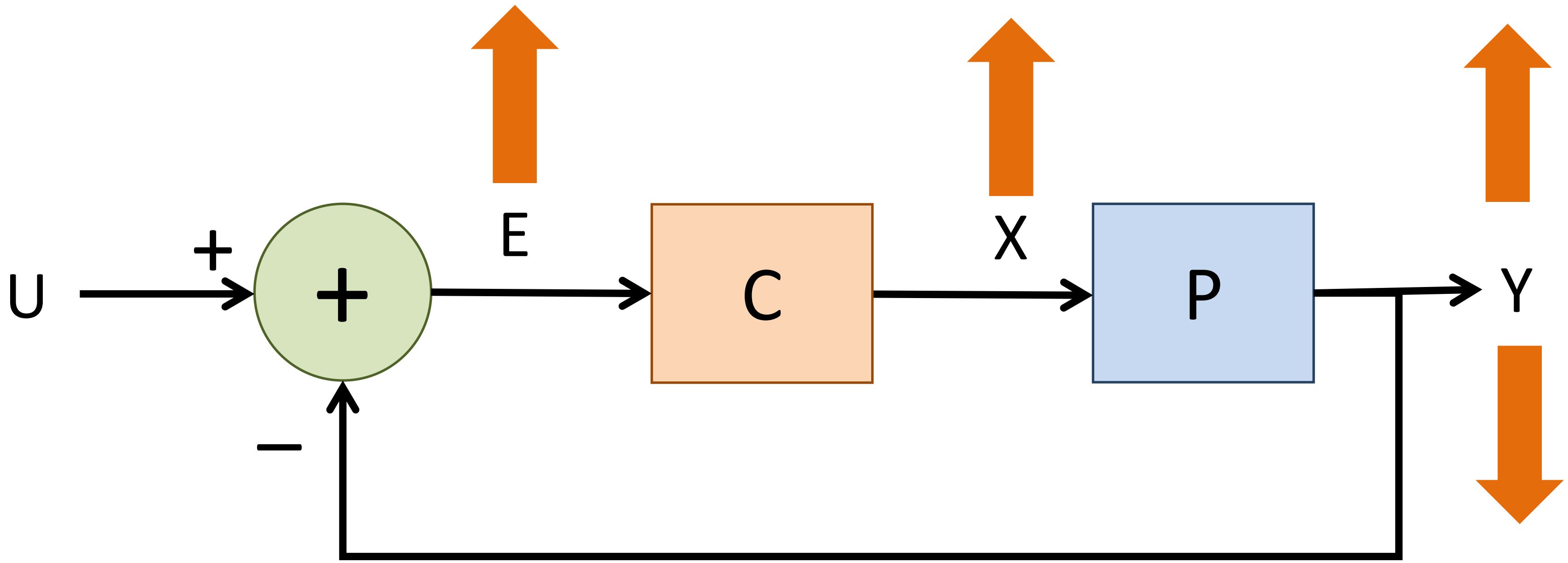
Basic Feedback Review

Step By Step Starting With Output Too High



Basic Feedback Review

Step By Step Starting With Output Too Low



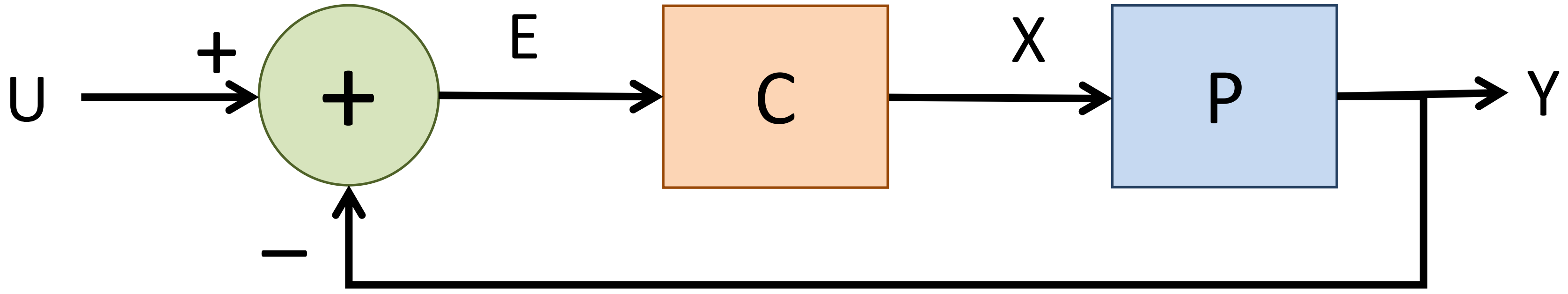
Embedded Power Labs

Basic Feedback: The Math

$$Y = P \cdot X$$
$$X = C \cdot E$$
$$E = U - Y$$

$$Y = P \cdot C \cdot E$$
$$= P \cdot C \cdot (U - Y)$$
$$= P \cdot C \cdot U - P \cdot C \cdot Y$$
$$Y + P \cdot C \cdot Y = P \cdot C \cdot U$$

$$Y = \frac{P \cdot C}{1 + P \cdot C} \cdot U$$



Basic Feedback: The Math

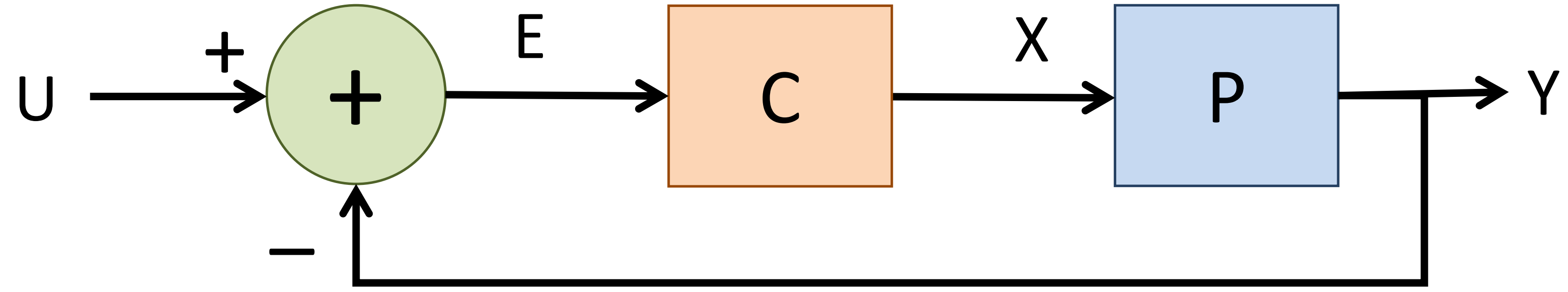
$$Y = P \cdot X$$

$$Y = P \cdot C \cdot E$$

Small Signal

$$Y(s) = \frac{P(s) \cdot C(s)}{1 + P(s) \cdot C(s)} \cdot U(s)$$

$$H_{CLOSED\_LOOP}(s) = \frac{Y(s)}{U(s)} = \frac{P(s) \cdot C(s)}{1 + P(s) \cdot C(s)}$$



Basic Feedback: The Math

$Y = P \cdot X$

$Y = P \cdot C \cdot E$

$(U - Y)$

$U - P \cdot C \cdot Y$

U

$\cdot U$

$\cdot C$

Small Signal

$$Y(s) = \frac{P(s) \cdot C(s)}{1 + P(s) \cdot C(s)} \cdot U(s)$$

$$H_{CLOSED\_LOOP}(s) = \frac{Y(s)}{U(s)} = \frac{P(s) \cdot C(s)}{1 + P(s) \cdot C(s)}$$

This Is The "Closed Loop Gain"

Basic Feedback: Open Loop Gain

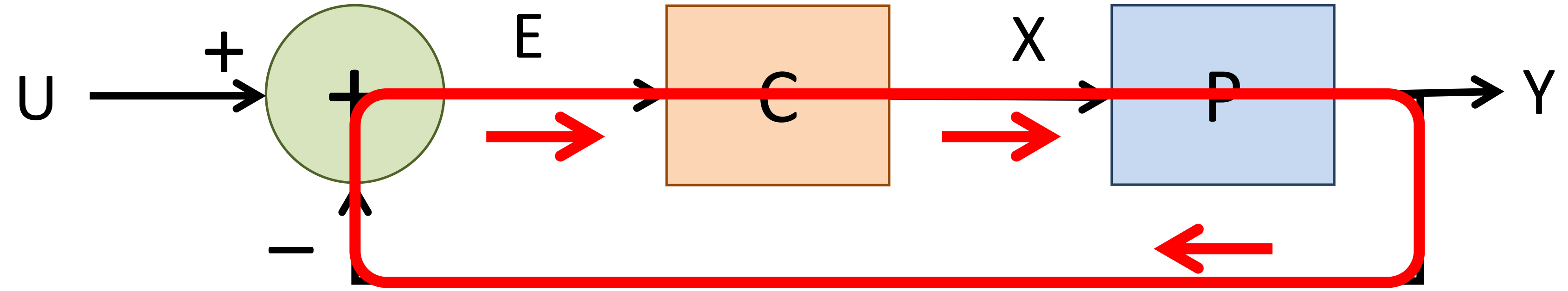
$$H_{CLOSED\_LOOP}(s) = \frac{Y(s)}{U(s)} = \frac{P(s)\cdot C(s)}{1+P(s)\cdot C(s)}$$

If Denominator Is Zero
Then The Loop Is Unstable
P(s) · C(s) = -1

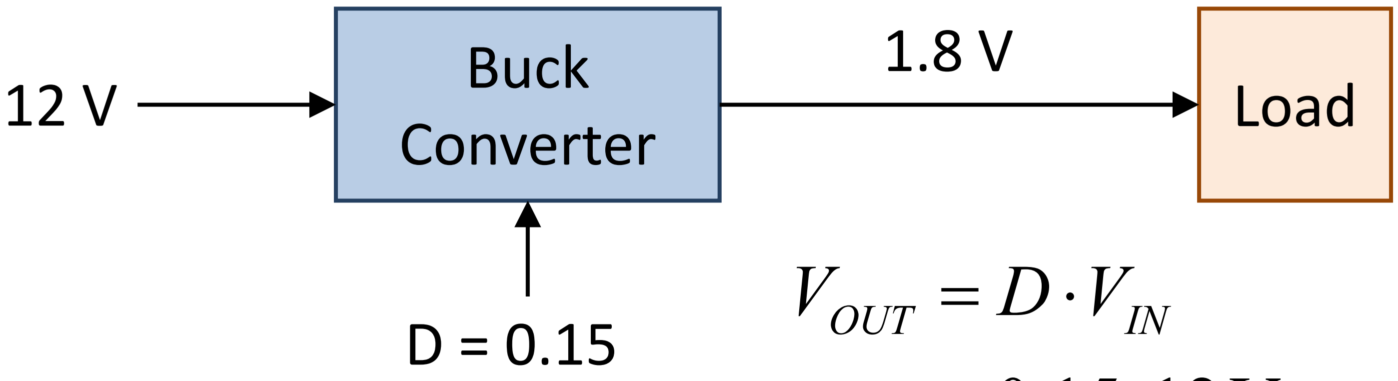
$$H_{OPEN\_LOOP}(s) = T(s) = C(s)\cdot P(s)$$

$$H_{CLOSED\_LOOP}(s) = \frac{Y(s)}{U(s)} = \frac{T(s)}{1+T(s)}$$

When Designing The Feedback
Loop We Will Focus On The
Open Loop Gain



Buck Converter

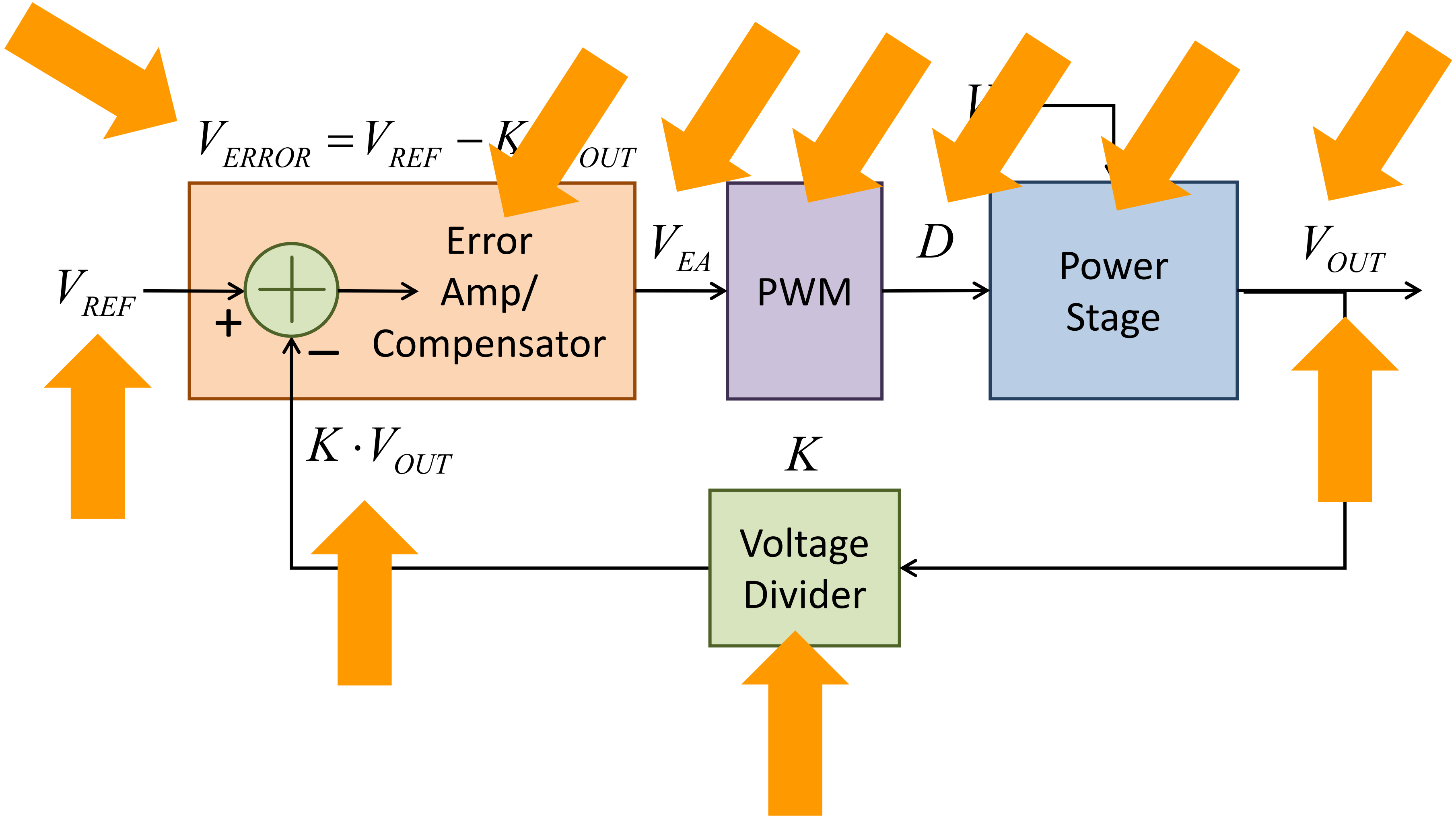


$$V_{OUT} = D \cdot V_{IN}$$
$$= 0.15 \cdot 12\,\mathrm{V}$$
$$= 1.8\,\mathrm{V}$$

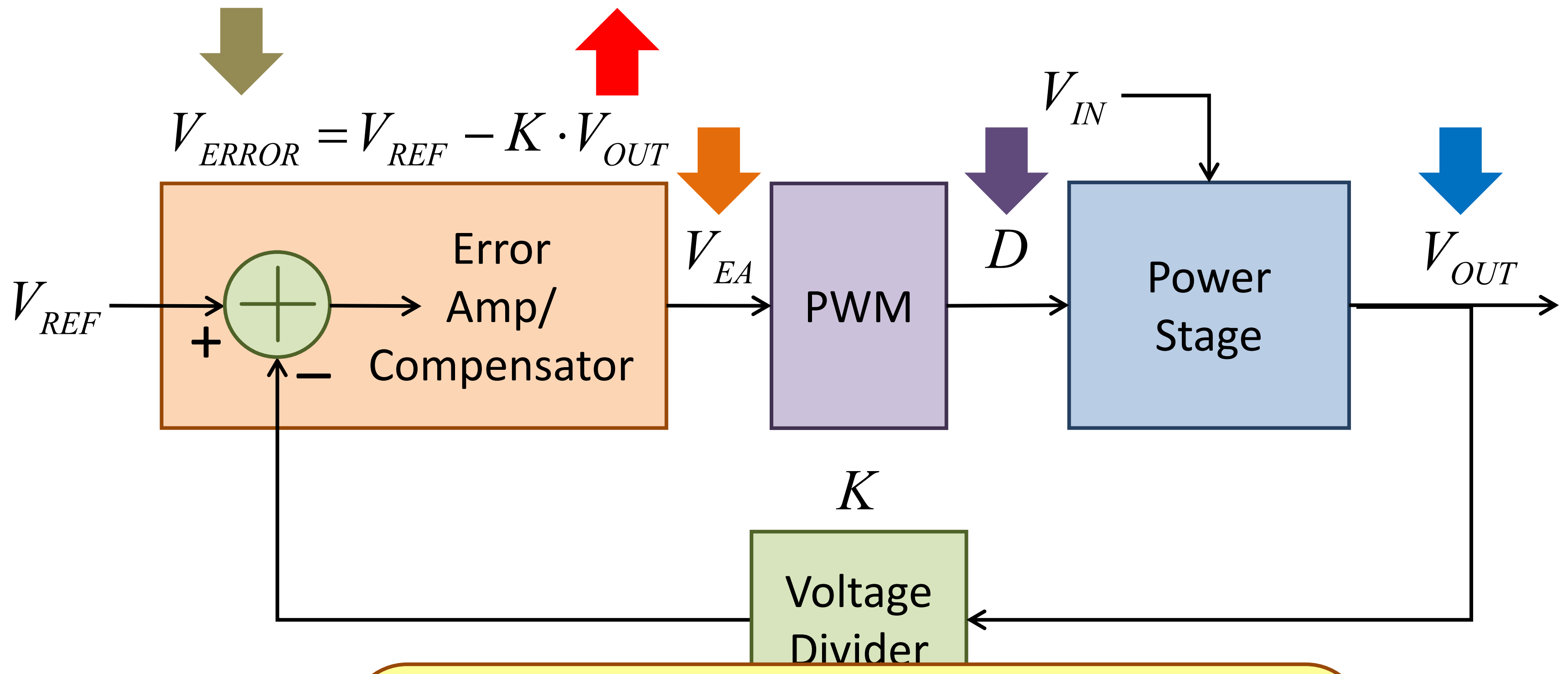
What Happens If:
Input Voltage Changes?
Load Changes?

Output Voltage Changes

Buck Converter With Feedback

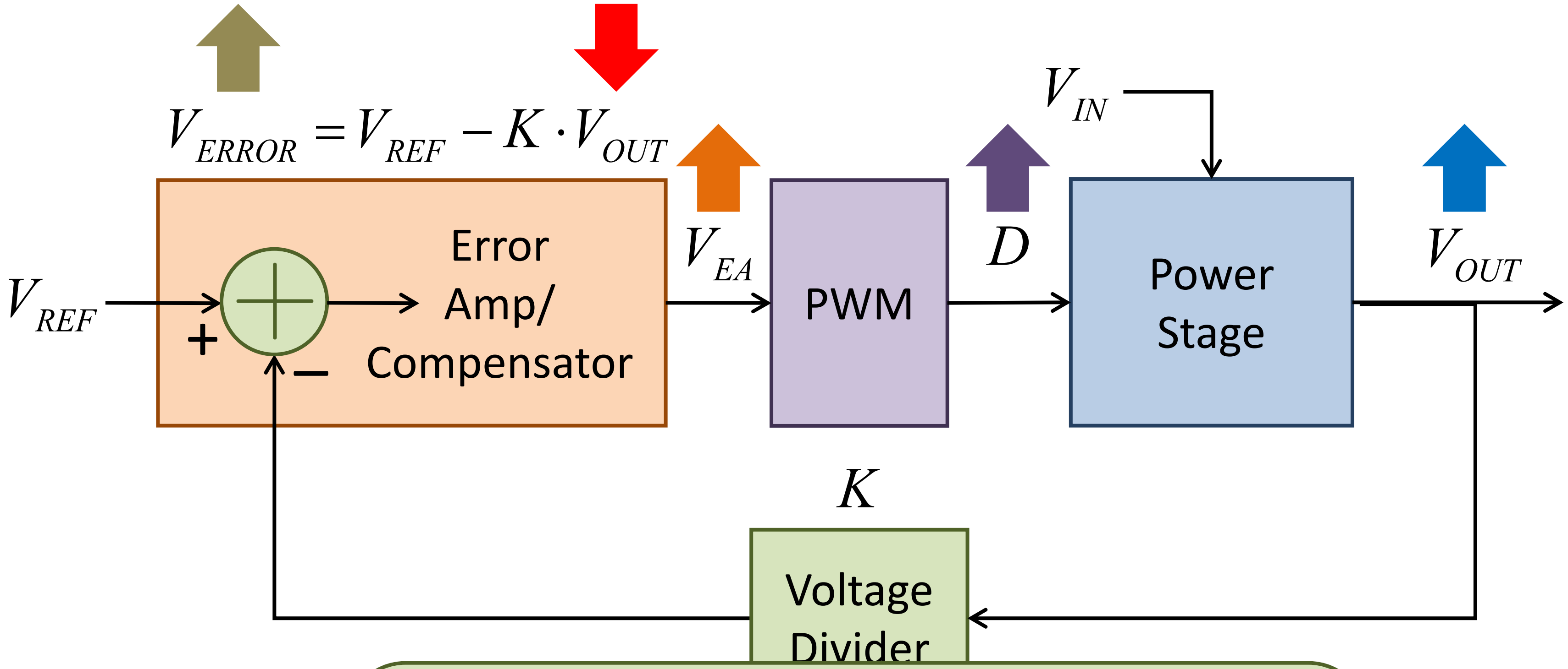


Buck Converter With Feedback



Output Voltage Too High ⇒
Error Voltage Is Negative ⇒
Duty Cycle Is Decreased ⇒
Output Voltage Is Reduced

Buck Converter With Feedback



Output Voltage Too Low ⇒
Error Voltage Is Positive ⇒
Duty Cycle Is Increased ⇒
Output Voltage Is Increased

Feedback Concerns

Tracking

Disturbance Rejection

Stability

Feedback Concerns

Tracking

Disturbance

bility

How Close Is The
Output
To The
Commanded Value?

Feedback Concerns

Tracking

Disturbance Rejection

If The Input Voltage Changes,
Or The Load Changes,
Or There Is Noise, How Well Does
The Output Return To The Proper Value?

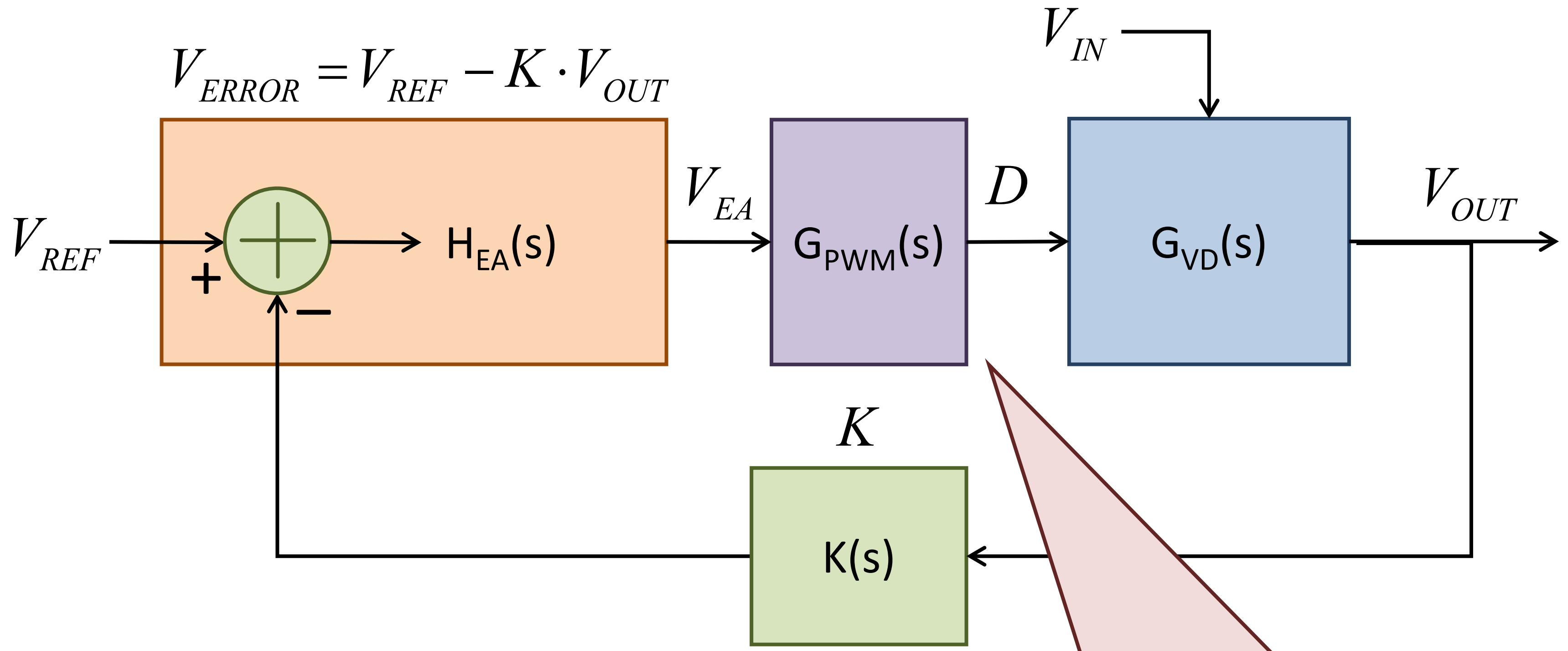
Feedback Concerns

Output Does Not “Run Away”

“Bounded Input, Bounded Output”
“Lyapunov”

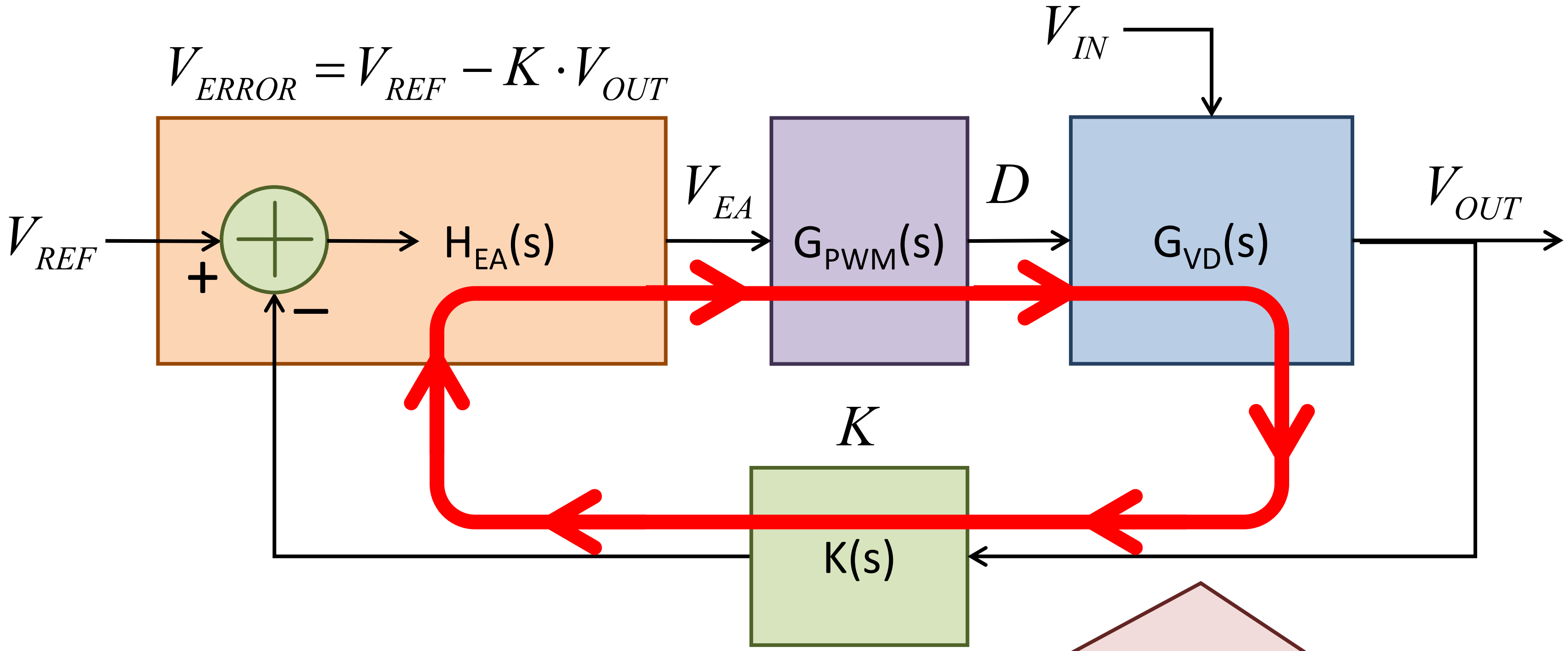
Stability

Loop Gain



Each Block Of The Control Loop Has Its Own Transfer Function

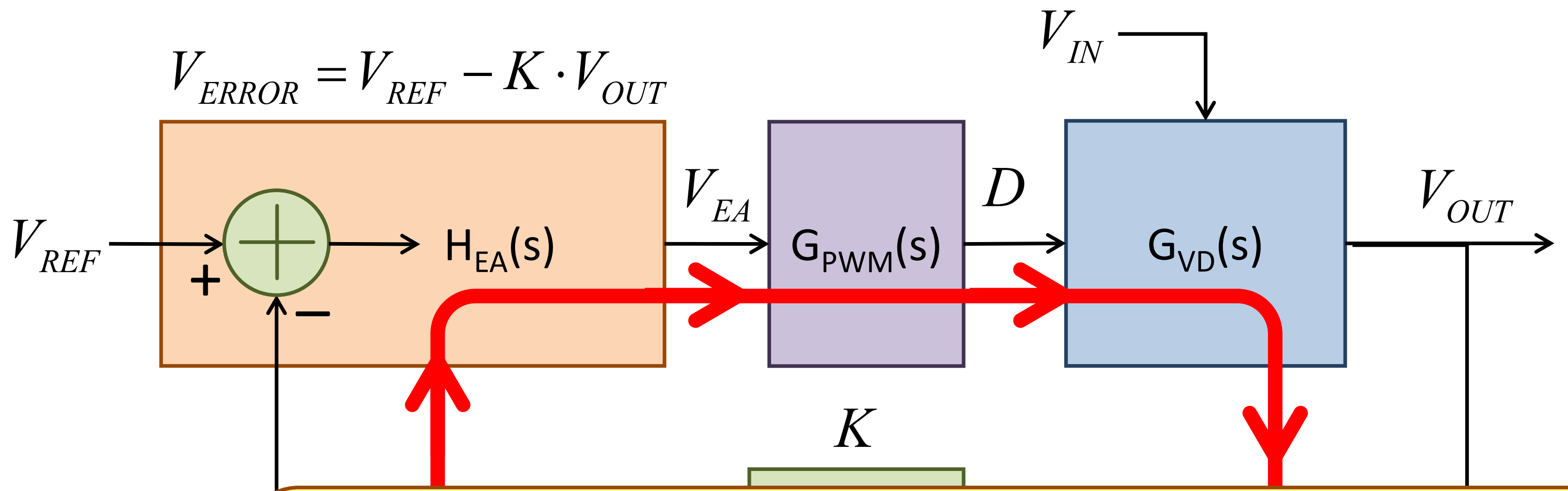
Loop Gain



What Happens To A Signal As It Goes Around The Control Loop?

EMBEDDED POWER LABS

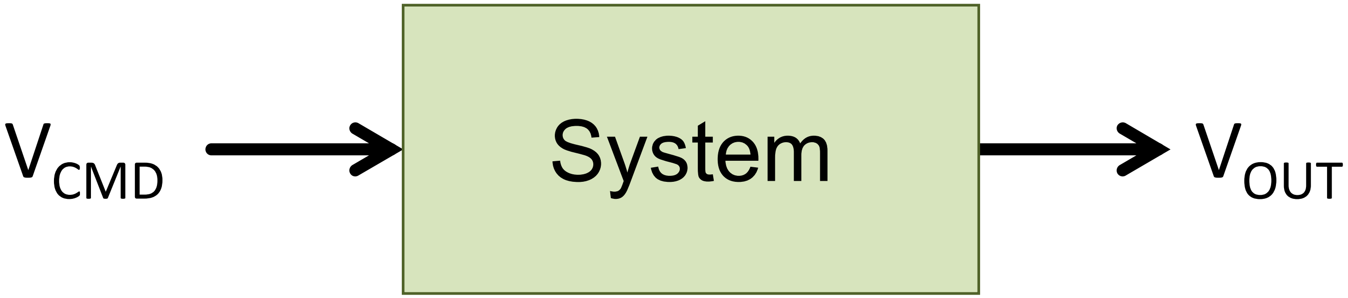
Loop Gain T(s)



Open Loop Gain

$$T(s) = K(s) \cdot H_{EA}(s) \cdot G_{PWM}(s) \cdot G_{VD}(s)$$

How To Get Good Tracking?

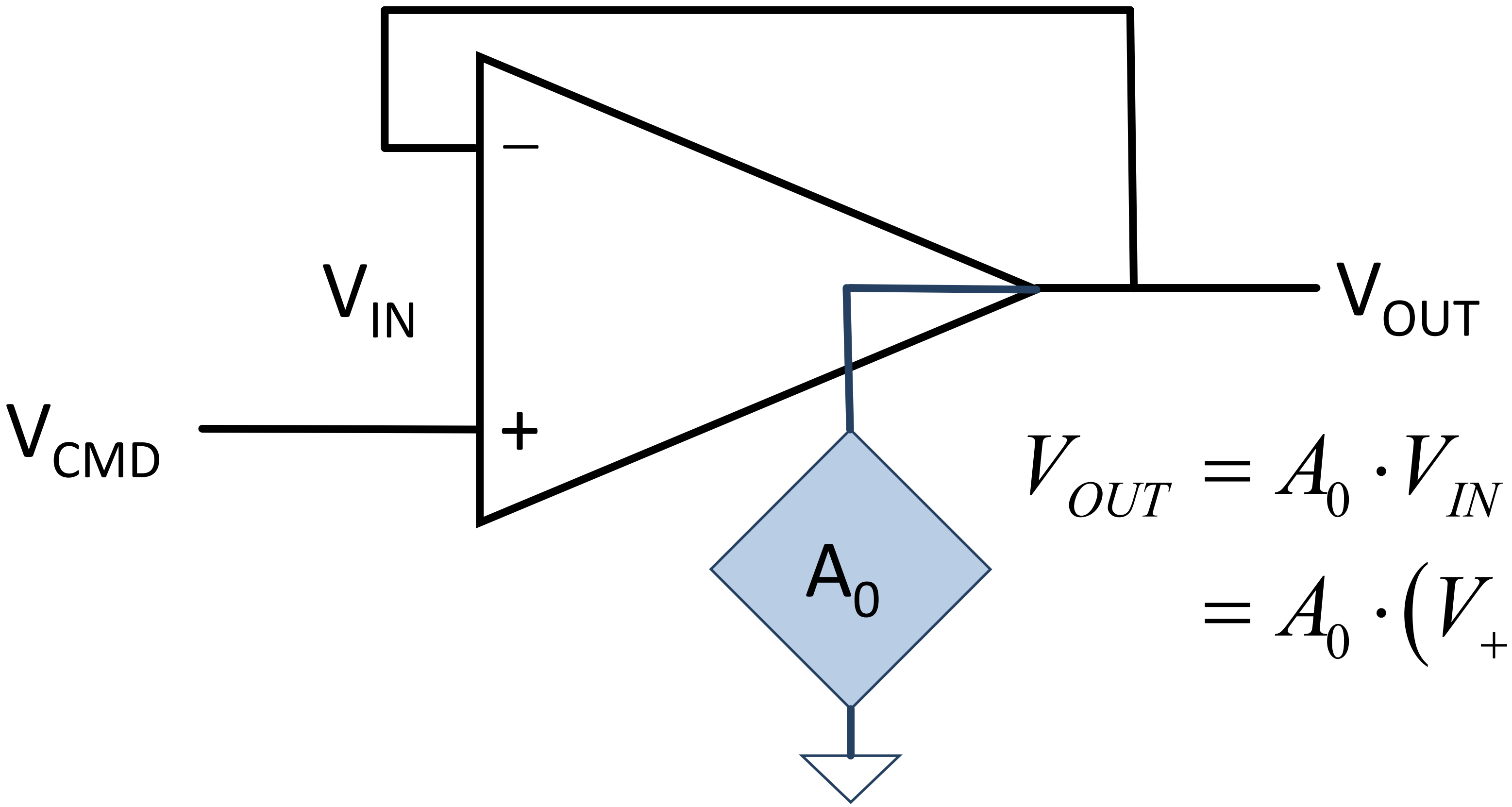


Good Tracking Means

$V_{OUT} = V_{CMD}$

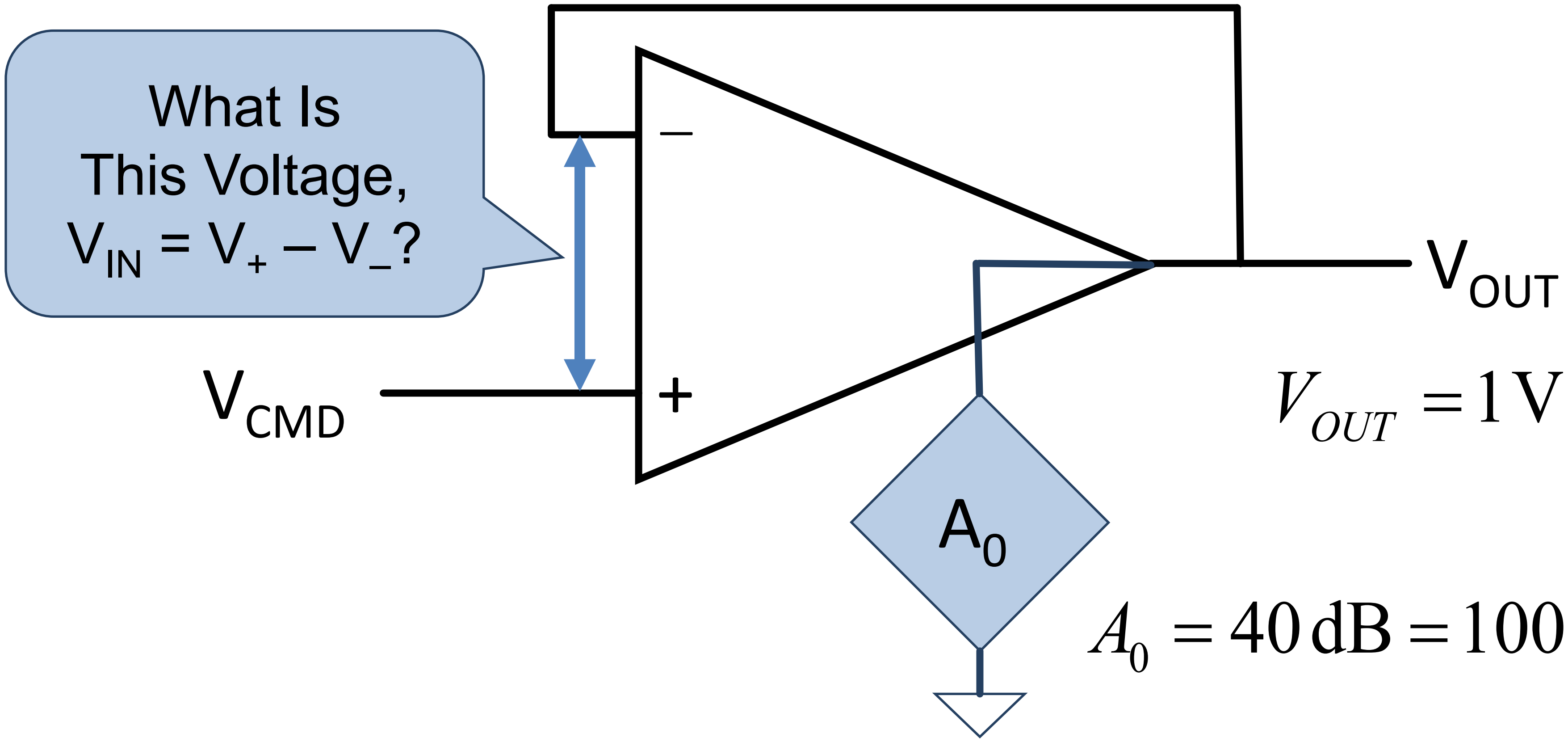
That Is, We Get The Output We Want
Based On The Input Command

Op-Amp Buffer/Follower



$$V_{OUT} = A_0 \cdot V_{IN}$$
$$= A_0 \cdot (V_+ - V_-)$$

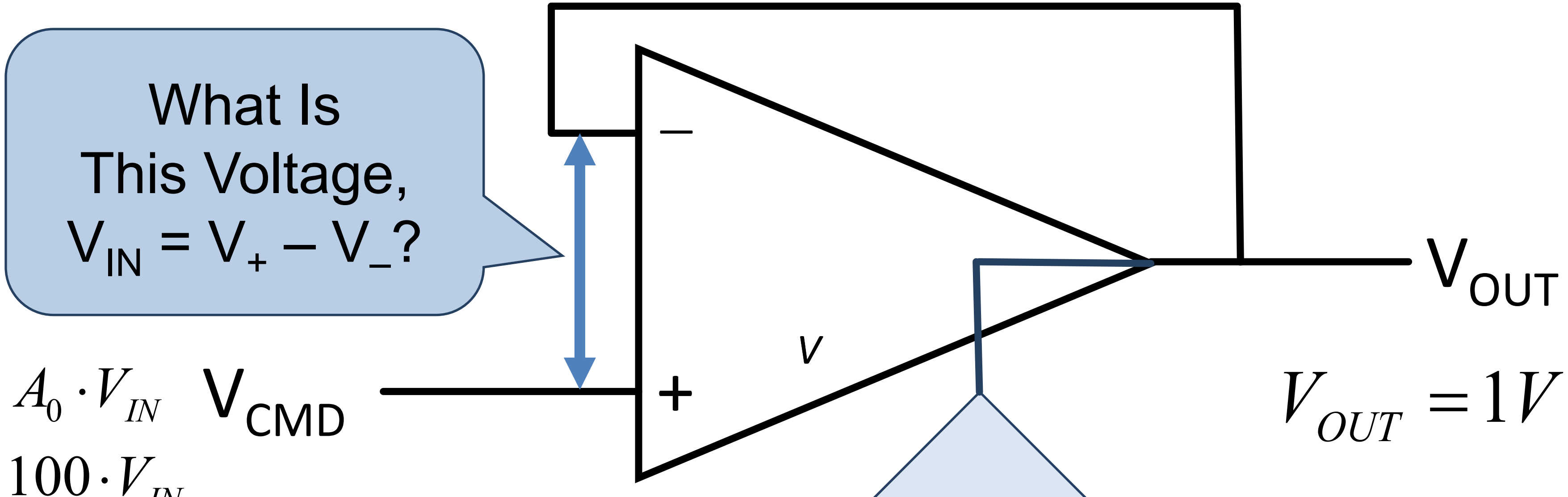
Op-Amp Buffer/Follower Example



$$V_{OUT} = 1\,\mathrm{V}$$

$$A_0 = 40\,\mathrm{dB} = 100$$

Op-Amp Buffer/Follower Example



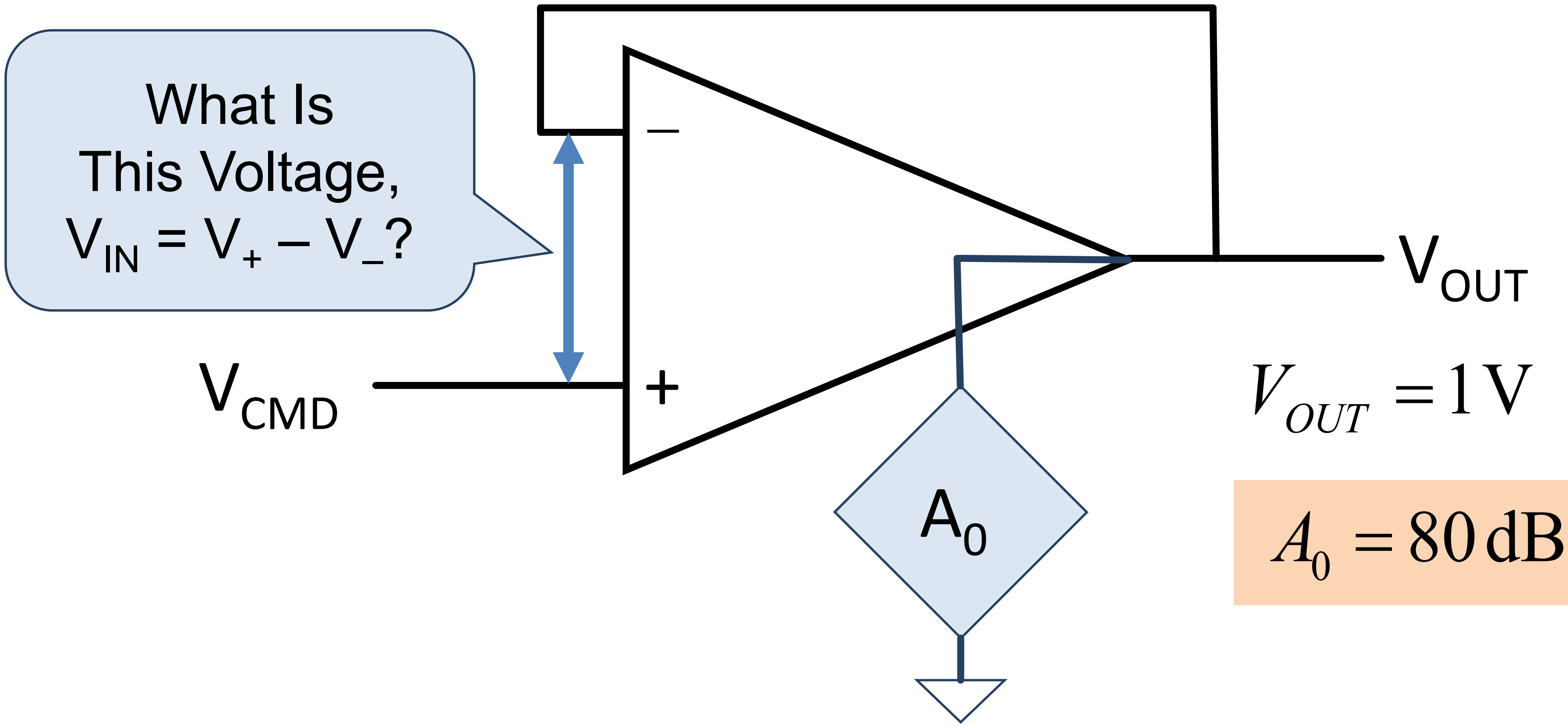
$$1V = A_0 \cdot V_{IN}$$
$$= 100 \cdot V_{IN}$$
$$= 100 \cdot \left(\frac{1}{100} \cdot \mathrm{V} \right)$$
$$V_{IN} = 10\,\mathrm{mV}$$

$$V_{OUT} = 1V$$

$$\mathrm{dB} = 100$$

$$\mathrm{V}_{CMD} = V_{OUT} + V_{IN}$$
$$= 1\,\mathrm{V} + 10\,\mathrm{mV}$$
$$= 1.01\,\mathrm{V} \approx \mathrm{V}_{OUT}$$

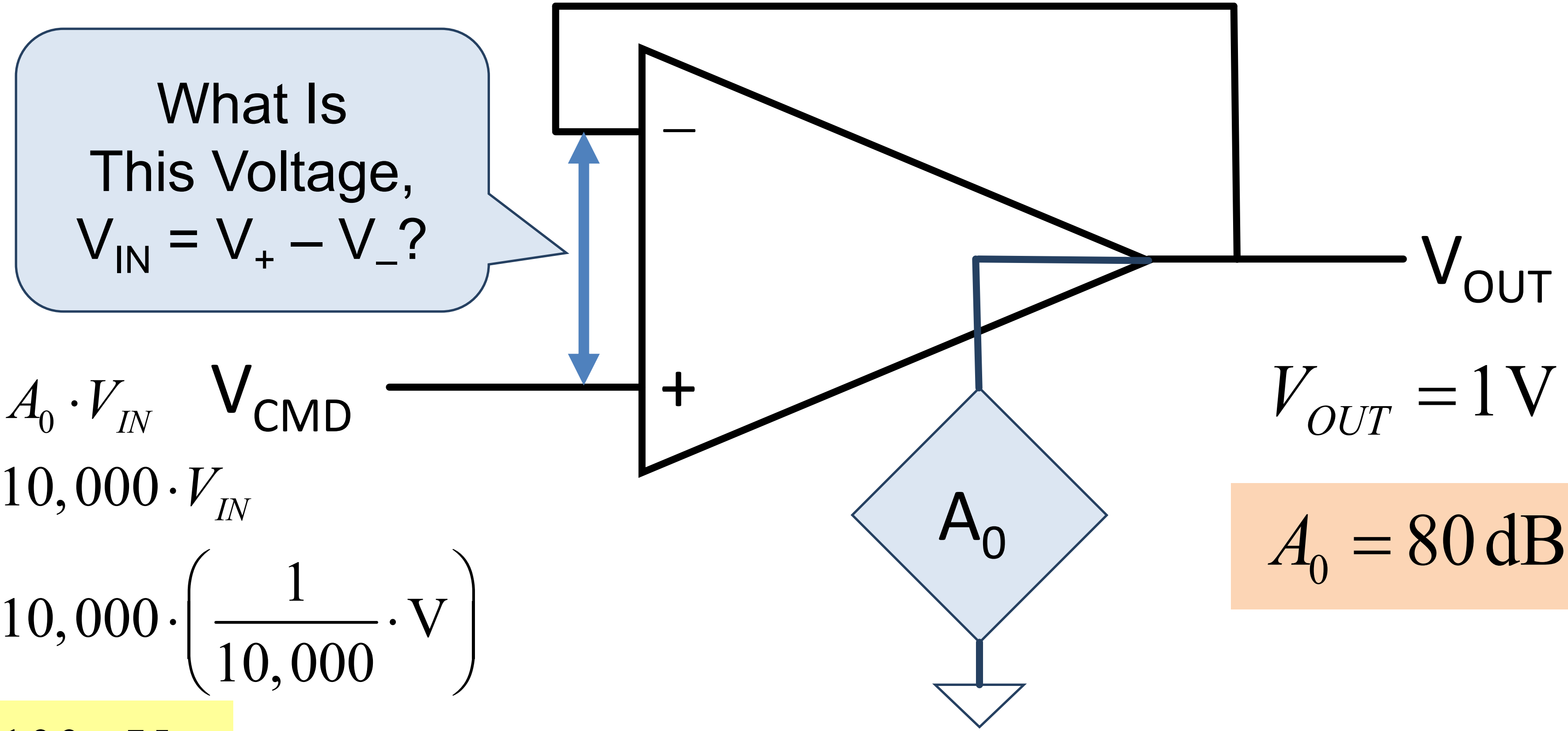
Op-Amp Buffer/Follower Example



$$V_{OUT} = 1\,\text{V}$$

$$A_0 = 80\,\text{dB} = 10,000$$

Op-Amp Buffer/Follower Example



$$1V = A_0 \cdot V_{IN}$$
$$= 10,000 \cdot V_{IN}$$
$$= 10,000 \cdot \left(\frac{1}{10,000} \cdot \text{V} \right)$$

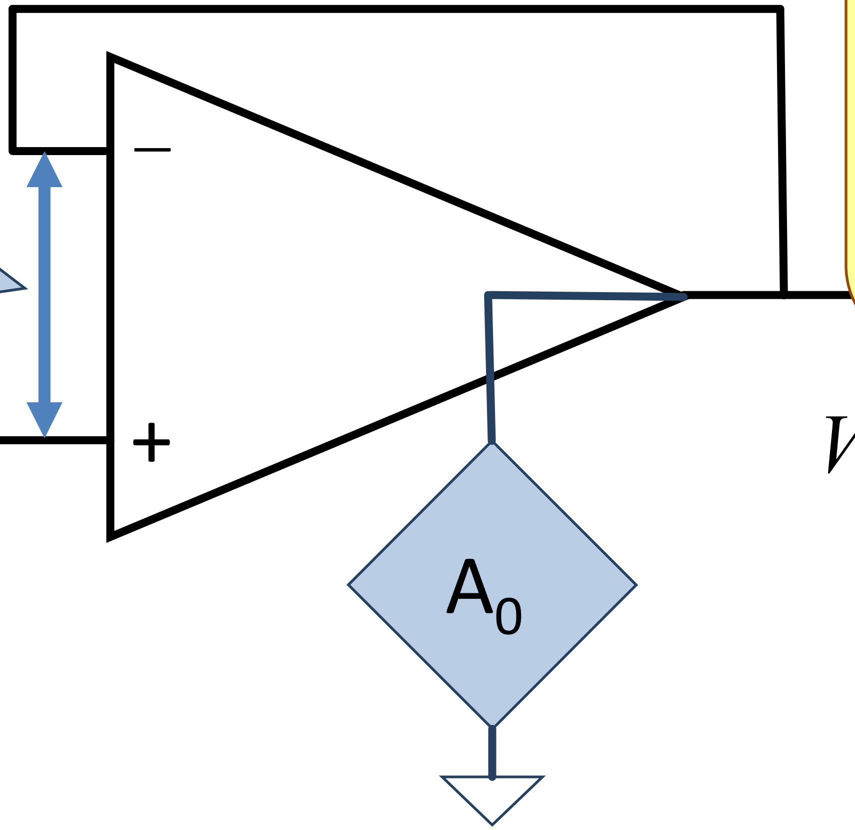
$$V_{IN} = 100\,\mu\text{V}$$

$$V_{OUT} = 1\,\text{V}$$

$$A_0 = 80\,\text{dB} = 10,000$$

Op-Amp Buffer/Follower Example

What Is
This Voltage,
$V_{IN} = V_+ - V_-$?



V_{CMD}

$$\begin{aligned} 1V &= A_0 \cdot V_{IN} \\ &= 10,000 \cdot V_{IN} \\ &= 10,000 \cdot \left(\frac{1}{10,000} \cdot \mathrm{V} \right) \end{aligned}$$

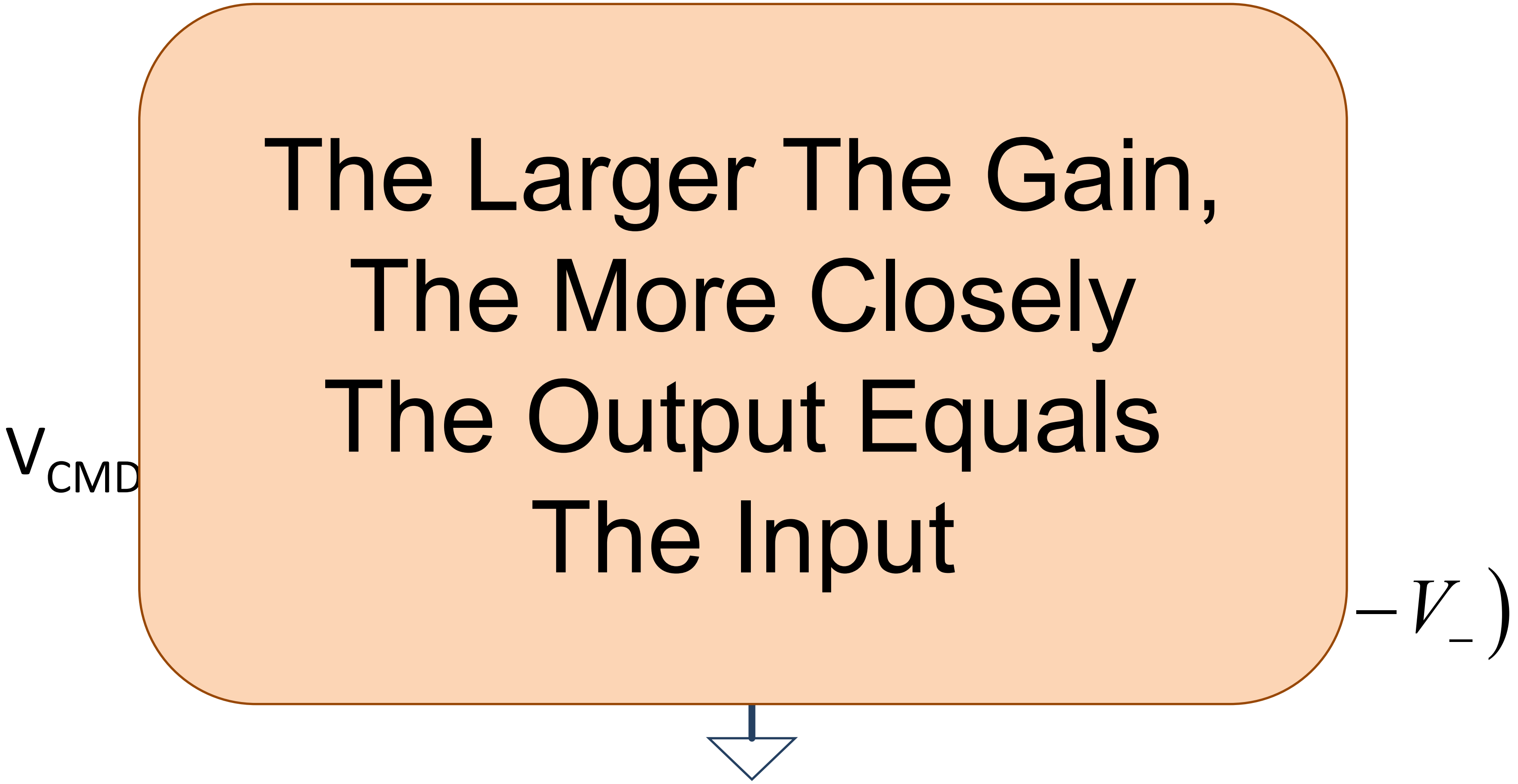
$$V_{IN} = 100\,\mu\mathrm{V}$$

$$\begin{aligned} \mathrm{V}_{CMD} &= V_{OUT} + V_{IN} \\ &= 1\,\mathrm{V} + 100\,\mu\mathrm{V} \\ &= 1.0001\,\mathrm{V} \approx \mathrm{V}_{OUT} \end{aligned}$$

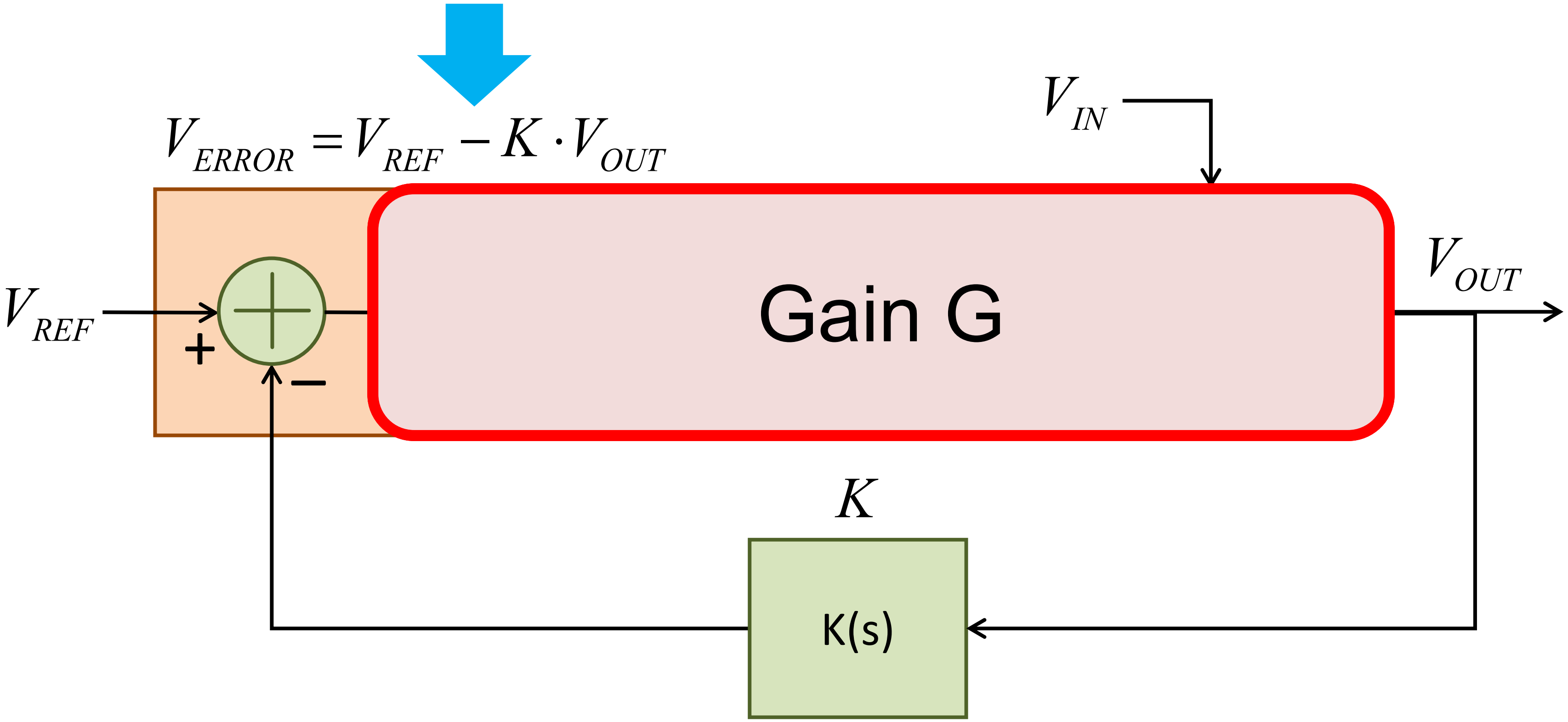
$$V_{OUT} = 1\,\mathrm{V}$$

$$A_0 = 80\,\mathrm{dB} = 10,000$$

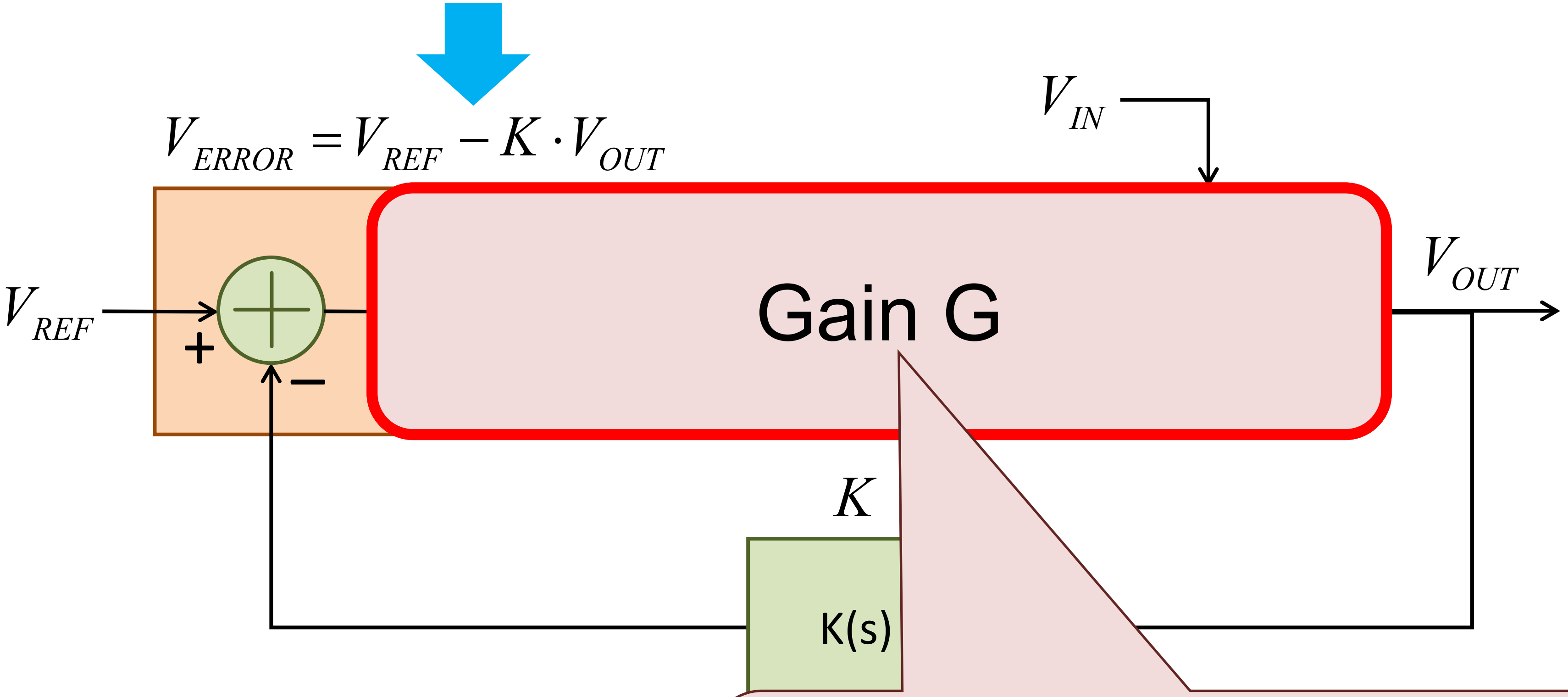
Op-Amp Buffer/Follower



How To Get Good Tracking?

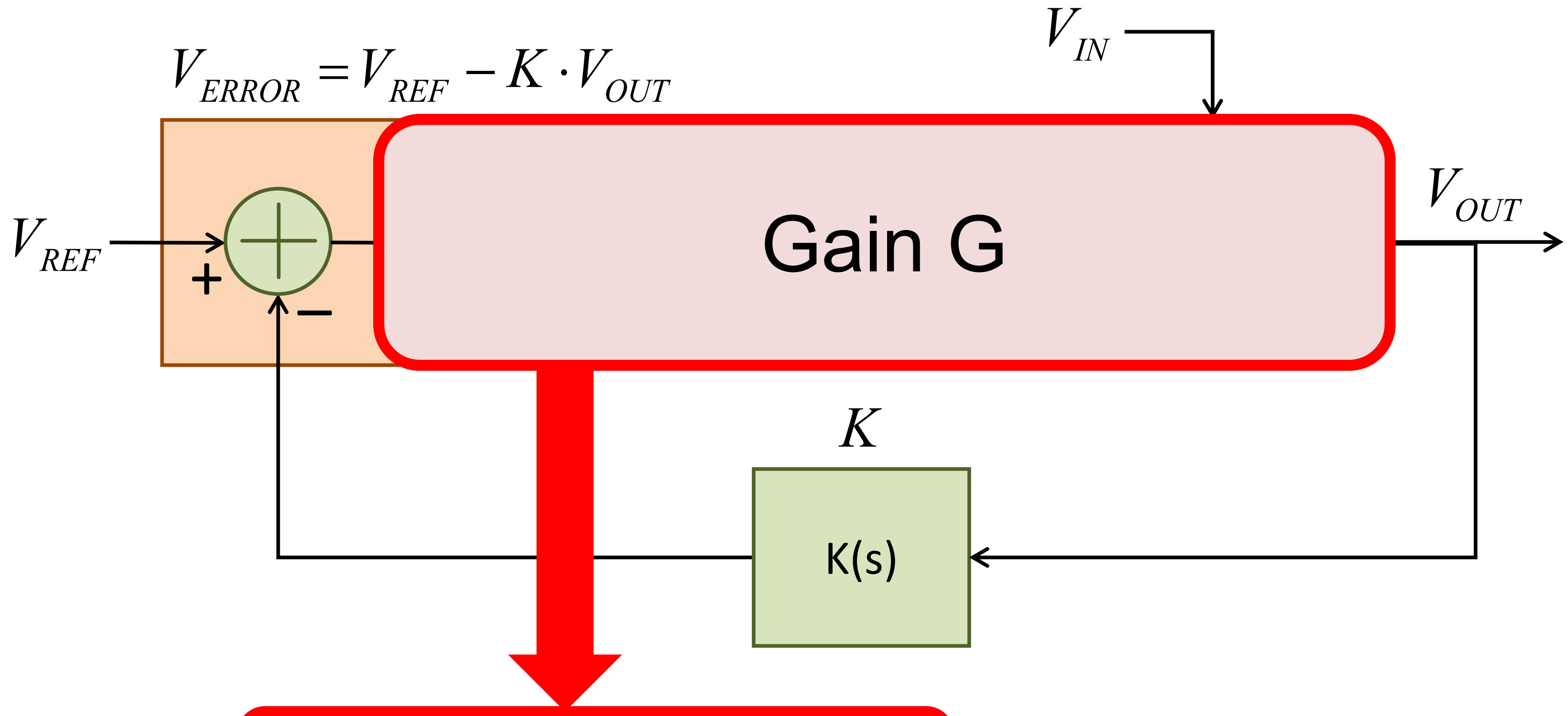


How To Get Good Tracking?



The Larger Gain G,
The More Closely The Output
Follows The Reference Input

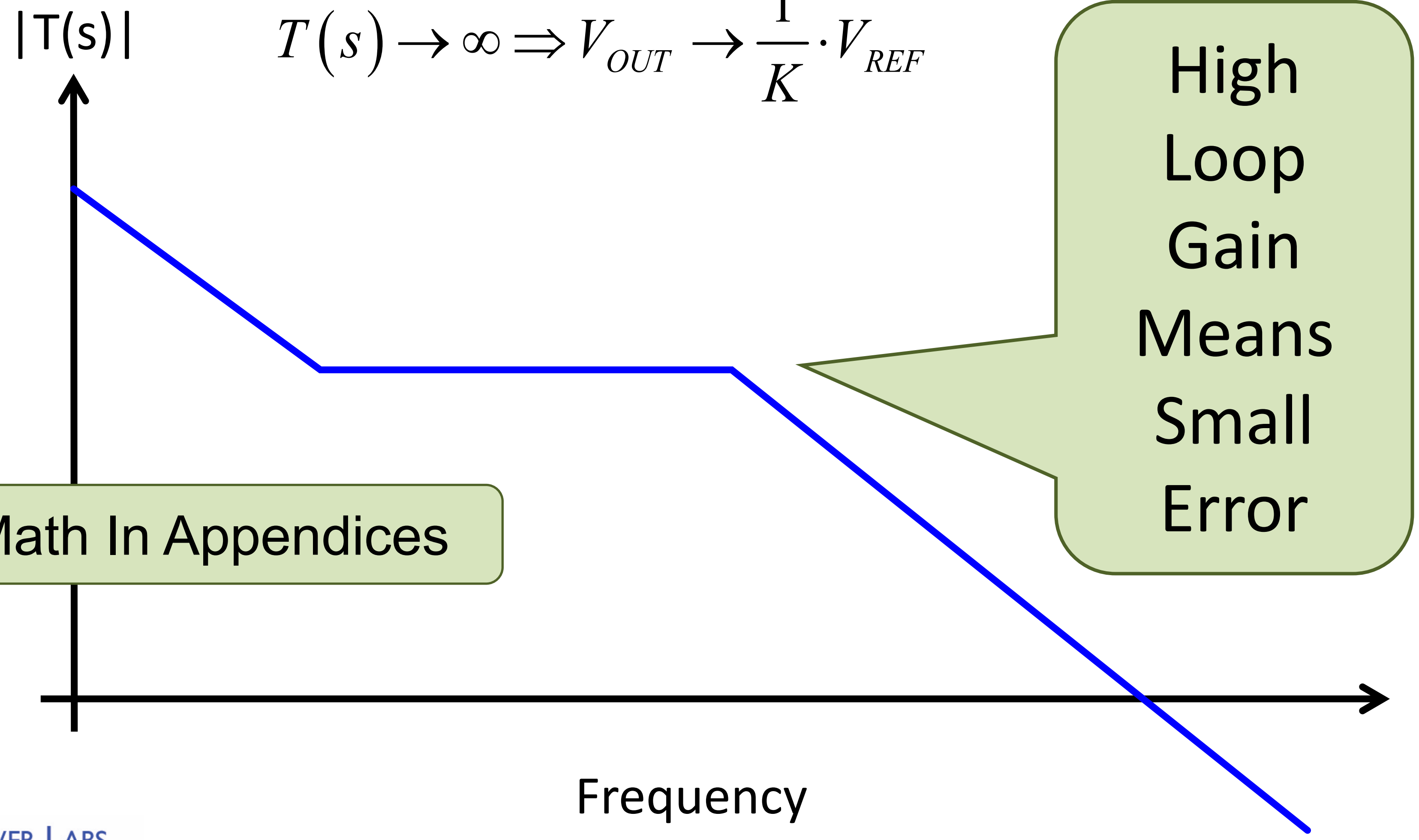
How To Get Good Tracking?



$$V_{OUT}(s) = G_{VD}(s) \cdot G_{PWM}(s) \cdot H_{EA}(s) \cdot \left(V_{REF} - K(s) \cdot V_{OUT}(s)\right)$$

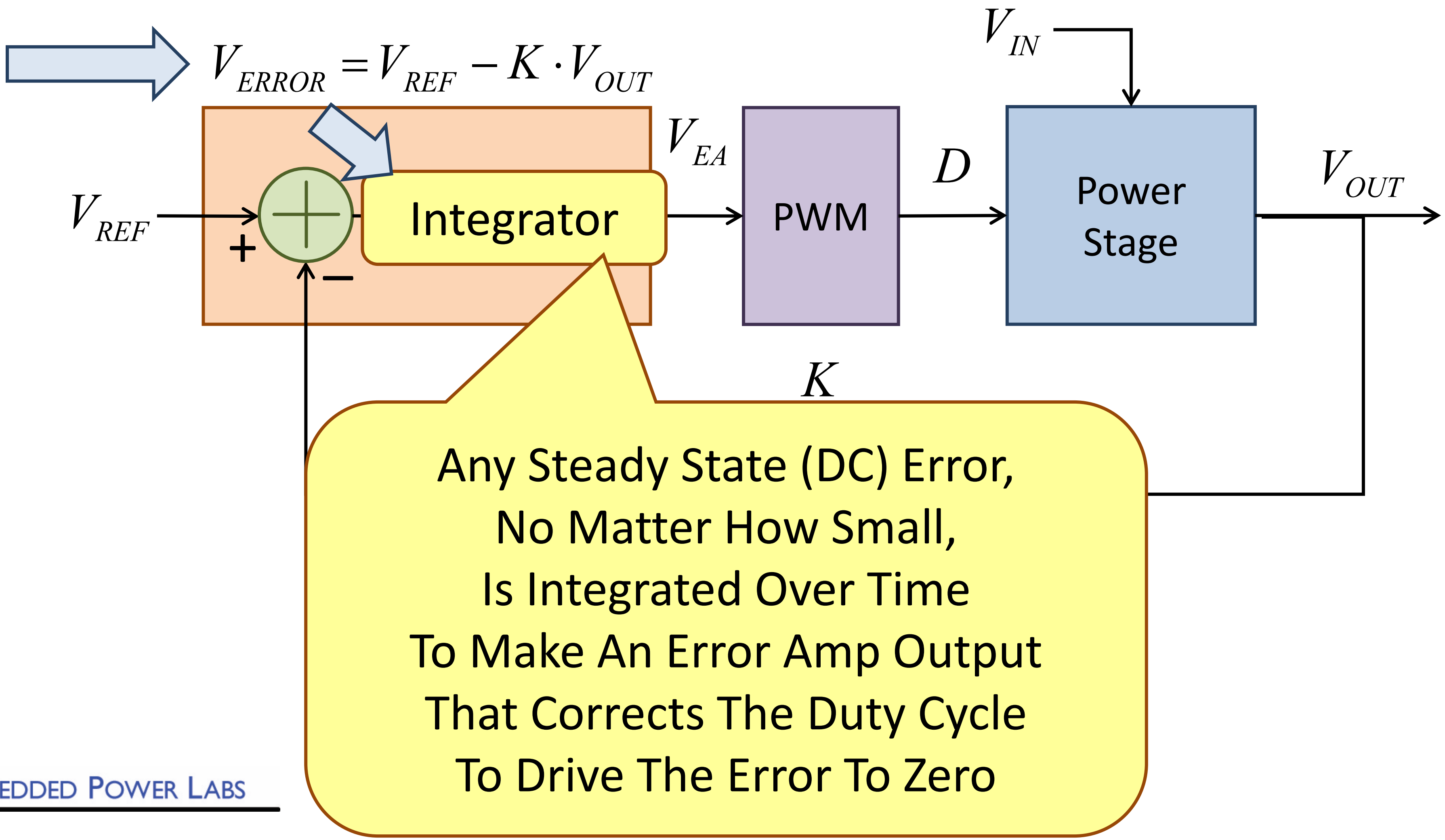
Loop Gain T(s)

$$T(s) \rightarrow \infty \Rightarrow V_{OUT} \rightarrow \frac{1}{K} \cdot V_{REF}$$

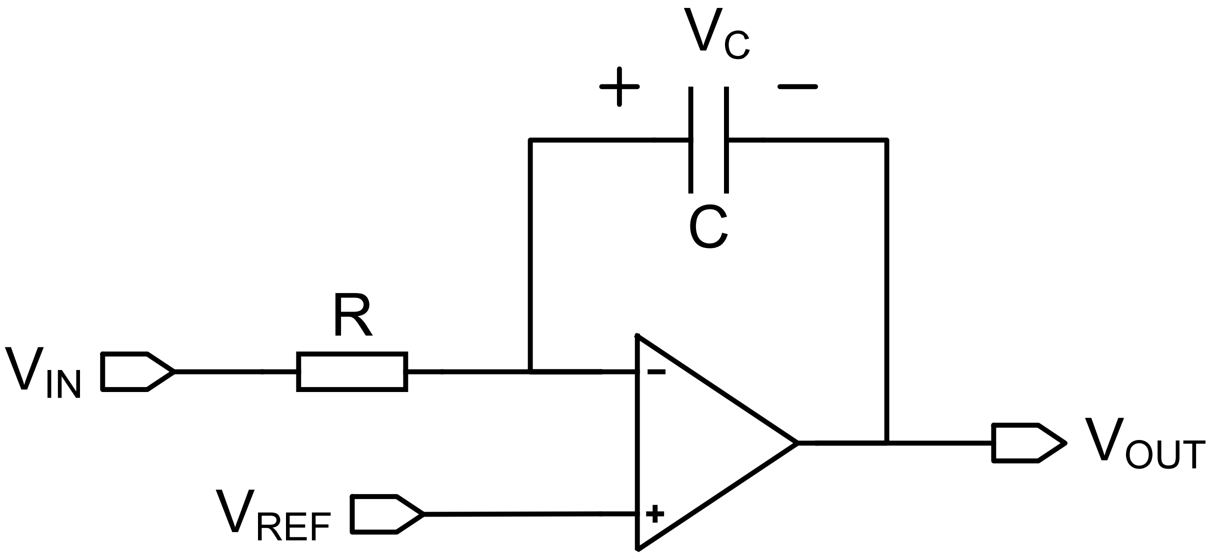


Full Math In Appendices

Assuring Steady State Tracking



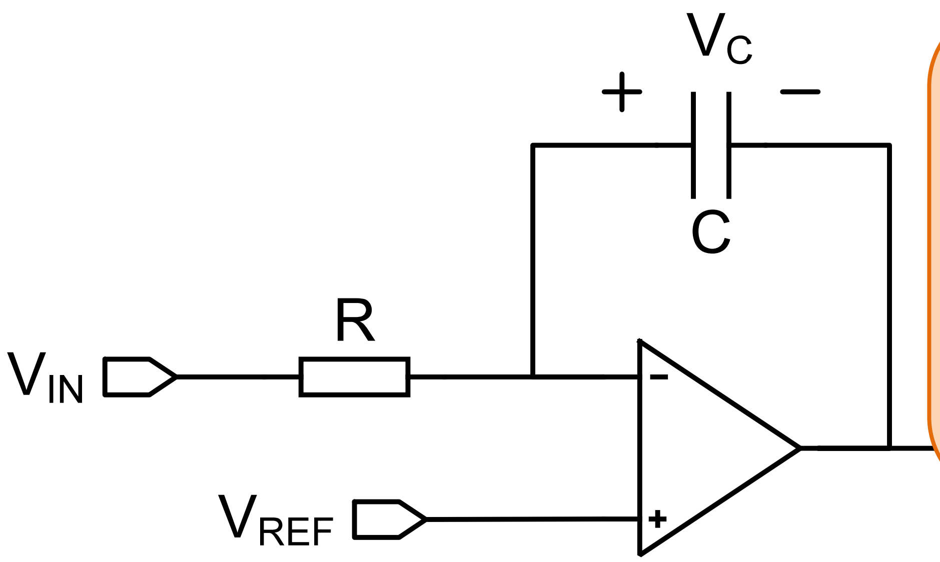
Analog Integrator



$$v_{OUT}(t) = V_{REF} - v_C(t)$$
$$= V_{REF} - \frac{1}{C} \cdot \int_0^t i_C(\tau)\, d\tau$$

$$v_{OUT}(t) = V_{REF} - \frac{1}{C} \cdot \int_0^t \frac{v_{in}(\tau) - V_{REF}}{R}\, d\tau$$
$$= V_{REF} - \frac{1}{R \cdot C} \cdot \int_0^t v_{ERROR}(\tau)\, d\tau$$

Analog Integrator



As

$s \rightarrow 0$

Gain $\rightarrow \infty$

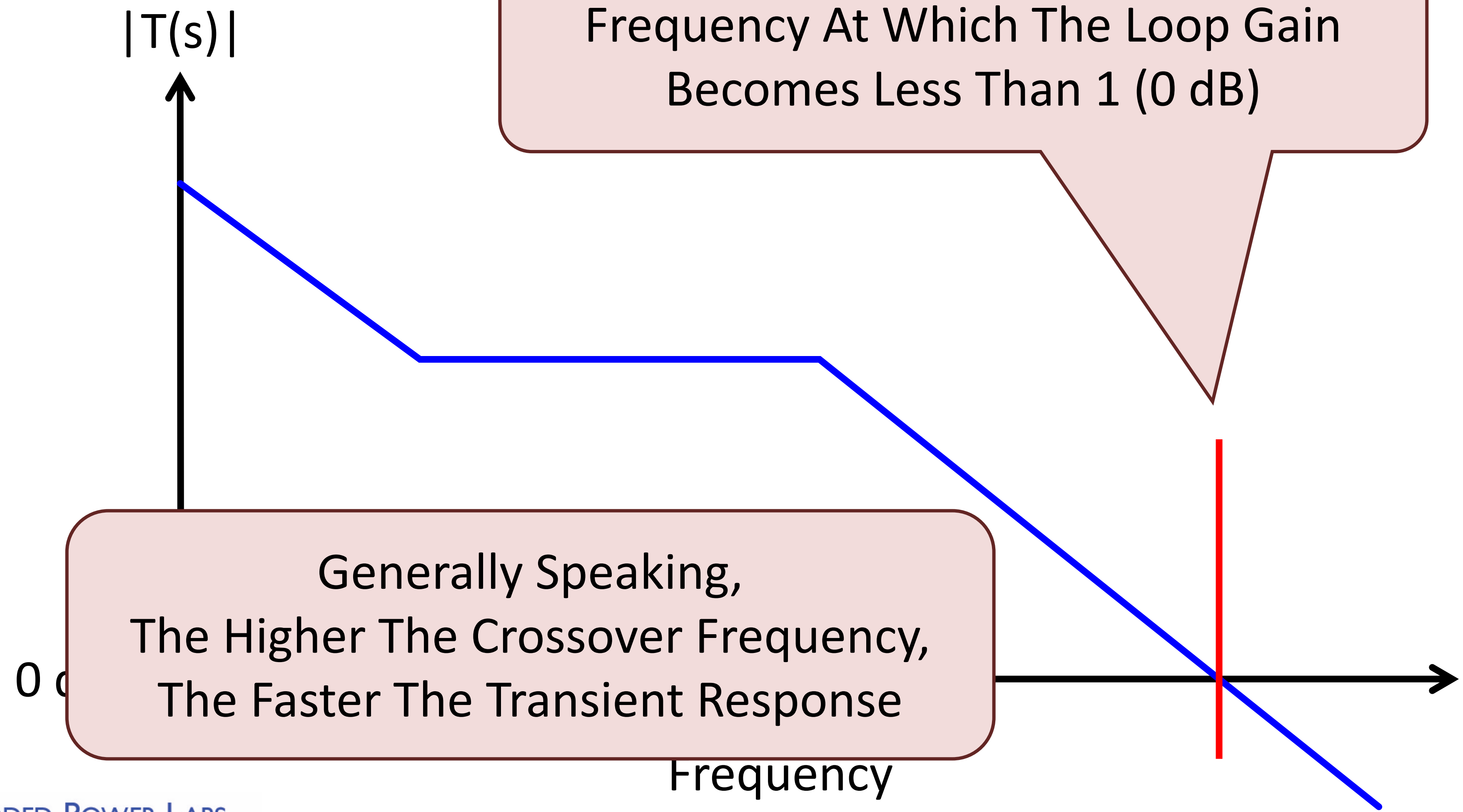
DC Error $\rightarrow 0$

$$\frac{v_{OUT}(s)}{v_{IN}(s)} = -\frac{Z_F(s)}{Z_S(s)} = -\frac{\frac{1}{s \cdot C}}{R} = -\frac{1}{s \cdot R \cdot C} = -\frac{1}{\frac{s}{\omega_P}} = -\frac{\omega_P}{s}$$

$$\omega_P = \frac{1}{R \cdot C}$$

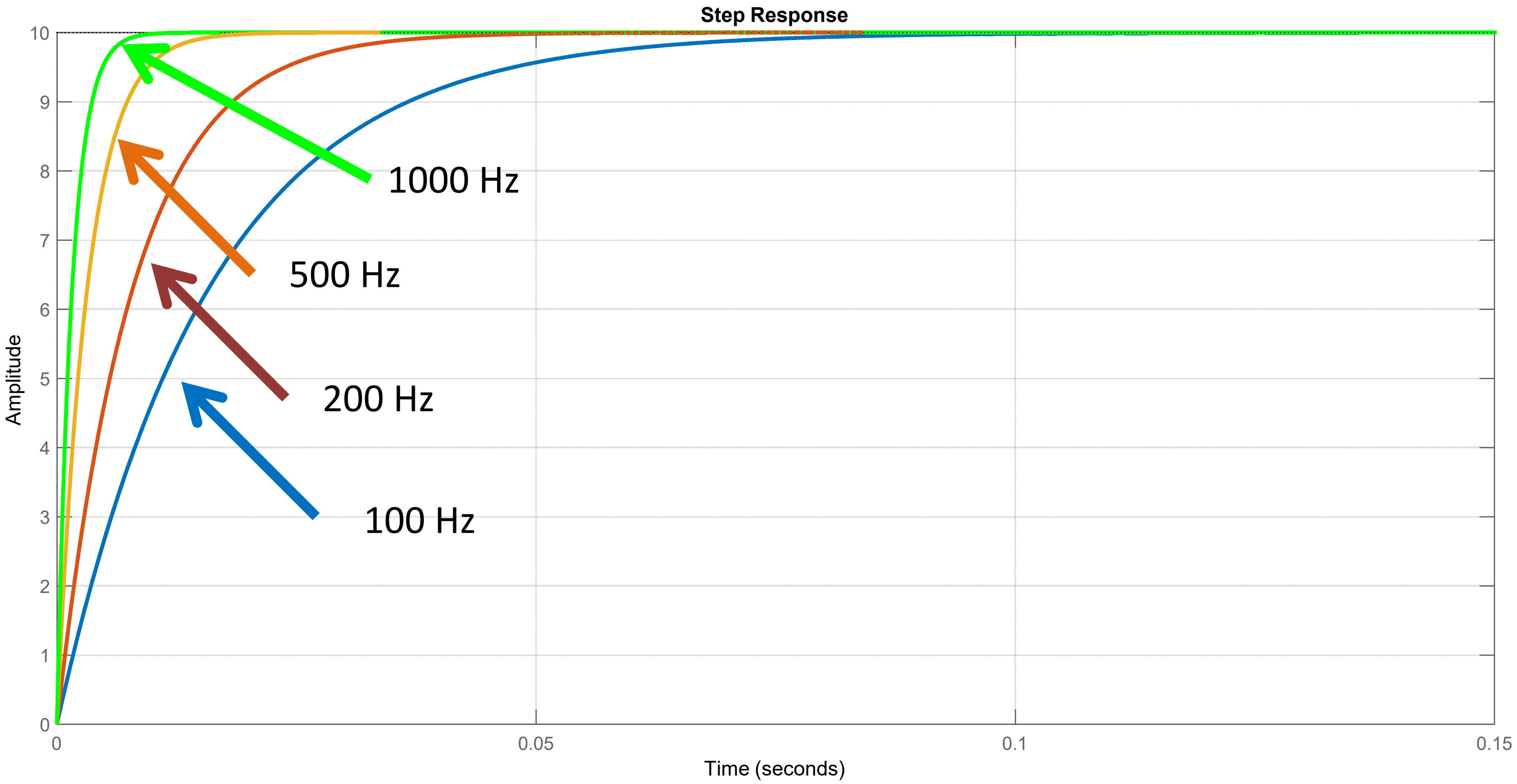
Crossover Frequency, F_C

Frequency At Which The Loop Gain Becomes Less Than 1 (0 dB)

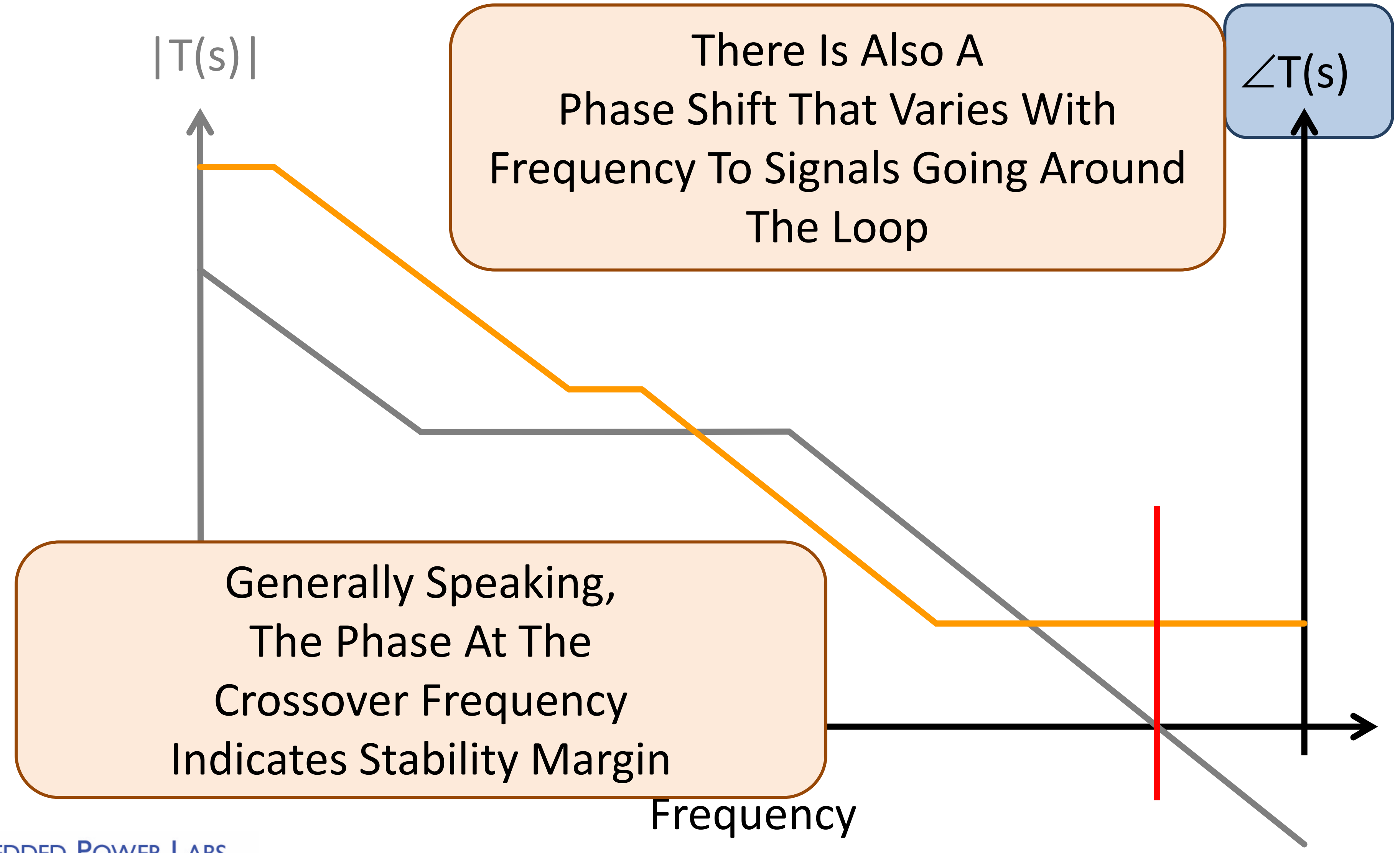


Generally Speaking,
The Higher The Crossover Frequency,
The Faster The Transient Response

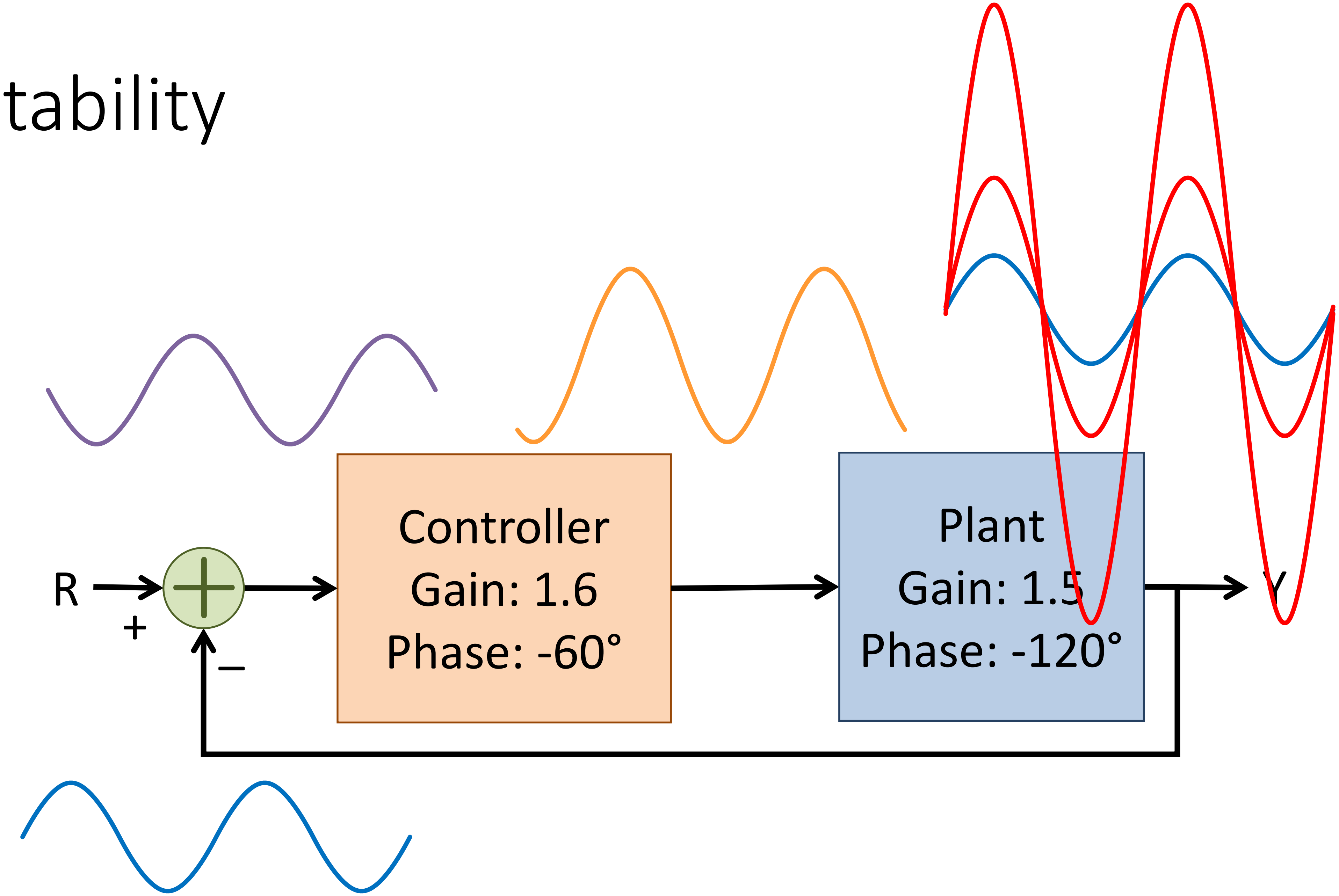
Response Time Vs. Bandwidth



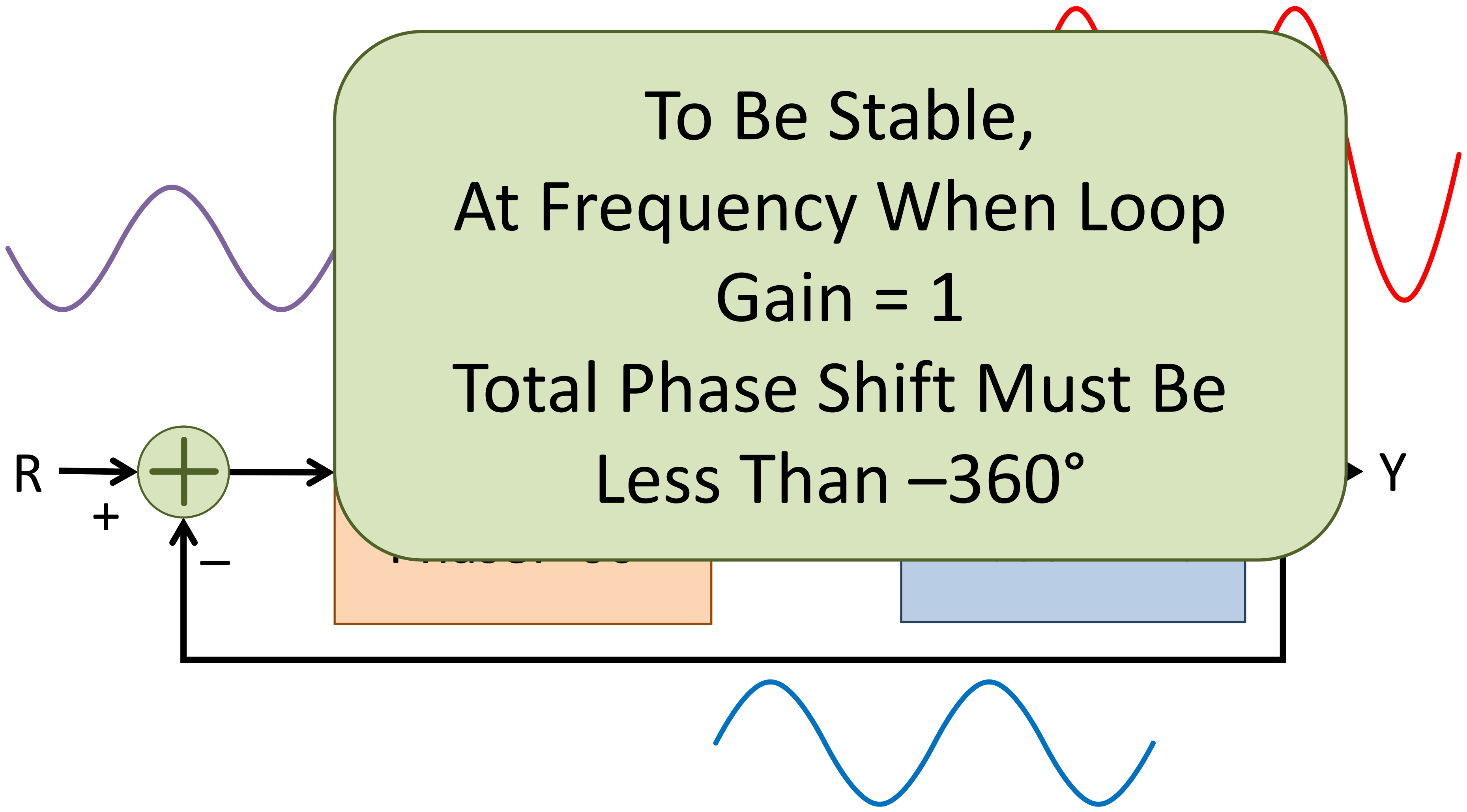
Loop Phase


|T(s)|
There Is Also A Phase Shift That Varies With Frequency To Signals Going Around The Loop
∠T(s)
Generally Speaking, The Phase At The Crossover Frequency Indicates Stability Margin
Frequency

Stability

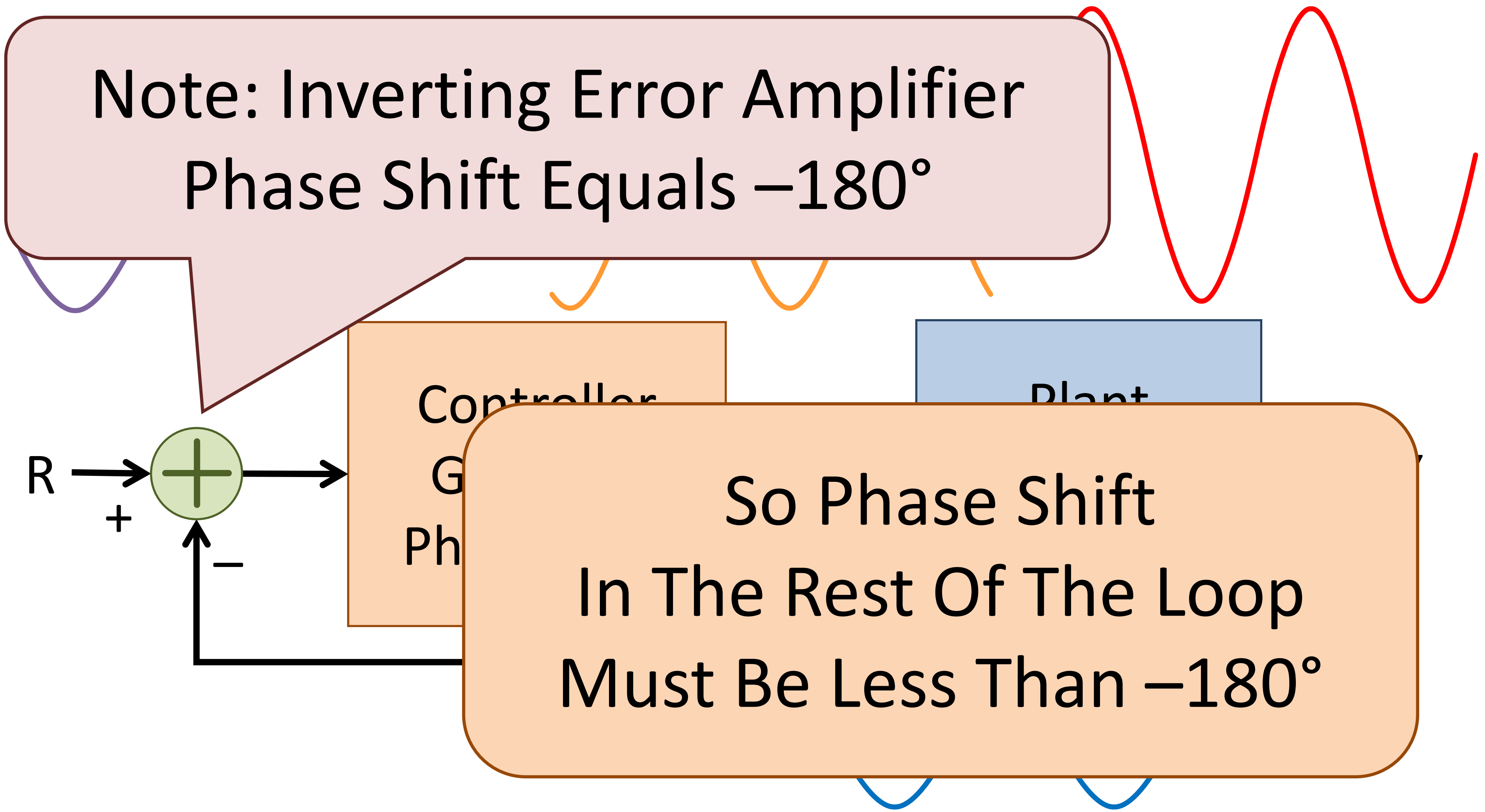


Stability



To Be Stable,
At Frequency When Loop Gain = 1
Total Phase Shift Must Be Less Than –360°

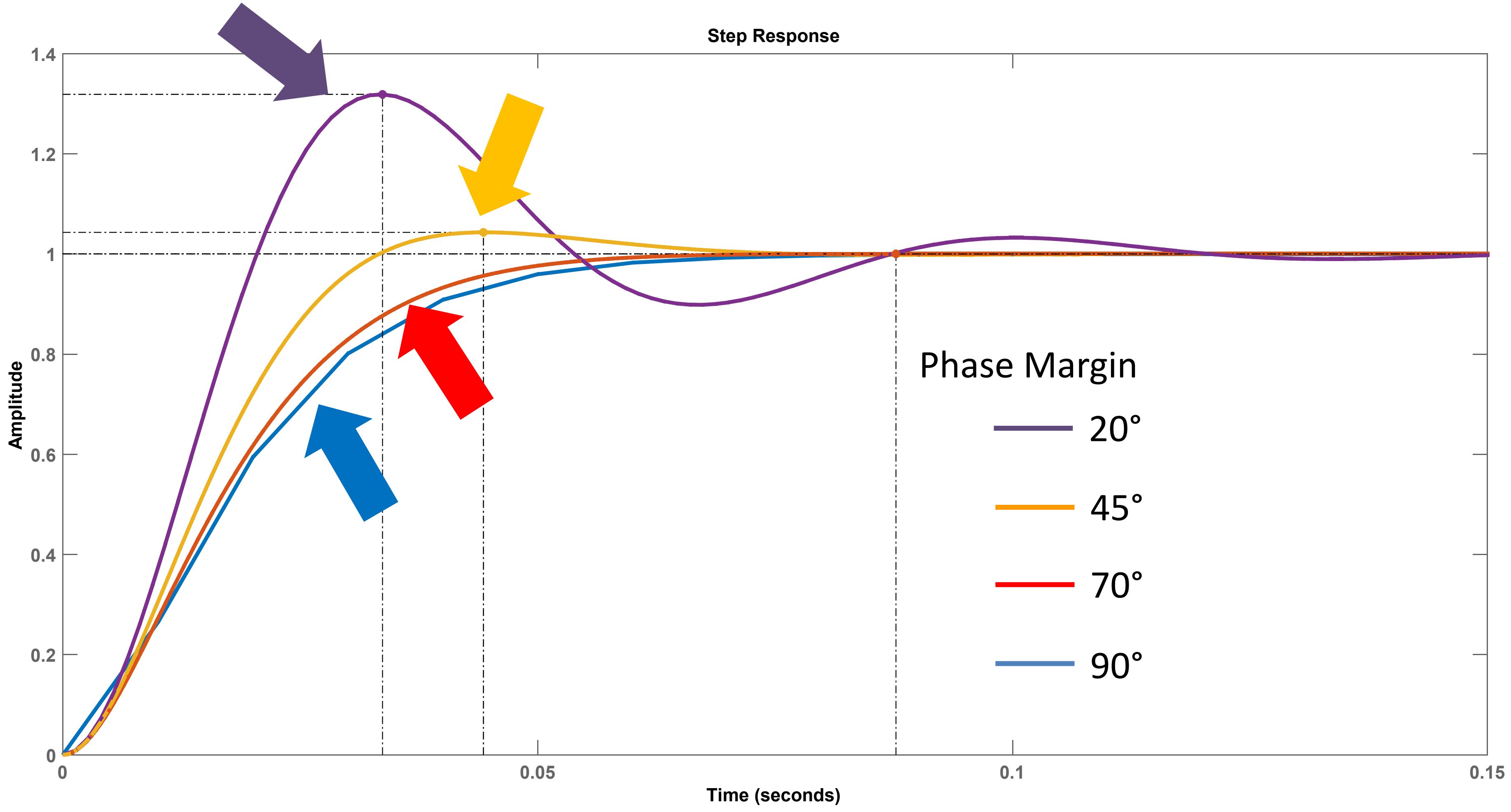
Stability



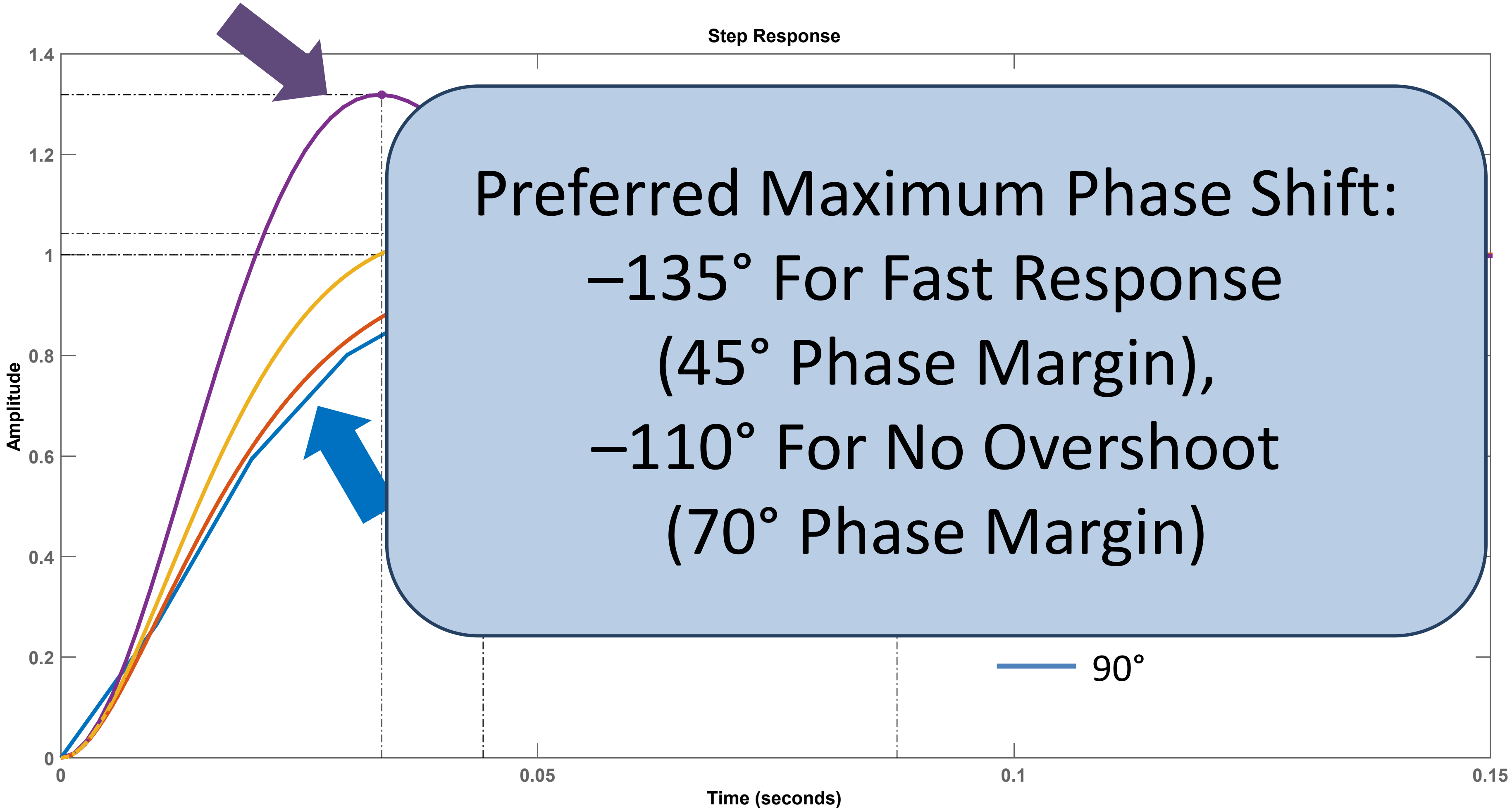
Note: Inverting Error Amplifier
Phase Shift Equals –180°

So Phase Shift
In The Rest Of The Loop
Must Be Less Than –180°

Step Response vs. Phase Margin



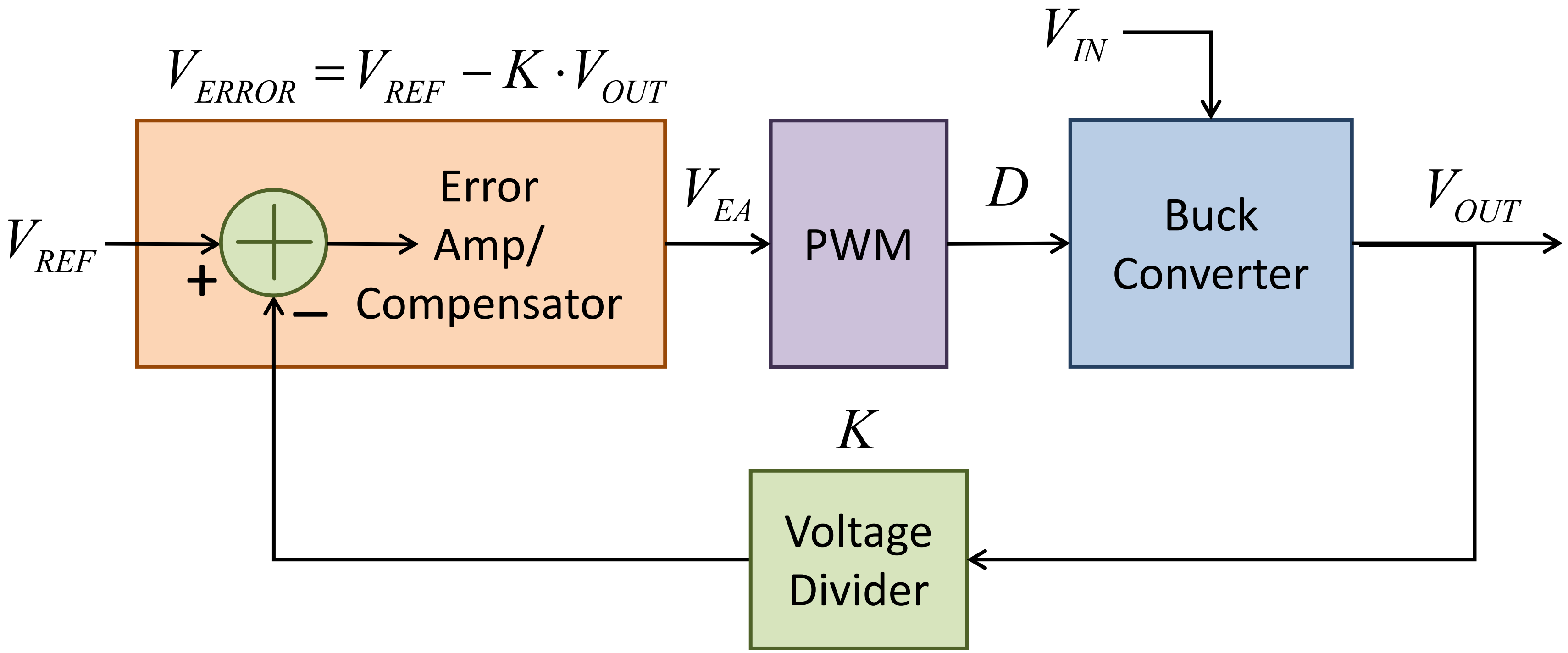
Stability vs. Phase Margin



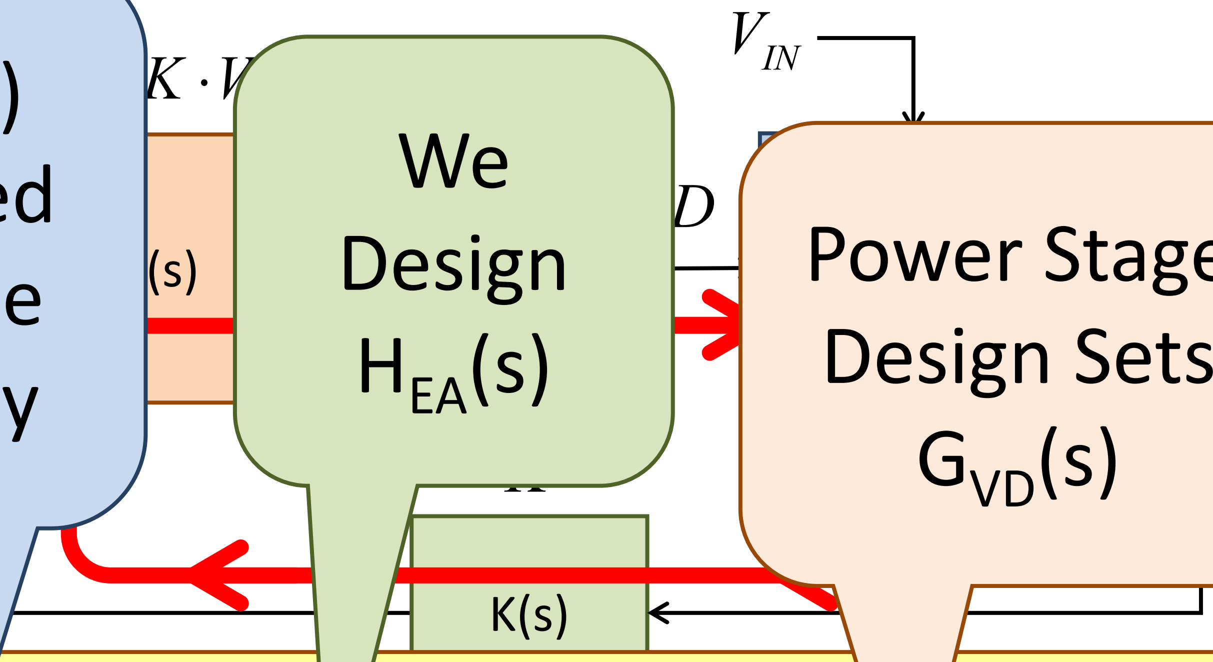
Preferred Maximum Phase Shift:
–135° For Fast Response
(45° Phase Margin),
–110° For No Overshoot
(70° Phase Margin)

Buck Converter
With Feedback Control

Buck Converter With Feedback



Loop Gain T(s)



So That T(s) Gives Desired Performance And Stability

We Design $H_{EA}(s)$

Power Stage Design Sets $G_{VD}(s)$

Loop Gain

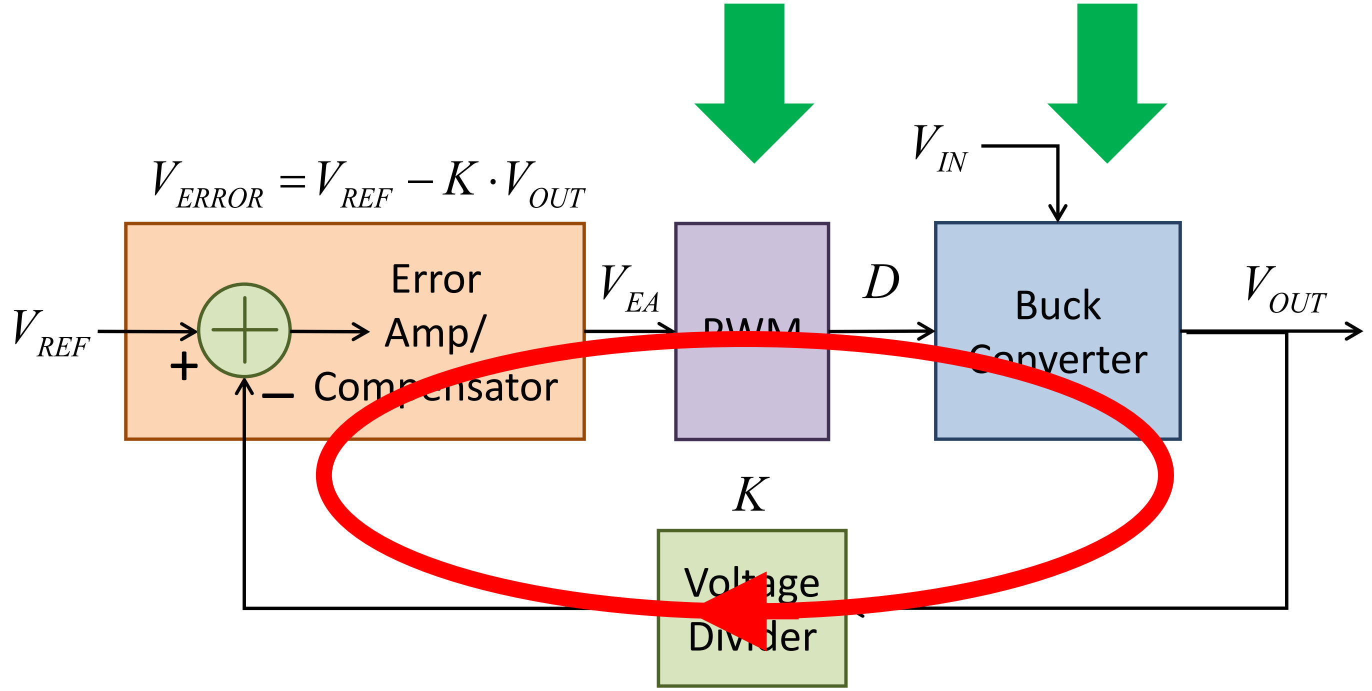
$$T(s) = K \cdot H_{EA}(s) \cdot G_{PWM} \cdot G_{VD}(s)$$

Compensator Design: Plan Of Attack

- Solve For $H_{EA}(s)$

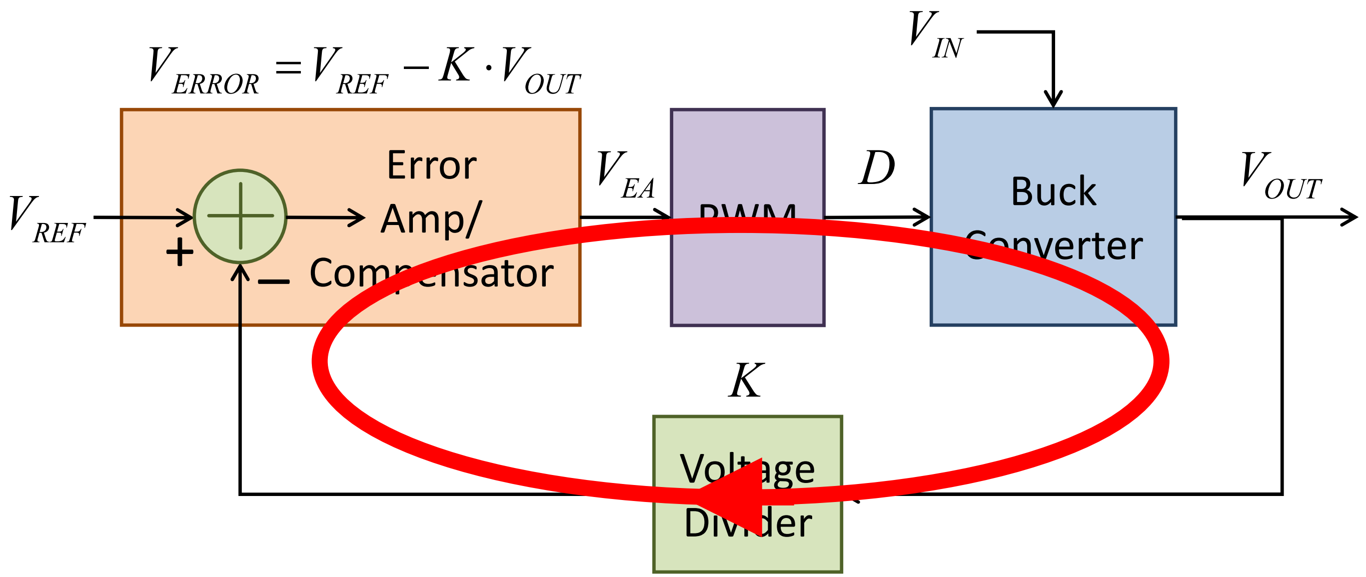
$$T(s) = K \cdot H_{EA}(s) \cdot G_{PWM} \cdot G_{VD}(s)$$

$$H_{EA}(s) = \frac{T(s)}{K \cdot G_{PWM} \cdot G_{VD}(s)}$$



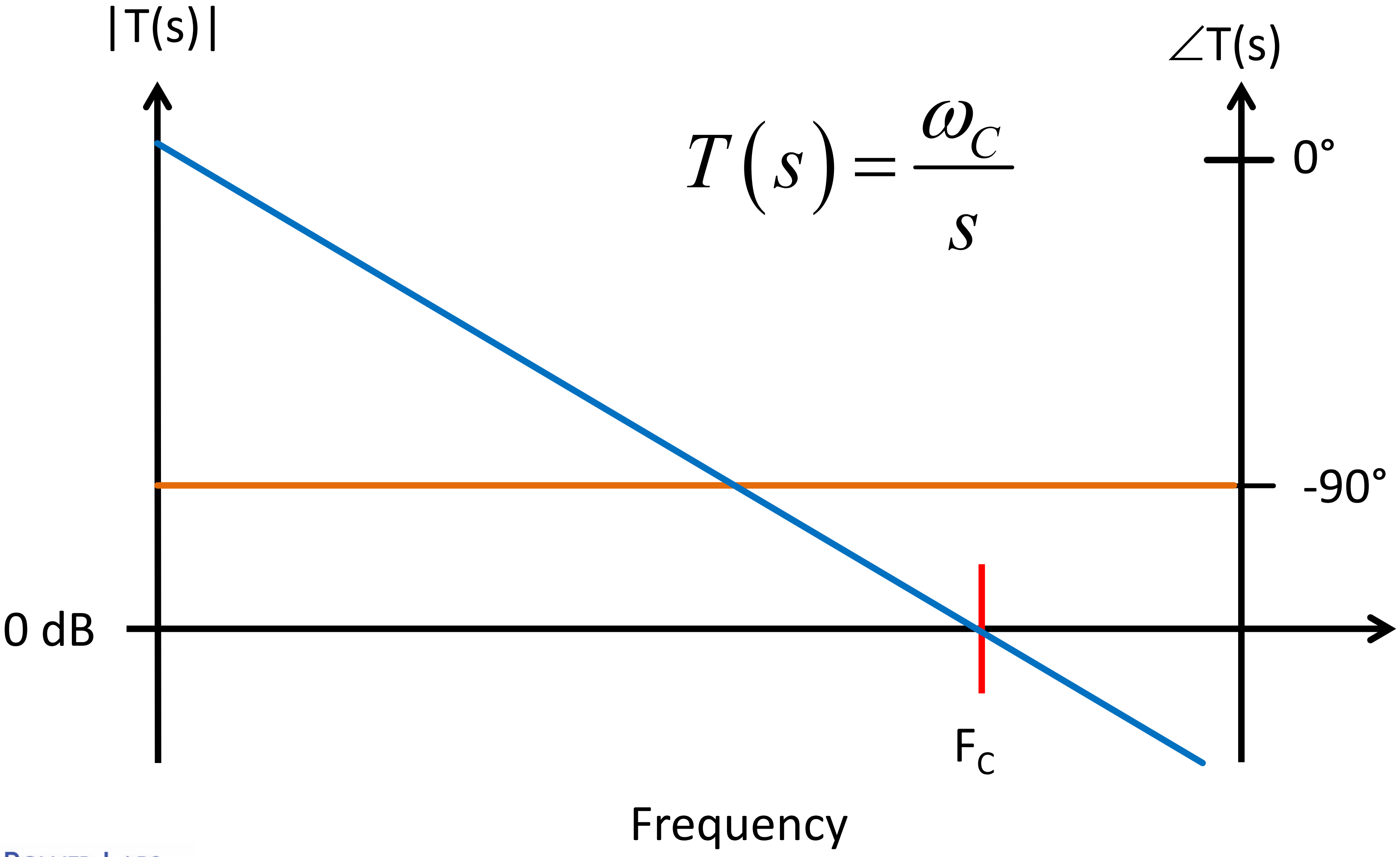
Choose T(s)

Loop Gain T(s)

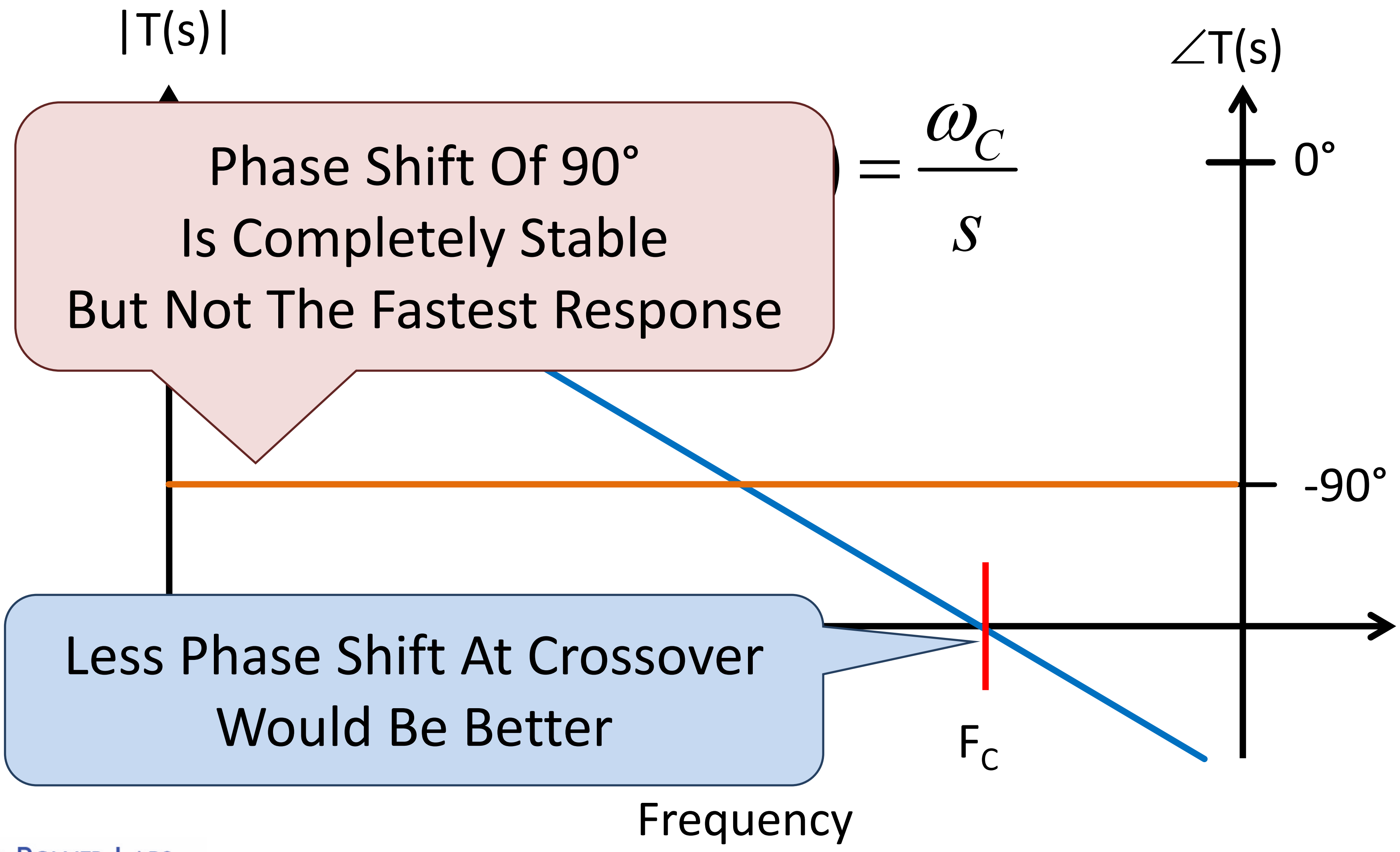


$V_{ERROR} = V_{REF} - K \cdot V_{OUT}$
V_{IN}
V_{REF}
+
–
Error Amp/ Compensator
V_{EA}
PWM
D
Buck Converter
V_{OUT}
K
Voltage Divider

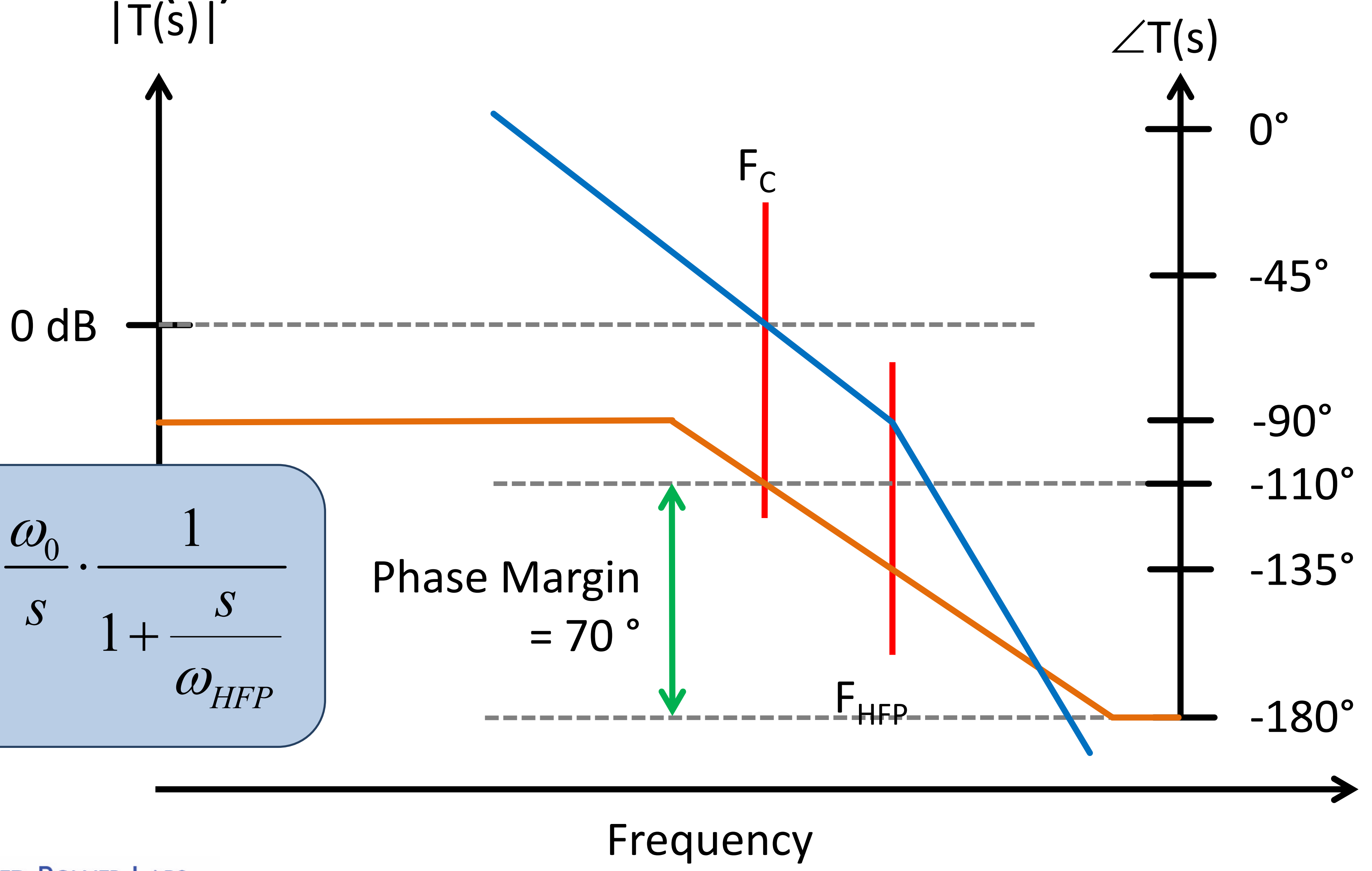
Ideal T(s): Simple Integrator?



Ideal T(s): Simple Integrator?

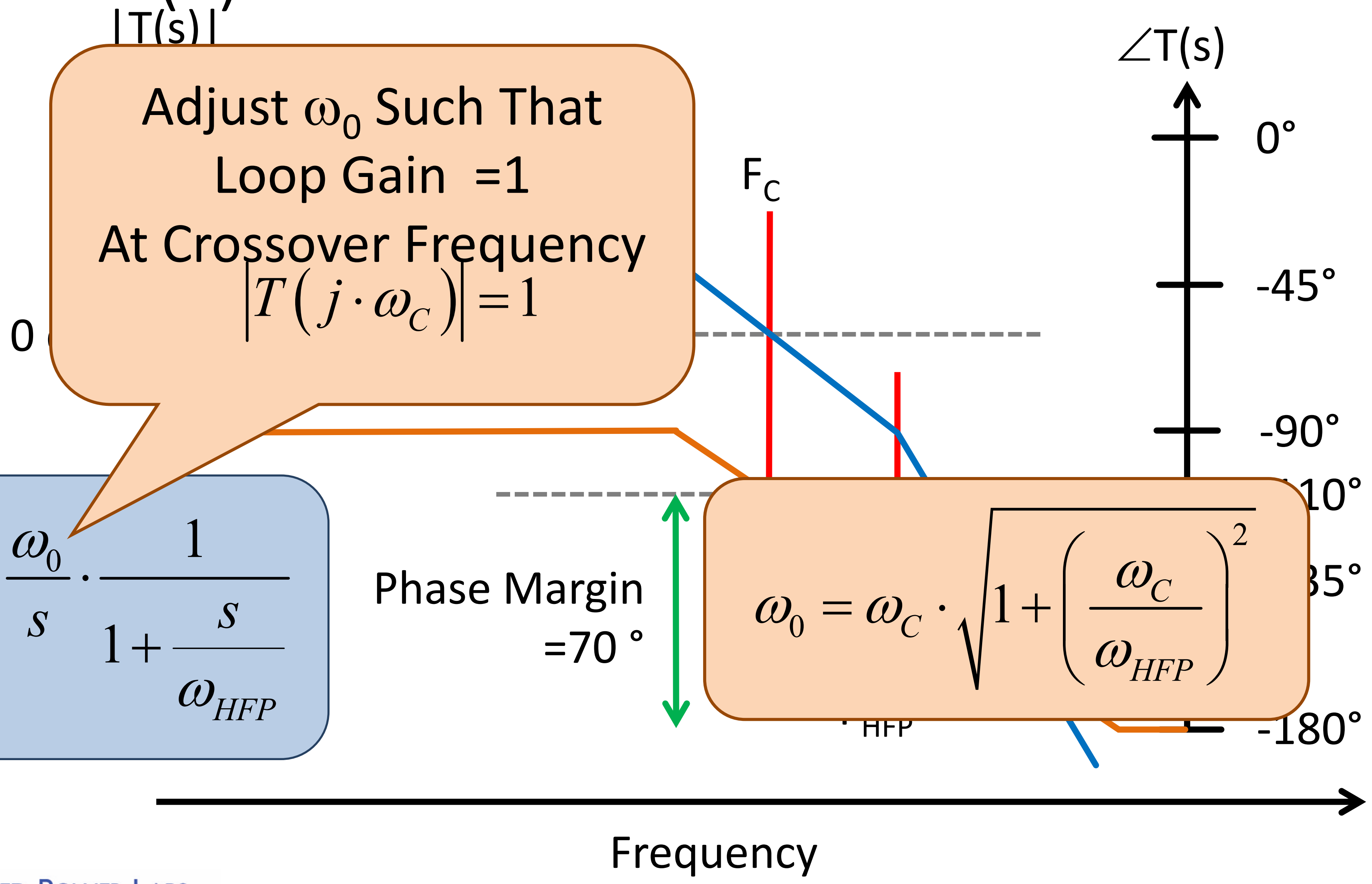


Better T(s)



$$T(s) = \frac{\omega_0}{s} \cdot \frac{1}{1 + \frac{s}{\omega_{HFP}}}$$

Better T(s)



Adjust ω_0 Such That
Loop Gain =1
At Crossover Frequency

$$\left|T\left(j\cdot\omega_C\right)\right|=1$$

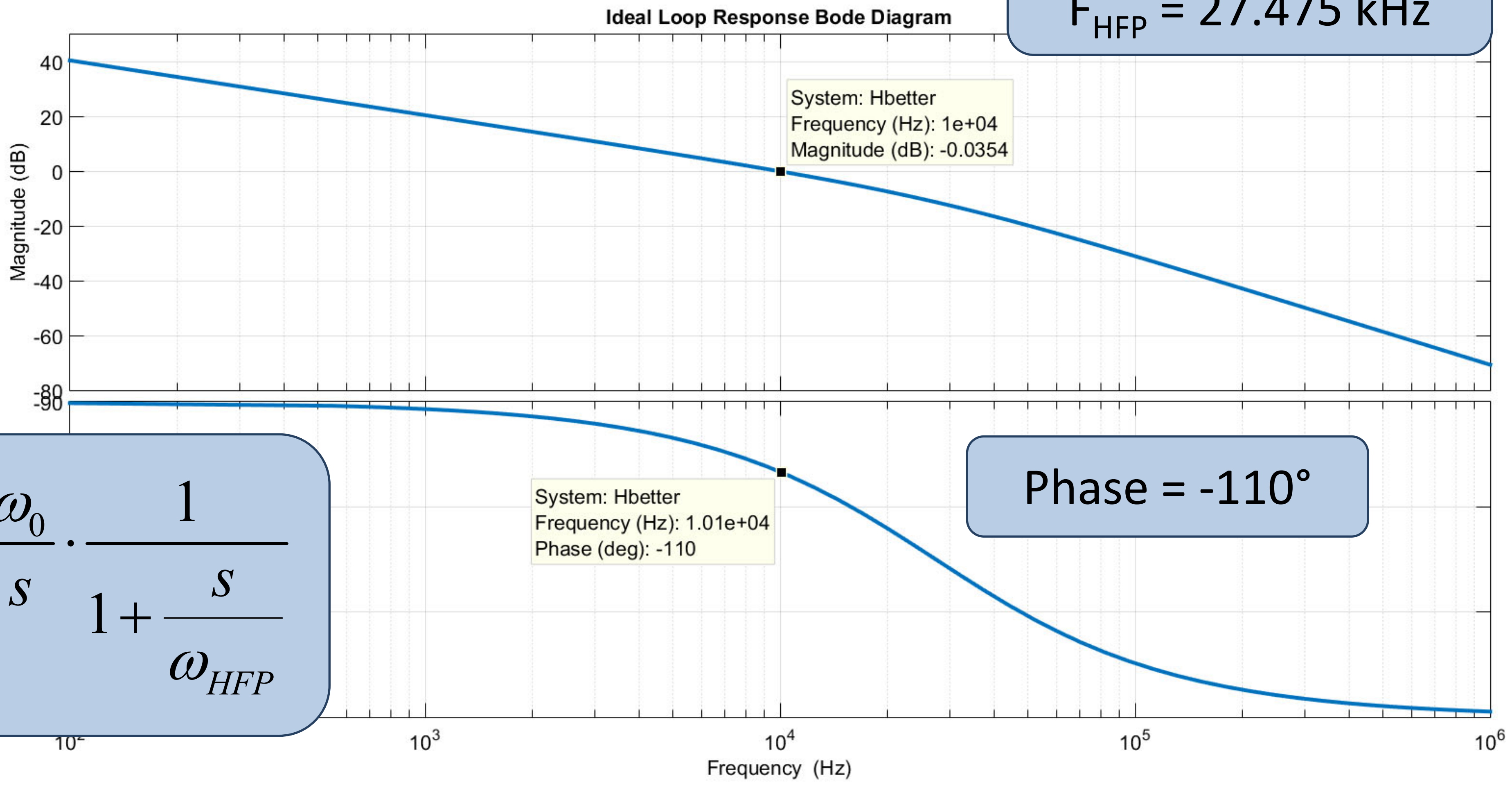
$$T(s)=\frac{\omega_0}{s}\cdot\frac{1}{1+\frac{s}{\omega_{HFP}}}$$

$$\omega_0=\omega_C\cdot\sqrt{1+\left(\frac{\omega_C}{\omega_{HFP}}\right)^2}$$

Better T(s)

F_C = 10 kHz

F_{HFP} = 27.475 kHz

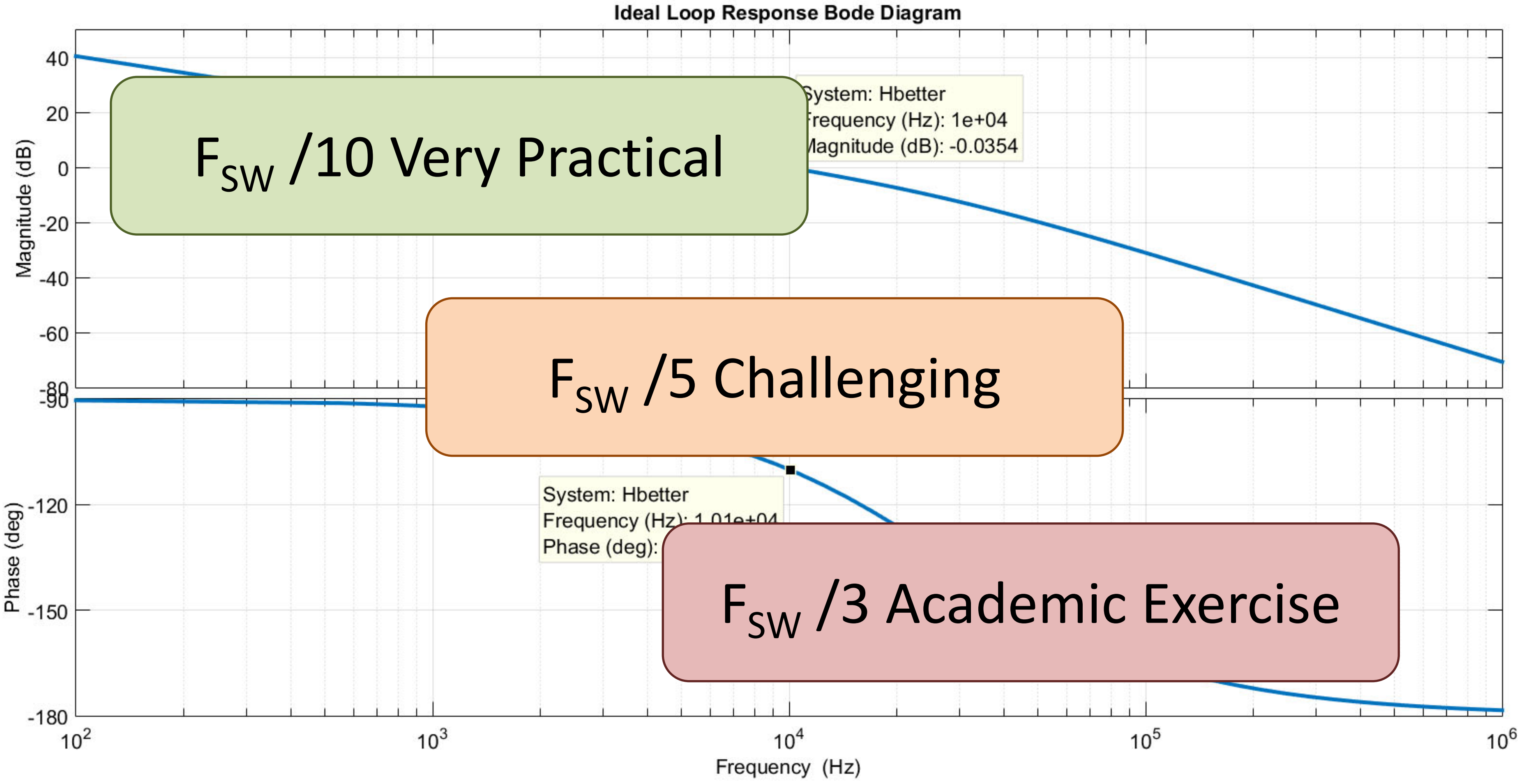


Phase = -110°

$$T(s) = \frac{\omega_0}{s} \cdot \frac{1}{1 + \frac{s}{\omega_{HFP}}}$$

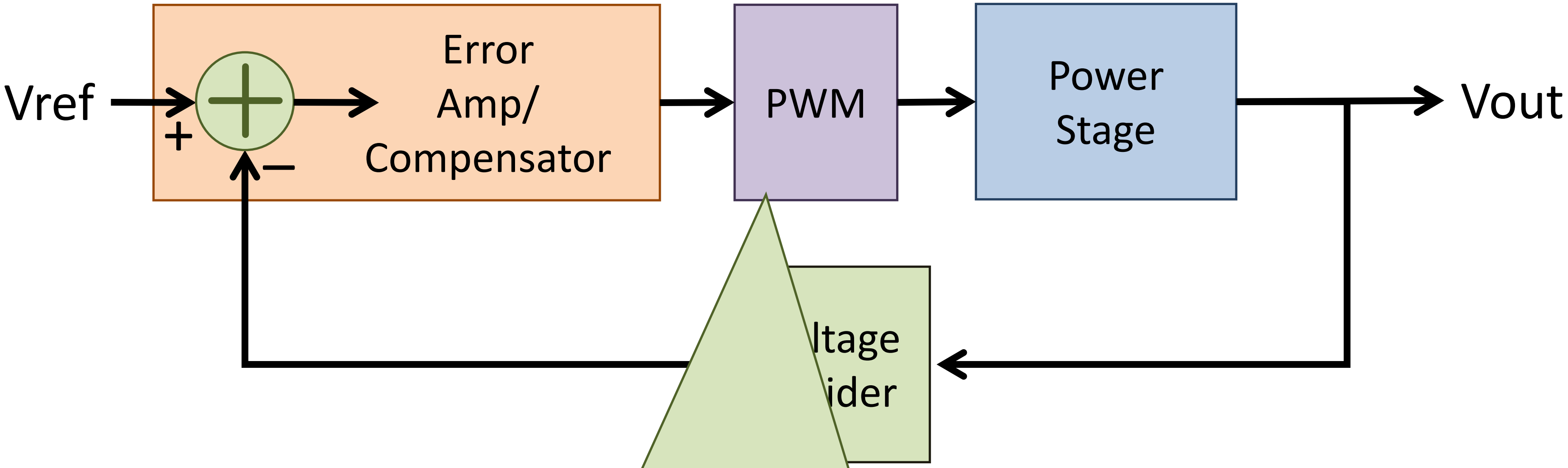
Embedded Power Labs

Better T(s): Practical Crossover Frequency



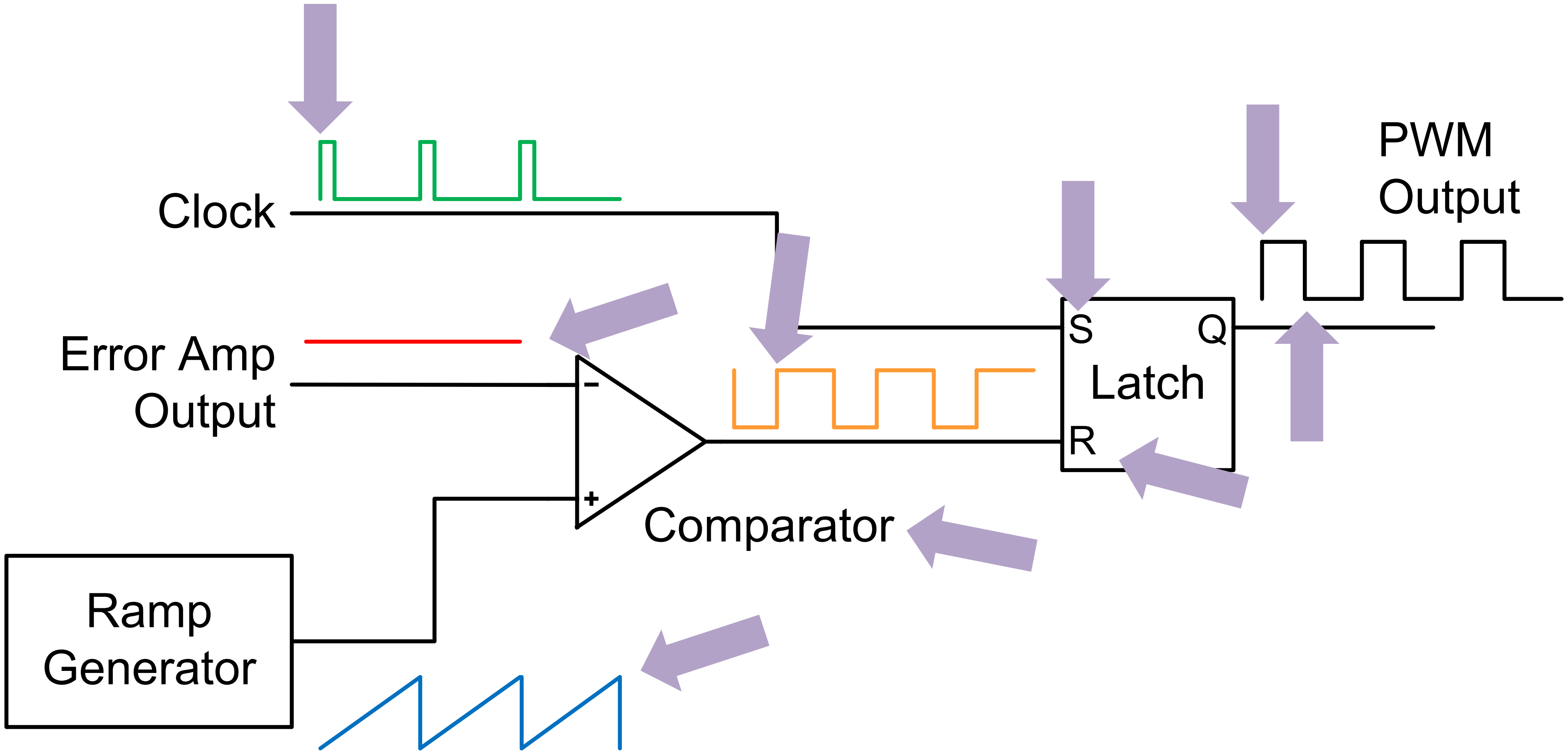
Small Signal Model Of The Pulse Width Modulator G_{PWM}

Small Signal Model Of Pulse-Width Modulator

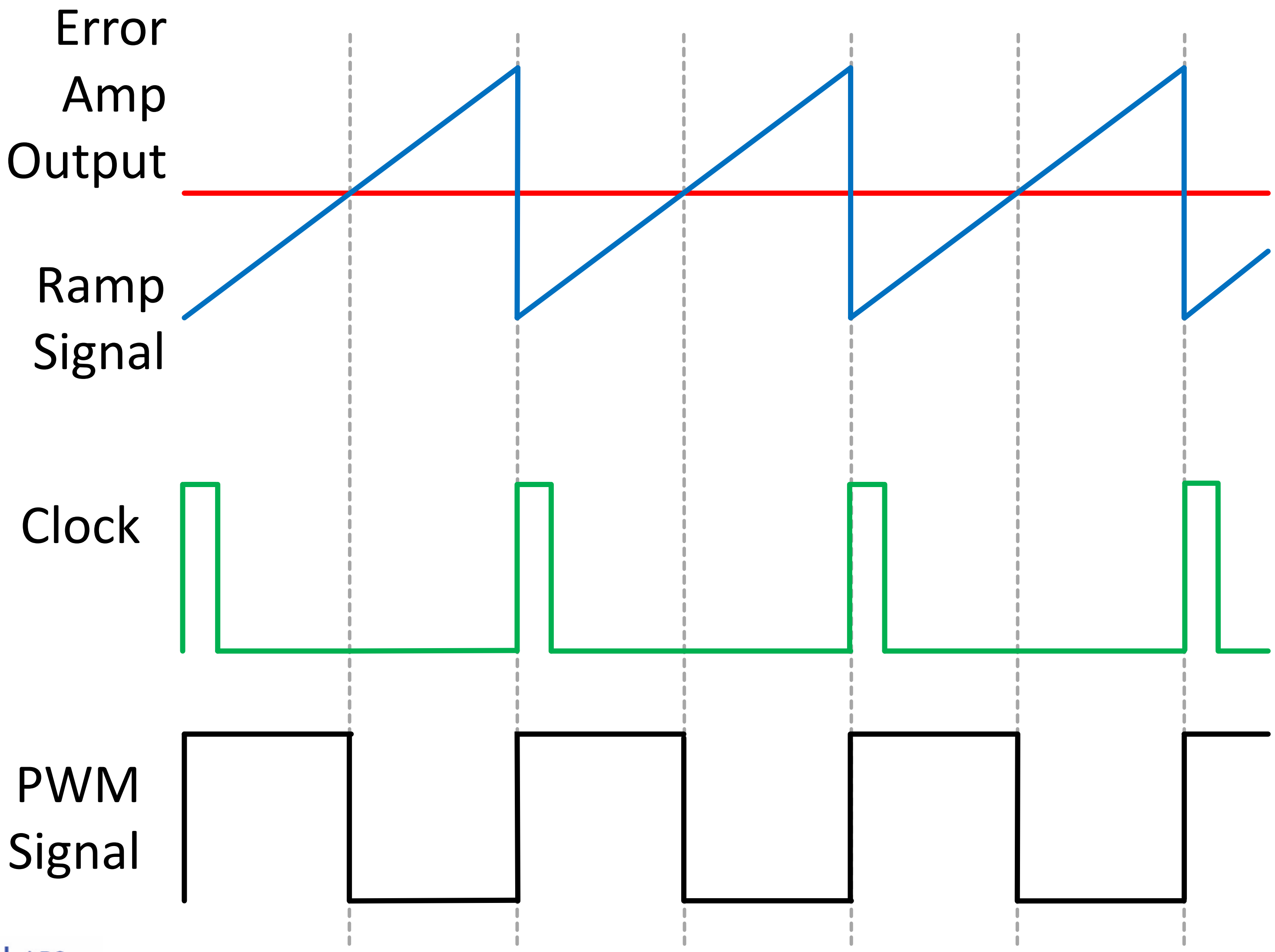


We Need To Calculate The Gain Of the Modulator: $G_{PWM}(s)$

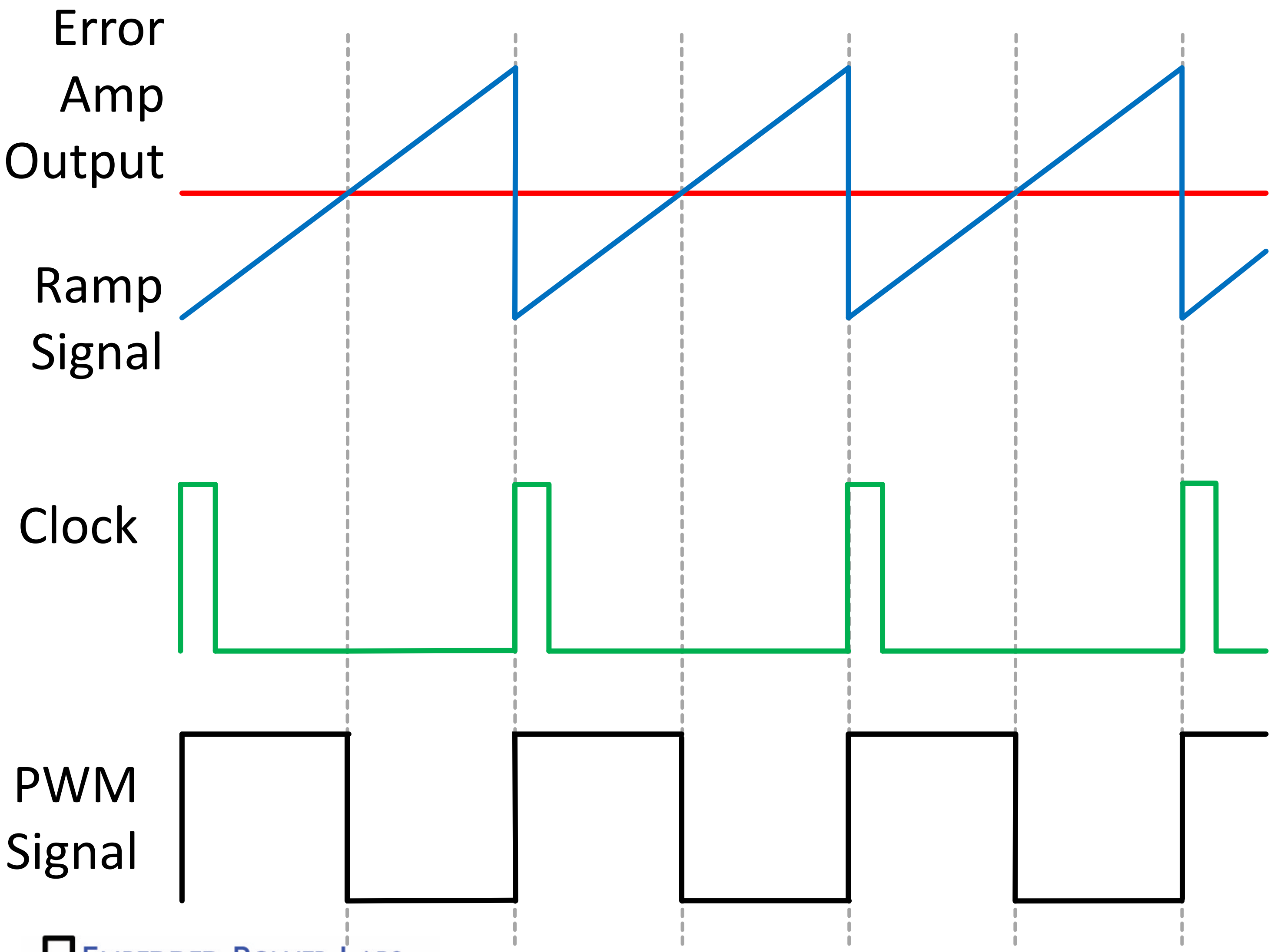
Pulse Width Modulator



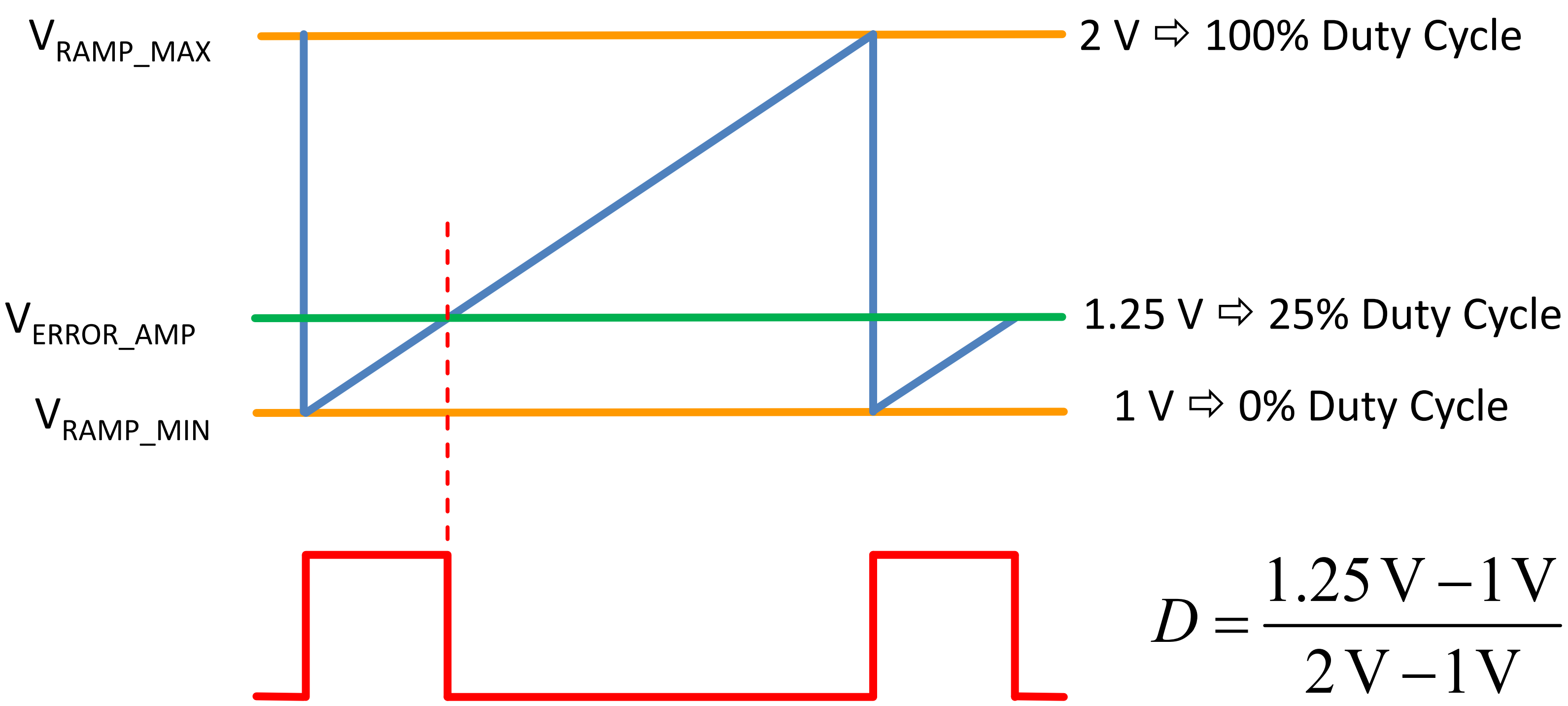
Analog Pulse Width Modulator



Video Lab 9: Trailing Edge PWM

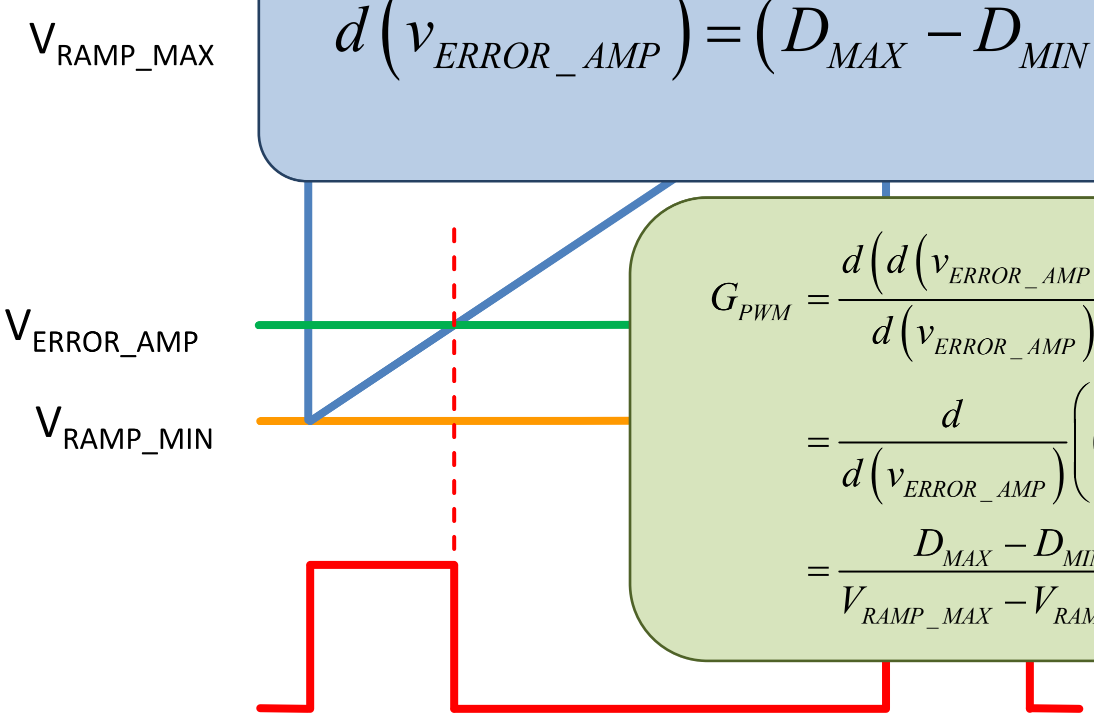


Small Signal Model Of Pulse-Width Modulator



$$D = \frac{1.25\,\text{V} - 1\,\text{V}}{2\,\text{V} - 1\,\text{V}} = 0.25$$

Small Signal Model Of Pulse-Width Modulator



$$d\left(v_{ERROR\_AMP}\right) = \left(D_{MAX} - D_{MIN}\right) \cdot \frac{v_{ERROR\_AMP} - V_{RAMP\_MIN}}{V_{RAMP\_MAX} - V_{RAMP\_MIN}}$$

$$G_{PWM} = \frac{d\left(d\left(v_{ERROR\_AMP}\right)\right)}{d\left(v_{ERROR\_AMP}\right)}$$

$$= \frac{d}{d\left(v_{ERROR\_AMP}\right)}\left(\left(D_{MAX} - D_{MIN}\right) \cdot \frac{v_{ERROR\_AMP} - V_{RAMP\_MIN}}{V_{RAMP\_MAX} - V_{RAMP\_MIN}}\right)$$

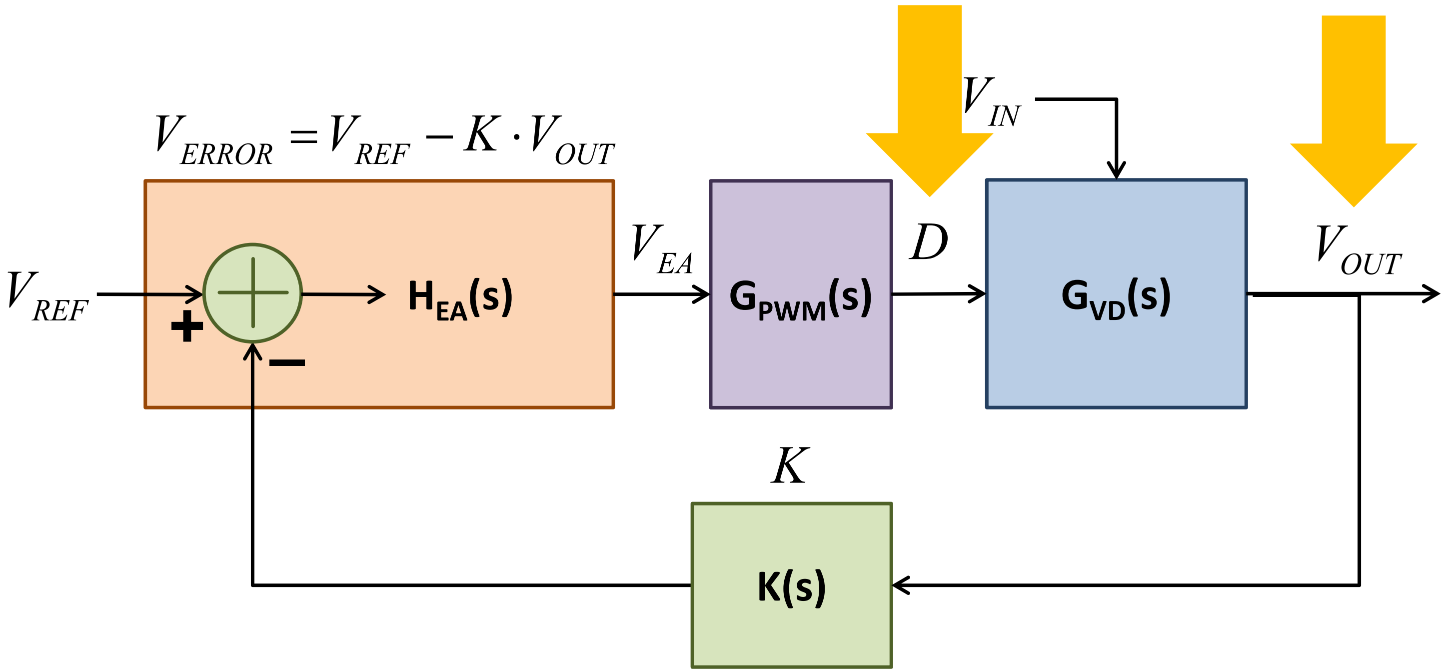
$$= \frac{D_{MAX} - D_{MIN}}{V_{RAMP\_MAX} - V_{RAMP\_MIN}}$$

$$D_{MAX} = 1 \qquad D_{MIN} = 0$$

$$G_{PWM} = \frac{1}{\Delta V_{RAMP}}$$

Control-To-Output Transfer Function G_{VD}

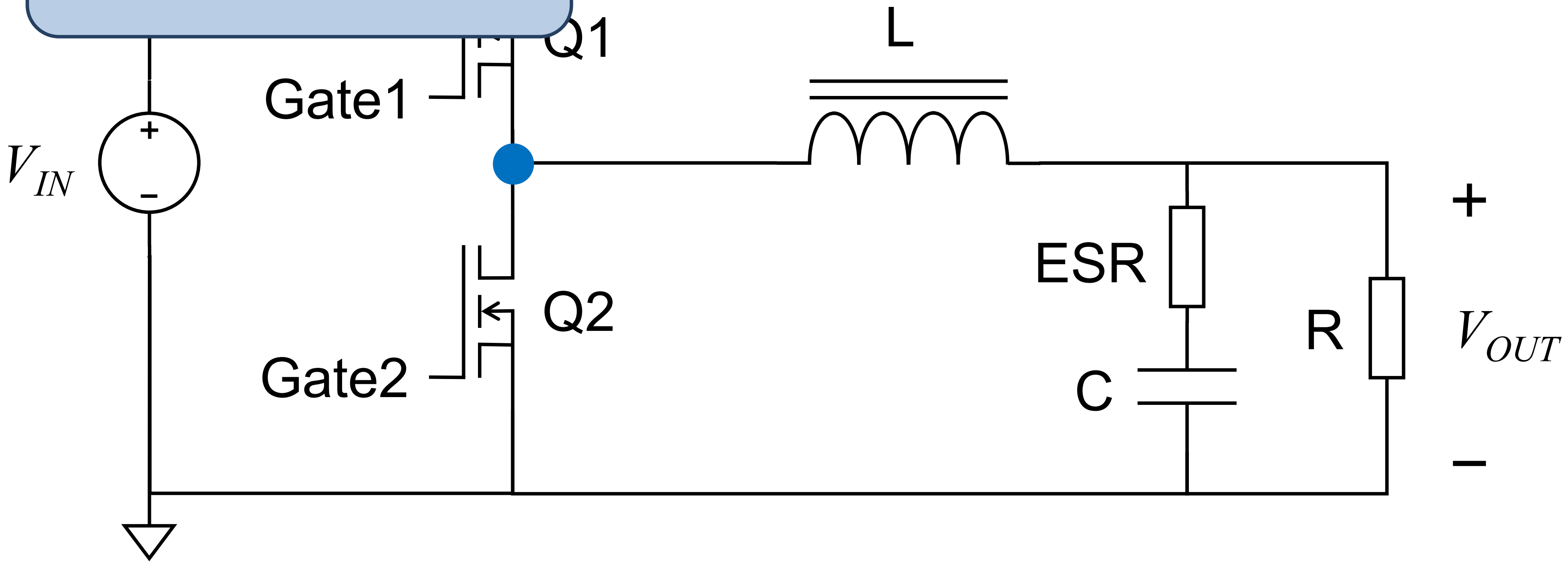
Buck Converter G_{VD}



- Derivation Of Control-To-Output Transfer Functions Generally Not Trivial
  - See Additional Resources To Learn More

Buck Converter

$$v_{SW}(t) = V_{IN} \cdot d(t)$$

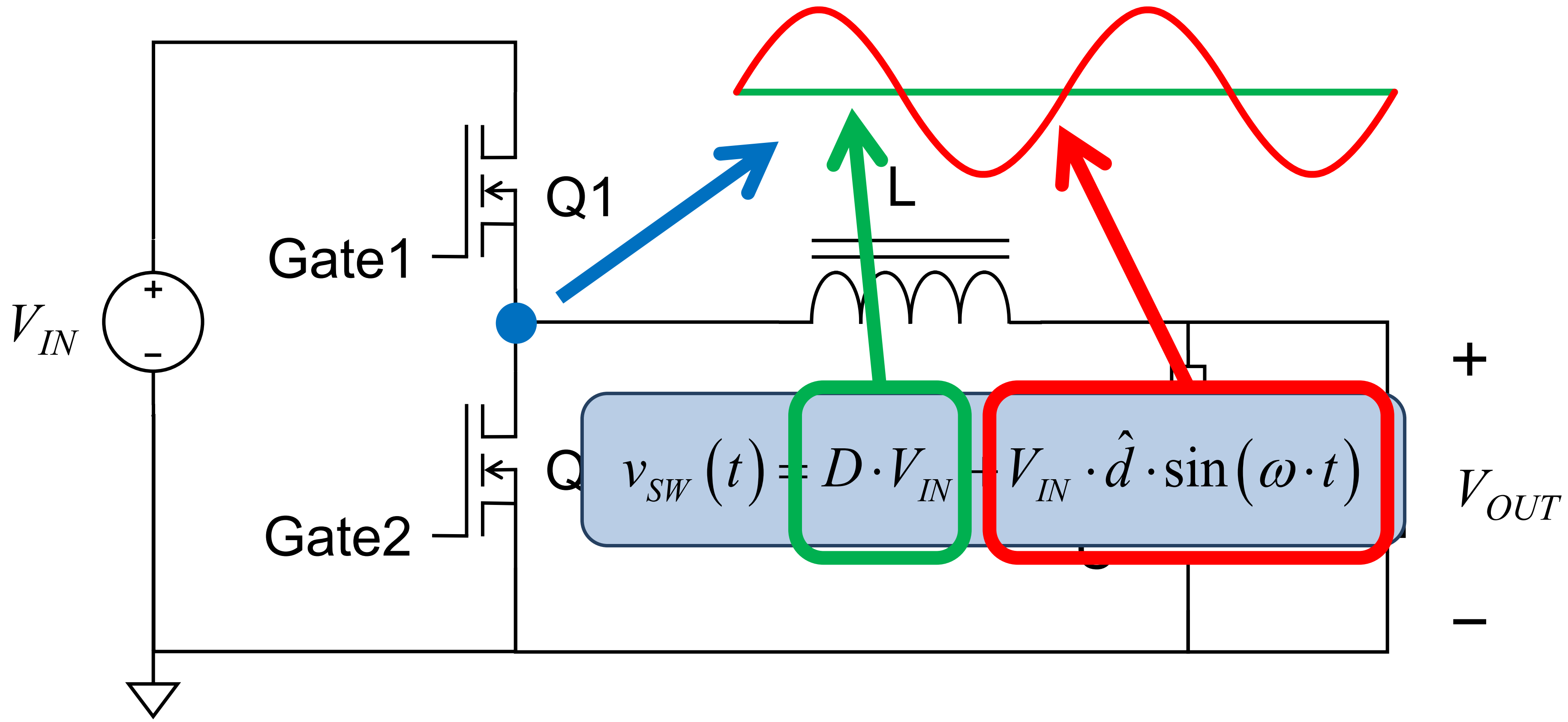


$$d(t) = D + \hat{d}(t)$$

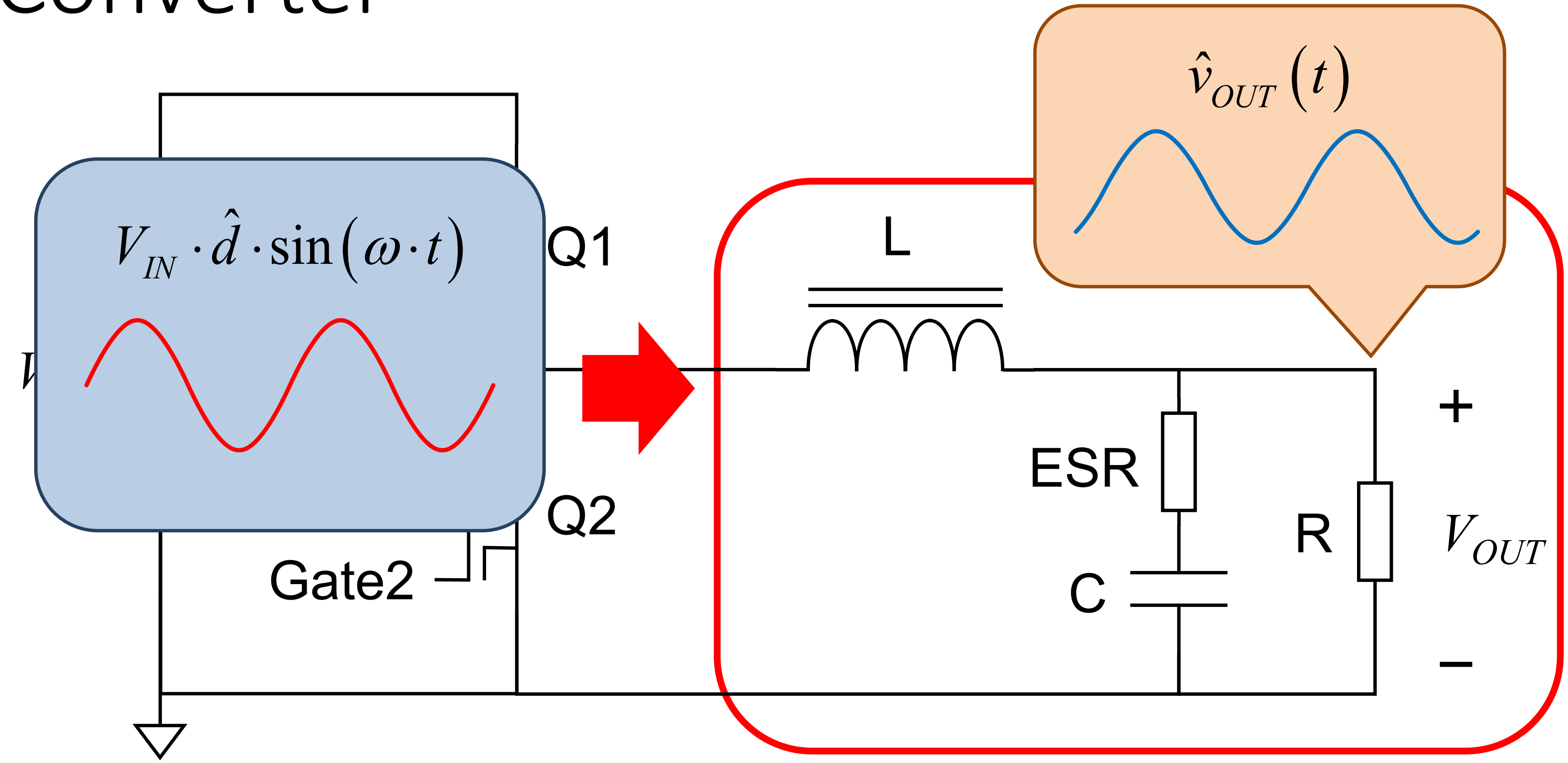
$$\hat{d} \ll D$$

$$\omega \ll \omega_{SW}$$

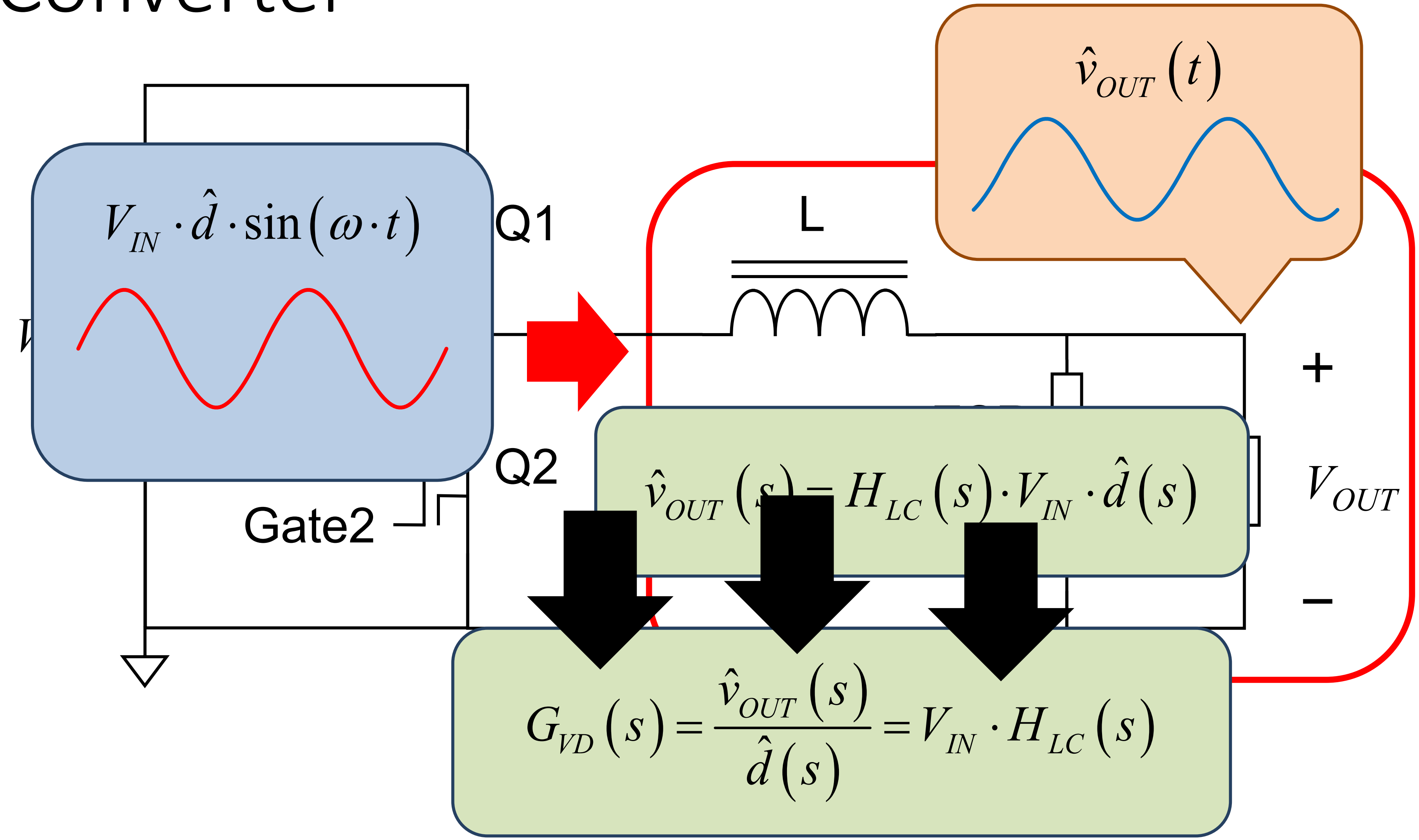
Buck Converter



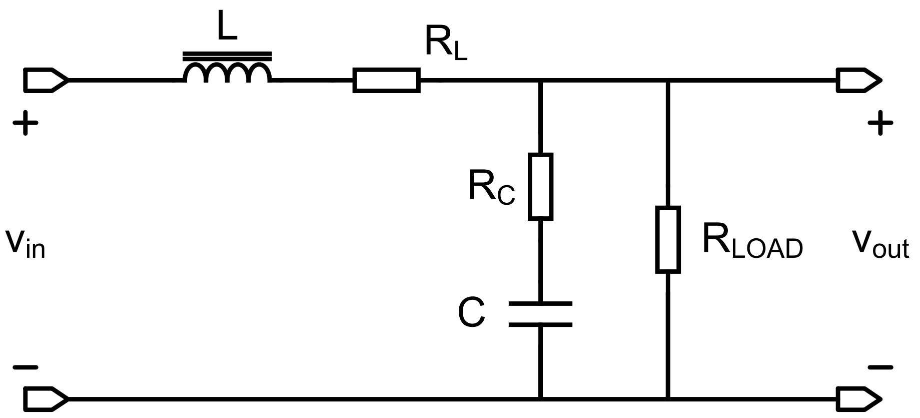
Buck Converter



Buck Converter



L-C Filter Transfer Function



$$v_{out}(s) = \frac{R_{LOAD} \parallel \left(R_C + \frac{1}{s \cdot C} \right)}{s \cdot L + R_L + R_{LOAD} \parallel \left(R_C + \frac{1}{s \cdot C} \right)} \cdot v_{in}(s)$$

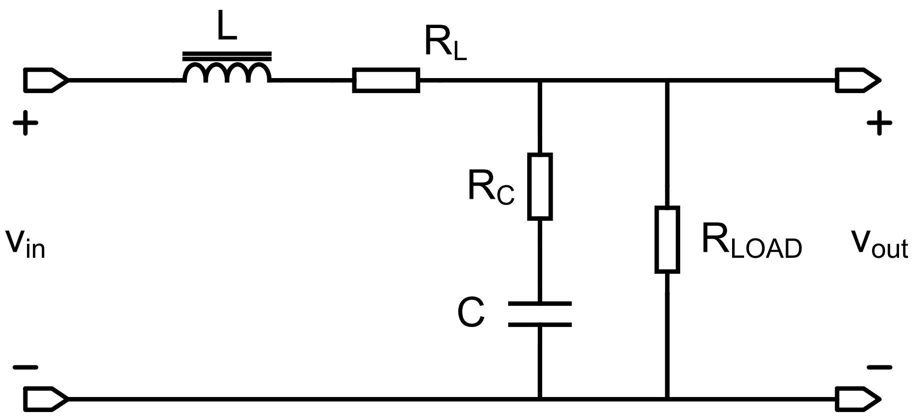
$$R_C \ll R_{LOAD} \qquad R_L \ll R_{LOAD}$$

ESR Zero

$$H_{LC}(s) \approx \frac{1 + s \cdot R_C \cdot C}{1 + \frac{L}{R_{LOAD}} \cdot s + L \cdot C \cdot s^2}$$

LC Double Pole

L-C Filter Transfer Function



$$v_{out}(s) = \frac{R_{LOAD} \parallel \left(R_C + \frac{1}{s \cdot C}\right)}{s \cdot L + R_L + R_{LOAD} \parallel \left(R_C + \frac{1}{s \cdot C}\right)} \cdot v_{in}(s)$$

$$R_C \ll R_{LOAD} \qquad R_L \ll R_{LOAD}$$

$$H_{LC}(s) \approx \frac{1 + s \cdot R_C \cdot C}{1 + \frac{L}{R_{LOAD}} \cdot s + L \cdot C \cdot s^2}$$

$$H_{LC}(s) \approx \frac{1 + \frac{s}{\omega_Z}}{1 + \frac{\omega_P \cdot L}{R_{LOAD}} \cdot \left(\frac{s}{\omega_P}\right) + \left(\frac{s}{\omega_P}\right)^2}$$

$$= \frac{1 + \frac{s}{\omega_Z}}{1 + \frac{1}{Q} \cdot \left(\frac{s}{\omega_P}\right) + \left(\frac{s}{\omega_P}\right)^2}$$

$$Q = \frac{R_{LOAD}}{\omega_P \cdot L}$$

$$\omega_Z = \frac{1}{R_C \cdot C}$$

$$\omega_P^2 = \frac{1}{L \cdot C}$$

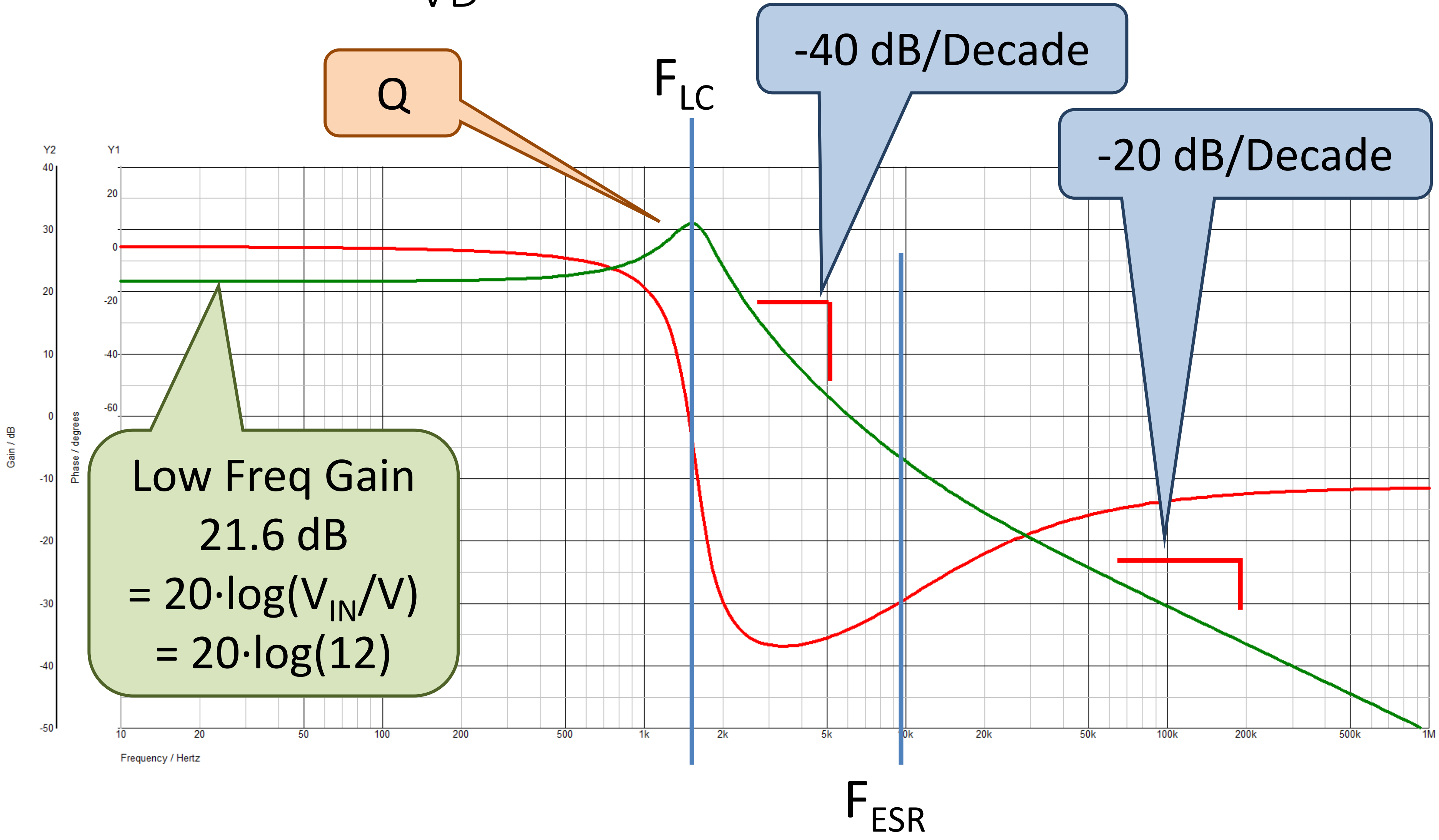
Buck Converter $G_{VD}(s)$

$$G_{VD}(s) = V_{IN} \cdot H_{LC}(s) \approx V_{IN} \cdot \frac{1 + s \cdot R_C \cdot C}{1 + \frac{L}{R_{LOAD}} \cdot s + L \cdot C \cdot s^2}$$

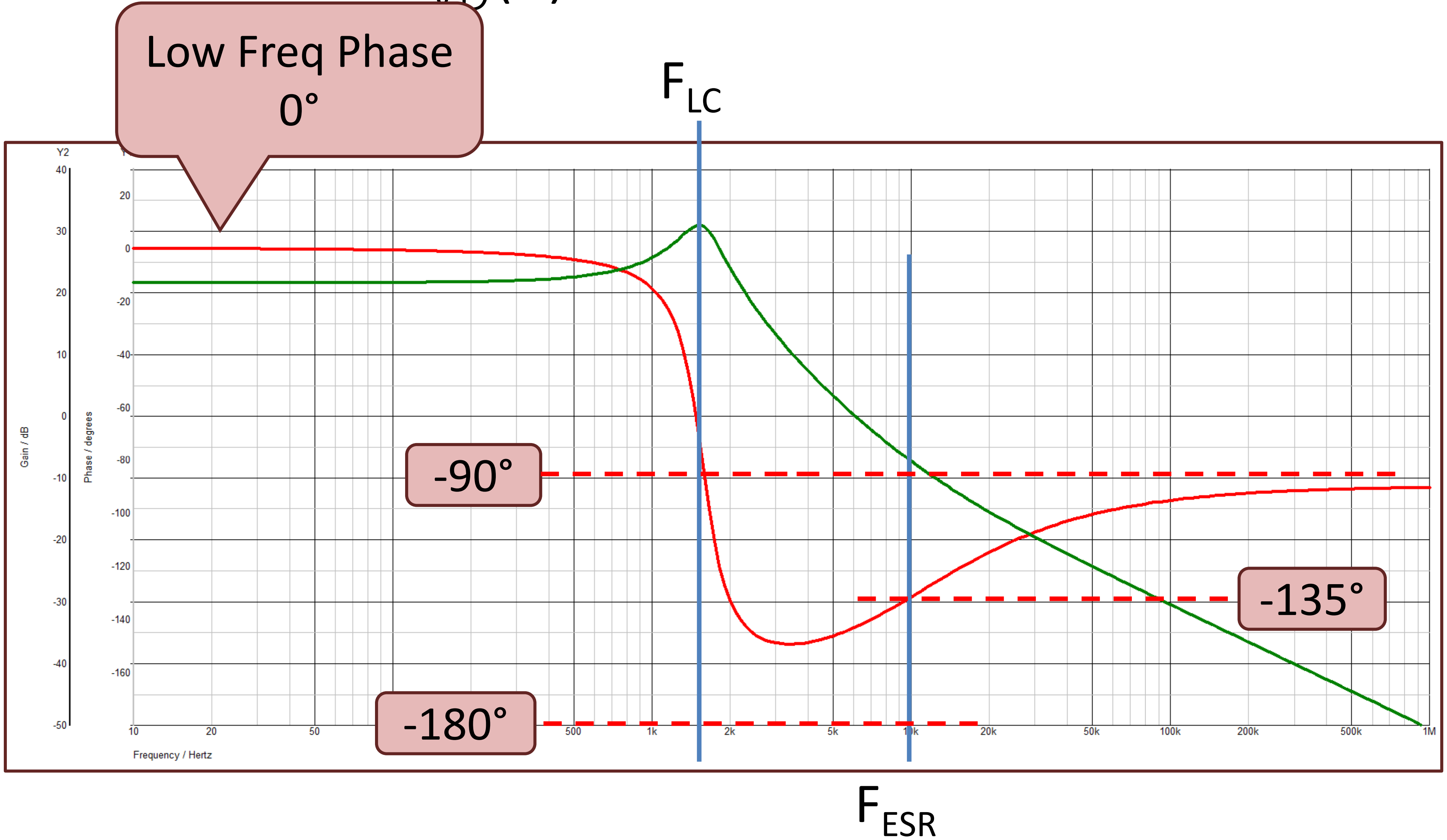
$$= V_{IN} \cdot \frac{1 + \frac{s}{\omega_{Z\_ESR}}}{1 + \frac{1}{Q} \cdot \left(\frac{s}{\omega_{LC}}\right) + \left(\frac{s}{\omega_{LC}}\right)^2}$$

$$\omega_{Z\_ESR} = \frac{1}{R_C \cdot C} \qquad \omega_{LC}^2 = \frac{1}{L \cdot C} \qquad Q = \frac{R_{LOAD}}{\omega_{LC} \cdot L}$$

Buck Converter $G_{VD}(s)$

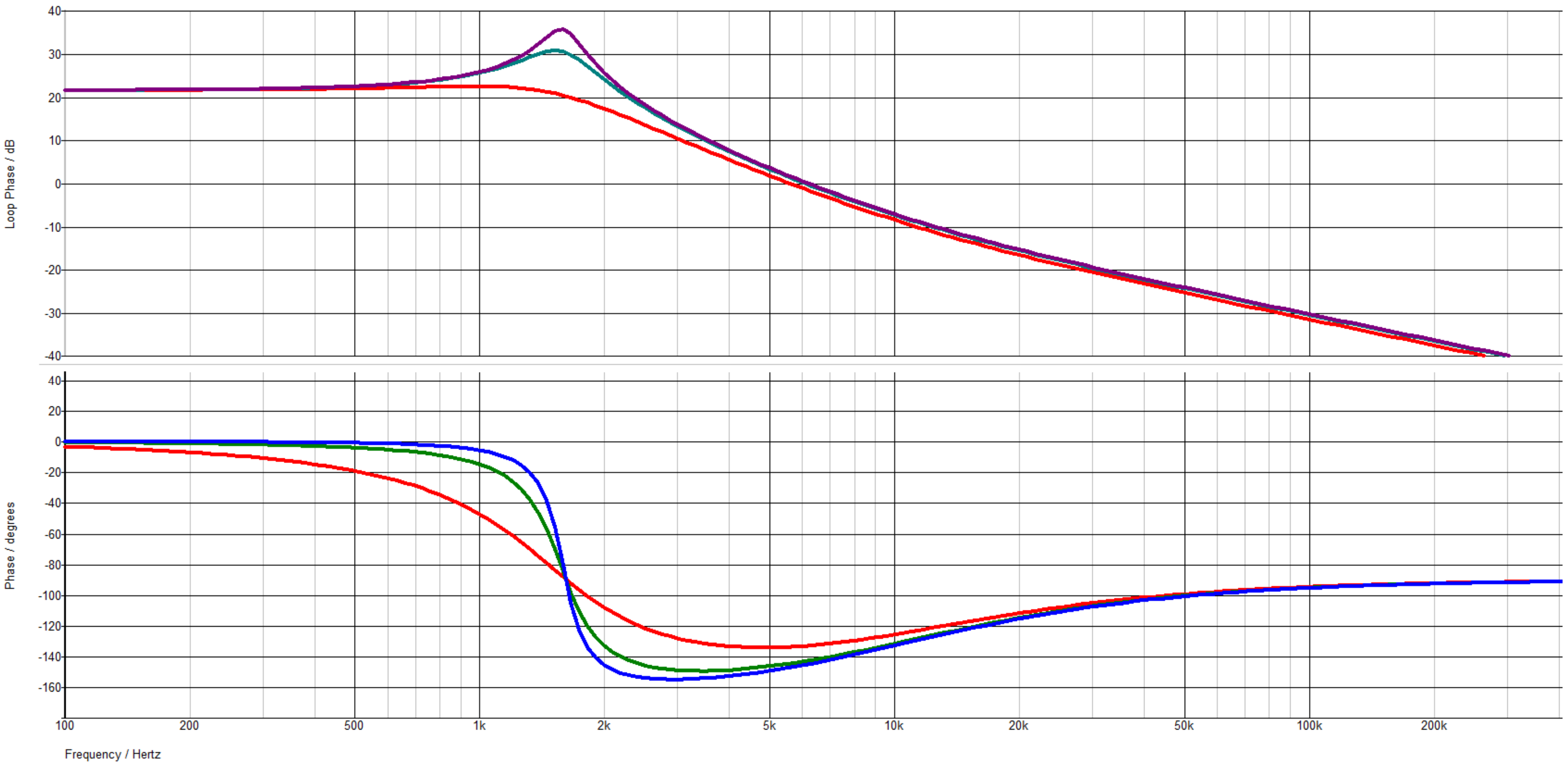


Buck Converter $G_{VD}(s)$



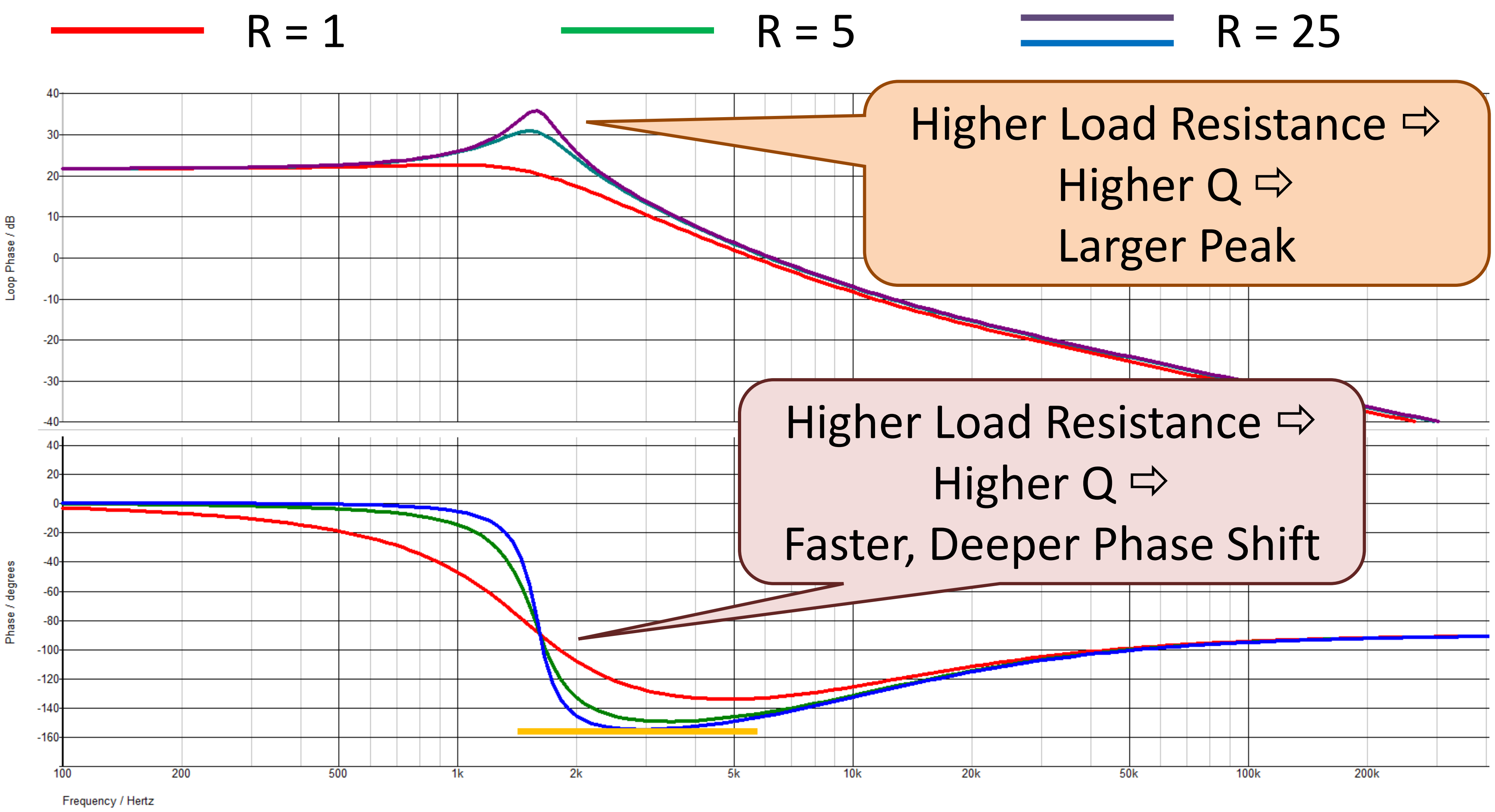
Buck Converter $G_{VD}(s)$

R = 1 R = 5 R = 25



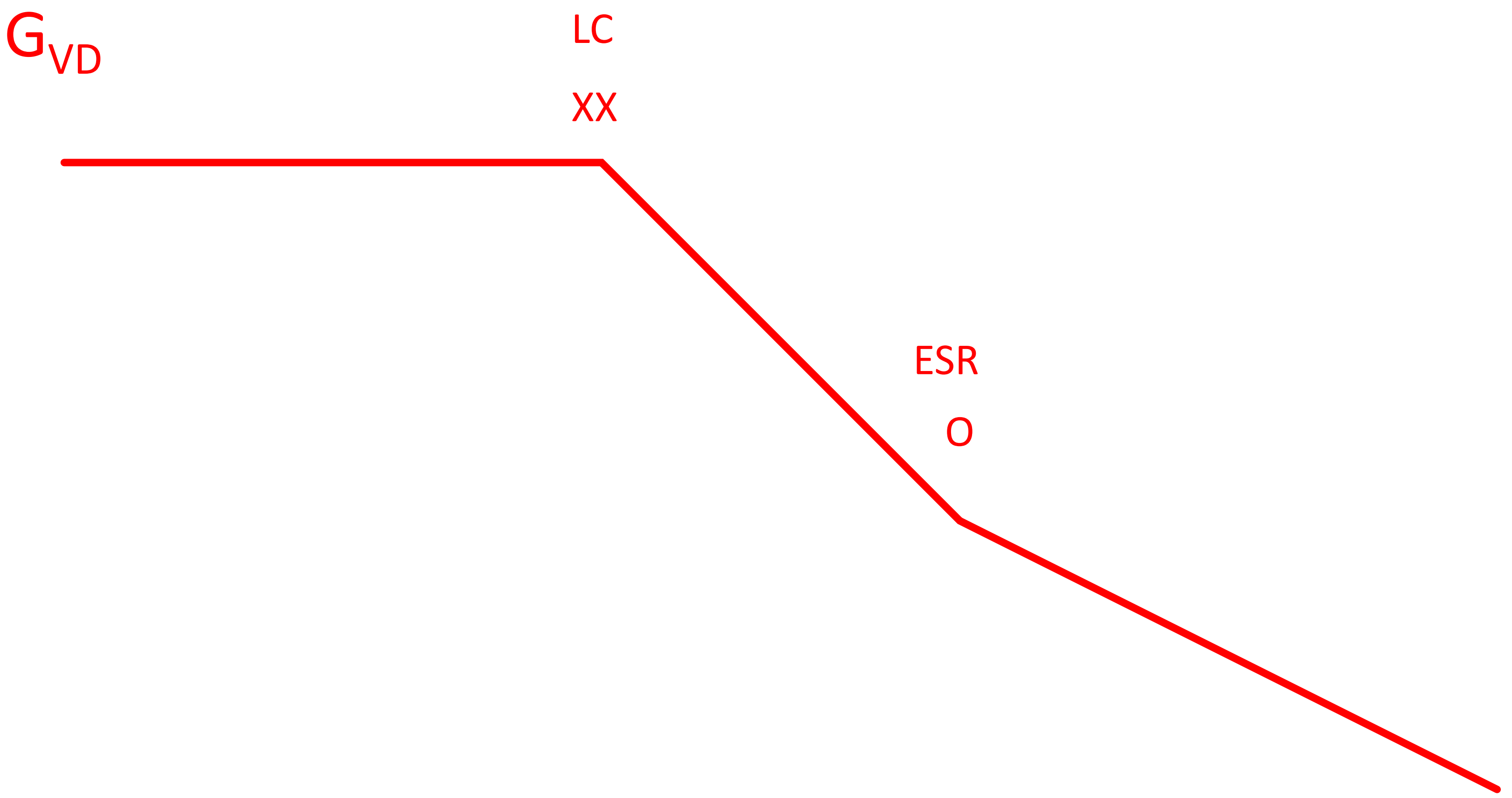
EMBEDDED POWER LABS

Buck Converter $G_{VD}(s)$

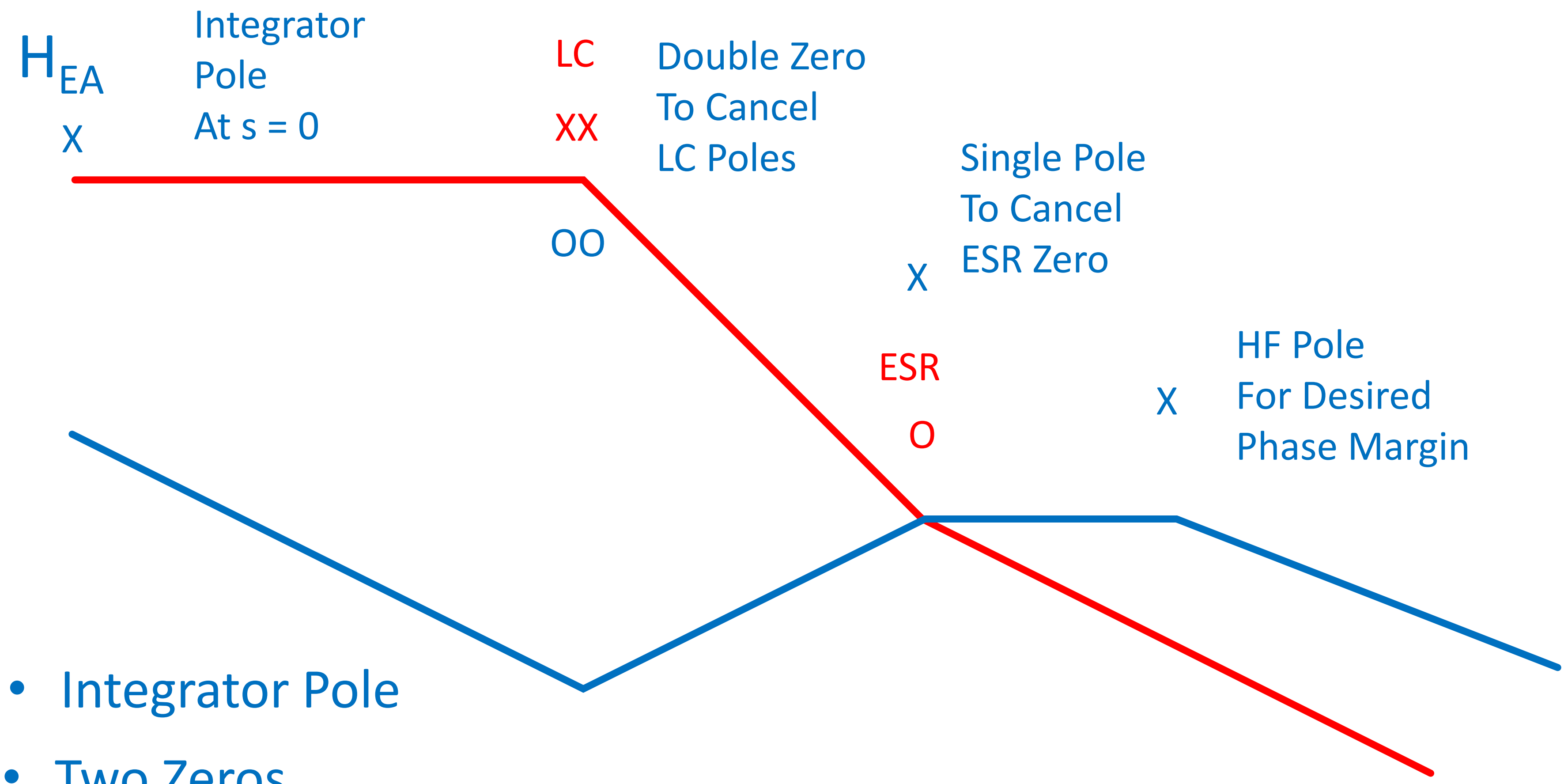


Solving For The Compensator

Compensator Poles And Zeros

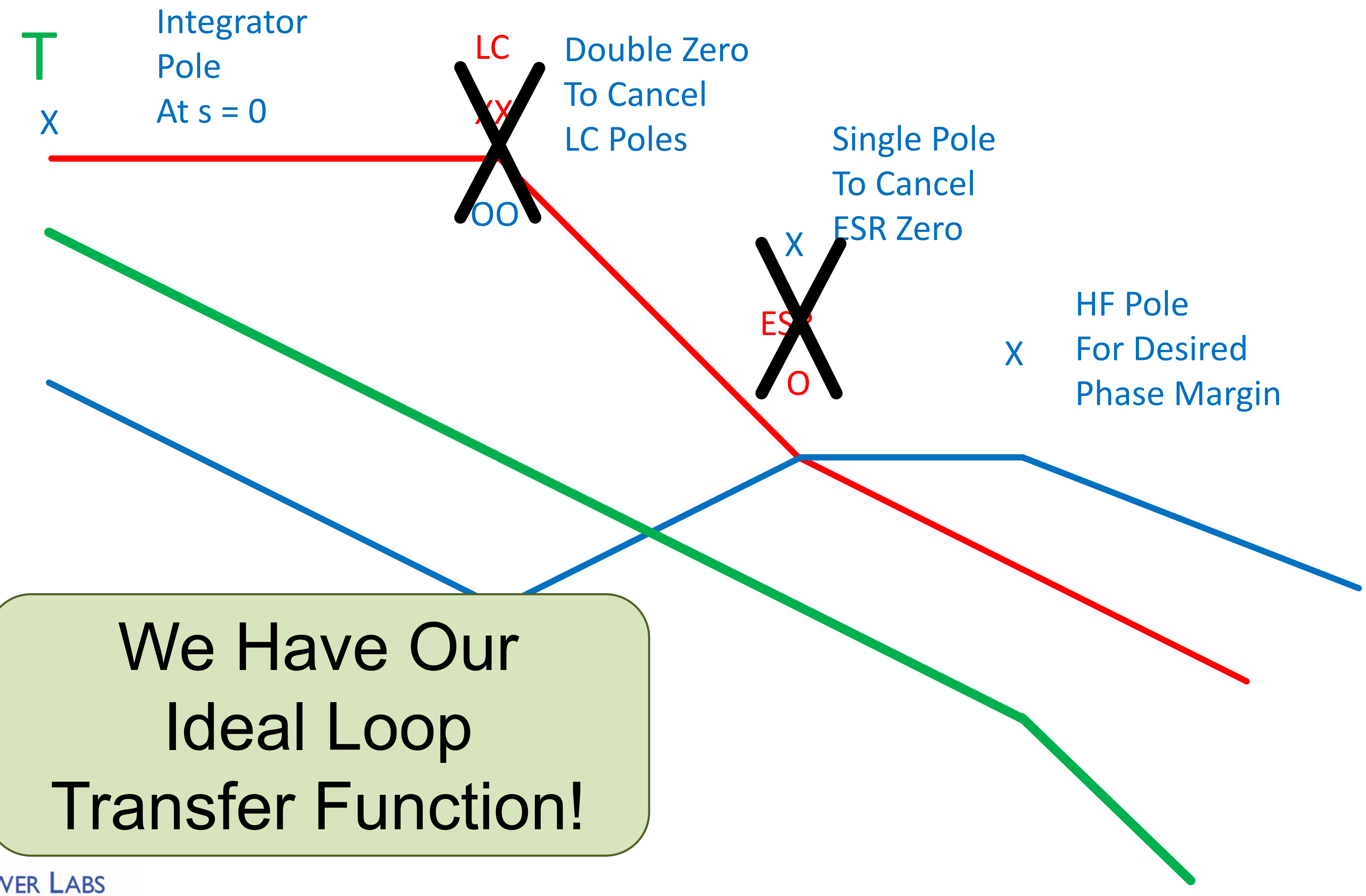


Compensator Poles And Zeros



- Integrator Pole
- Two Zeros
- Two Poles

Compensator Poles And Zeros



We Have Our
Ideal Loop
Transfer Function!

Solving For The Compensator

$$T(s) = K(s) \cdot H_{EA}(s) \cdot G_{PWM}(s) \cdot G_{VD}(s)$$

$$T(s) = \frac{\omega_0}{s} \cdot \frac{1}{1 + \frac{s}{\omega_{HFP}}}$$

$$H_{EA}(s) = \frac{T(s)}{K \cdot G_{PWM} \cdot G_{VD}(s)}$$

$$\omega_0 = \omega_C \cdot \sqrt{1 + \left(\frac{\omega_C}{\omega_{HFP}}\right)^2}$$

$$\omega_{HFP} = \frac{\omega_C}{\tan(90 - PM)}$$

$$K = 1$$

$$G_{PWM} = \frac{1}{V_{RAMP}}$$

$$G_{VD}(s) = V_{IN} \cdot \frac{1 + \frac{s}{\omega_{Z\_ESR}}}{1 + \frac{1}{Q} \cdot \left(\frac{s}{\omega_{LC}}\right) + \left(\frac{s}{\omega_{LC}}\right)^2}$$

Solving For The Compensator

Ideal Compensator Transfer Fu[nction]

$$H_{EA}(s)=\frac{\dfrac{\omega_0}{s}\cdot\dfrac{1}{1+\dfrac{s}{\omega_{HFP}}}}{\dfrac{1}{V_{RAMP}}\cdot\left(V_{IN}\cdot\dfrac{1+\dfrac{s}{\omega_{Z\_ESR}}}{1+\dfrac{1}{Q}\cdot\left(\dfrac{s}{\omega_{LC}}\right)+\left(\dfrac{s}{\omega_{LC}}\right)^2}\right)}$$

$$=\frac{\omega_{P0}}{s}\frac{1+\dfrac{1}{Q}\cdot\left(\dfrac{s}{\omega_{LC}}\right)+\left(\dfrac{s}{\omega_{LC}}\right)^2}{\left(1+\dfrac{s}{\omega_{Z\_ESR}}\right)\cdot\left(1+\dfrac{s}{\omega_{HFP}}\right)}$$

Two Zeros

Integrator Pole

Two Poles

$$\omega_{P0}=\frac{V_{RAMP}\cdot\omega_0}{V_{IN}}=\frac{V_{RAMP}}{V_{IN}}\cdot\omega_C\cdot\sqrt{1+\left(\frac{\omega_C}{\omega_{HFP}}\right)^2}$$

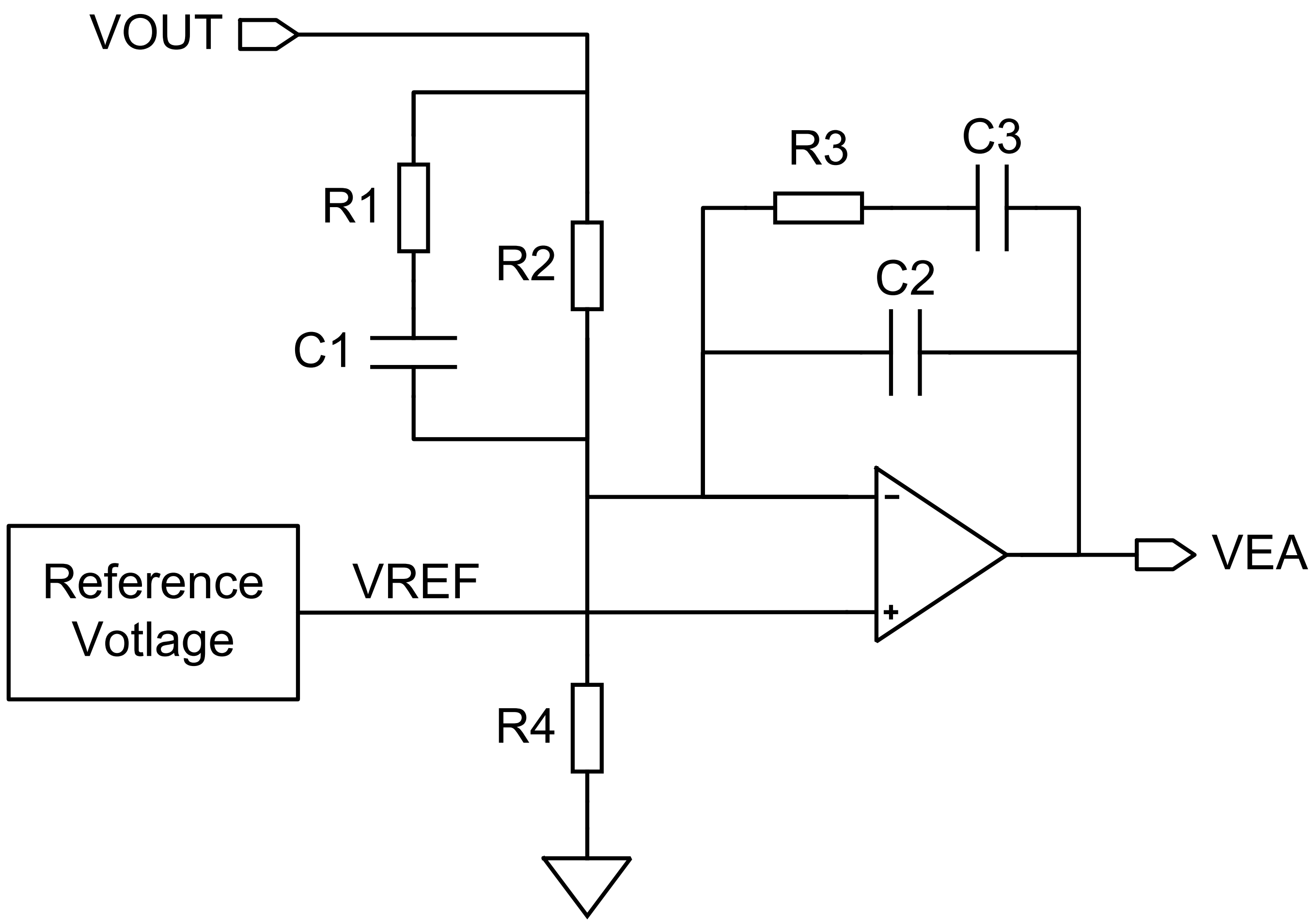
Practical Compensator Transfer Function

$$H_{EA}(s) = \frac{\omega_{P0}}{s} \cdot \frac{\left(1+\frac{s}{\omega_{Z1}}\right)\cdot\left(1+\frac{s}{\omega_{Z2}}\right)}{\left(1+\frac{s}{\omega_{P1}}\right)\cdot\left(1+\frac{s}{\omega_{P2}}\right)}$$

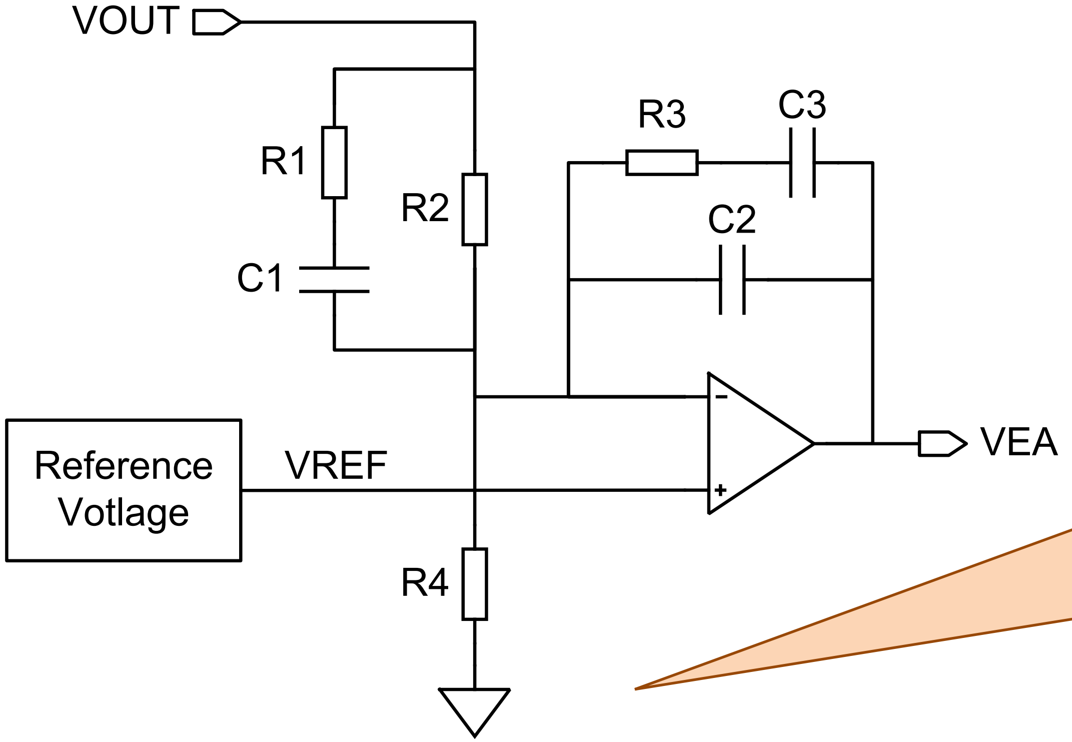
How Do We Implement This In A Circuit?

How Do We Choose The Pole And Zero Frequencies?

Type III Compensator



Type III Compensator



$$H_{EA}(s) = \frac{v_{EA}(s)}{v_{OUT}(s)}$$

$$= -\frac{1}{s \cdot R_2 \cdot (C_2 + C_3)} \cdot \frac{1 + s \cdot R_3 \cdot C_3}{1 + s \cdot R_1 \cdot C_1} \cdot \frac{1 + s \cdot (R_1 + R_2) \cdot C_1}{1 + s \cdot R_3 \cdot \frac{C_2 \cdot C_3}{C_2 + C_3}}$$

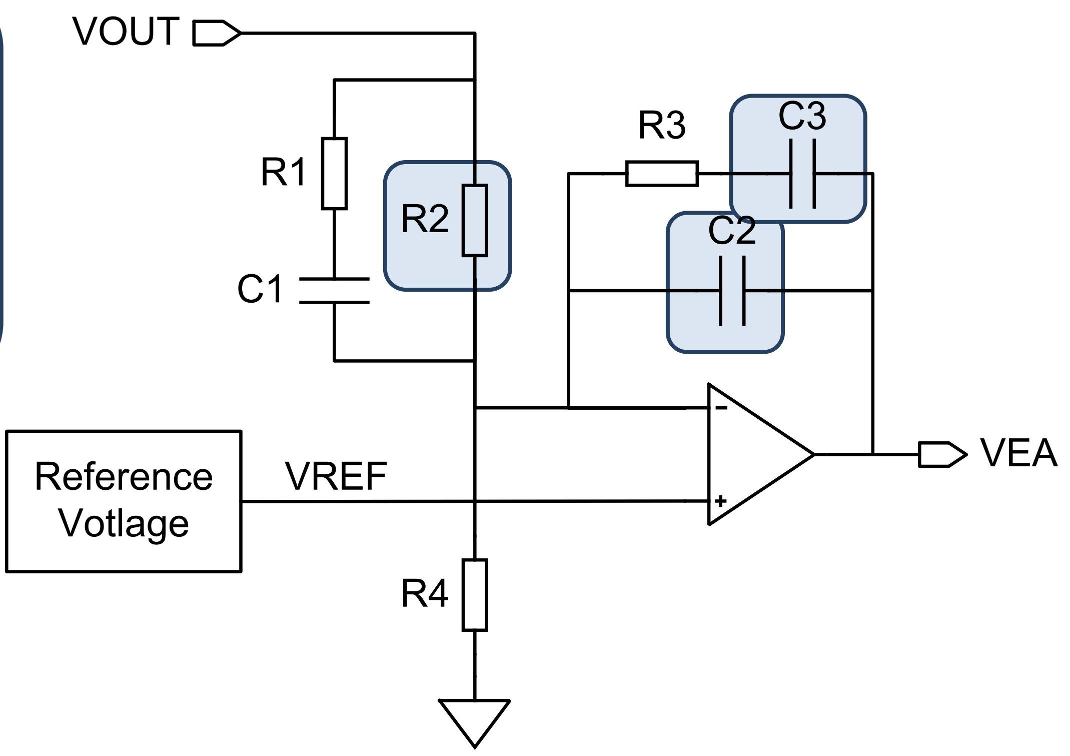
Note
No Voltage Divider Term!

Note
R4 Does NOT Appear!

Type III Compensator

Integrator Pole

$$\omega_{P0} = \frac{1}{R_2 \cdot (C_2 + C_3)}$$



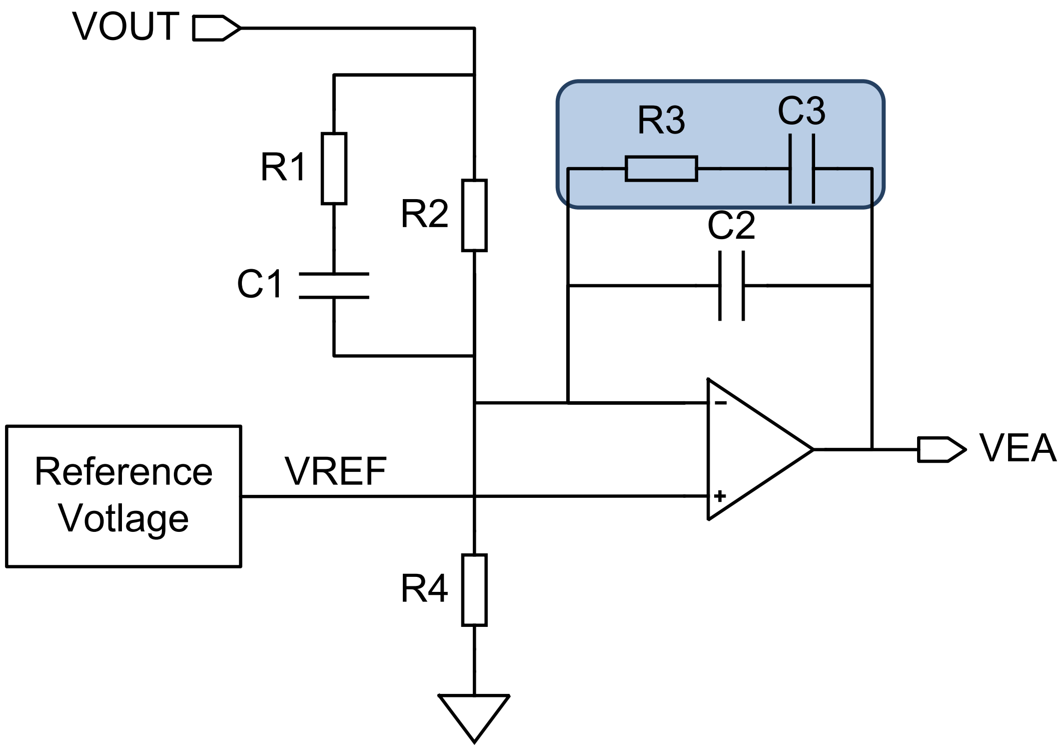
$$H_{EA}(s) = \frac{v_{EA}(s)}{v_{OUT}(s)}$$

$$= -\frac{1}{s \cdot R_2 \cdot (C_2 + C_3)} \cdot \frac{1 + s \cdot R_3 \cdot C_3}{1 + s \cdot R_1 \cdot C_1} \cdot \frac{1 + s \cdot (R_1 + R_2) \cdot C_1}{1 + s \cdot R_3 \cdot \frac{C_2 \cdot C_3}{C_2 + C_3}}$$

Type III Compensator

First Zero

$$\omega_{Z1} = \frac{1}{R_3 \cdot C_3}$$



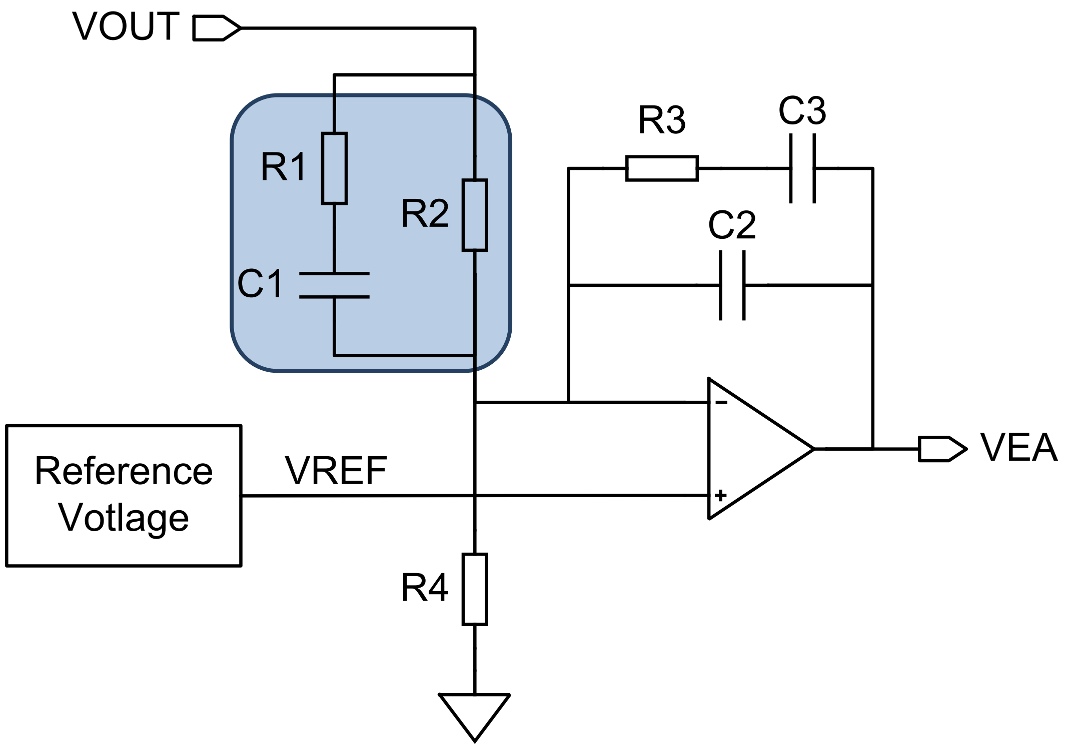
$$H_{EA}(s) = \frac{v_{EA}(s)}{v_{OUT}(s)}$$

$$= -\frac{1}{s \cdot R_2 \cdot (C_2 + C_3)} \cdot \frac{1 + s \cdot R_3 \cdot C_3}{1 + s \cdot R_1 \cdot C_1} \cdot \frac{1 + s \cdot (R_1 + R_2) \cdot C_1}{1 + s \cdot R_3 \cdot \frac{C_2 \cdot C_3}{C_2 + C_3}}$$

Type III Compensator

Second Zero

$$\omega_{Z2} = \frac{1}{(R_1 + R_2) \cdot C_1}$$



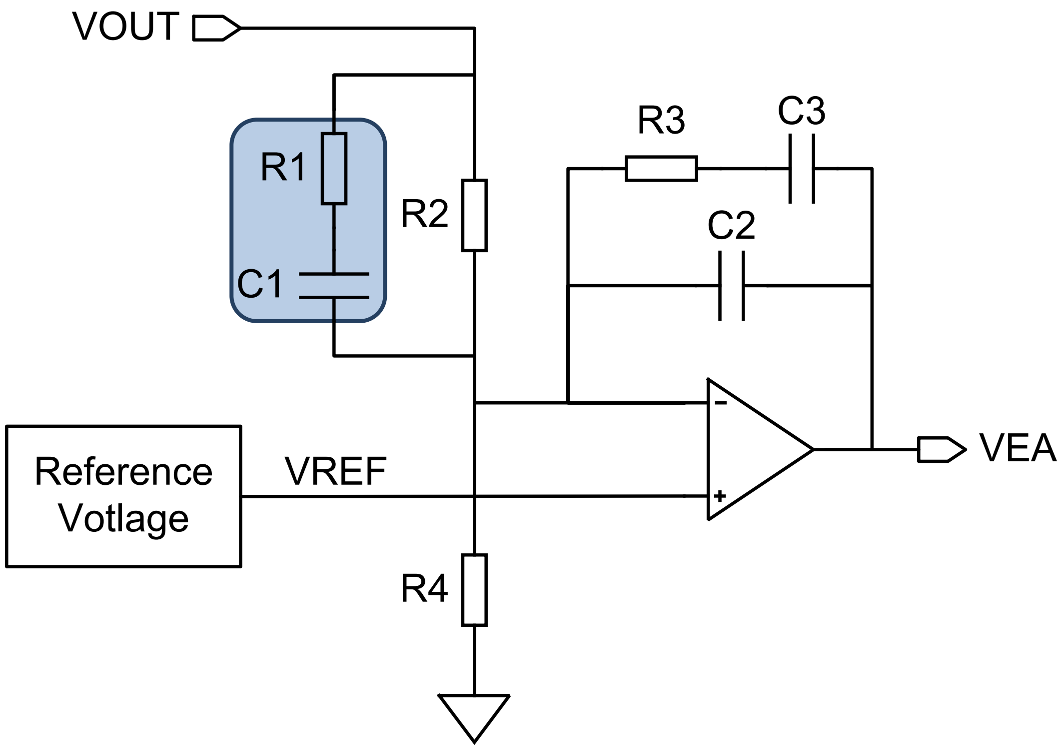
$$H_{EA}(s) = \frac{v_{EA}(s)}{v_{OUT}(s)}$$

$$= -\frac{1}{s \cdot R_2 \cdot (C_2 + C_3)} \cdot \frac{1 + s \cdot R_3 \cdot C_3}{1 + s \cdot R_1 \cdot C_1} \cdot \frac{1 + s \cdot (R_1 + R_2) \cdot C_1}{1 + s \cdot R_3 \cdot \frac{C_2 \cdot C_3}{C_2 + C_3}}$$

Type III Compensator

Second Pole

$$\omega_{P1} = \frac{1}{R_1 \cdot C_1}$$



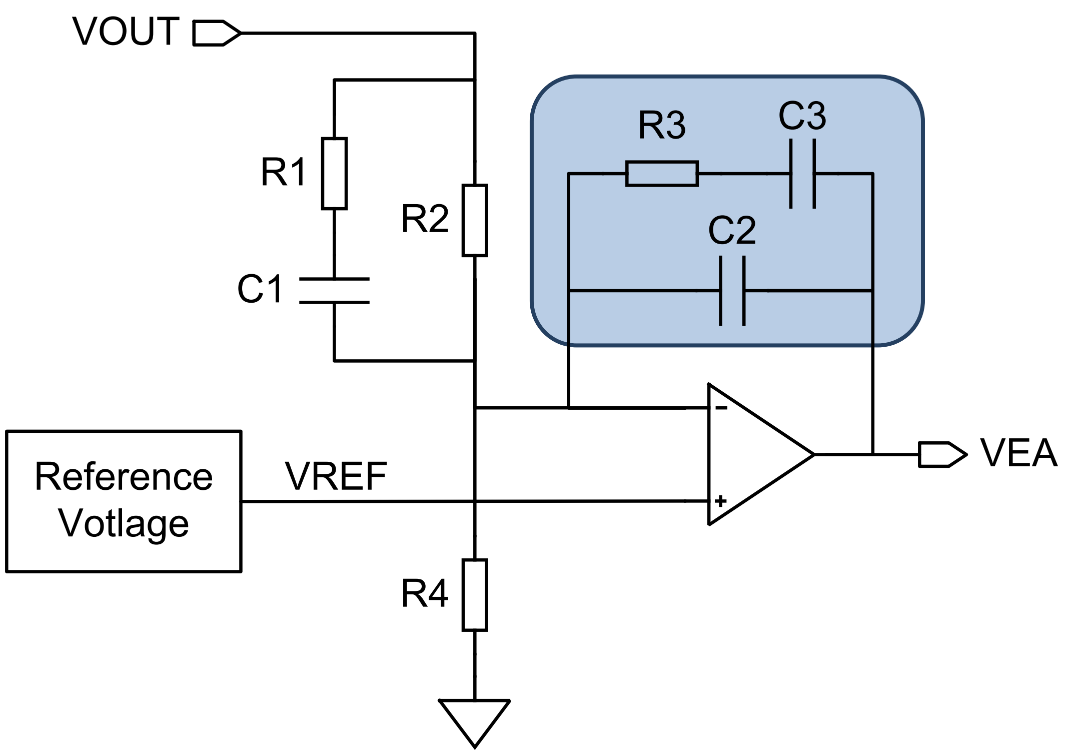
$$H_{EA}(s) = \frac{v_{EA}(s)}{v_{OUT}(s)}$$

$$= -\frac{1}{s \cdot R_2 \cdot (C_2 + C_3)} \cdot \frac{1 + s \cdot R_3 \cdot C_3}{1 + s \cdot R_1 \cdot C_1} \cdot \frac{1 + s \cdot (R_1 + R_2) \cdot C_1}{1 + s \cdot R_3 \cdot \frac{C_2 \cdot C_3}{C_2 + C_3}}$$

Type III Compensator

Third Pole

$$\omega_{P2} = \frac{1}{R_3 \cdot \frac{C_2 \cdot C_3}{C_2 + C_3}}$$

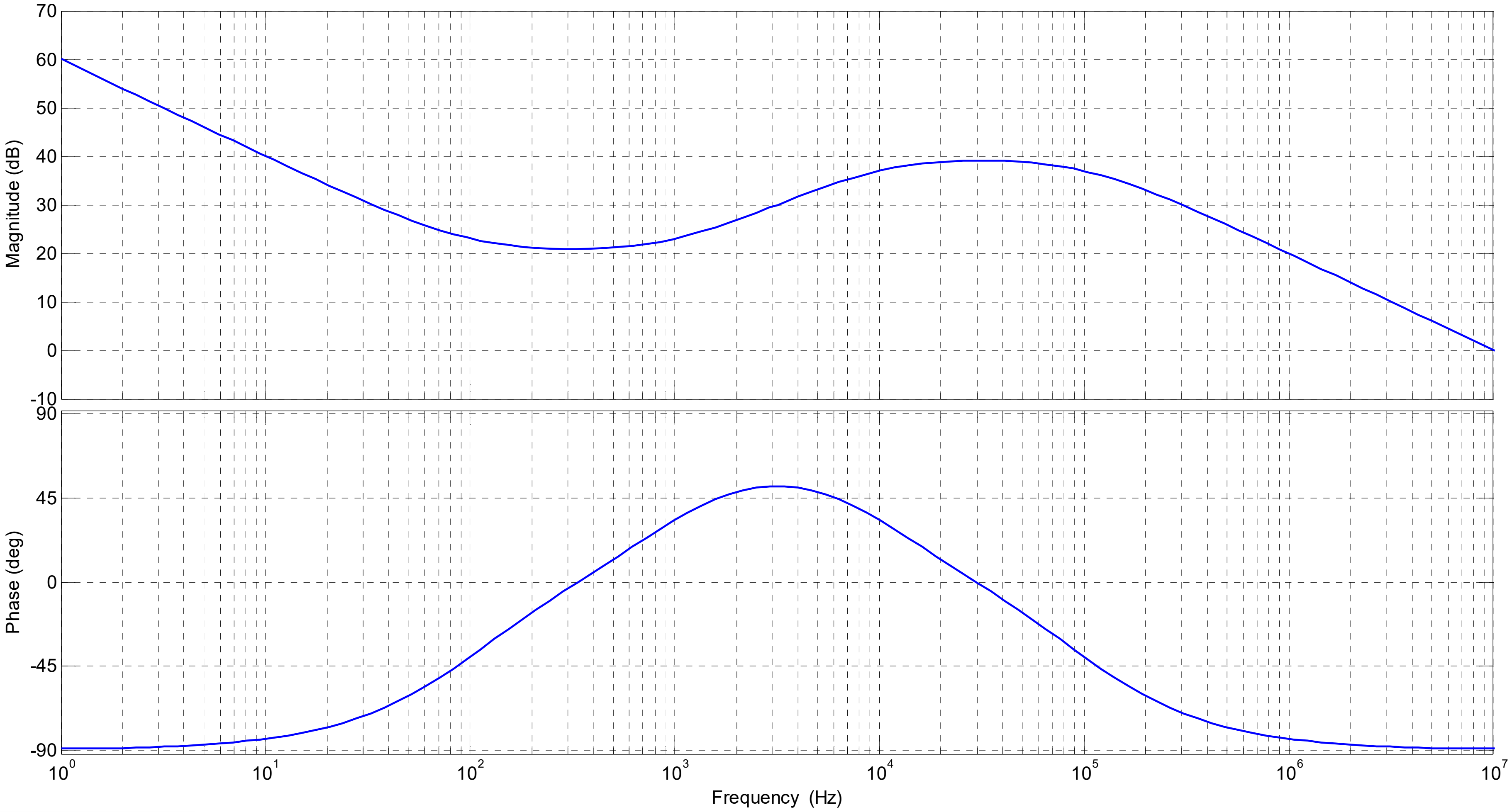


$$H_{EA}(s) = \frac{v_{EA}(s)}{v_{OUT}(s)}$$

$$= -\frac{1}{s \cdot R_2 \cdot (C_2 + C_3)} \cdot \frac{1 + s \cdot R_3 \cdot C_3}{1 + s \cdot R_1 \cdot C_1} \cdot \frac{1 + s \cdot (R_1 + R_2) \cdot C_1}{1 + s \cdot R_3 \cdot \frac{C_2 \cdot C_3}{C_2 + C_3}}$$

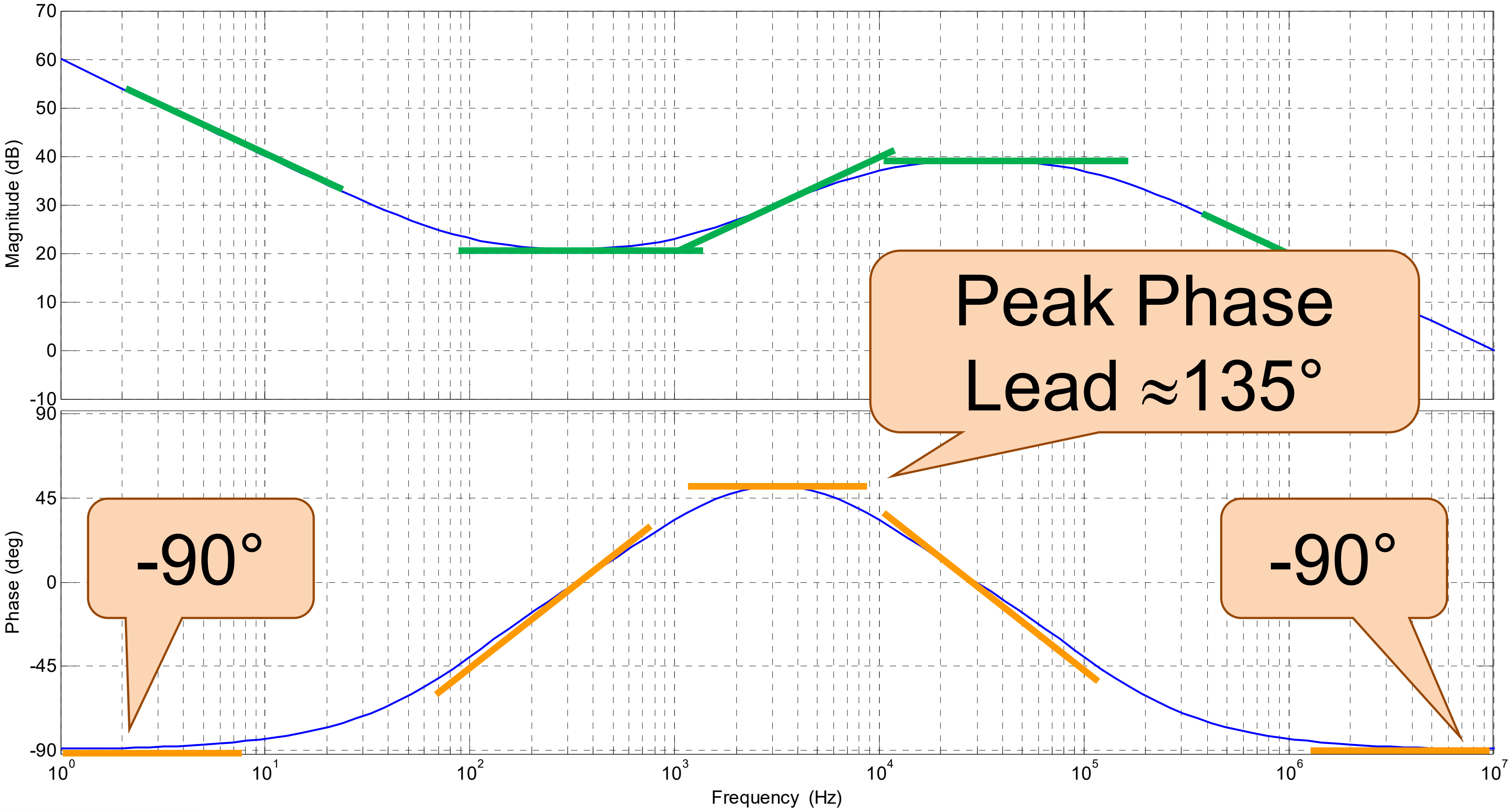
Type III Bode Plot

$$f_{P0} = 1000\,\text{Hz} \quad f_{Z1} = 100\,\text{Hz} \quad f_{Z2} = 1000\,\text{Hz} \quad f_{P1} = 10\,\text{kHz} \quad f_{P2} = 100\,\text{kHz}$$



Type III Bode Plot

$f_{P0} = 1000\,\text{Hz} \quad f_{Z1} = 100\,\text{Hz} \quad f_{Z2} = 1000\,\text{Hz} \quad f_{P1} = 10\,\text{kHz} \quad f_{P2} = 100\,\text{kHz}$



Choosing Compensator Poles And Zeroes

$$T(s) = K(s) \cdot H_{EA}(s) \cdot G_{PWM}(s) \cdot G_{VD}(s)$$

$$T(s) = \frac{\omega_0}{s} \cdot \frac{1}{1+\frac{s}{\omega_{HFP}}} = \left[\frac{\omega_{P0}}{s} \cdot \frac{\left(1+\frac{s}{\omega_{Z1}}\right)\cdot\left(1+\frac{s}{\omega_{Z2}}\right)}{\left(1+\frac{s}{\omega_{P1}}\right)\cdot\left(1+\frac{s}{\omega_{P2}}\right)} \right] \cdot \frac{1}{V_{RAMP}} \cdot \left[V_{IN} \cdot \frac{1+\frac{s}{\omega_{Z\_ESR}}}{1+\frac{1}{Q}\cdot\left(\frac{s}{\omega_{LC}}\right)+\left(\frac{s}{\omega_{LC}}\right)^2} \right]$$

To Design The Compensator We Need These Five Frequencies

Choosing Compensator Poles And Zeroes

$$T(s) = K(s) \cdot H_{EA}(s) \cdot G_{PWM}(s) \cdot G_{VD}(s)$$

$$T(s) = \frac{\omega_0}{s} \cdot \frac{1}{1 + \frac{s}{\omega_{HFP}}} = \left[\frac{\omega_{P0}}{s} \cdot \frac{\left(1 + \frac{s}{\omega_{Z1}}\right) \cdot \left(1 + \frac{s}{\omega_{Z2}}\right)}{\left(1 + \frac{s}{\omega_{P1}}\right) \cdot \left(1 + \frac{s}{\omega_{P2}}\right)} \right] \cdot \frac{1}{V_{RAMP}} \cdot \left[V_{IN} \cdot \frac{1 + \frac{s}{\omega_{Z\_ESR}}}{1 + \frac{1}{Q} \cdot \left(\frac{s}{\omega_{LC}}\right) + \left(\frac{s}{\omega_{LC}}\right)^2} \right]$$

$$\omega_{Z1} = \omega_{Z2} = \omega_{LC}$$

$$\omega_{P1} = \omega_{Z\_ESR}$$

$$\omega_{P2} = \omega_{HFP}$$

Choosing Compensator Poles And Zeroes

$$T(s) = K(s) \cdot H_{EA}(s) \cdot G_{PWM}(s) \cdot G_{VD}(s)$$

$$T(s) = \frac{\omega_0}{s} \cdot \frac{1}{1+\frac{s}{\omega_{HFP}}} = \left[\frac{\omega_{P0}}{s} \cdot \frac{\left(1+\frac{s}{\omega_{Z1}}\right)\cdot\left(1+\frac{s}{\omega_{Z2}}\right)}{\left(1+\frac{s}{\omega_{P1}}\right)\cdot\left(1+\frac{s}{\omega_{P2}}\right)} \right] \cdot \frac{1}{V_{RAMP}} \cdot \left[V_{IN} \cdot \frac{1+\frac{s}{\omega_{Z\_ESR}}}{1+\frac{1}{Q}\cdot\left(\frac{s}{\omega_{LC}}\right)+\left(\frac{s}{\omega_{LC}}\right)^2} \right]$$

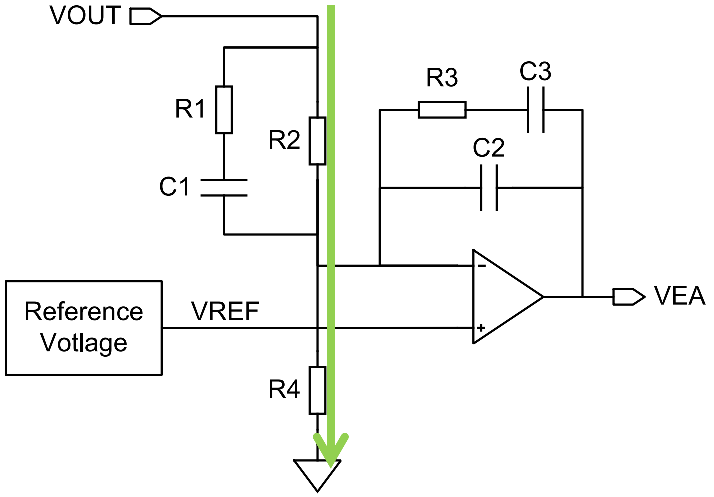
DC Gain Term/Integrator Pole Frequency: $\omega_{P0} = \frac{V_{RAMP} \cdot \omega_0}{V_{IN}} = \frac{V_{RAMP}}{V_{IN}} \cdot \omega_C \cdot \sqrt{1+\left(\frac{\omega_C}{\omega_{HFP}}\right)^2}$

Calculating Component Values

- If $V_{OUT} > V_{REF}$ Choose Current (I_{BIAS}) Through R2 And R4
  - 100 µA – 1 mA Typical

$$R_4 = \frac{V_{REF}}{I_{BIAS}}$$

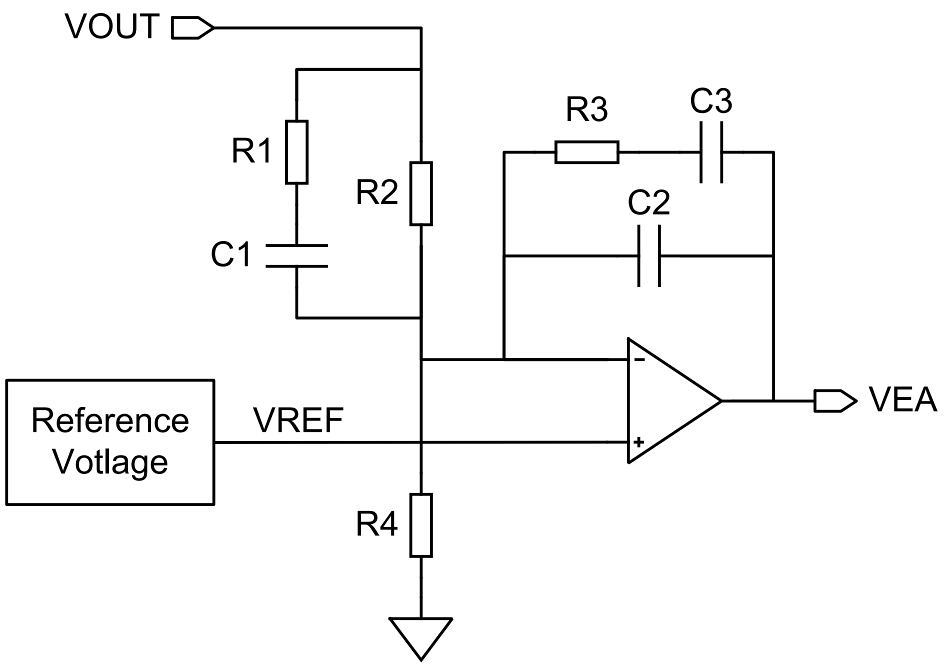
$$R_2 = \frac{V_{OUT} - V_{REF}}{V_{REF}} \cdot R_4$$



- If $V_{OUT} = V_{REF}$:
  - R4 Not Used
  - Choose Convenient Value For R2 (10 kΩ)

Calculating Component Values

$$R_1 = \frac{f_{Z2}}{f_{P1} - f_{Z2}} \cdot R2$$



$$C_1 = \frac{1}{2 \cdot \pi \cdot f_{P1} \cdot R_1}$$

$$C_2 = \frac{f_{Z1}}{2 \cdot \pi \cdot f_{P2} \cdot f_{P0} \cdot R_2}$$

$$C_3 = \frac{1}{2 \cdot \pi \cdot f_{P0} \cdot R_2} - C_2$$

$$R_3 = \frac{1}{2 \cdot \pi \cdot f_{Z1} \cdot C_3}$$

Buck Converter Example

$$V_{IN} = 12\,Vdc$$

$$V_{OUT} = 3.3\,Vdc$$

$$I_{OUT} = 1\,Adc$$

$$V_{REF} = 2.5\,Vdc$$

$$F_{SW} = 300\,\text{kHz}$$

$$L_{OUT} = 18\,\mu\text{H}$$

$$C_{OUT} = 47\,\mu\text{F}$$

$$R_{COUT} = 20\,\text{m}\Omega$$

$$F_C = \frac{1}{10} F_{SW} = 30\,\text{kHz}$$

$$PM = 70°$$

$$F_{LC} = \frac{1}{2 \cdot \pi \cdot \sqrt{L_{OUT} \cdot C_{OUT}}} = 5.472\,\text{kHz}$$

$$F_{ESR} = \frac{1}{2 \cdot \pi \cdot R_{COUT} \cdot C_{OUT}} = 169.3\,\text{kHz}$$

Buck Converter Example

Choose Compensator Zero Frequencies And One Pole Frequency

$$F_{Z1} = F_{Z2} = F_{LC} = 5.472\,\text{kHz}$$
$$F_{P1} = F_{ESR} = 169.3\,\text{kHz}$$

Calculate Other Pole Frequency To Give Desired Phase Margin At Crossover

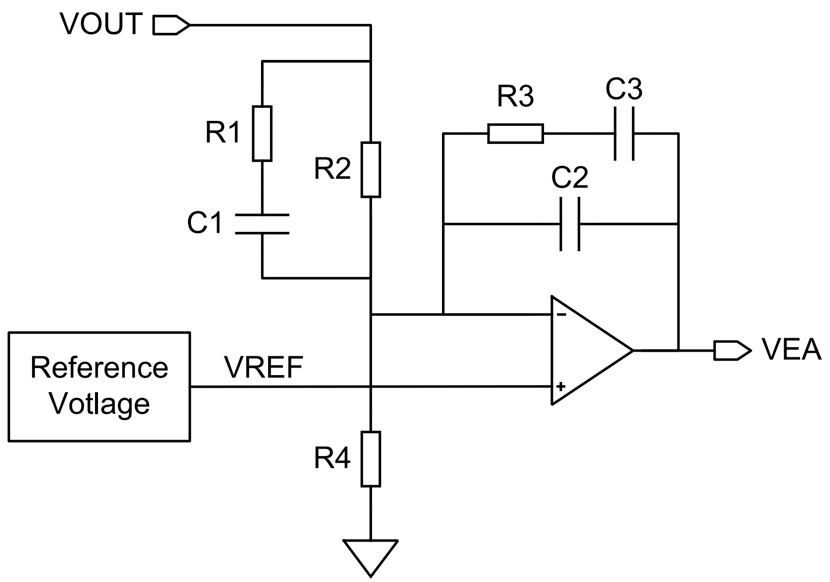
$$PM = 70°$$
$$Phase\_Lag(F_C) = PM - 90° = -20°$$
$$F_{P2} = \frac{F_C}{-\tan(Phase\_Lag)} = 82.4\,\text{kHz}$$

Calculate F_{P0}

$$F_{P0} = \frac{V_{RAMP}}{V_{IN}} \cdot F_C \cdot \sqrt{1 + \left(\frac{\omega_C}{\omega_{HFP}}\right)^2}$$
$$= \frac{1\,\text{V}}{12\,\text{V}} \cdot 30\,\text{kHz} \cdot \sqrt{1 + \left(\frac{2 \cdot \pi \cdot 30\,\text{kHz}}{2 \cdot \pi \cdot 82.4\,\text{kHz}}\right)^2}$$
$$= 2661\,\text{kHz}$$

Calculating Component Values

- $V_{OUT} > V_{REF}$ Choose Current (I_{BIAS}) Through R2 And R4
- $I_{BIAS} = 100\ \mu A$



$$R_4 = \frac{V_{REF}}{I_{BIAS}} = \frac{2.5V}{100\,\mu A} = 25\,k\Omega \Rightarrow 24.9\,k\Omega$$

$$R_2 = \frac{V_{OUT} - V_{REF}}{V_{REF}} \cdot R_4 = \frac{3.3V - 2.5V}{2.5V} \cdot 24.9\,k\Omega = 7.968\,k\Omega \Rightarrow 8.06\,k\Omega$$

$$V_{OUT} = \frac{R_2 + R_4}{R_4} \cdot V_{REF} = 3.309V$$

Calculate Component Values

$$R_1 = \frac{F_{Z2}}{F_{P1} - F_{Z2}} \cdot R_2 = 269.18\,\Omega \Rightarrow 267\,\Omega$$

$$C_2 = \frac{F_{Z1}}{2 \cdot \pi \cdot F_{P2} \cdot F_{P0} \cdot R_2} = 492.7\,pF \Rightarrow 510\,pF$$

$$C_1 = \frac{1}{2 \cdot \pi \cdot F_{P1} \cdot R_1} = 3.521\,nF \Rightarrow 3.6\,nF$$

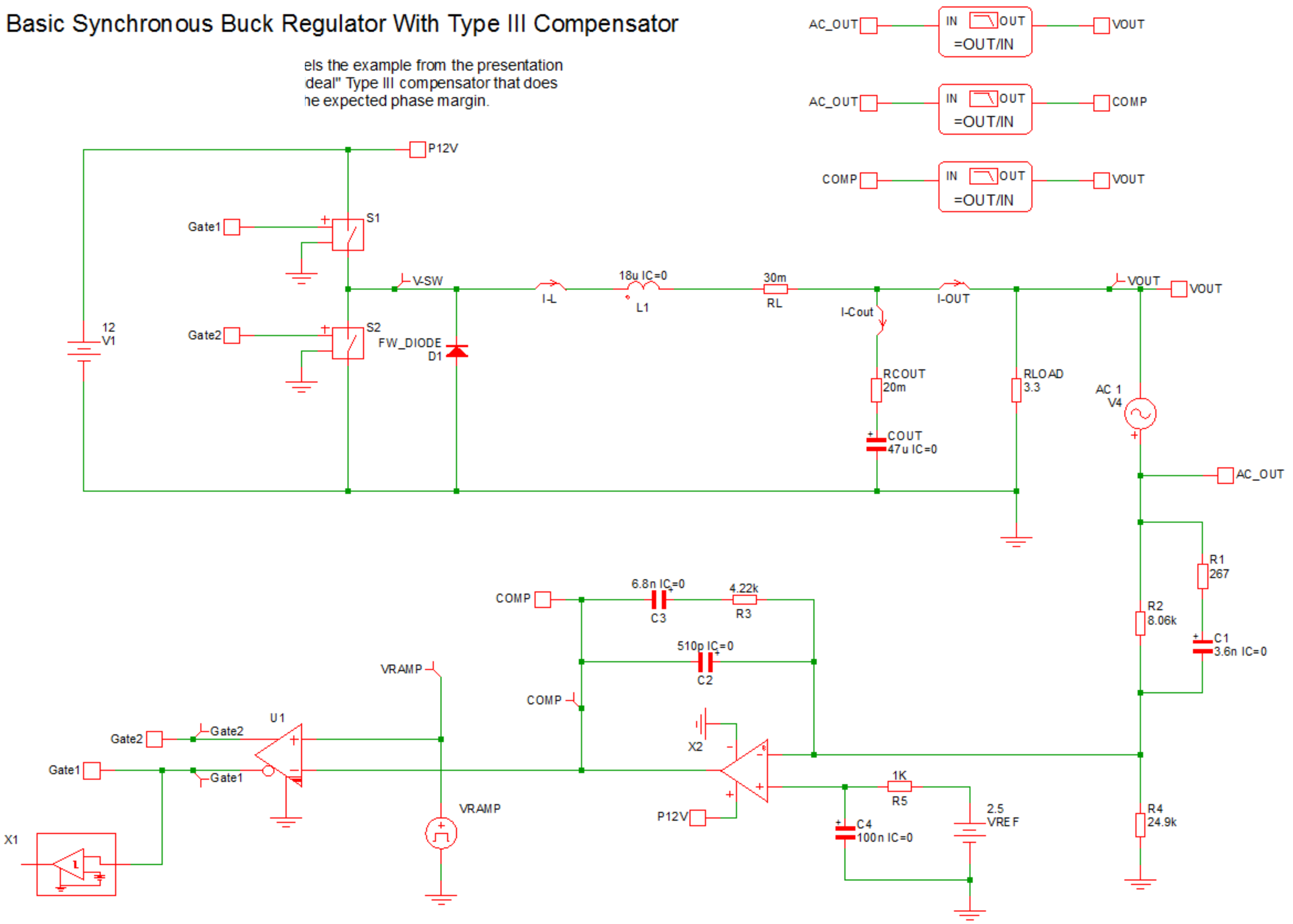
$$C_3 = \frac{1}{2 \cdot \pi \cdot F_{P0} \cdot R_2} = 6.912\,nF \Rightarrow 6.8\,nF$$

$$R_3 = \frac{1}{2 \cdot \pi \cdot F_{Z1} \cdot C_3} = 4.277\,k\Omega \Rightarrow 4.22\,k\Omega$$

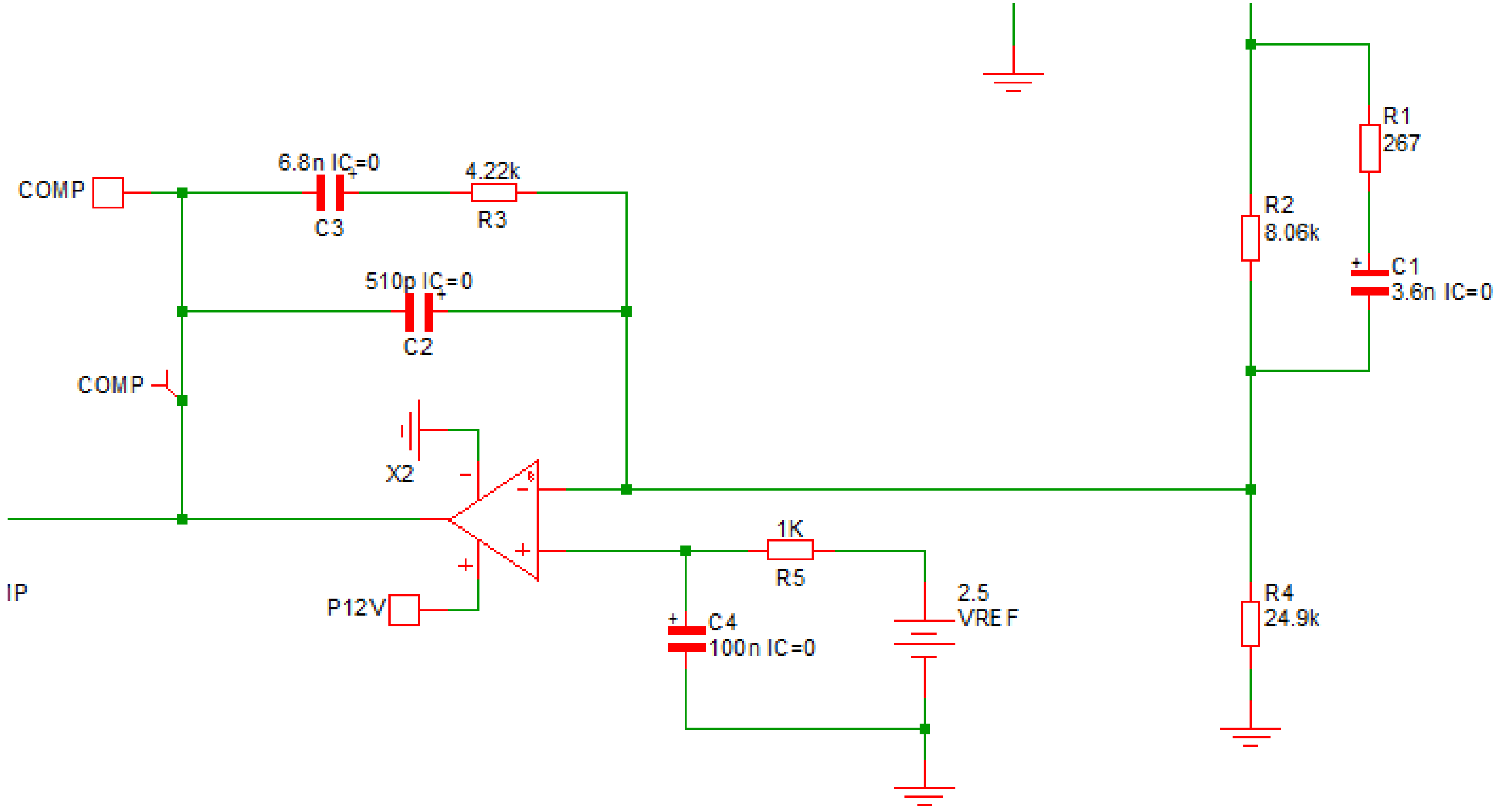
Check Actual Compensator Pole And Zero Frequencies

Desired	As Designed
$F_{P0} = 2.661\,\text{kHz}$	$F_{P0} = 2.701\,\text{kHz}$
$F_{Z1} = 5.472\,\text{kHz}$	$F_{Z1} = 5.546\,\text{kHz}$
$F_{Z2} = 5.472\,\text{kHz}$	$F_{Z2} = 5.309\,\text{kHz}$
$F_{P1} = 169.314\,\text{kHz}$	$F_{P1} = 165.6\,\text{kHz}$
$F_{P2} = 82.424\,\text{kHz}$	$F_{P2} = 79.50\,\text{kHz}$

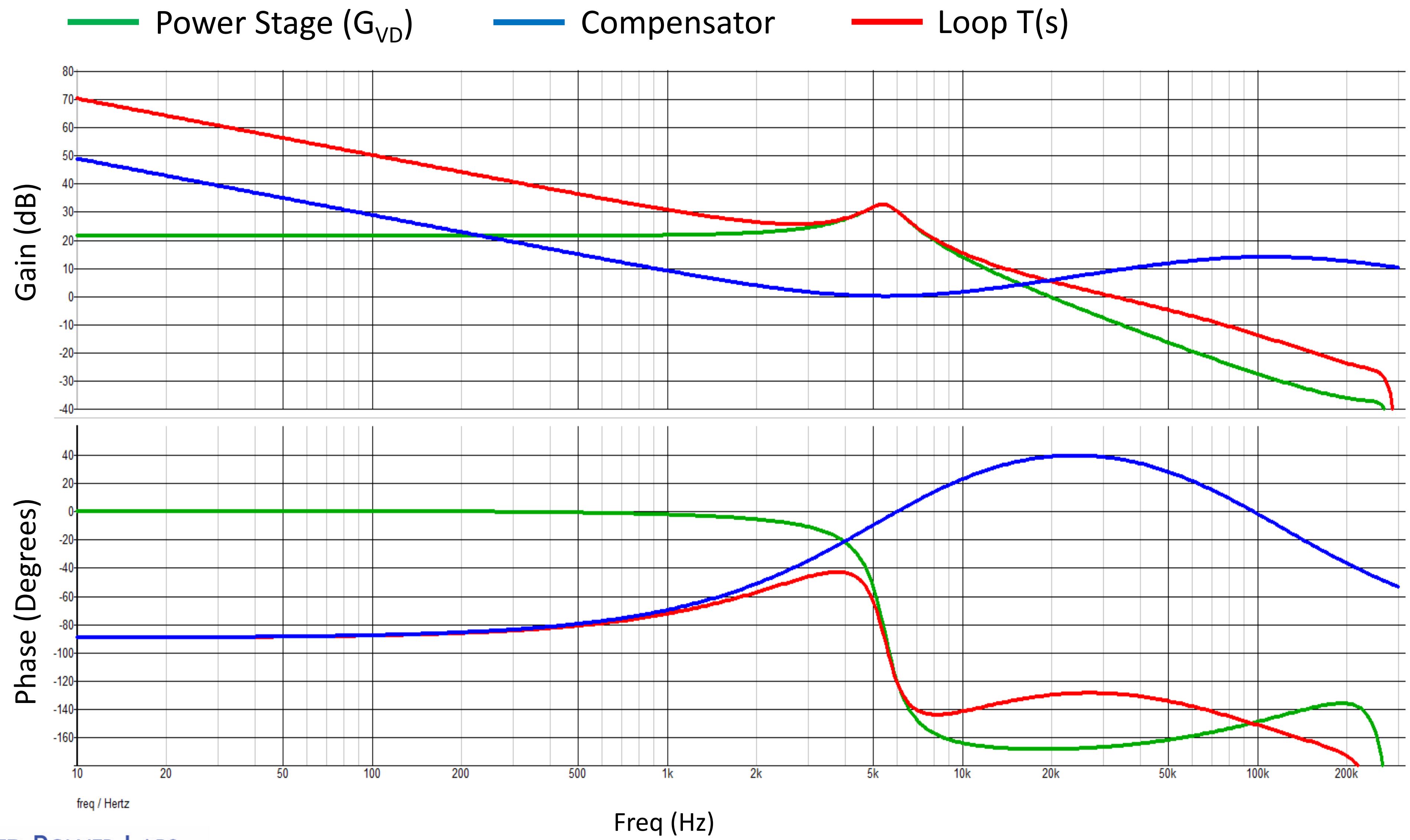
Simulation Schematic



Simulation Schematic

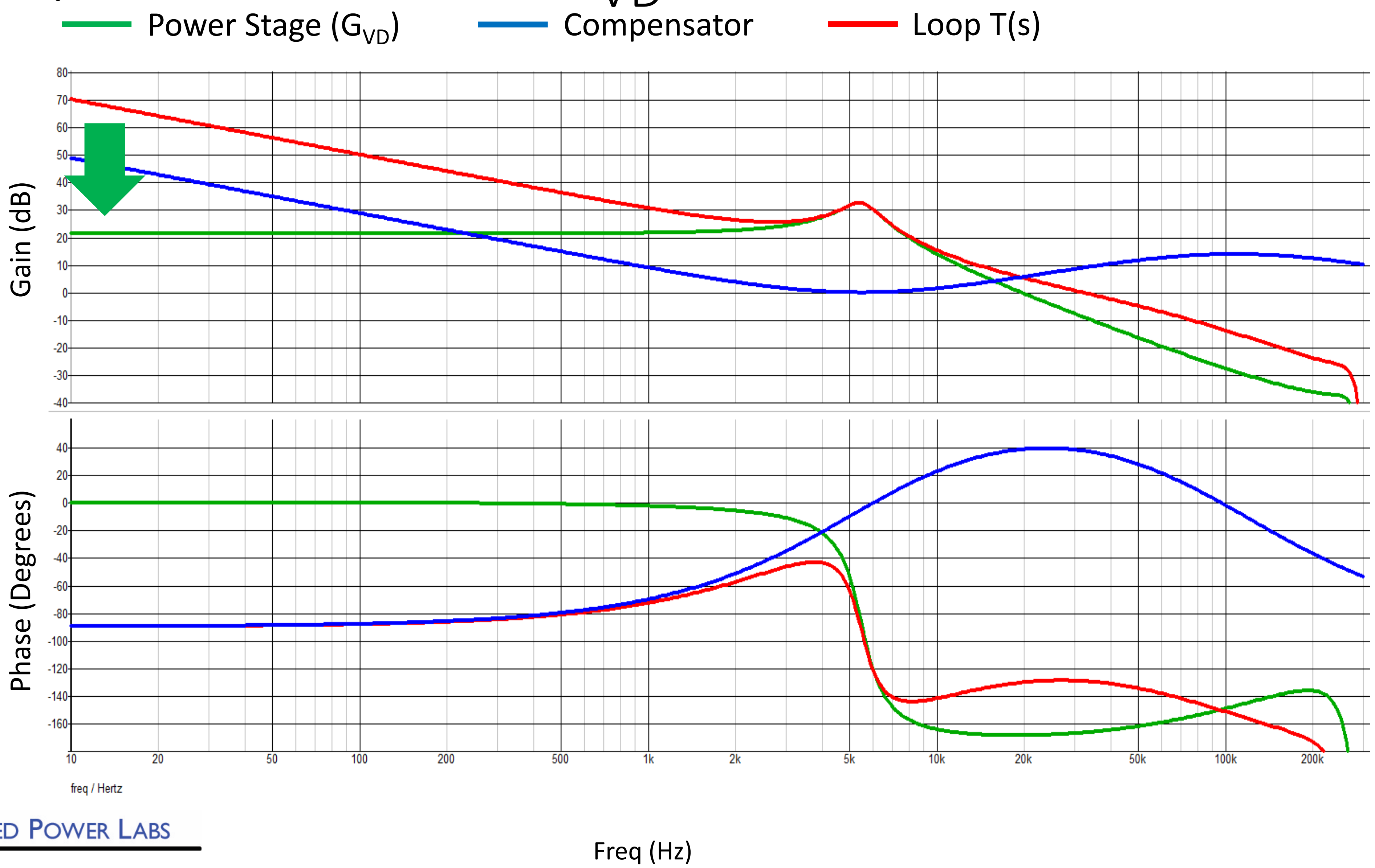


Example Bode Plot



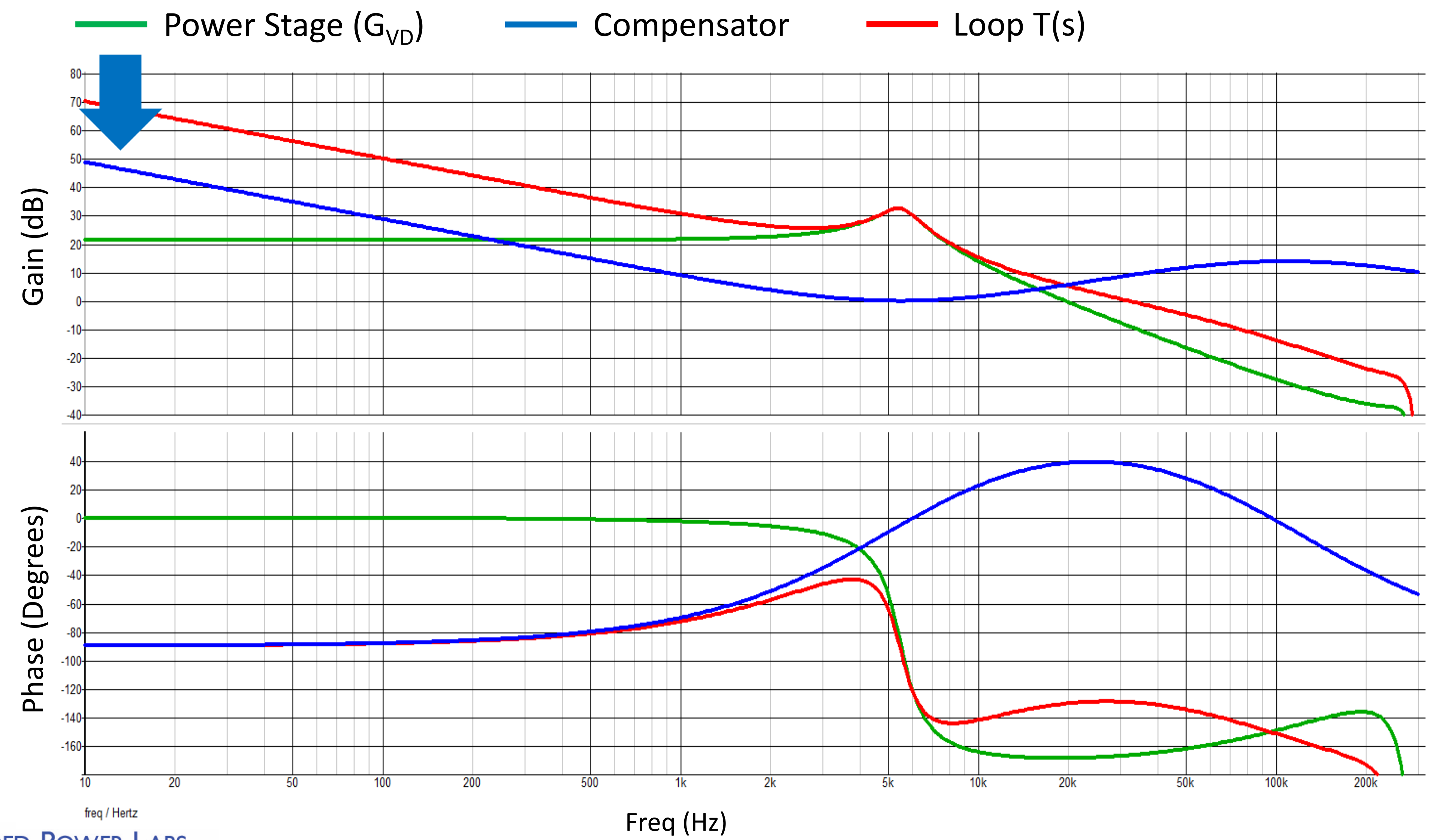
EMBEDDED POWER LABS

Example: Bode Plot $G_{VD}(s)$



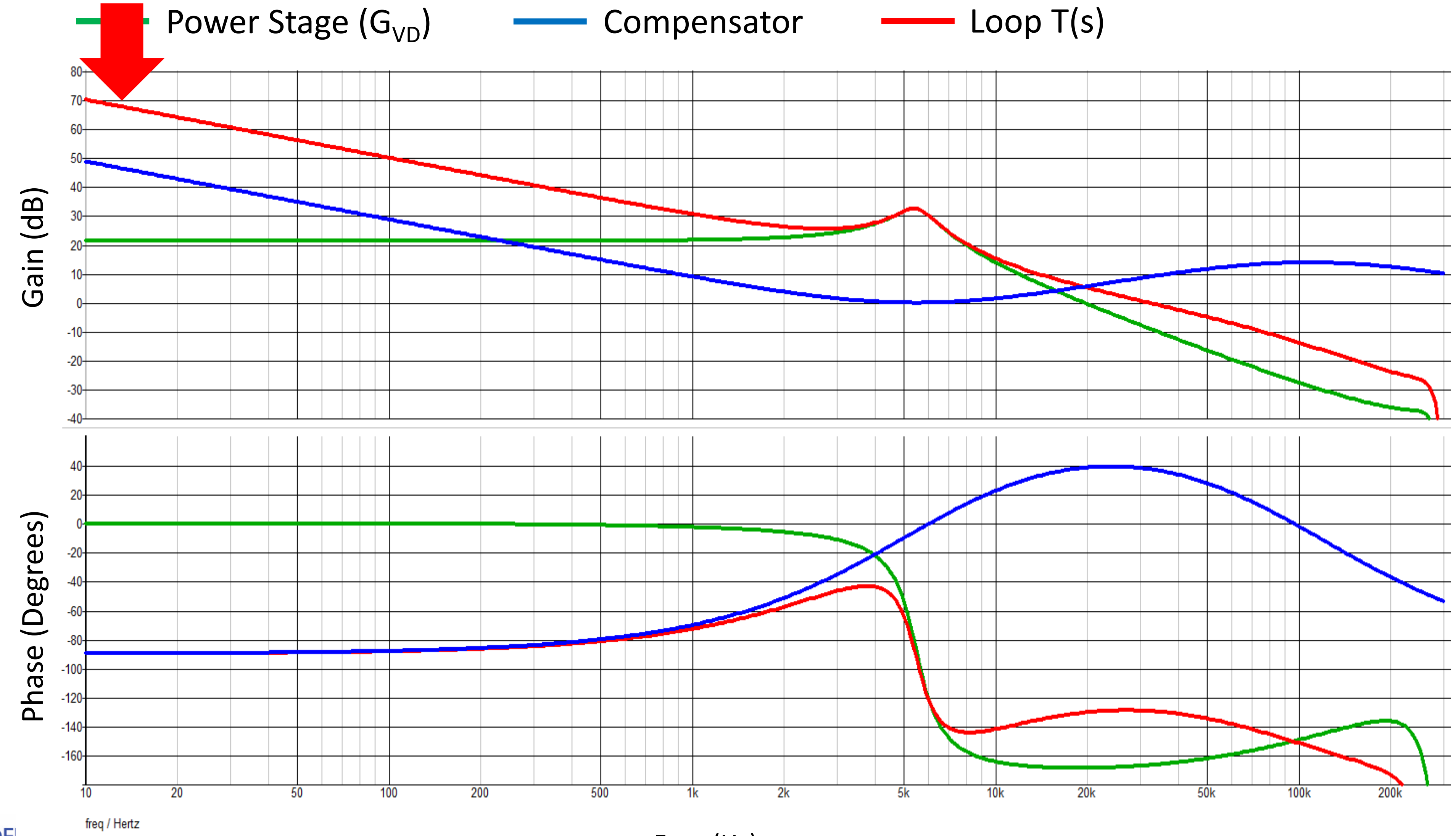
EMBEDDED POWER LABS

Example: Compensator Bode Plot



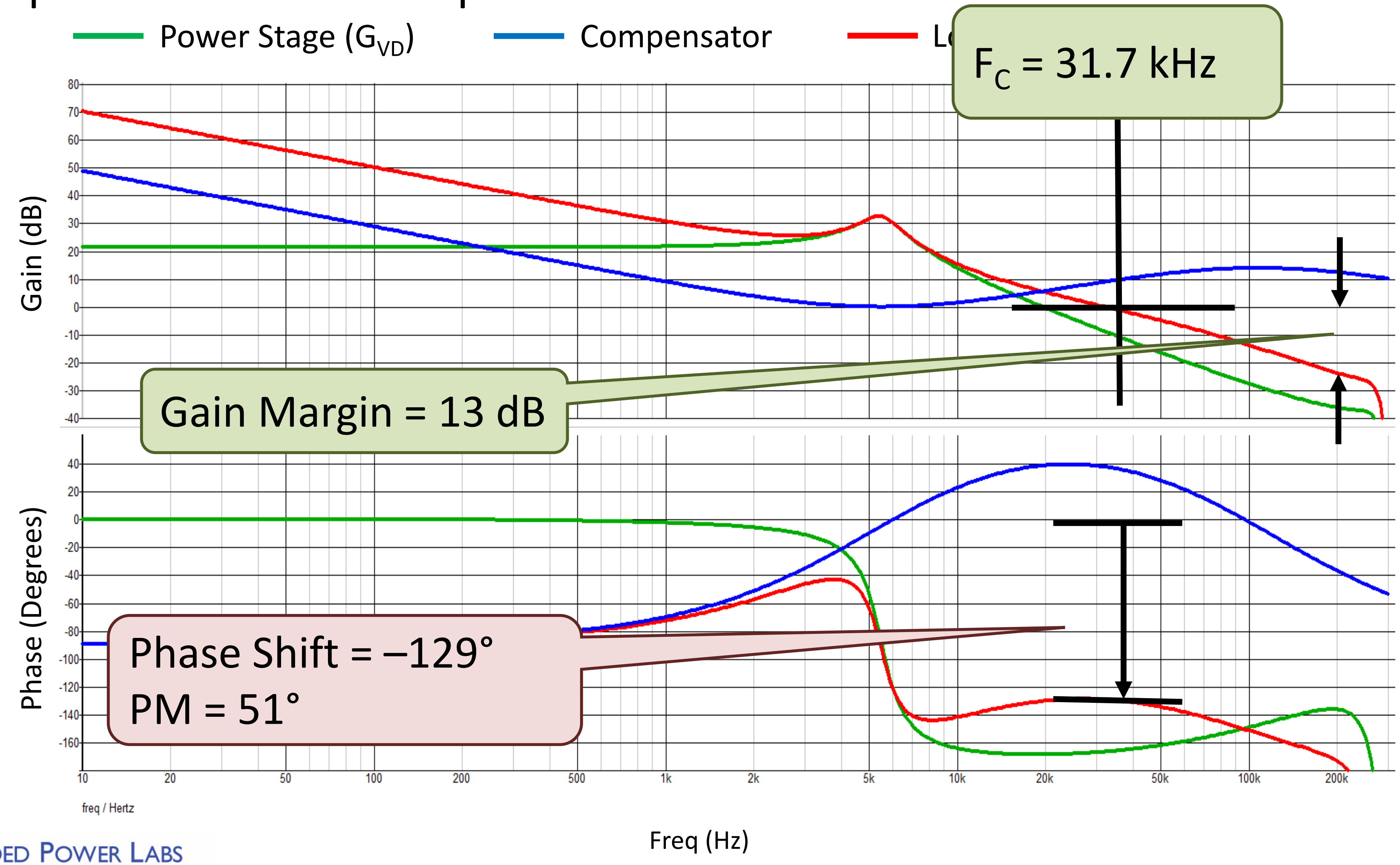
EMBEDDED POWER LABS

Example: Closed Loop Bode Plot



Freq (Hz)

Example: Closed Loop Bode Plot



Where This Went Wrong

$$T(s) = K(s)$$

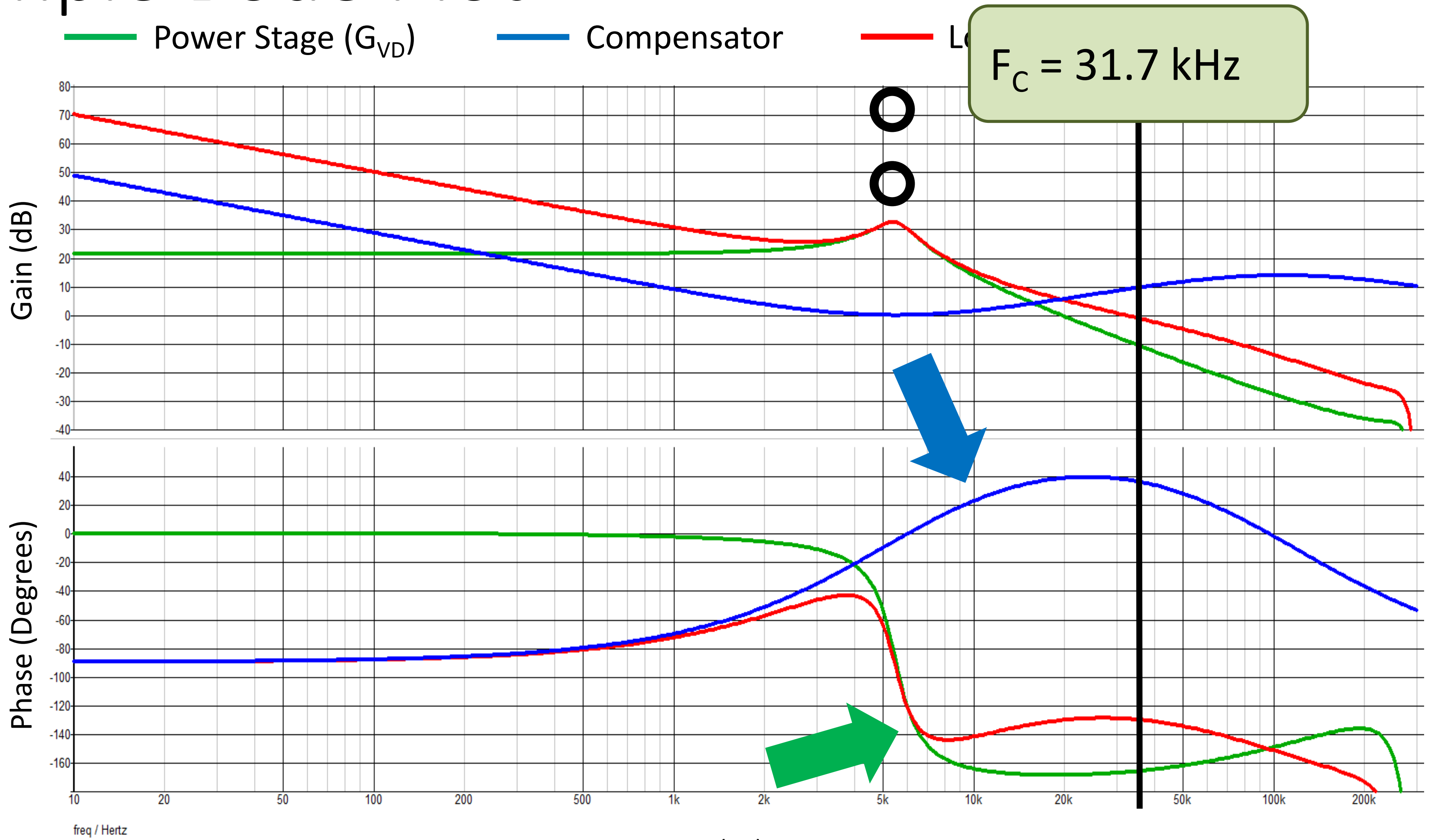
We Tried To Cancel Two Complex Poles..

$$T(s) = \frac{\omega_0}{s} \cdot \frac{1}{1+\frac{s}{\omega_{HFP}}} = \left[\frac{\omega_{P0}}{s} \cdot \frac{\left(1+\frac{s}{\omega_{Z1}}\right)\cdot\left(1+\frac{s}{\omega_{Z2}}\right)}{\left(1+\frac{s}{\omega_{P1}}\right)\left(1+\frac{s}{\omega_{P2}}\right)} \right] \cdot \frac{1}{V_{RAMP}} \cdot \left[V_{IN} \cdot \frac{1+\frac{s}{\omega_{Z\_ESR}}}{1+\frac{1}{Q}\cdot\left(\frac{s}{\omega_{LC}}\right)+\left(\frac{s}{\omega_{LC}}\right)^2} \right]$$

With Two Real Zeros

$$\ldots \frac{\ldots}{V_{RAMP}} \left(\frac{V_{RAMP} \cdot \omega_0}{V_{IN}} \right) \cdot \frac{V_{IN}}{V_{RAMP}}$$

Example Bode Plot



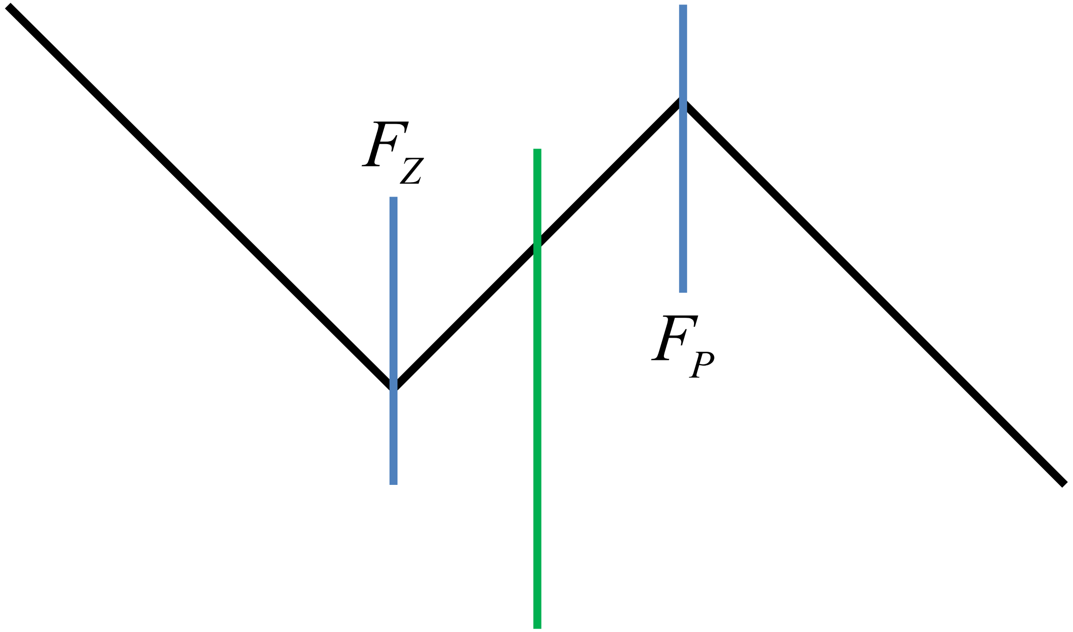
Freq (Hz)

EMBEDDED POWER LABS

K-Factor Method

- A Popular Cookbook Method For Choosing Compensator Pole And Zero Locations
- Published By Dean Venable In 1983
  - "The K Factor: A New Mathematical Tool for Stability Analysis and Synthesis”; Proceedings of Powercon 10, March 1983
  - Reprints And App Notes Widely Available Through A Web Search

K-Factor Method



$$F_{MAX\_BOOST} = \sqrt{F_Z \cdot F_P} = F_C$$

$$F_Z = \frac{F_C}{\sqrt{K}} \qquad F_P = F_C \cdot \sqrt{K} \qquad K = \left[\tan\left(\frac{Phase\_Boost}{4} + 45°\right)\right]^2$$

K-Factor Method Summary

- Choose Desired F_C And Phase Margin (PM)
- Examine $G_{VD}(s)$
  - Determine Phase Lag At F_C
  - Determine Gain At F_C
- Calculate Required Phase Boost At F_C
  - Phase Boost = PM – Phase $(G_{VD}(_{FC}))$ - 90°
- Calculate K-Factor
- Calculate F_Z And F_P
- Calculate Component Values As Function Of F_Z, F_P, And Required Gain

K-Factor Example

- F_C = 30 kHz, PM = 70°
- $G_{VD}(F_C)$: Phase = –167°,
  Gain = -7.7 dB (0.412)
- Phase Boost = 70° – (–167°) – 90° = 147°
- Calculate K-Factor

$$K = \left[\tan\left(\frac{147^\circ}{4} + 45^\circ\right)\right]^2 = 47.6$$

- Calculate F_Z And F_P

$$F_Z = \frac{F_C}{\sqrt{K}} = \frac{30\,\text{kHz}}{\sqrt{47.6}} = 4.350\,\text{kHz}$$

$$F_P = F_C \cdot \sqrt{K} = 30\,\text{kHz} \cdot \sqrt{47.6} = 206.9\,\text{kHz}$$

K-Factor Example

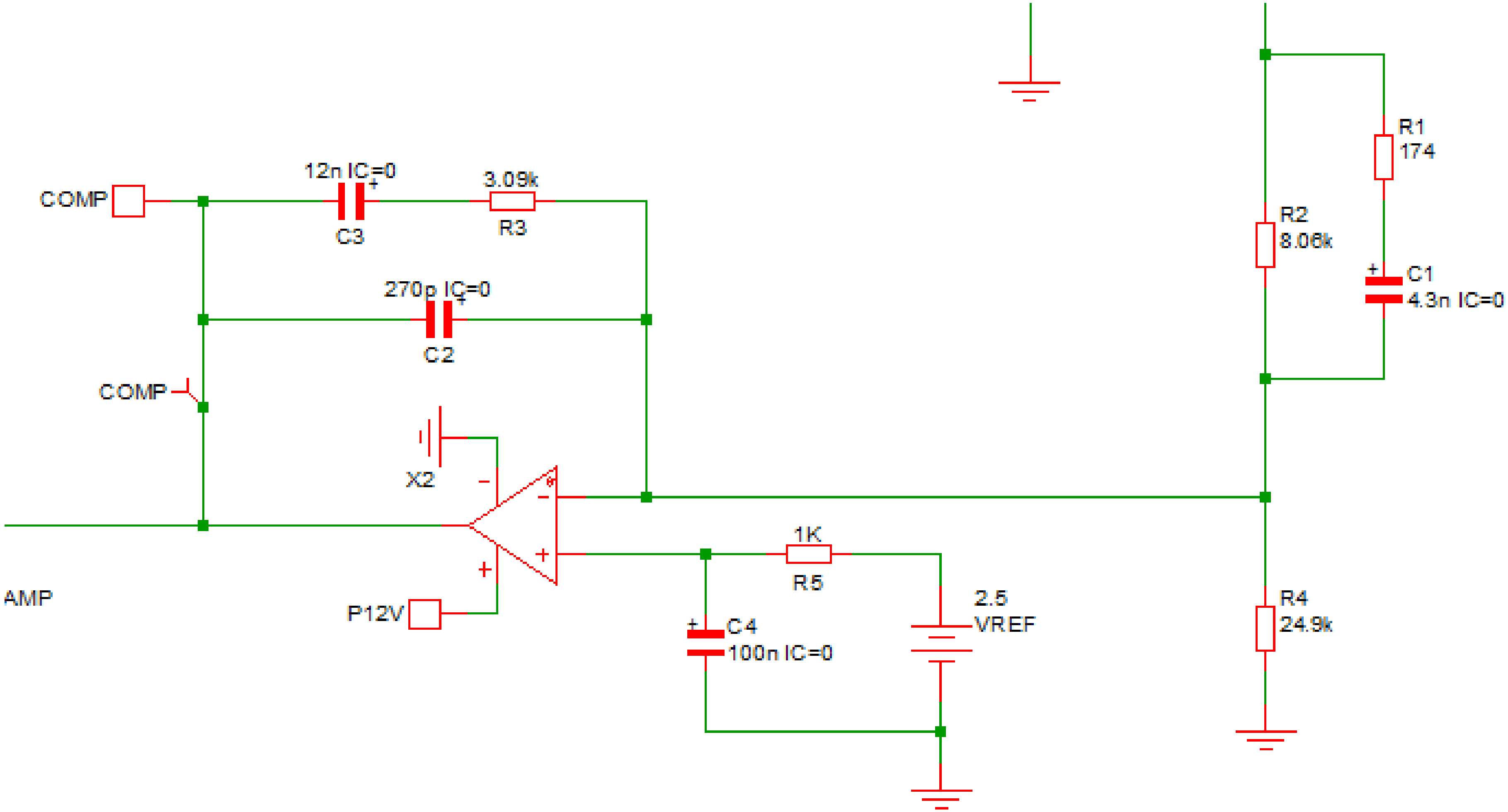
- Component Values Are:

$$R2 = 8.04\,\text{k}\Omega$$
$$R4 = 24.9\,\text{k}\Omega$$
$$R1 = 174\,\Omega$$
$$C1 = 4.3\,\text{nF}$$
$$C2 = 270\,\text{pF}$$
$$R3 = 3.09\,\text{k}\Omega$$
$$C3 = 12\,\text{nF}$$

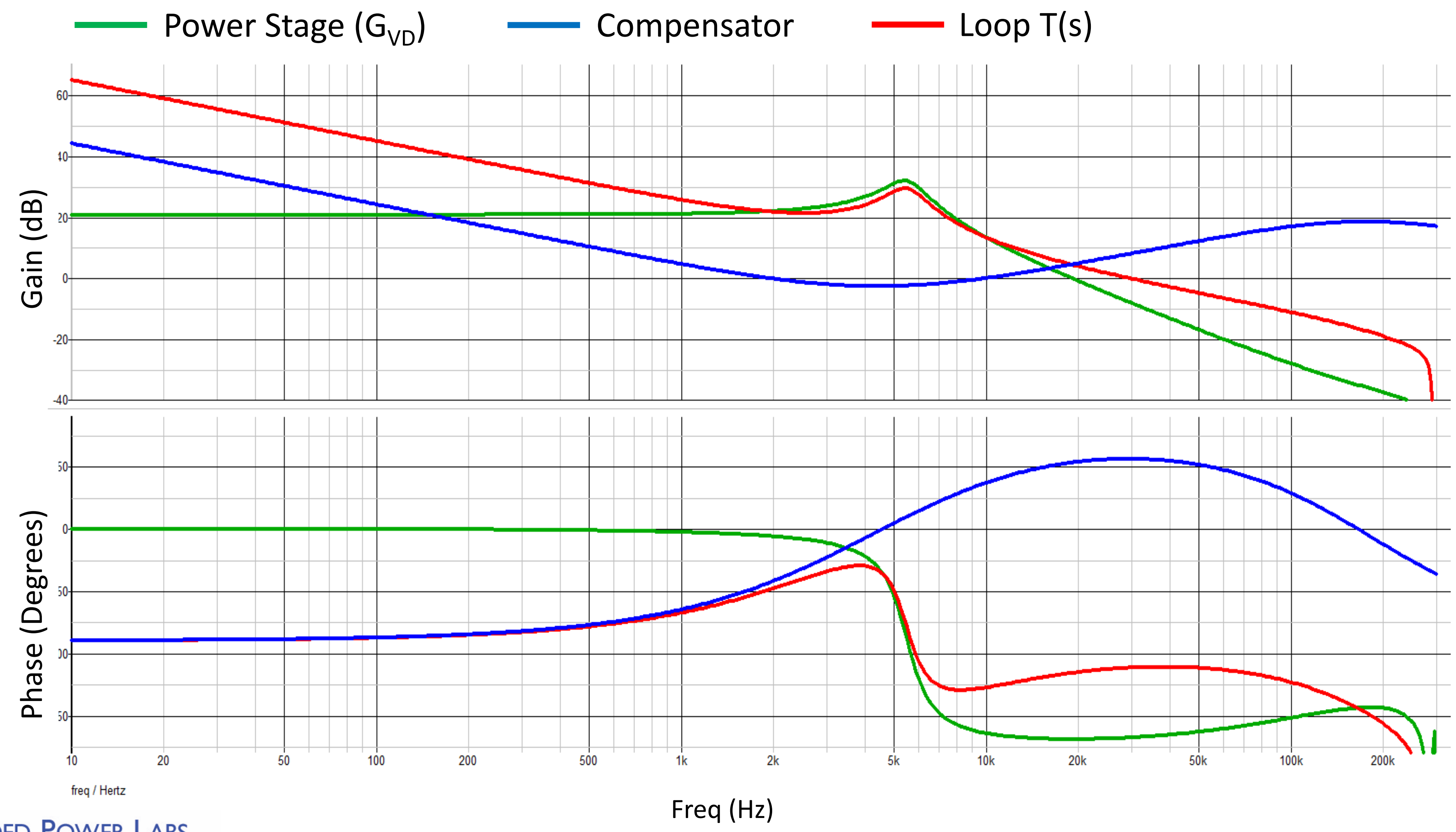
- Desired And Actual Frequencies

$F_{P0} = 1.527\,\text{kHz}$	$F_{P0} = 1.609\,\text{kHz}$
$F_Z = 4.35\,\text{kHz}$	$F_{Z1} = 4.292\,\text{kHz}$
$F_P = 206.9\,\text{kHz}$	$F_{Z2} = 4.495\,\text{kHz}$
	$F_{P1} = 212.7\,\text{kHz}$
	$F_{P2} = 195.1\,\text{kHz}$

K-Factor Example: Simulated Compensator

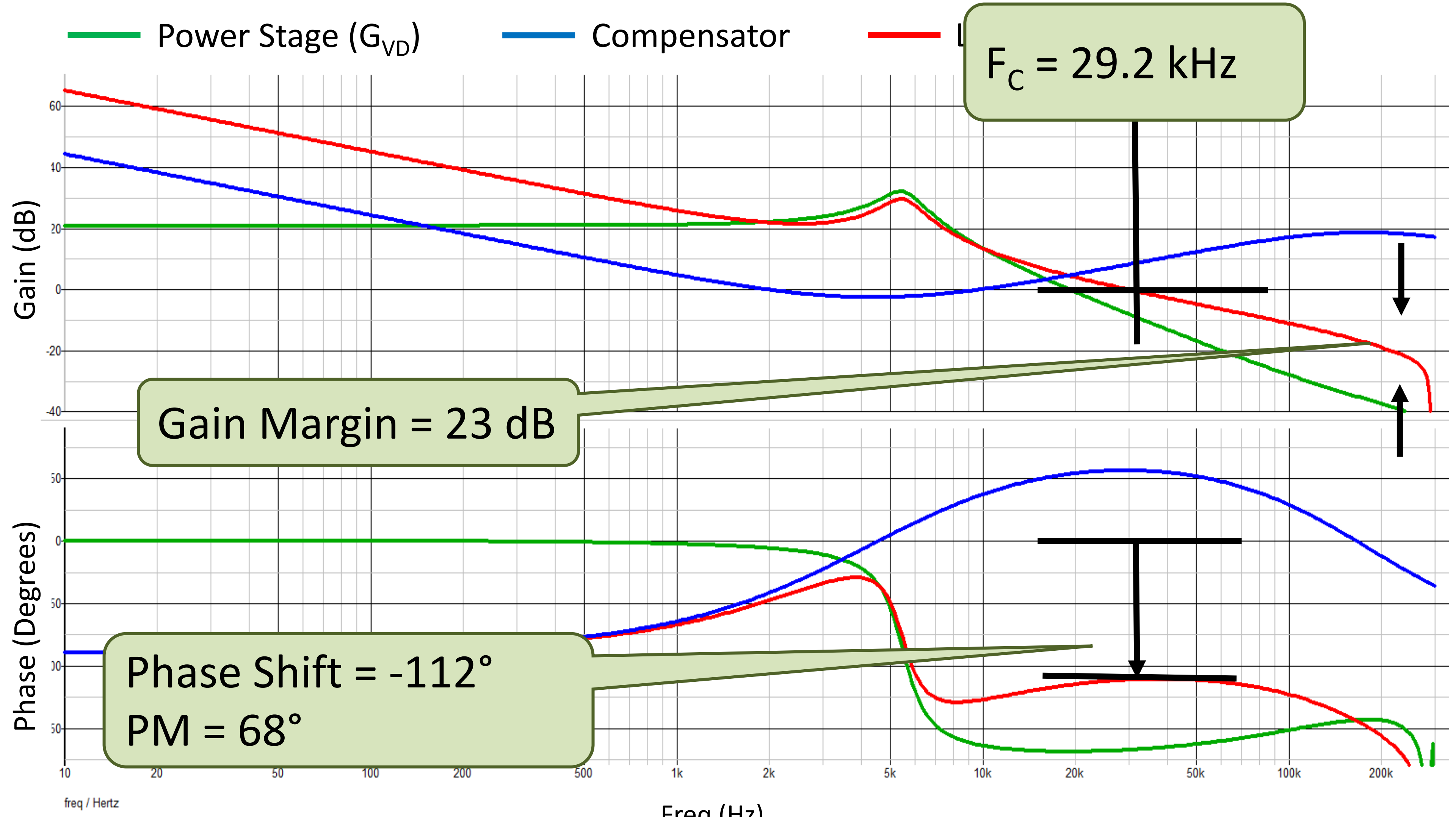


K-Factor Example: Simulation Bode Plot



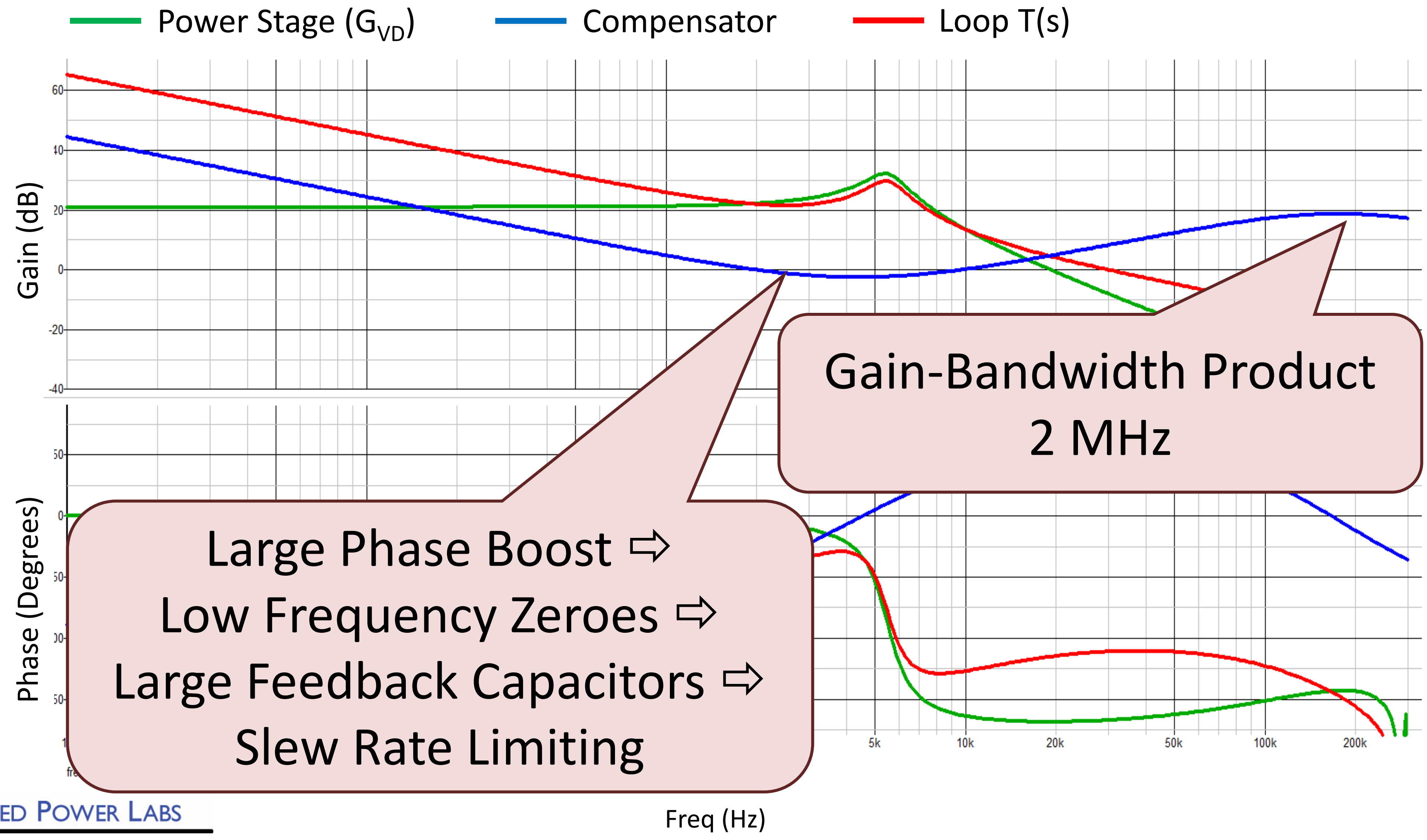
EMBEDDED POWER LABS

K-Factor Example: Simulation Bode Plot



EMBEDDED POWER LABS

K-Factor Example: Caveats



Loop Design Considerations

- Loop Characteristics Typically Vary A Lot!
  - Variations In G_{VD} With Input Voltage And Load
  - Tolerance Of Components In Output Filter And Compensator
  - Variation In Crossover Frequency Of One Octave Common

Loop Design Considerations

- Loop Characteristics Typically Vary A Lot!
  - Variations In G_{VD} With Input Voltage And Load
  - Tolerance Of Components In Output Filter And Compensator
  - Variation In F_C Of One Octave Common

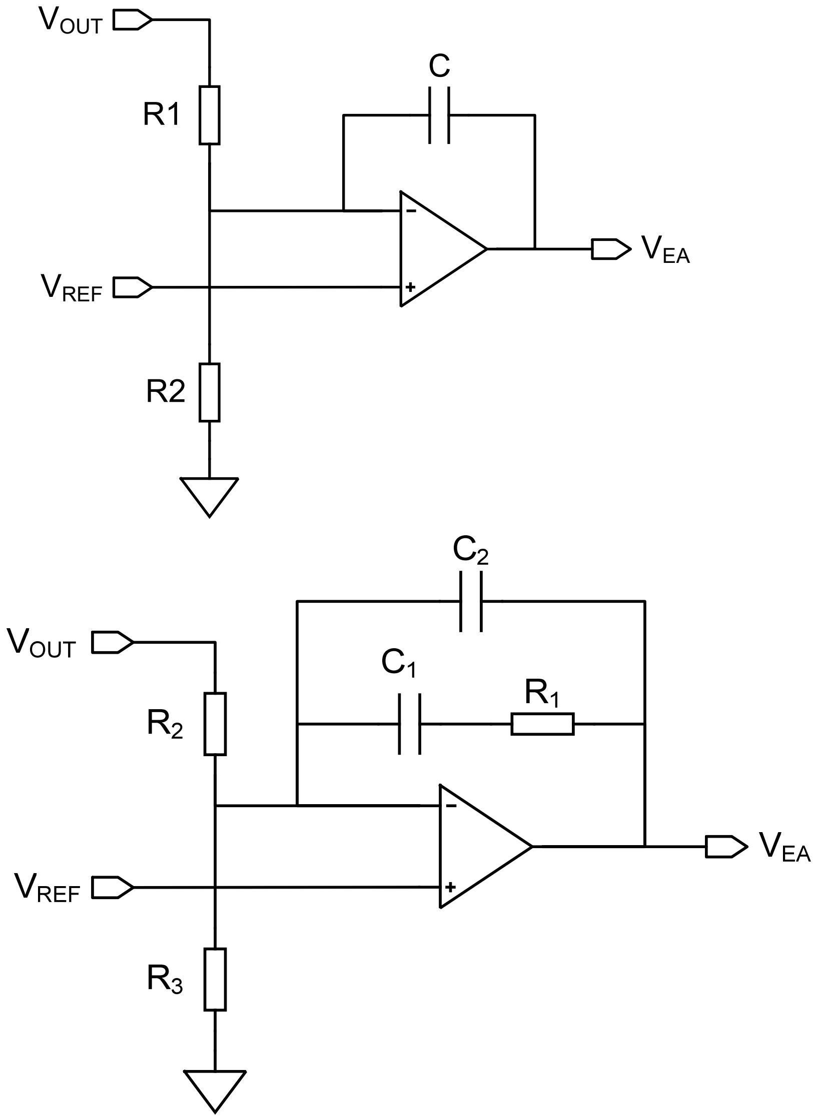
You MUST Do Some Kind Of Worst Case Analysis

You MUST Measure The Loop

Other Compensators And Modulators

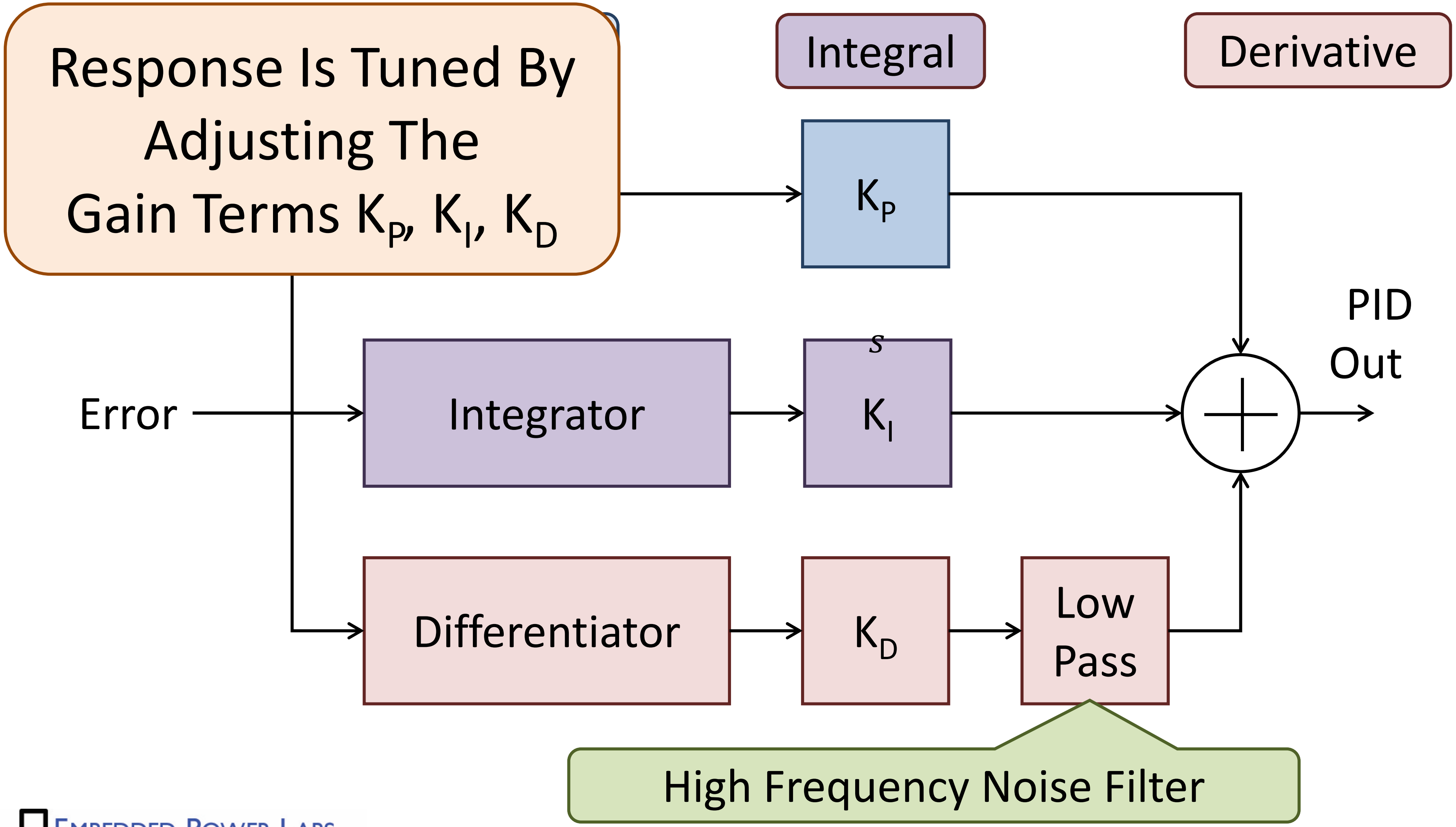
Other Standard Compensators

- How Much Phase Boost Needed?
  - Type I: No Phase Boost
    - Integrator Pole Only
  - Type II: Up To 90° Phase Boost
    - Integrator Pole
    - One Zero And One Pole
  - Type III: Up To 180° Phase Boost
    - Integrator Pole
    - Two Zeros And Two Poles
- See Appendices For Details



PID Compensator

$$H_{PID}(s) = K_P + K_I \cdot \frac{1}{s} + K_D \cdot s$$

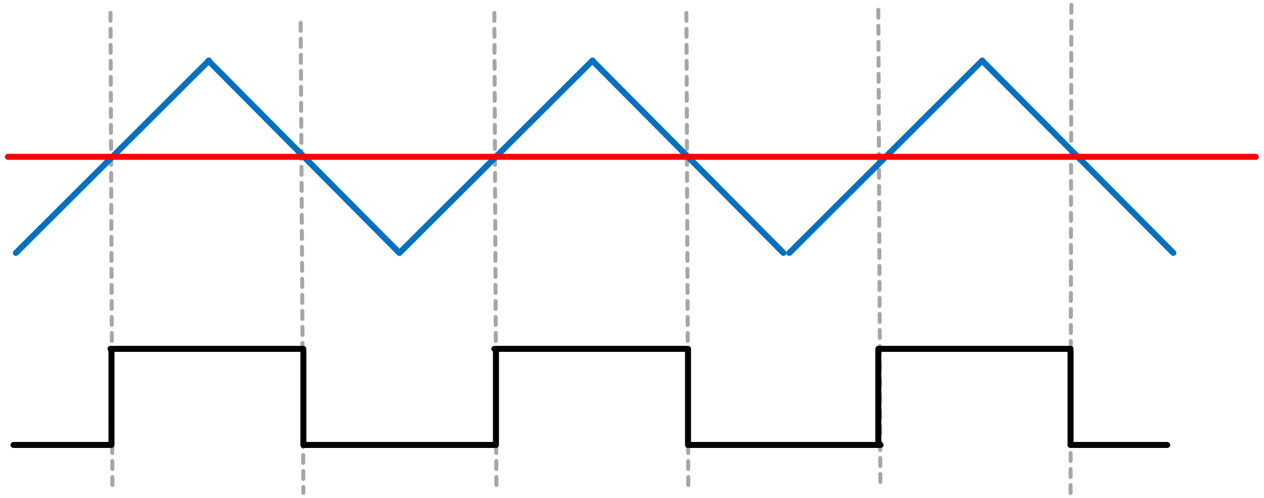


PID Compensator

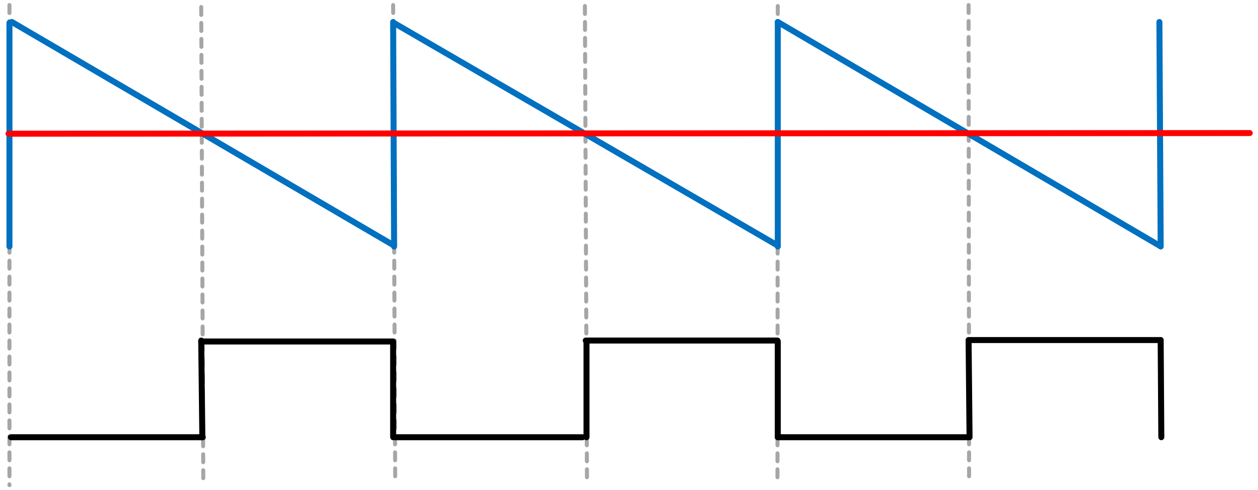
- PID Loved By Academics
  - Good Commercial Applications Are Limited
- Good For “One Of” Designs Where Plant Characteristics Are Not Well Known
  - Examples: Motor Drive, Industrial Processes
- Various Tuning Algorithms
  - e.g. Ziegler-Nichols

Analog Pulse Width Modulator

Double Edge Modulation

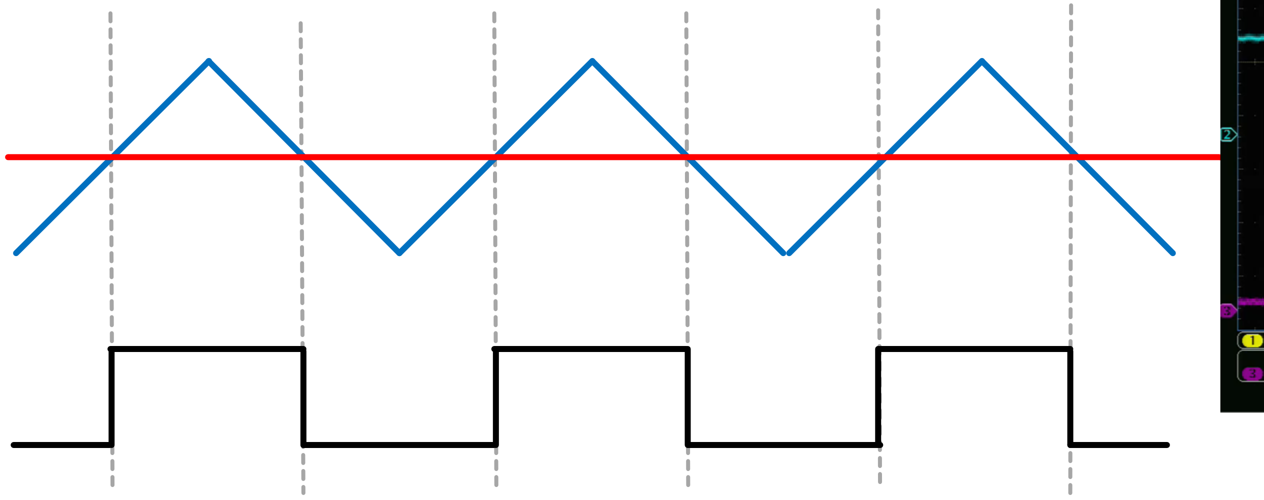


Leading Edge Modulation

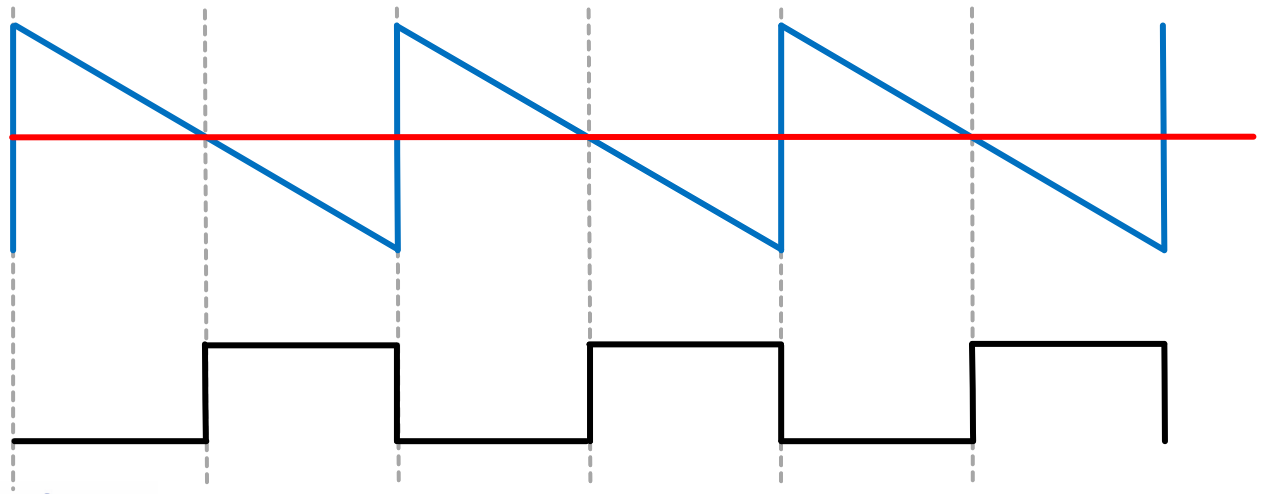


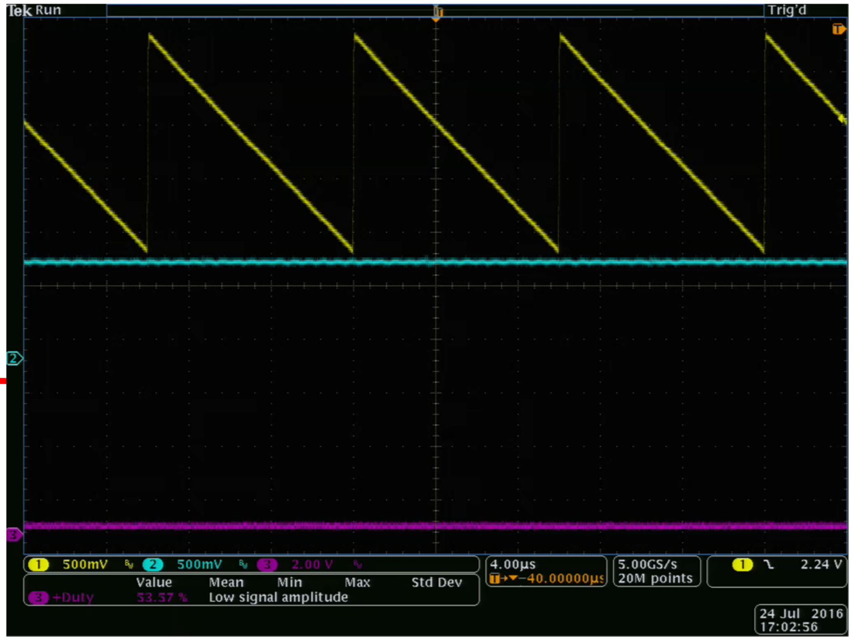
Video Lab 10: Leading Edge And Double Edge PWM

Double Edge Modulation



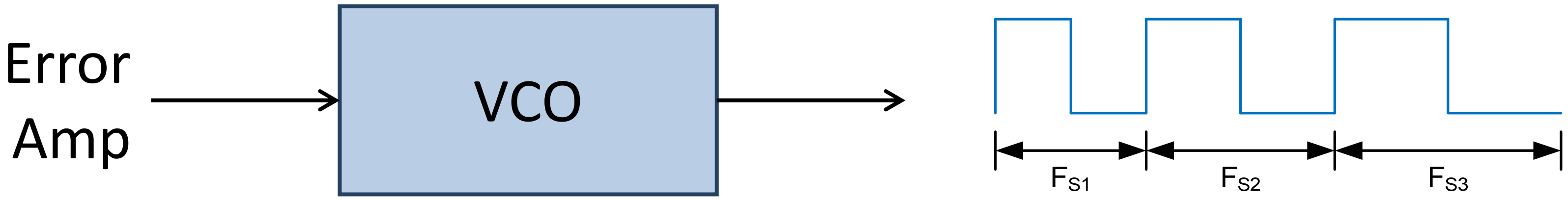
Leading Edge Modulation



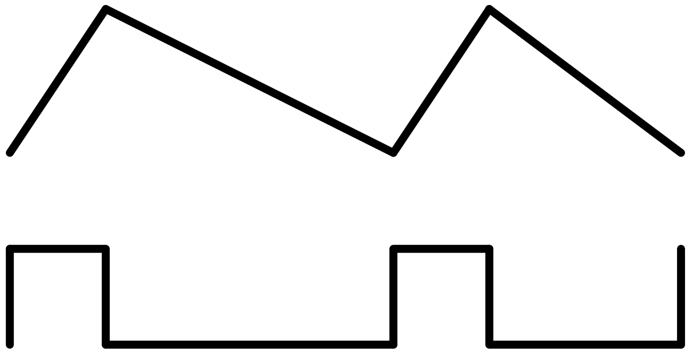


Variable Frequency Modulators

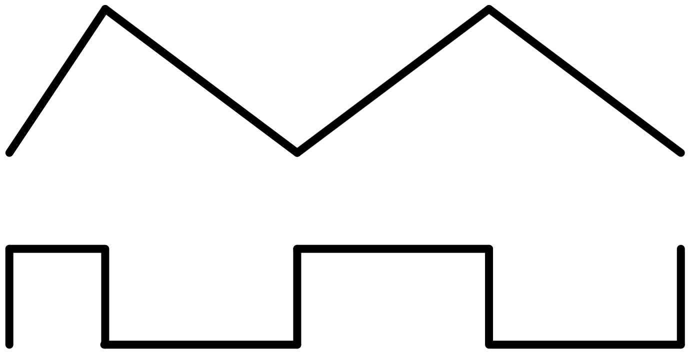
Constant Duty Cycle, Variable Frequency



Constant On Time

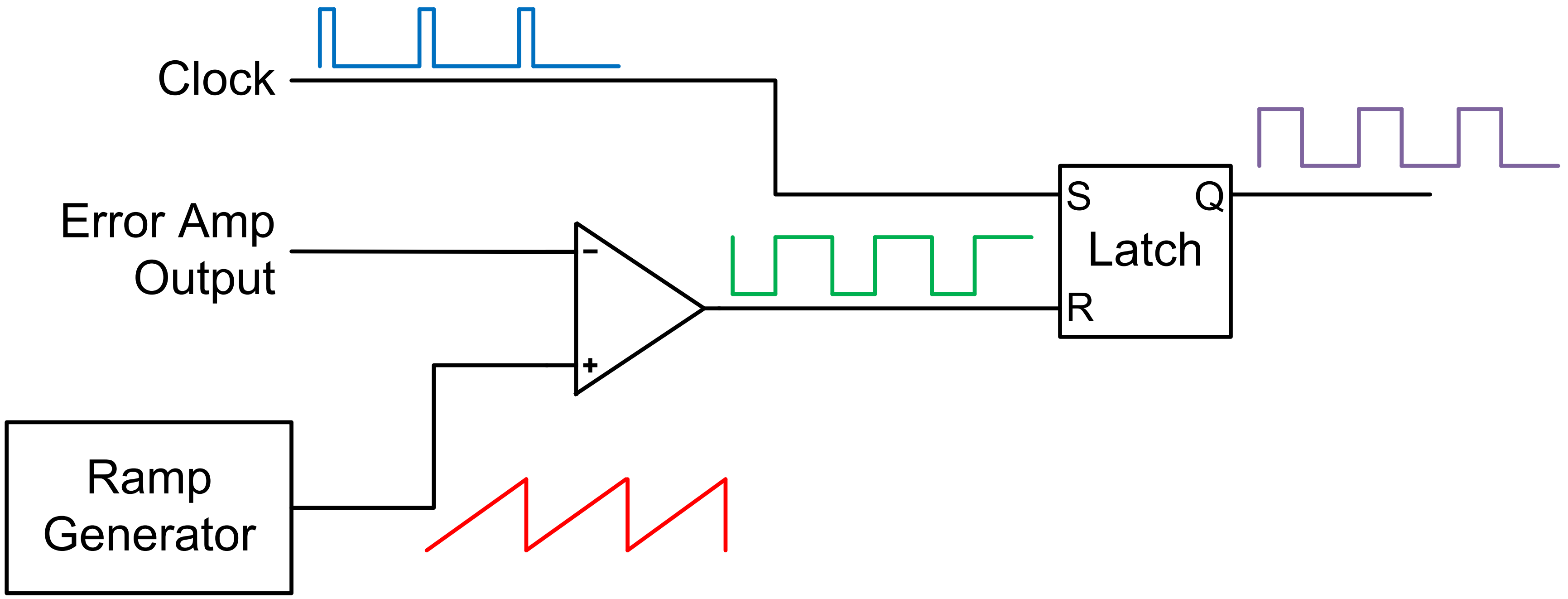


Constant Off Time

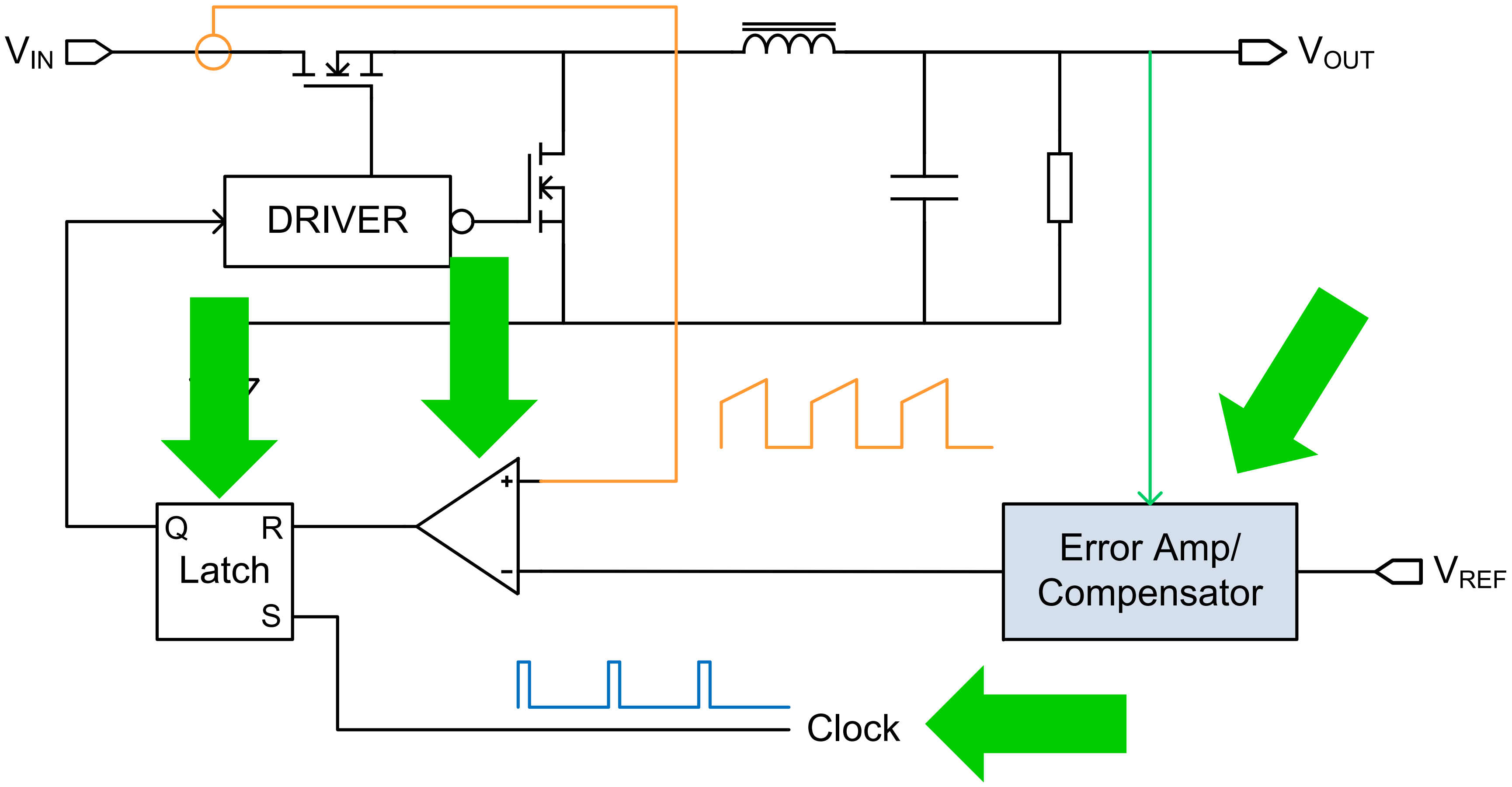


Current Mode Control Overview

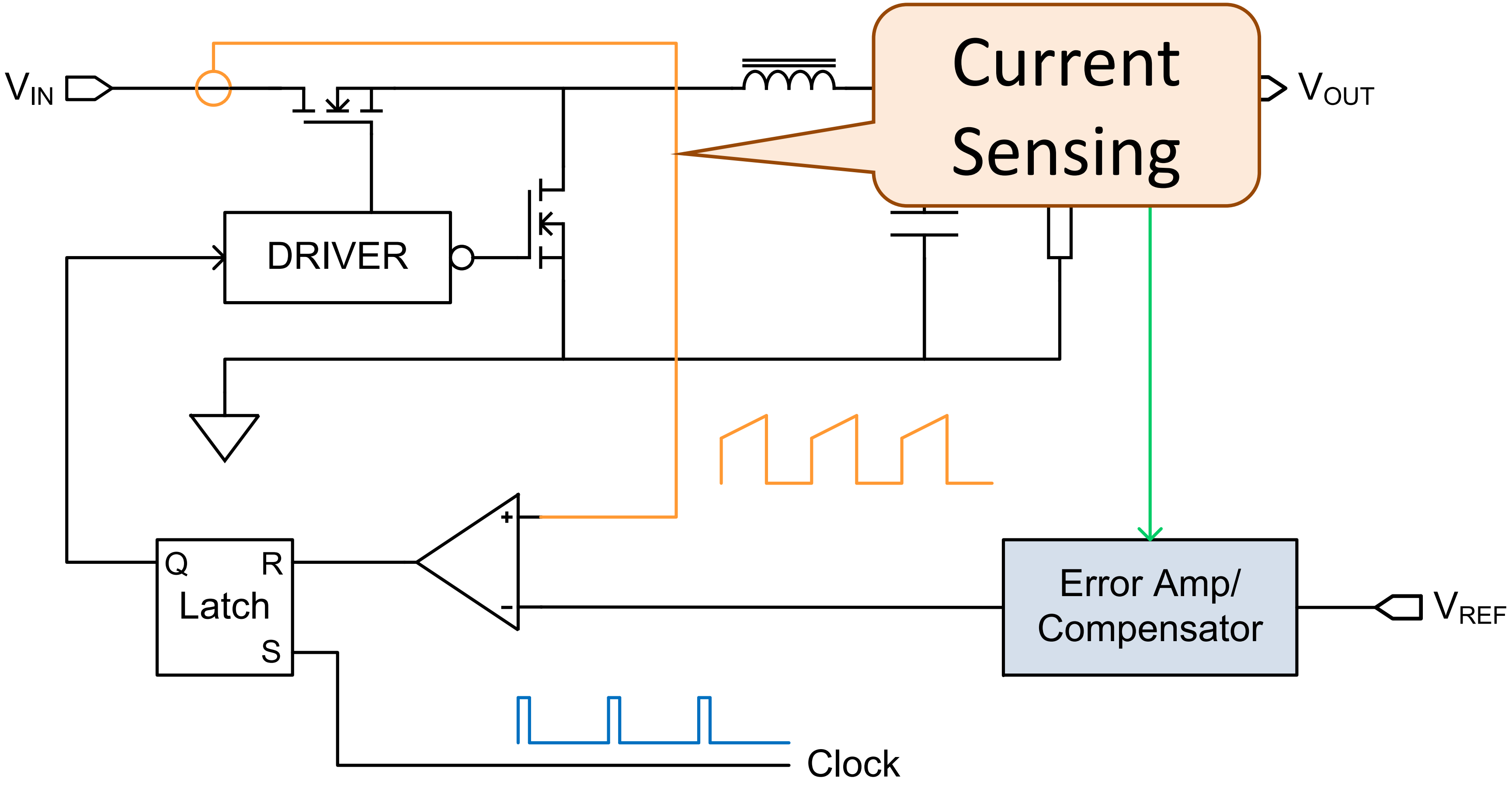
Analog PWM Reminder



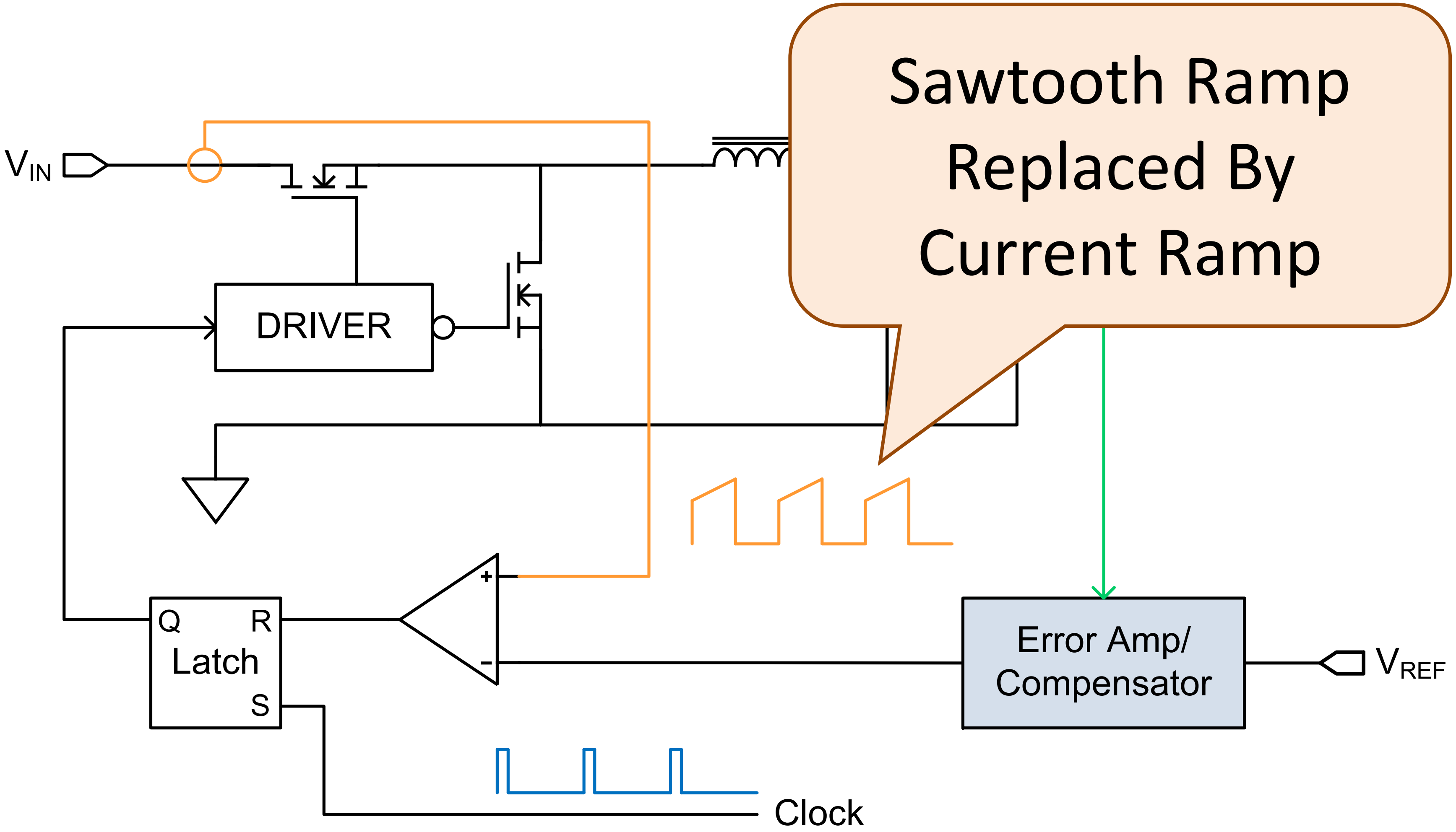
Peak Current Mode Control



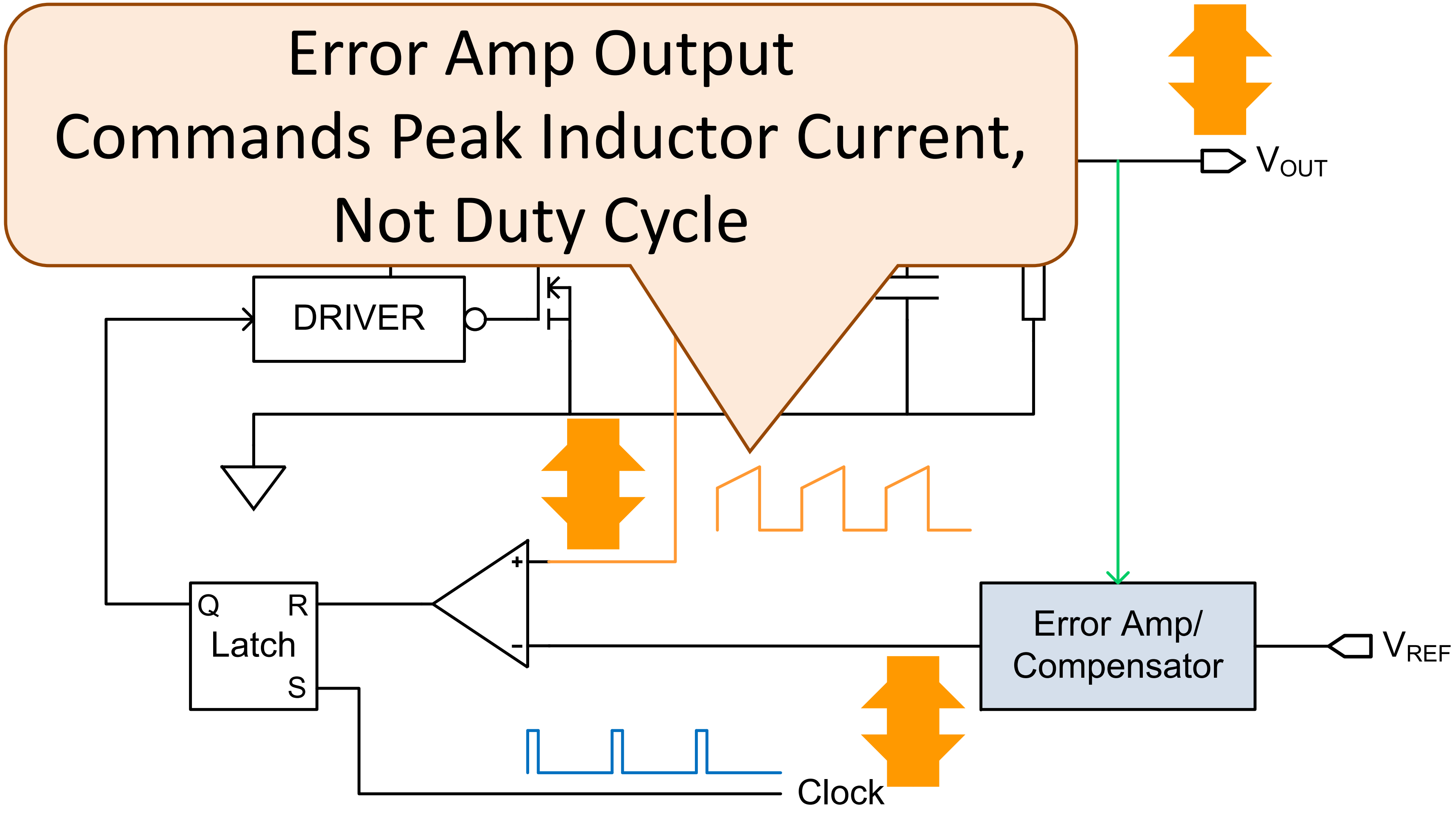
Peak Current Mode Control



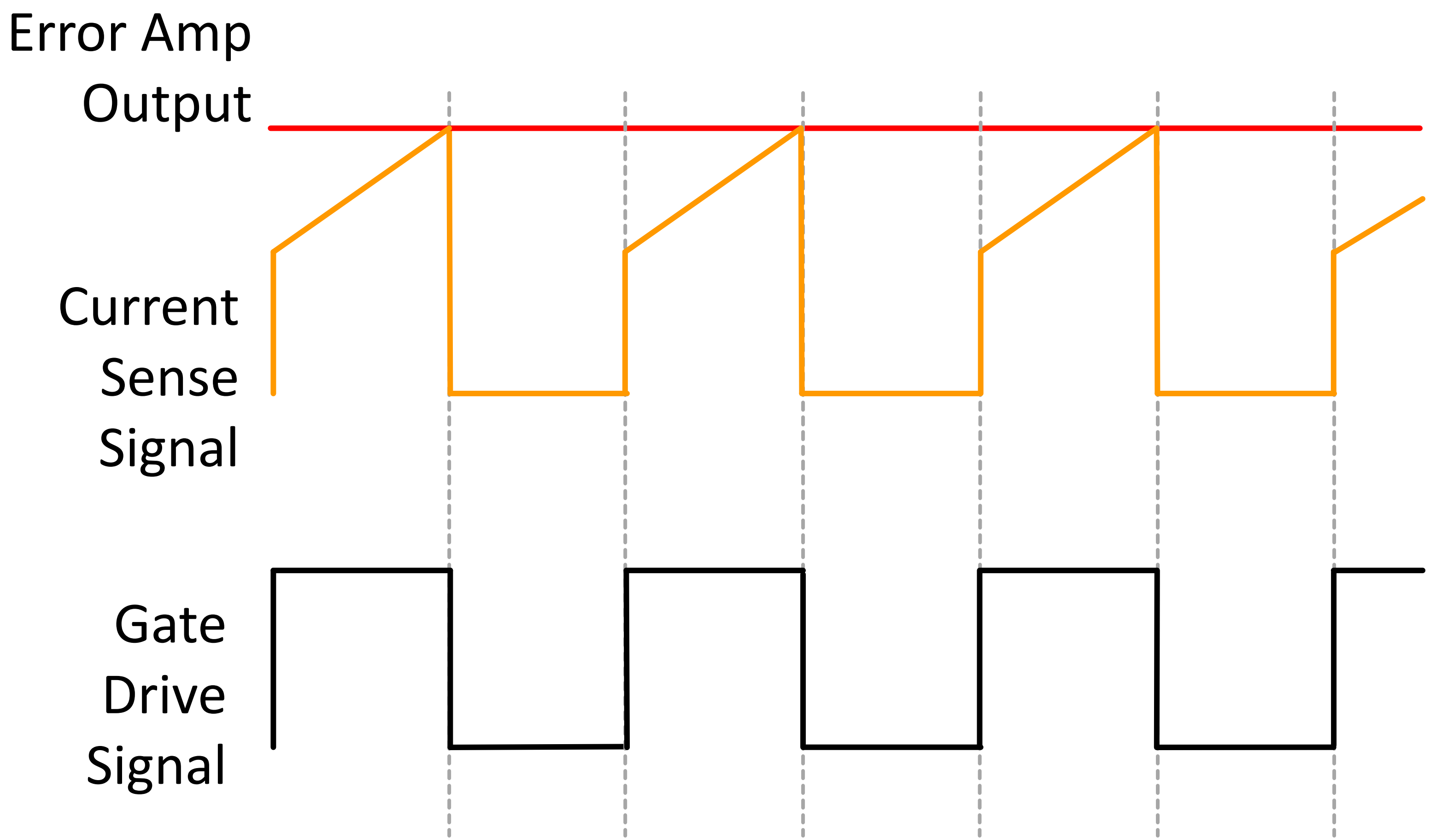
Peak Current Mode Control



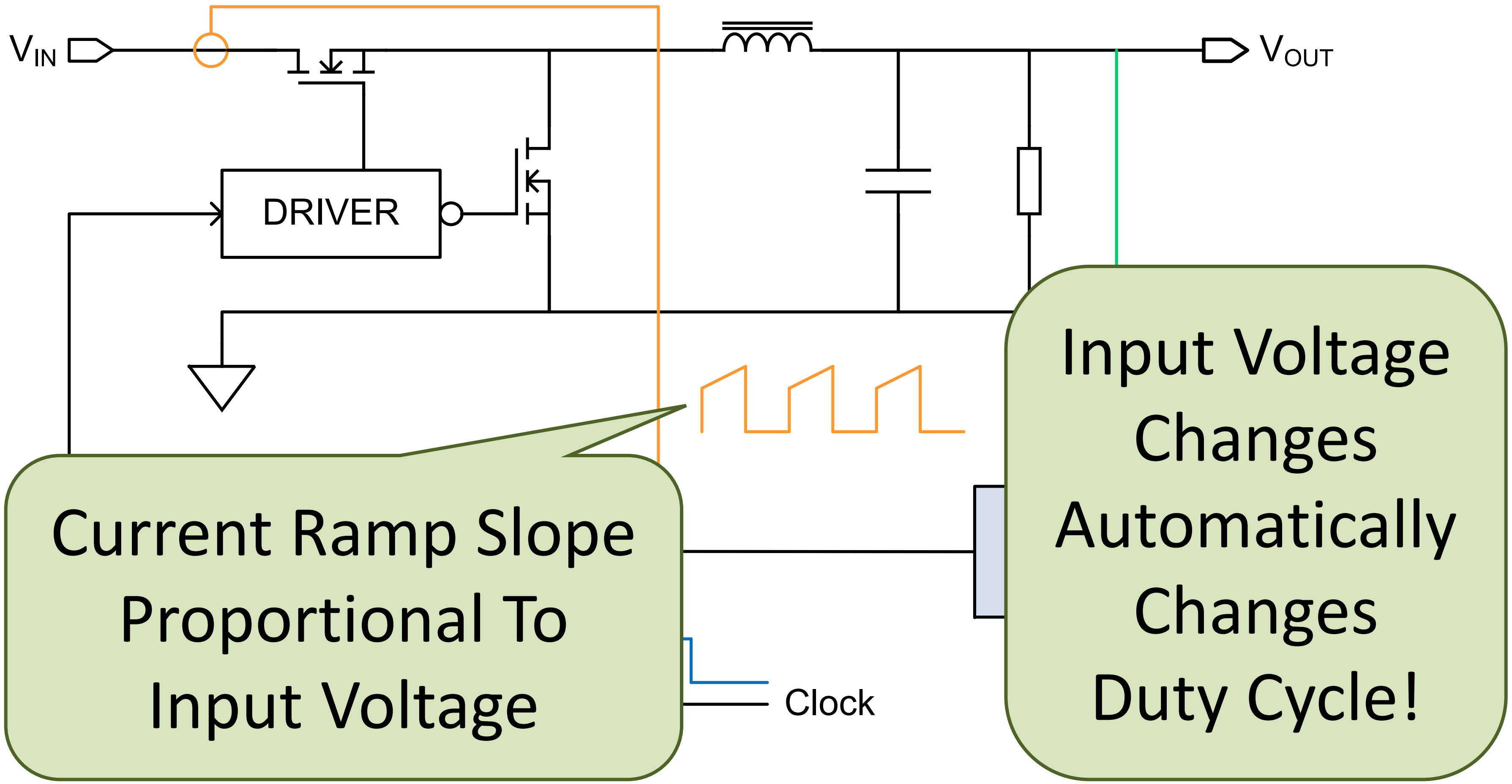
Peak Current Mode Control



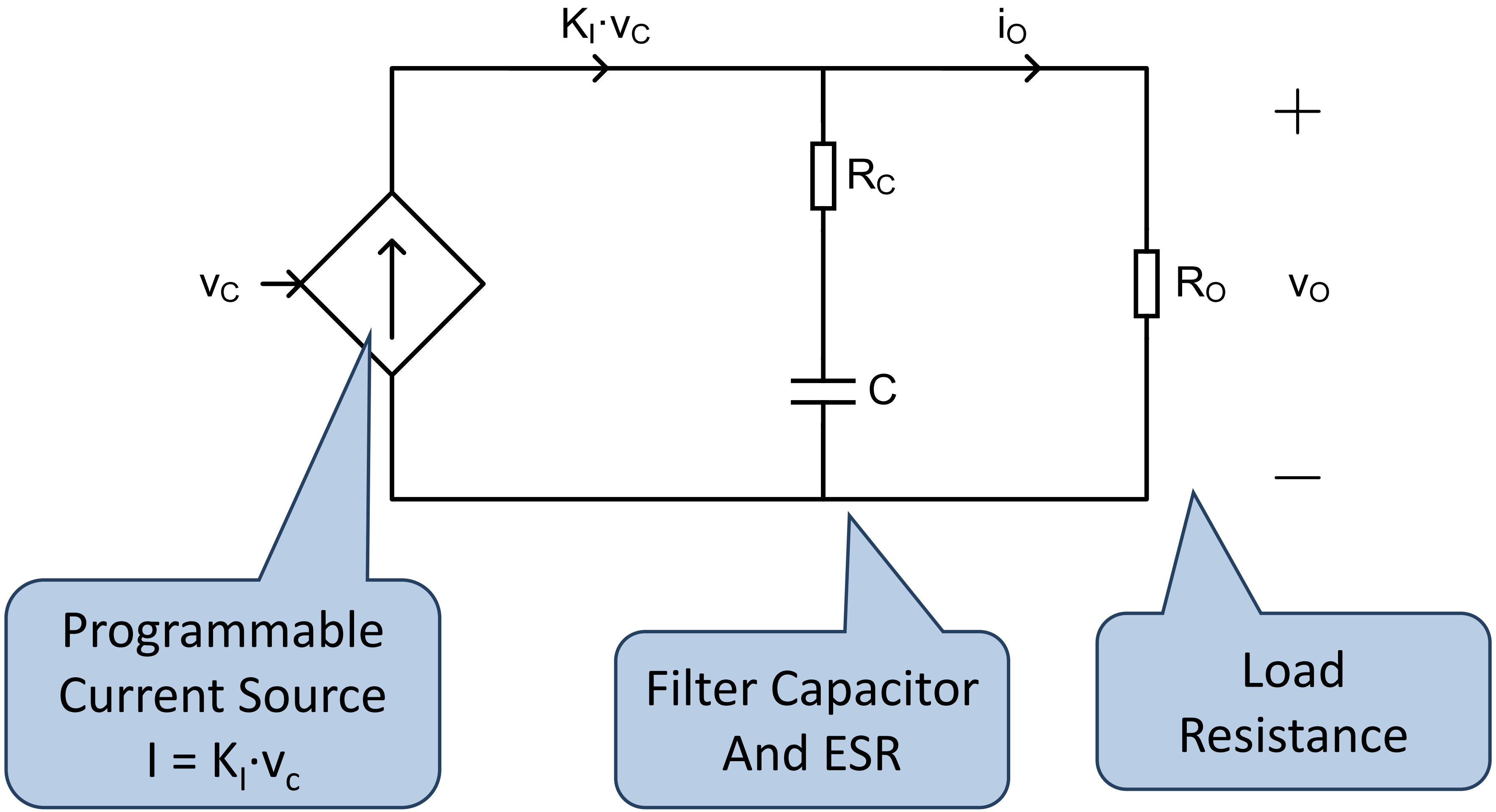
Peak Current Mode Control



Feed Forward



Control Characteristic G_{VC}



Control Characteristic G_{VC}

$$v_O(s) = K_I \cdot v_C(s) \cdot Z_O(s)$$

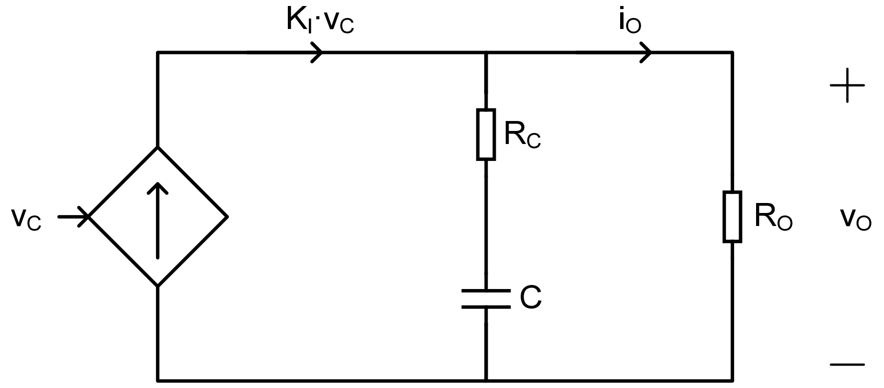
$$G_{VC}(s) = \frac{v_O(s)}{v_C(s)} = K_I \cdot Z_O(s)$$

$$G_{VC}(s) = K_I \cdot R_O \cdot \frac{1 + s \cdot R_C \cdot C}{1 + s \cdot (R_C + R_O) \cdot C}$$

$$G_{VC}(s) = G_O \cdot \frac{1 + \frac{s}{\omega_Z}}{1 + \frac{s}{\omega_P}}$$

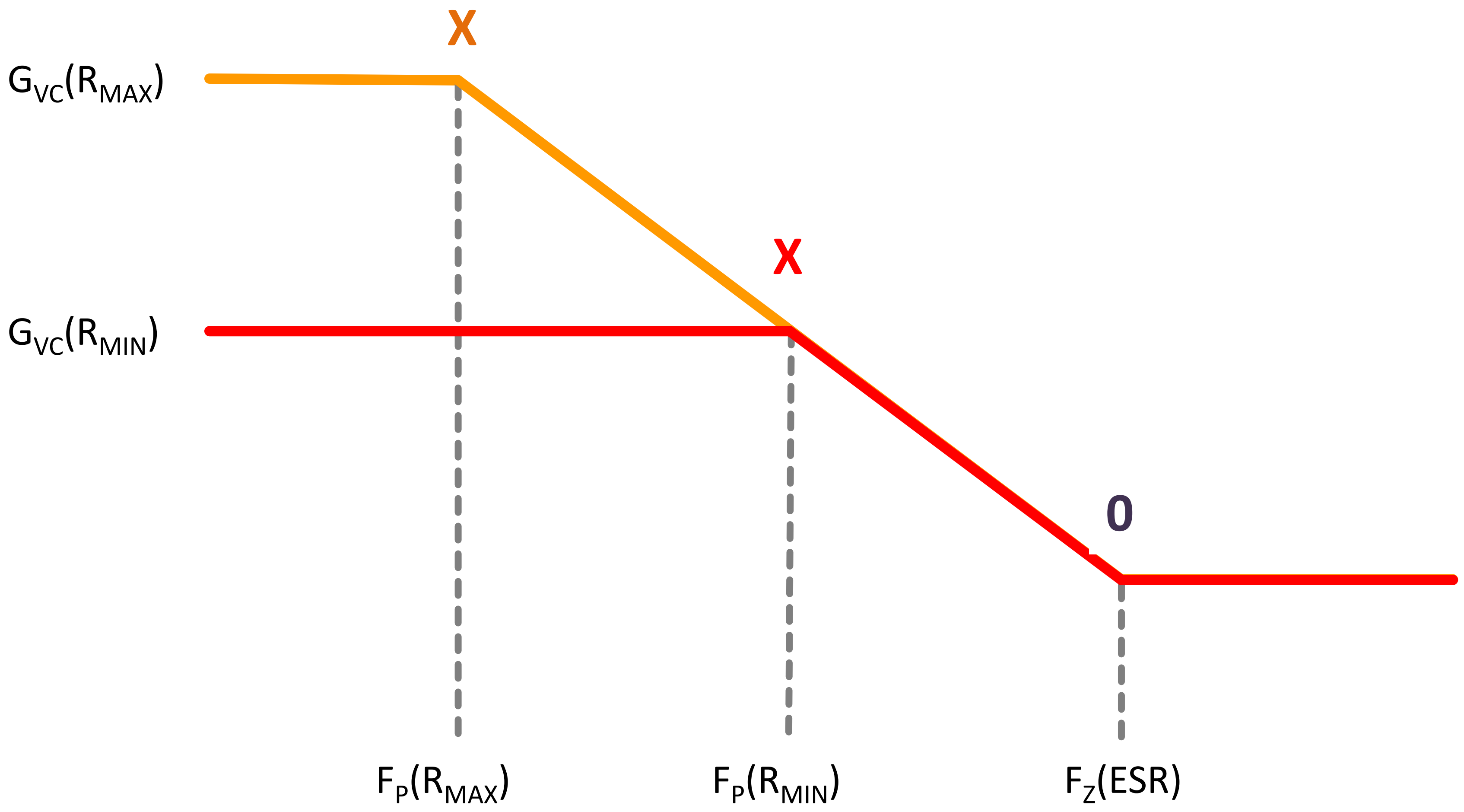
$$\omega_Z = \frac{1}{R_C \cdot C}$$

$$\omega_P = \frac{1}{(R_C + R_O) \cdot C} \approx \frac{1}{R_O \cdot C}$$

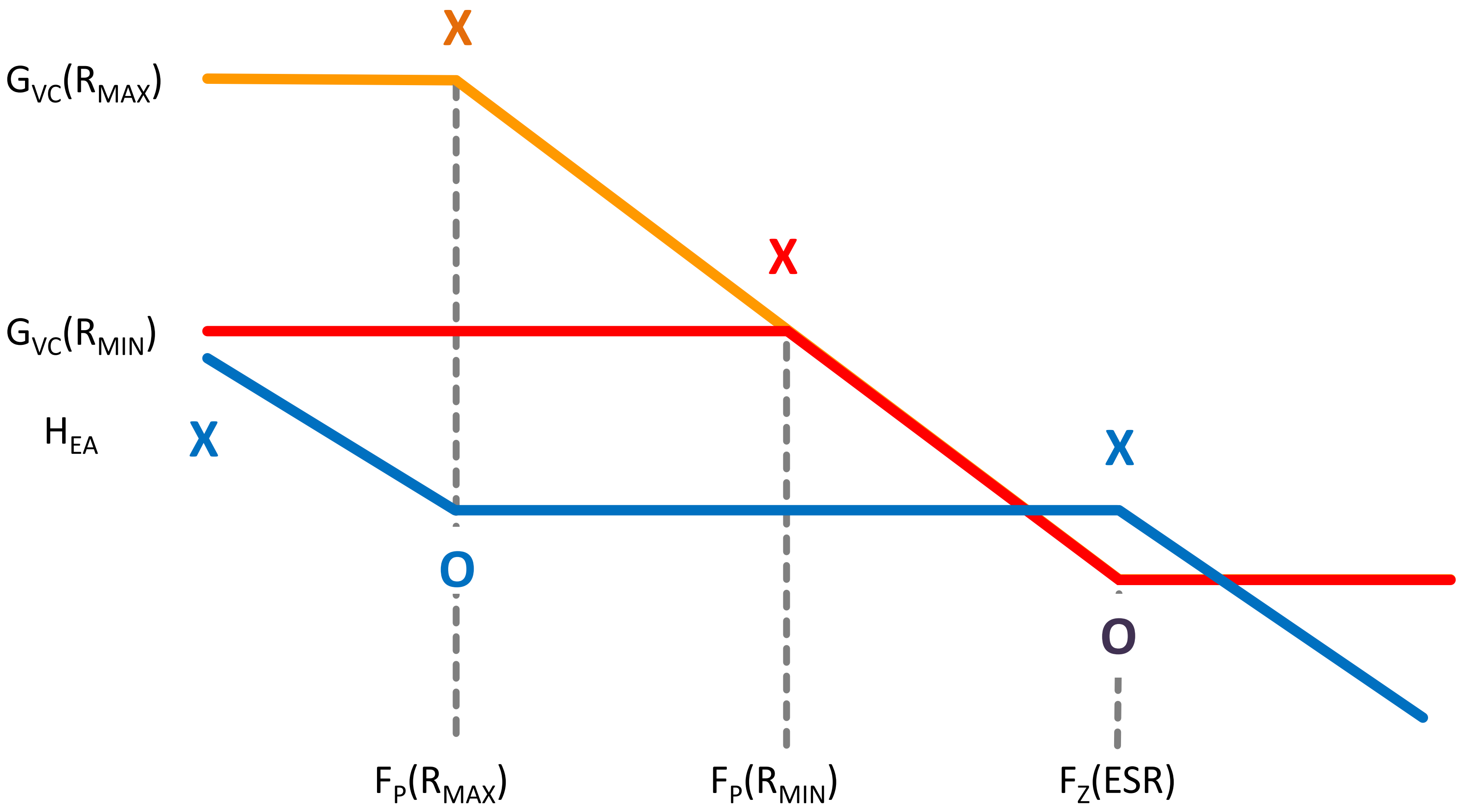


Control Characteristic G_{VC}

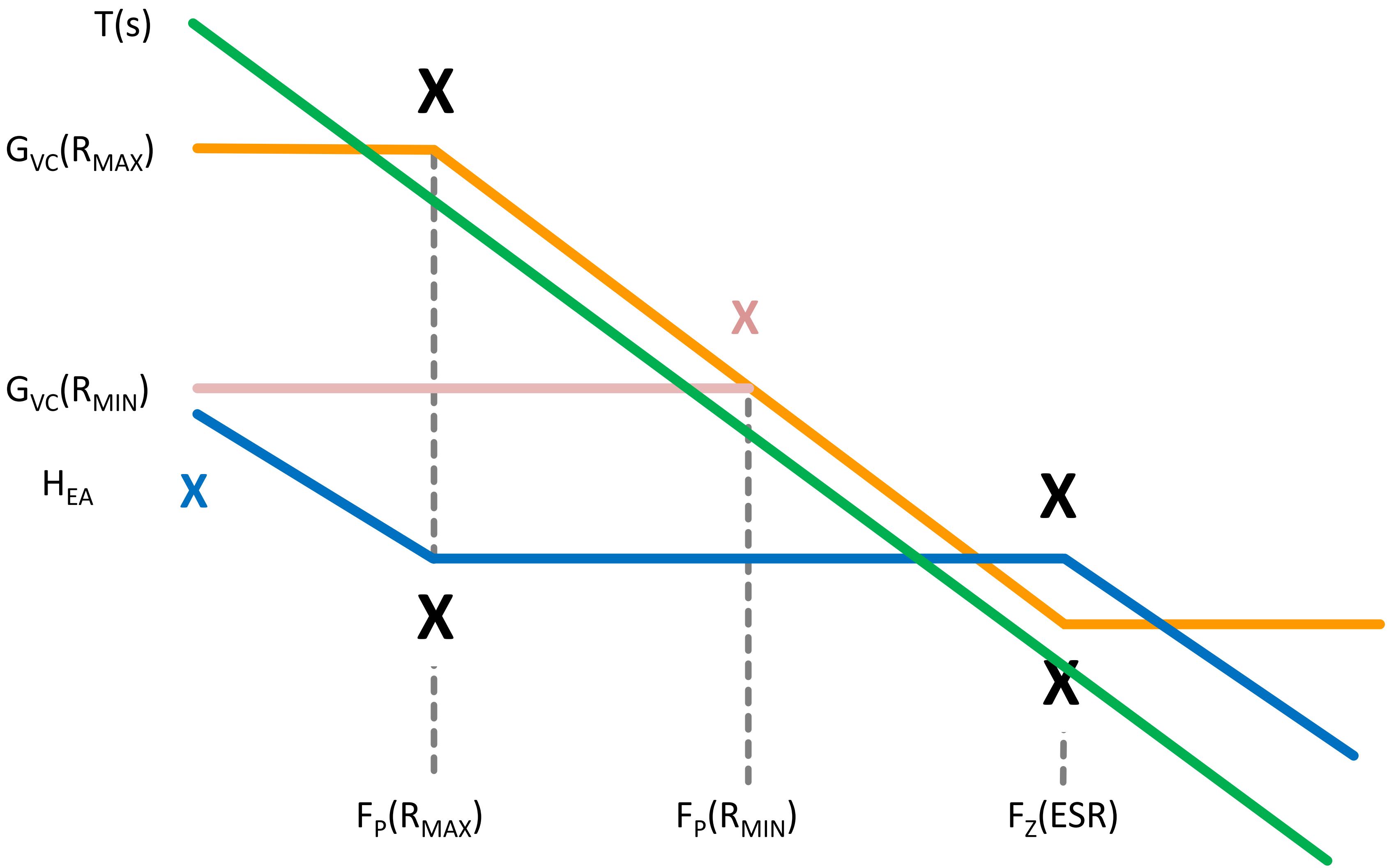
$$G_{VC}(s) = G_O \cdot \frac{1+\frac{s}{\omega_Z}}{1+\frac{s}{\omega_P}} = K_I \cdot R_O \cdot \frac{1+\frac{s}{\omega_Z}}{1+\frac{s}{\omega_P}}$$



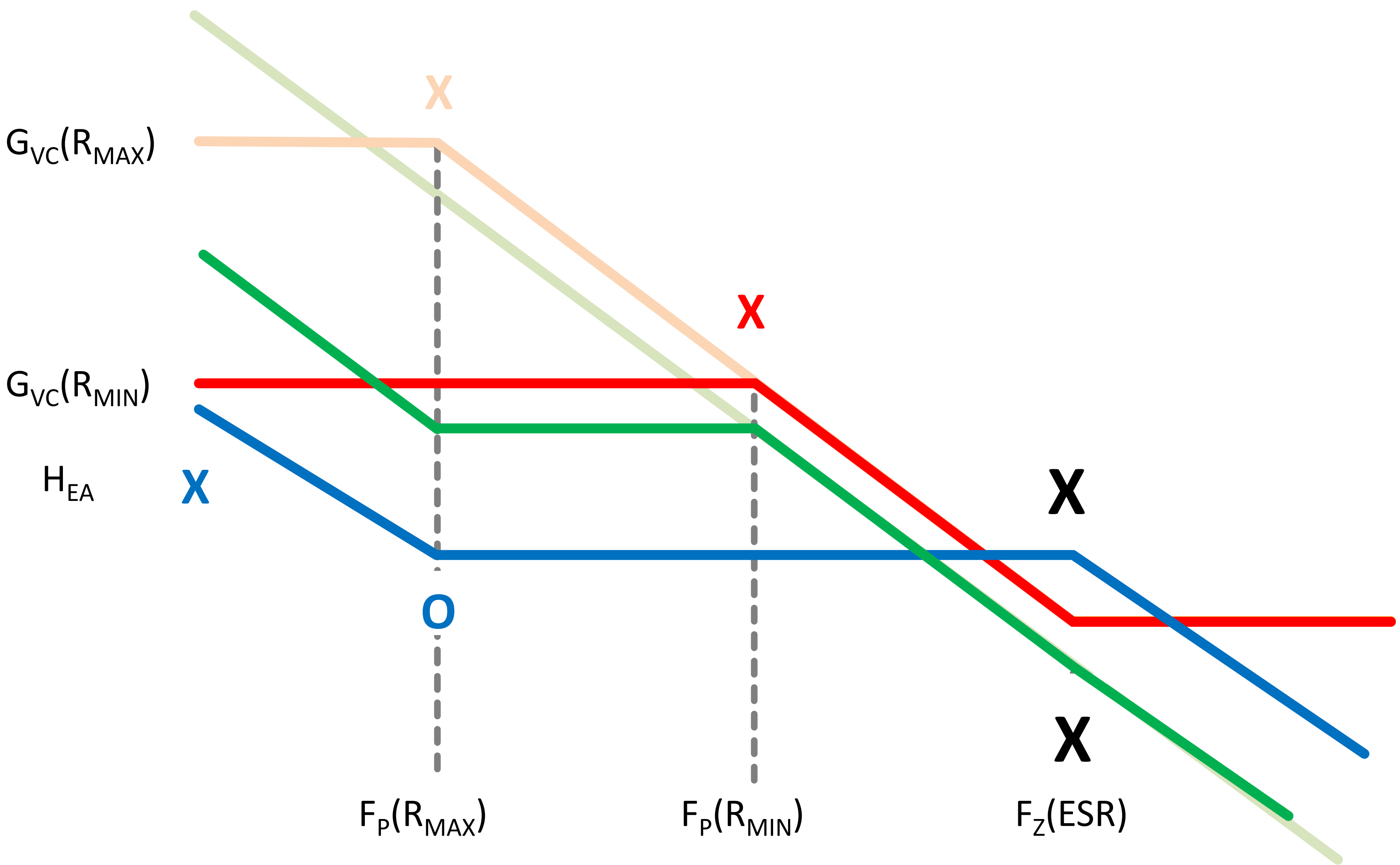
Compensator H_{EA}



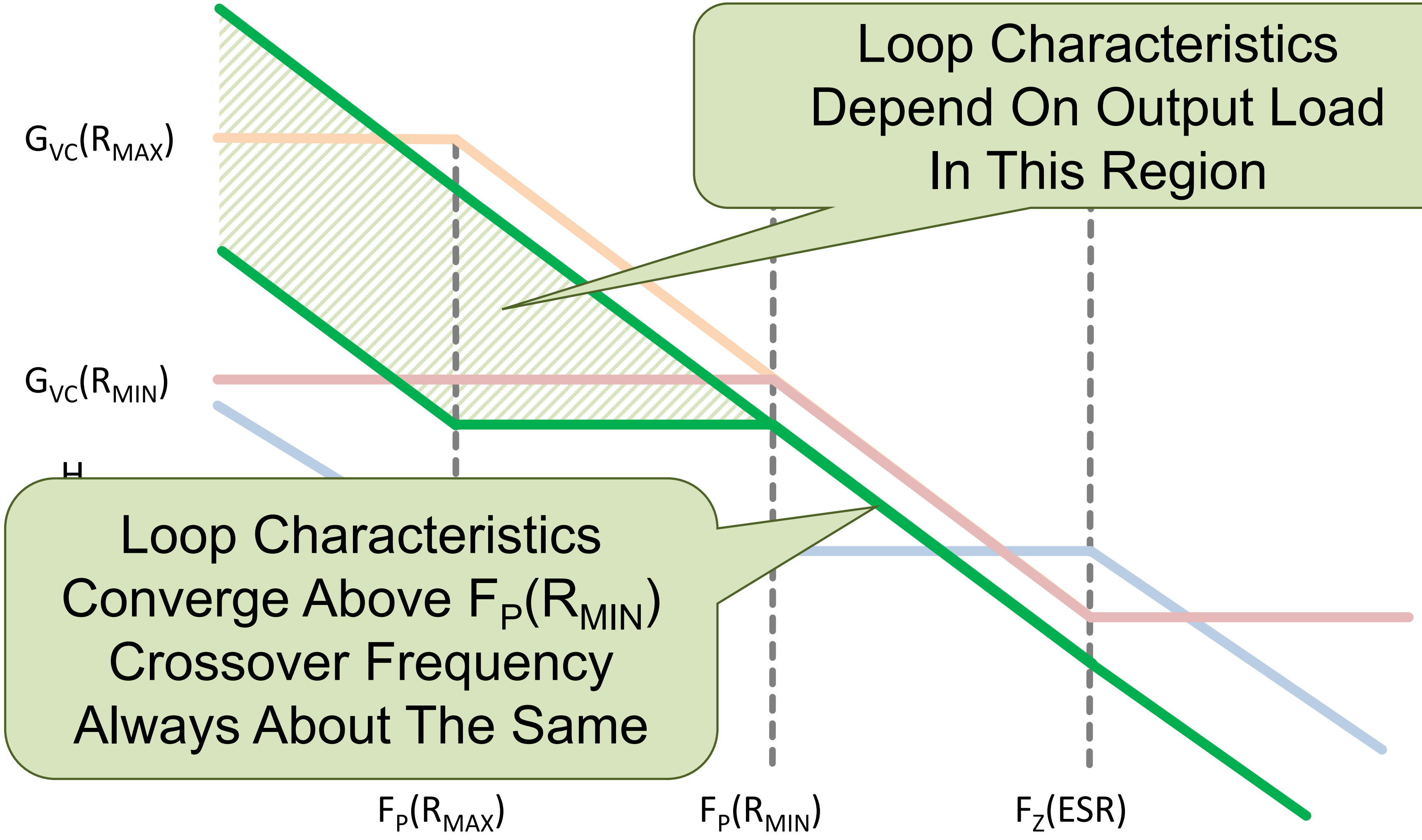
Loop Gain T(s)



Loop Gain T(s)

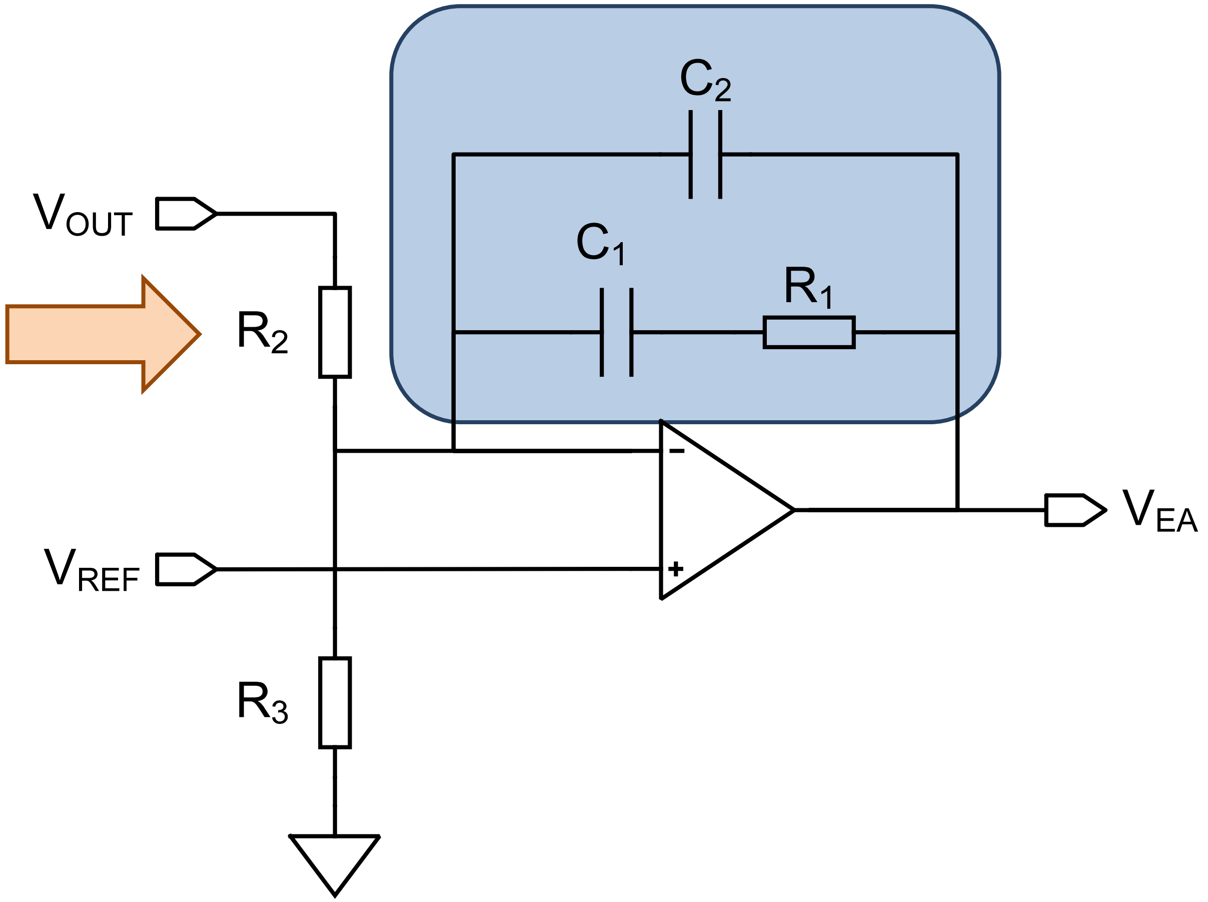


Loop Gain T



Loop Characteristics Depend On Output Load In This Region
$G_{VC}(R_{MAX})$
$G_{VC}(R_{MIN})$
Loop Characteristics Converge Above $F_P(R_{MIN})$ Crossover Frequency Always About The Same
$F_P(R_{MAX})$
$F_P(R_{MIN})$
$F_Z(ESR)$

Type II Compensator



Type II Compensator

$$H_2(s) = -\frac{Z_F}{Z_S}$$

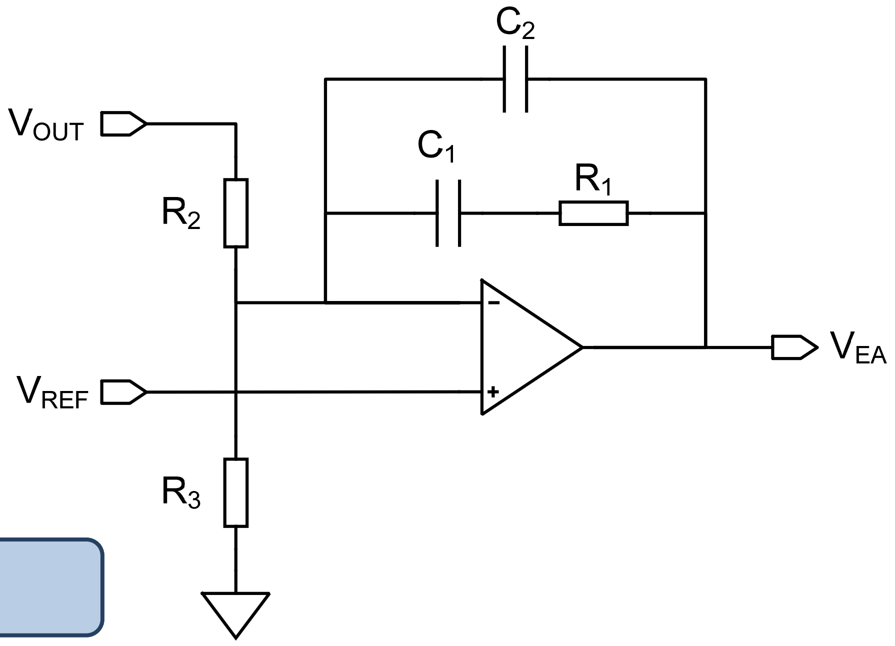
$$= -\frac{\frac{1}{s\cdot(C_1+C_2)}\cdot\frac{1+s\cdot R_1\cdot C_1}{1+s\cdot R_1\cdot\frac{C_1\cdot C_2}{C_1+C_2}}}{R_2}$$

$$= -\frac{1}{s\cdot R_2\cdot(C_1+C_2)}\cdot\frac{1+s\cdot R_1\cdot C_1}{1+s\cdot R_1\cdot\frac{C_1\cdot C_2}{C_1+C_2}}$$

Zero

Integrator Pole

Second Pole



Compensator Frequencies

$$T(s) = \frac{\omega_c}{s} = H_{EA}(s) \cdot G_{VC}(s)$$

$$T(s) = \frac{\omega_c}{s} = \left(\frac{\omega_P}{s} \cdot \frac{1 + \frac{s}{\omega_{ZC}}}{1 + \frac{s}{\omega_{PC}}} \right) \cdot \left(G_O \cdot \frac{1 + \frac{s}{\omega_Z}}{1 + \frac{s}{\omega_P}} \right)$$

$$T(s) = \frac{\omega_c}{s} = \left(\frac{\omega_P}{s} \cdot \frac{1 + \frac{s}{\omega_{ZC}}}{1 + \frac{s}{\omega_{PC}}} \right) \cdot \left(K_I \cdot R_O \cdot \frac{1 + \frac{s}{\omega_Z}}{1 + \frac{s}{\omega_P}} \right)$$

Compensator Frequencies

$$T(s) = \frac{\omega_c}{s} = H_{EA}(s) \cdot G_{VC}(s)$$

$$T(s) = \frac{\omega_c}{s} = \left(\frac{\omega_P}{s} \cdot \frac{1 + \frac{s}{\omega_{ZC}}}{1 + \frac{s}{\omega_{PC}}} \right) \cdot \left(G_O \cdot \frac{1 + \frac{s}{\omega_Z}}{1 + \frac{s}{\omega_P}} \right)$$

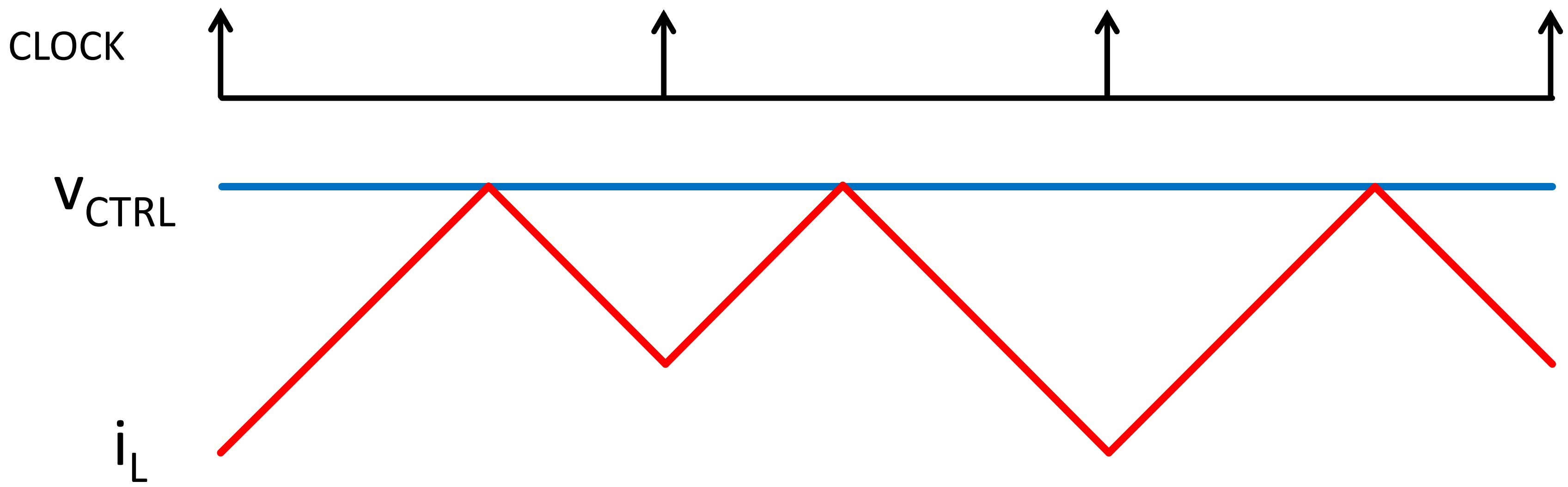
$$\omega_{ZC} = \omega_P \approx \frac{1}{R_O \cdot C}$$

$$\omega_{PC} = \omega_Z$$

$$T(s) = \frac{\omega_c}{s} = \left(\frac{\omega_P}{s} \cdot \frac{1 + \frac{s}{\omega_{ZC}}}{1 + \frac{s}{\omega_{PC}}} \right) \cdot \left(K_I \cdot R_O \cdot \frac{1 + \frac{s}{\omega_Z}}{1 + \frac{s}{\omega_P}} \right)$$

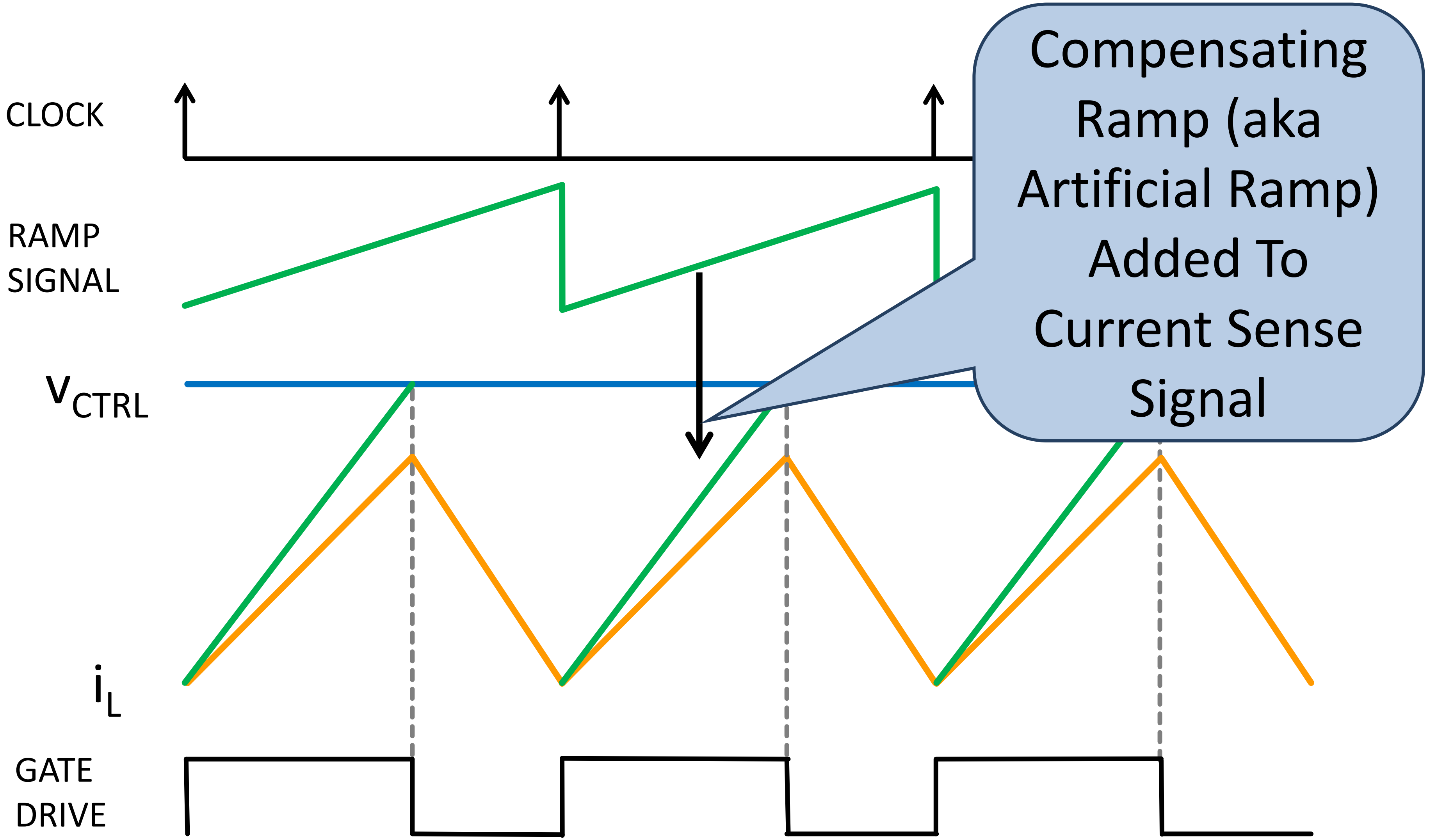
$$\omega_P = \frac{\omega_c}{K_I \cdot R_O}$$

Sub-Harmonic Oscillation

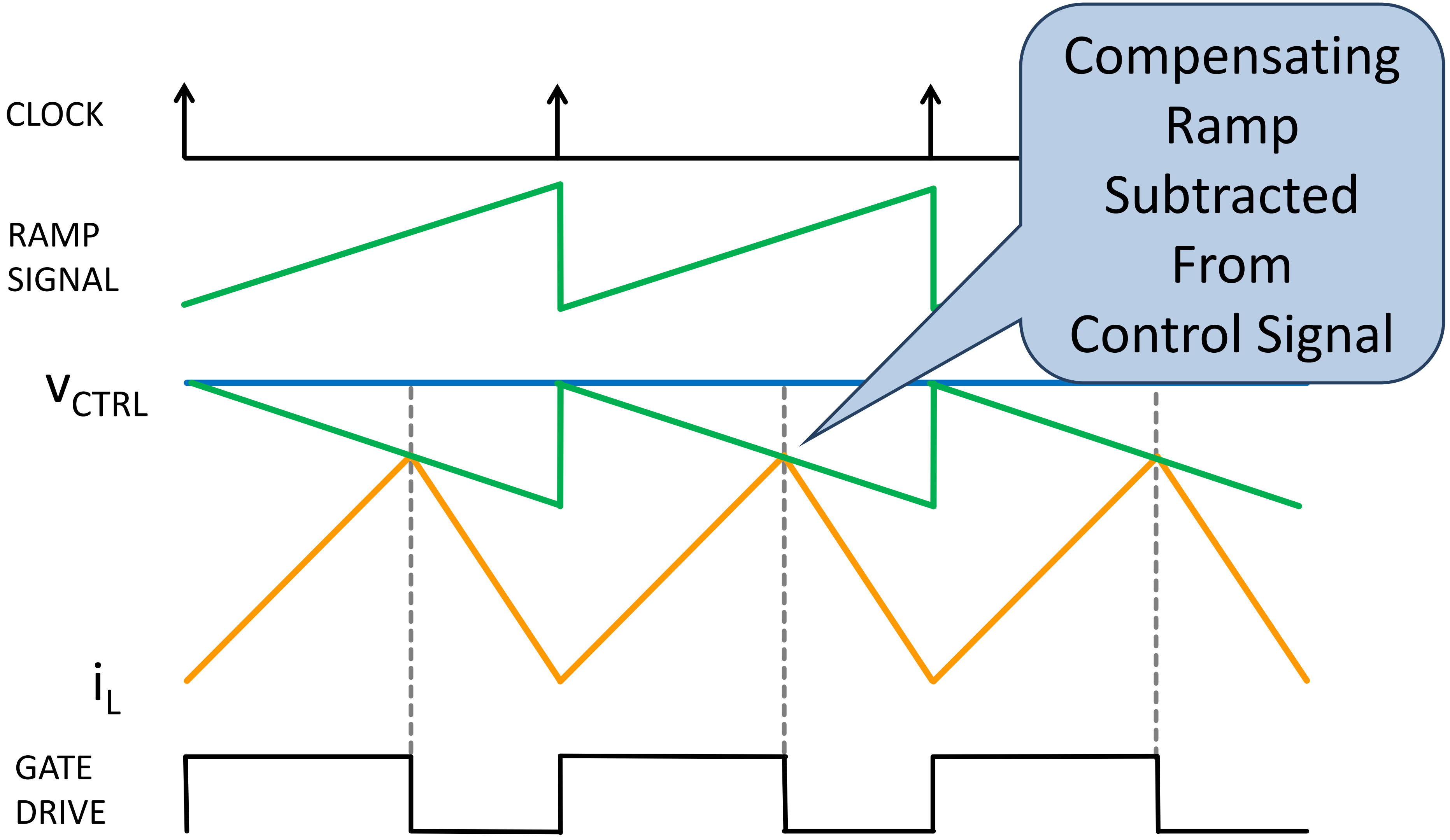


This Oscillation Happens For Duty Cycles Greater Than 50%

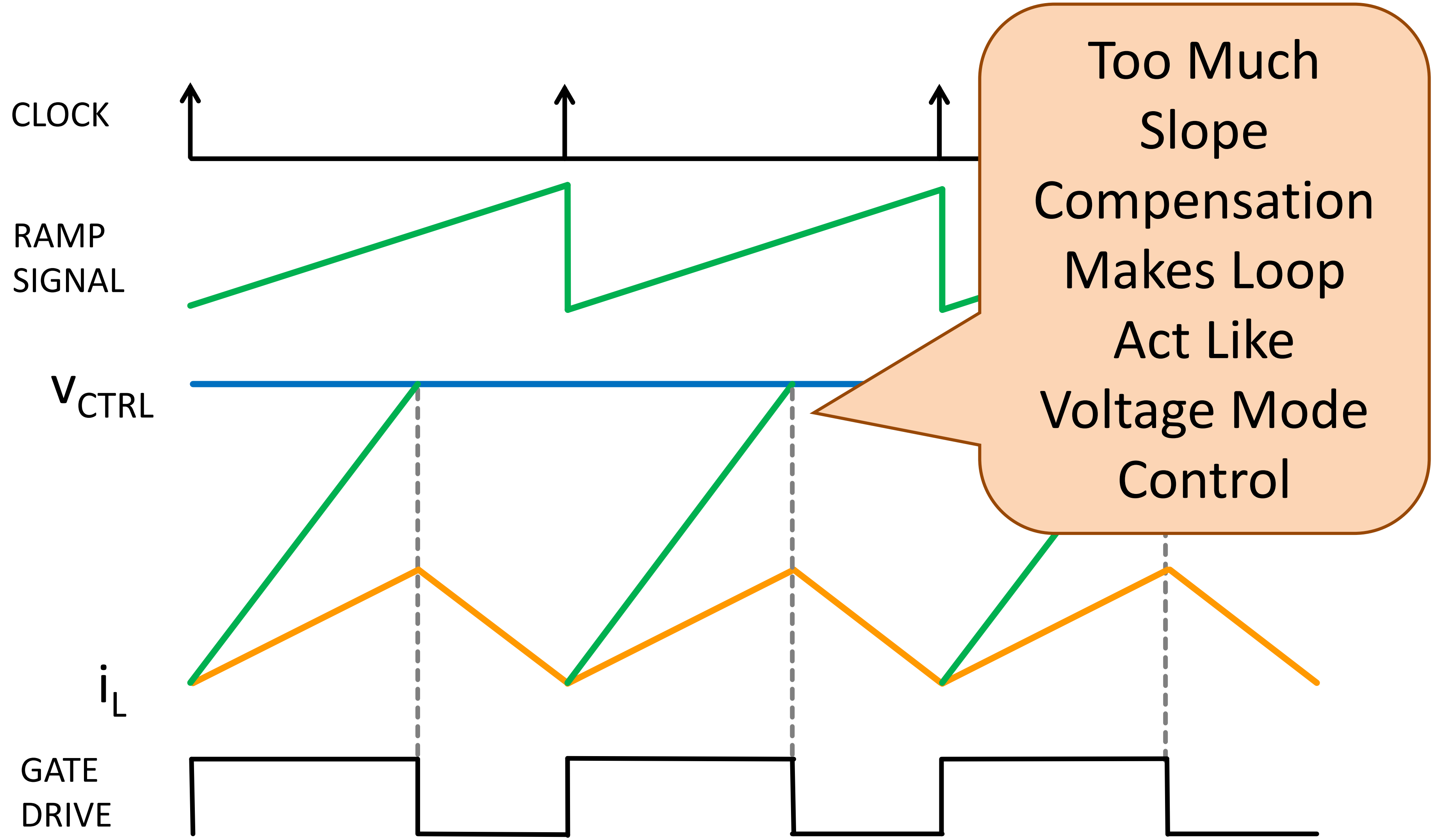
Slope Compensation



Slope Compensation



Slope Compensation



Why Peak Current Mode Control?

Advantages

- Faster Control Loop
- Feed Forward
- Simpler Compensator As Inductor Essentially Removed From Loop
- Build In Current Limiting

Disadvantages

- Must Sense The Current
- Noise Sensitivity
- “Slope Compensation”
  - Must Add Additional Ramp Signal To Current Sense Signal To Avoid Subharmonic Oscillations

Why Peak Current Mode Control?

Advantages

- Faster Control Loop
- Feed Forward
- Simpler Compensator As Inductor Essentially Removed From Loop
- Build In Current Limiting

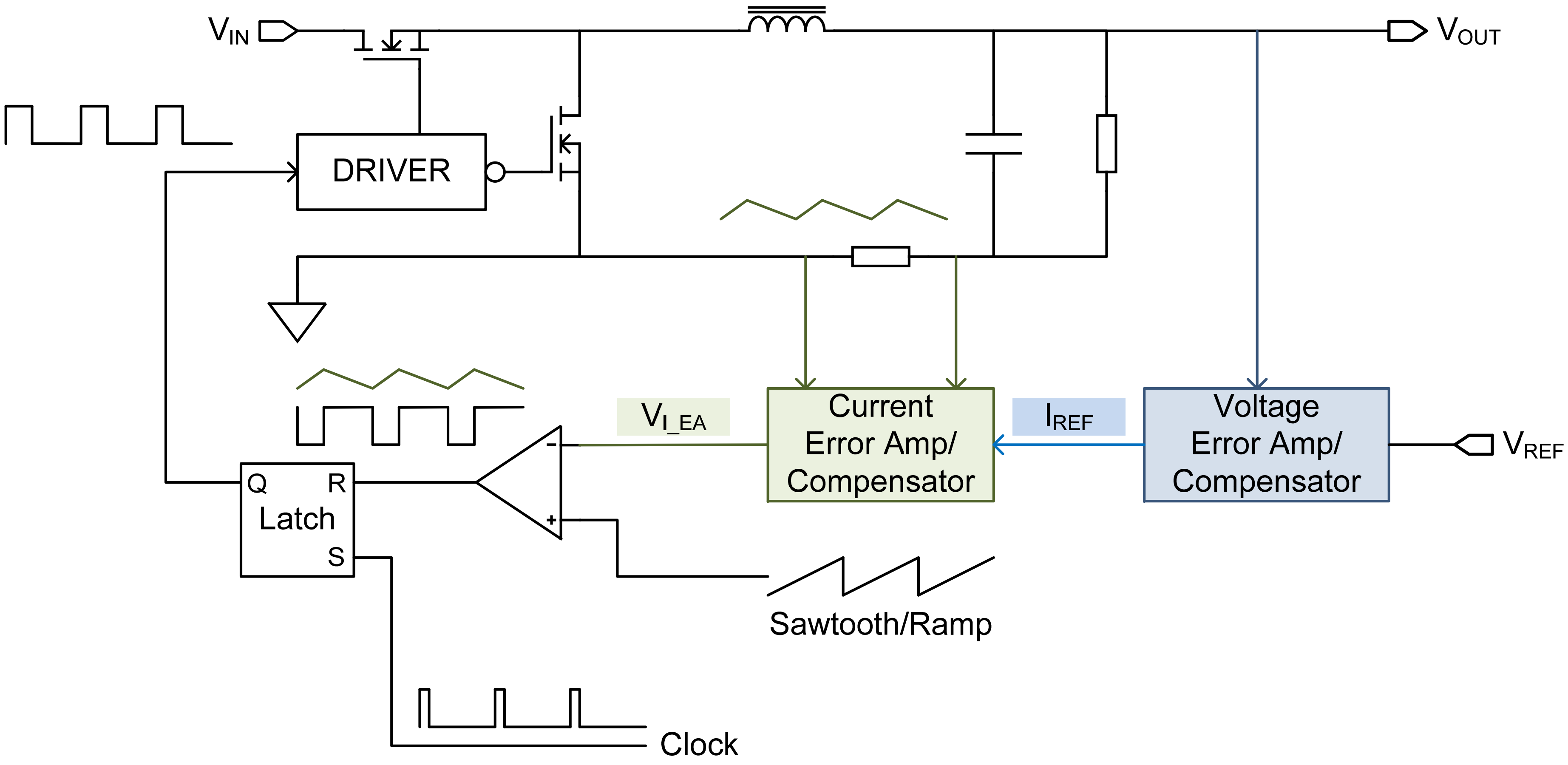
Are Generally Heavily Outweighed By These Advantages!

Disadvantages

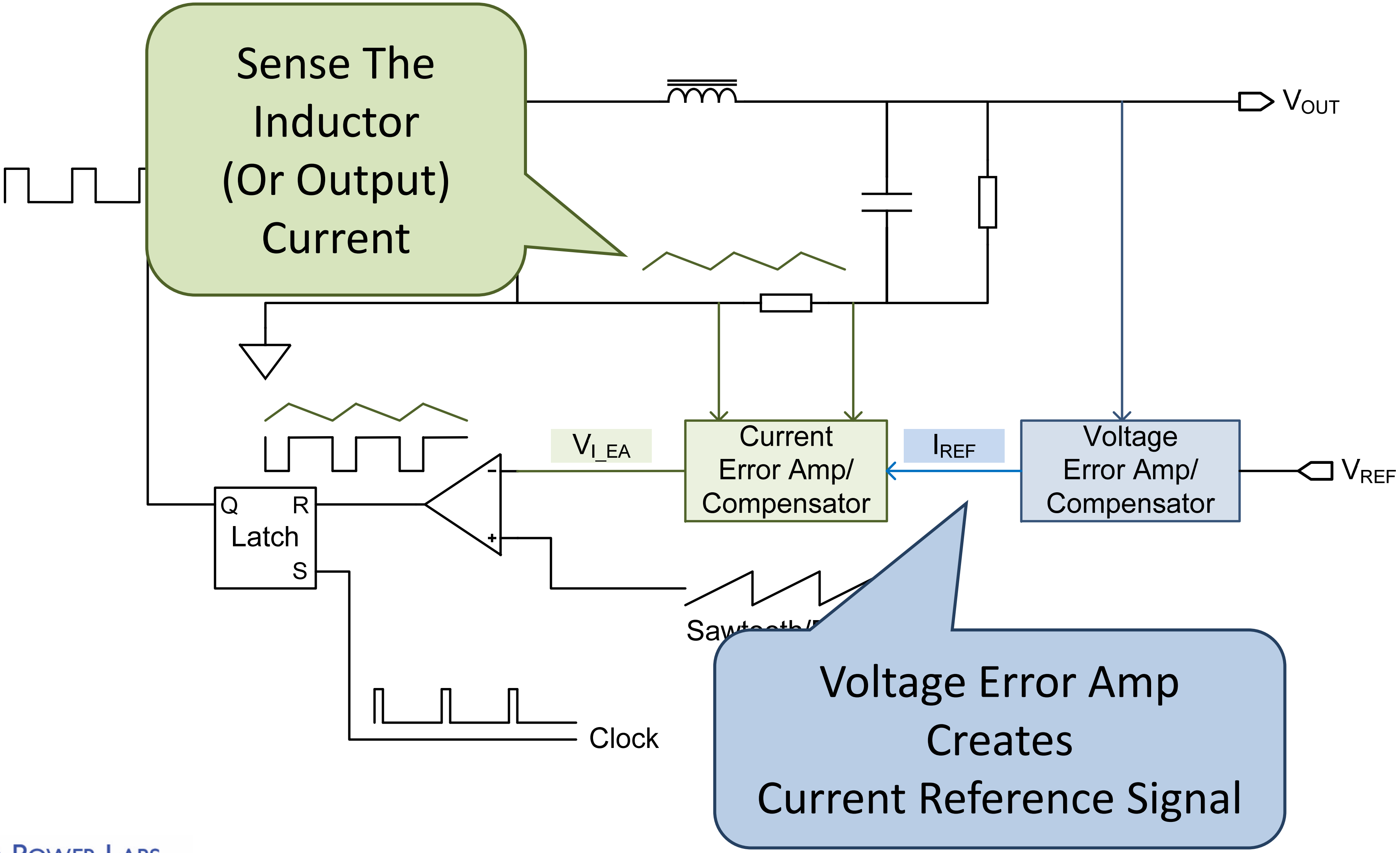
- Must Sense The Current
- Noise Sensitivity
- “Slope Compensation”
  - Must Add Additional Ramp Signal To Current Sense Signal To Avoid Subharmonic Oscillations

These Disadvantages

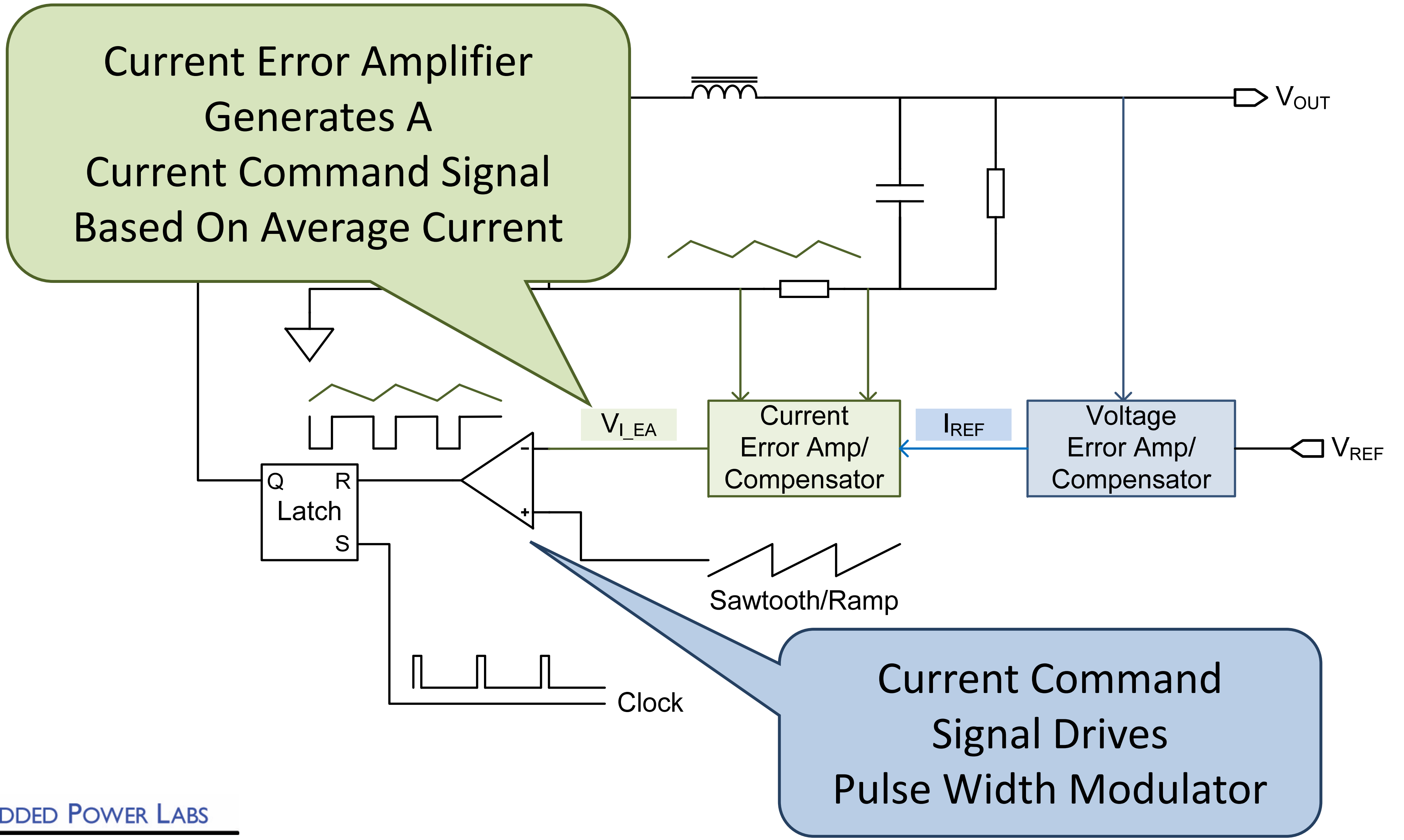
Average Current Mode Control



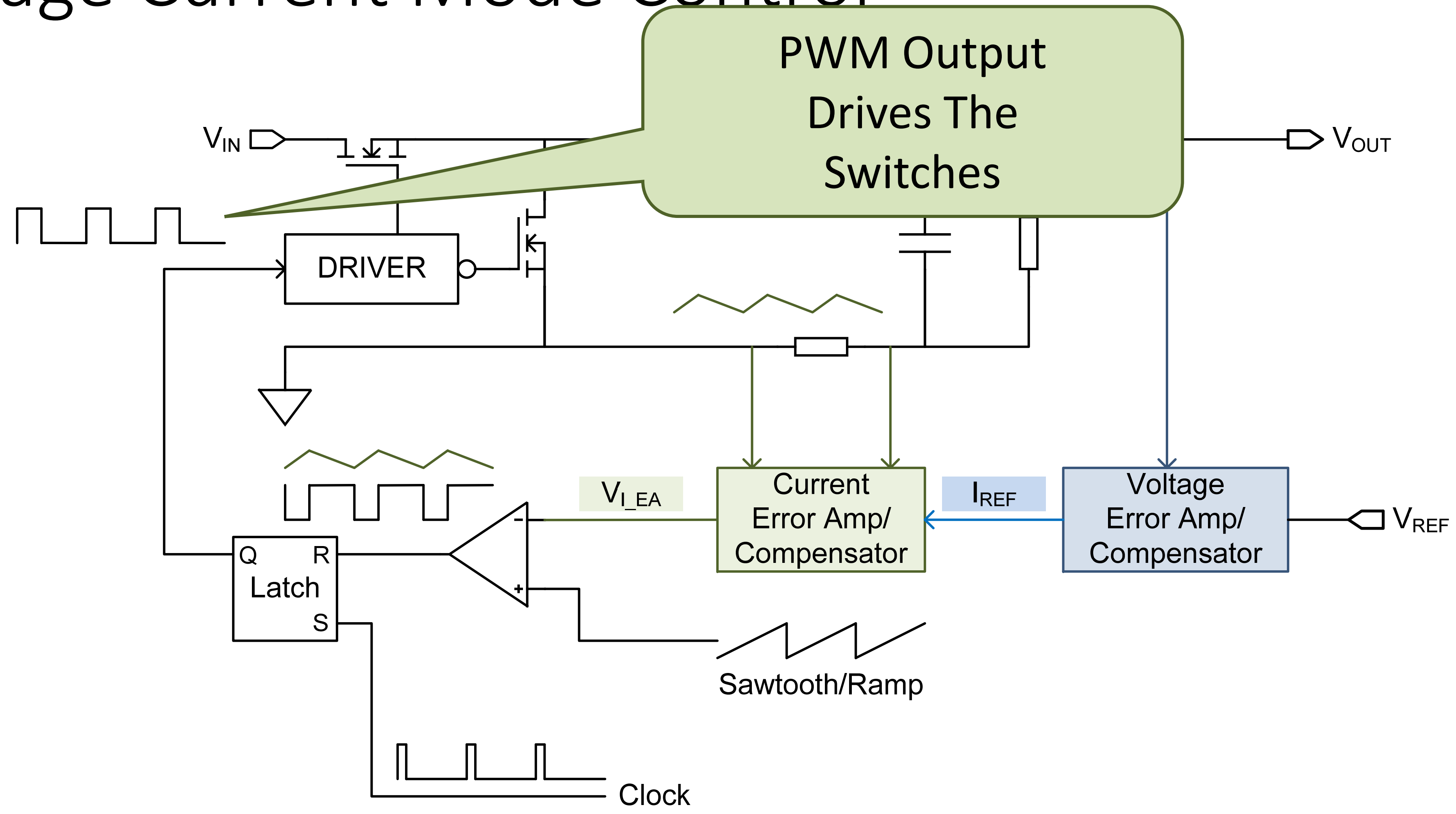
Average Current Mode Control



Average Current Mode Control



Average Current Mode Control



Why Average Current Mode Control?

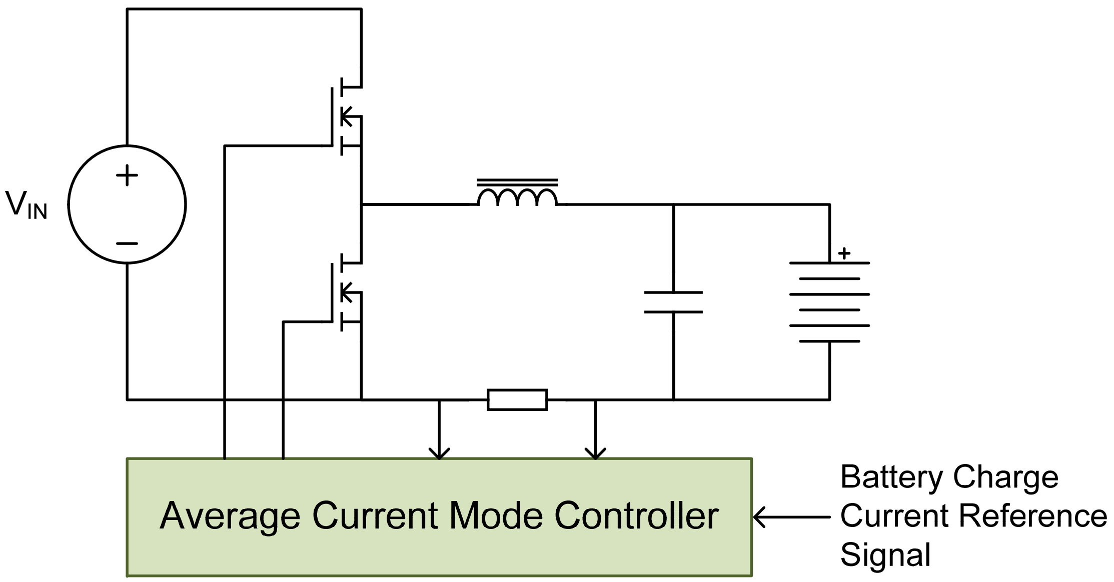
Advantages

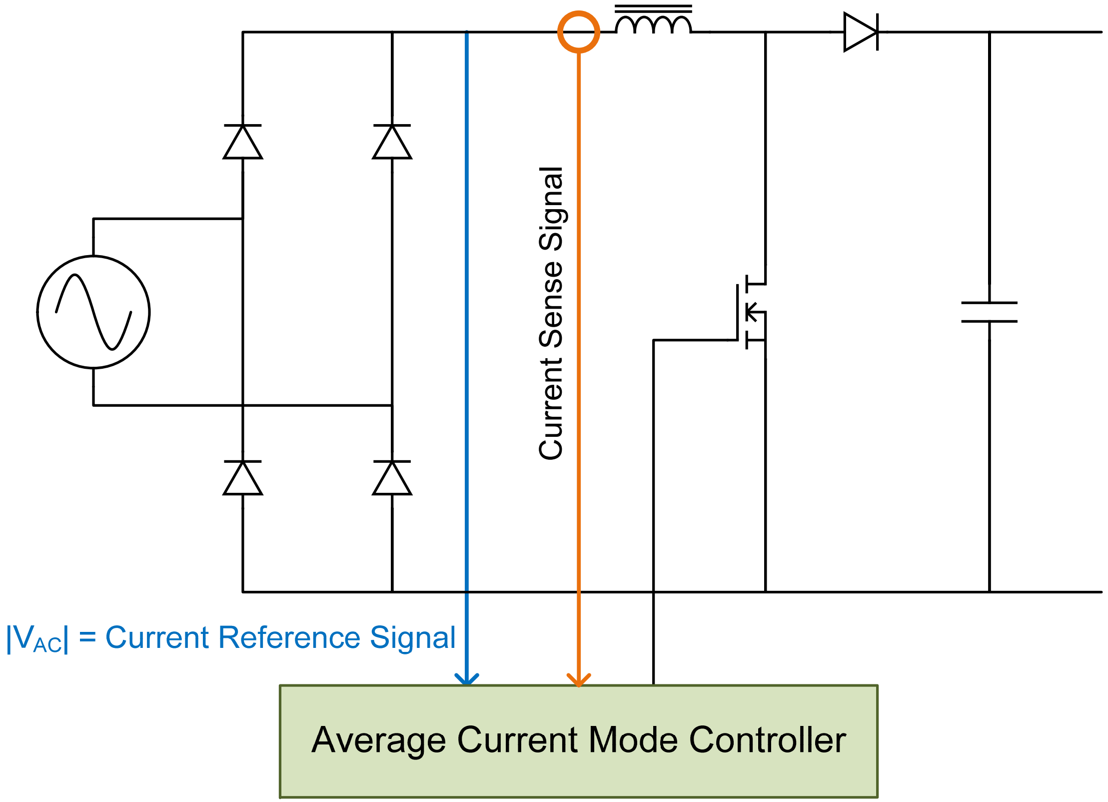
- Direct Control Of Average Output Current
- No Slope Compensation
- Excellent Noise Immunity

Disadvantages

- More Components

Applications Of Average Current Mode Control





Summary

In Part II We Covered

- Basics Of Feedback Control
- Buck Converter Design Example
  - Determining The Transfer Function Of Each Block In The Control Loop
  - Solving For The Compensator
  - Designing The Compensator
  - K-Factor Method
- Overview Of Current Mode Control

Additional Resources

- Great Book For The Basics

  "Fundamentals Of Power Supply Design"

  Robert A. Mammano

- My Favorite Textbook

  "Fundamentals Of Power Electronics"

  2nd Edition, Erickson & Maksimovic

- For References Of Control To Output Transfer Functions

  "Switch-Mode Power Supplies – SPICE Simulations And Practical Designs"

  2nd Edition, 2014, Christophe P. Basso

- Best Paper On Modeling Current Mode Control

  R. B. Ridley, "A new, continuous-time model for current-mode control," IEEE Transactions on Power Electronics, vol. 6, pp. 271-280, 1991

Additional Resources: Selected Tools

Commercial	Free
SIMPLIS/SIMetrix	SIMPLIS/SIMetrix Elements Microchip Technology MPLAB® MINDI™
PSIM	
MATLAB/Simulink/ Control Toolboxes	Scilab/Xcos GNU Octave
Mathcad	Smath Studio
Pspice®	LTspice

Survey Forms!

Questions?

About The Speaker

Bob White has over 40 years experience in power electronics. He has held managerial and individual contributor positions in product development, technology development, applications and systems engineering, and technical marketing. His areas of expertise include power systems for computing and telecommunications systems, digital power, and applications of wide bandgap power semiconductor devices. Bob is currently the president and chief engineer of Embedded Power Labs, a power electronics consulting company.

Bob has been very active in the IEEE Power Electronics Society and the APEC committees, including twice serving as the APEC General Chair.

He is a Fellow of the IEEE, has a BSEE from MIT, a MSEE from Worcester Polytechnic Institute and is currently pursuing a Ph.D. in power electronics at the University of Colorado-Boulder.

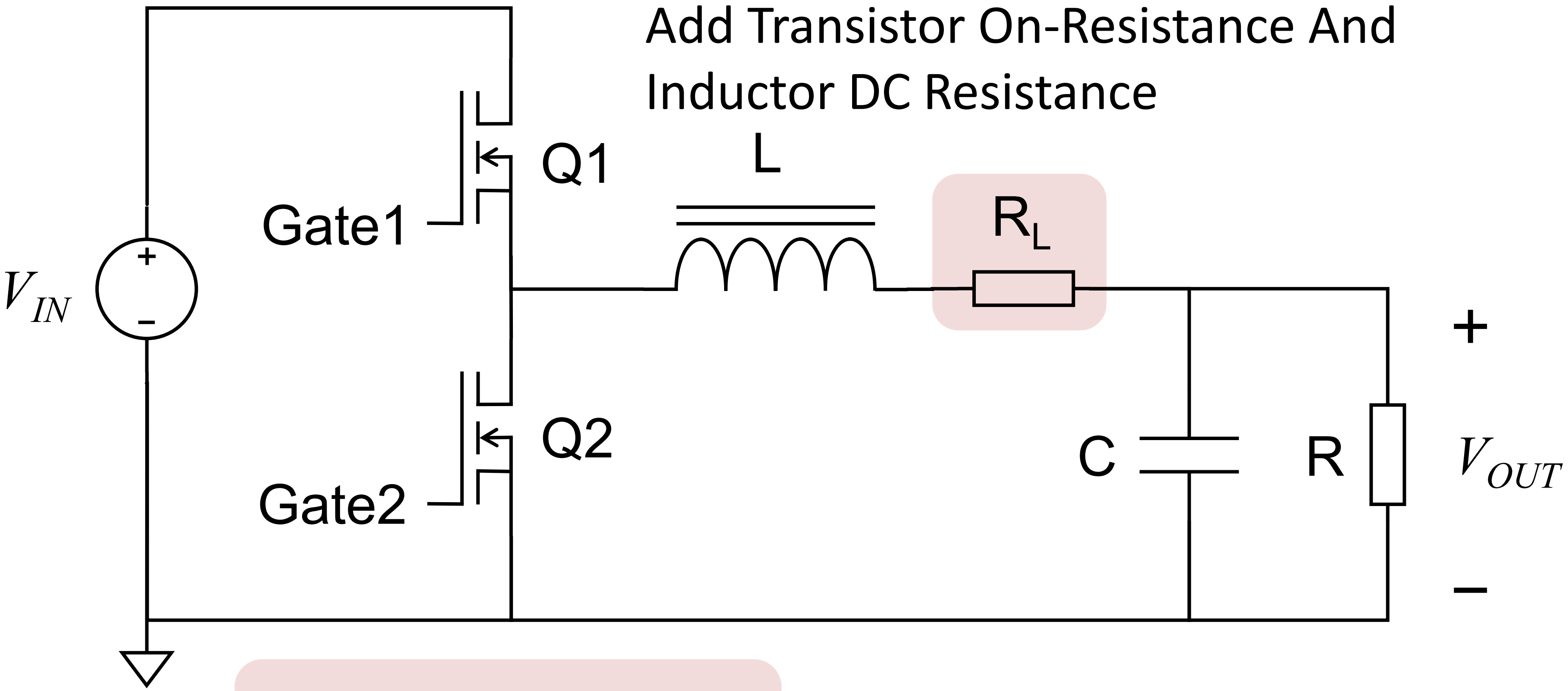
He can be reached by email at bob.white@embeddedpowerlabs.com

Appendices

I. Non-Ideal Buck Converter
II. Buck Converter Discontinuous Conduction Mode
III. Isolated Buck Converters: Forward, Half-Bridge, Full-Bridge
IV. Boost Converter Discontinuous Conduction And Critical Conduction Modes
V. Non-Ideal Boost Converter
VI. Buck-Boost Converter Analysis
VII. More Realistic Flyback Converter Simulation
VIII. Flyback Converter Continuous Conduction Mode
IX. Four Switch Noninverting Buck-Boost
X. Simple Pole And Zero Transfer Functions
XI. Loop Gain And Tracking
XII. Ideal Loop Calculations
XIII. Analog Compensators

APPENDIX I.
Non-Ideal Buck Converter

Non-Ideal Buck Converter



Q1 On Resistance: Q_{R1}
Q2 On Resistance: Q_{R2}

Conversion Ratio Derivation

$$V_L\left(T_{ON}\right)=V_{IN}-R_{Q1}\cdot I_L-R_L\cdot I_L-V_{OUT}$$

$$V_L\left(T_{OFF}\right)=-V_{OUT}-R_{Q2}\cdot I_L-R_L\cdot I_L$$

$$V_L\left(T_{ON}\right)\cdot T_{ON}+V_L\left(T_{OFF}\right)\cdot T_{OFF}=0$$

$$\left(V_{IN}-R_{Q1}\cdot I_L-R_L\cdot I_L-V_{OUT}\right)\cdot T_{ON}$$
$$+\left(-V_{OUT}-R_{Q2}\cdot I_L-R_L\cdot I_L\right)\cdot T_{OFF}=0$$

$$\left(V_{IN}-R_{Q1}\cdot I_L-R_L\cdot I_L-V_{OUT}\right)\cdot D\cdot T_{SW}$$
$$+\left(-V_{OUT}-R_{Q2}\cdot I_L-R_L\cdot I_L\right)\cdot\left(1-D\right)\cdot T_{SW}=0$$

$$\left(V_{IN}-R_{Q1}\cdot I_L-R_L\cdot I_L-V_{OUT}\right)\cdot D$$
$$+\left(-V_{OUT}-R_{Q2}\cdot I_L-R_L\cdot I_L\right)\cdot\left(1-D\right)=0$$

Conversion Ratio Derivation

$$D \cdot V_{IN} - D \cdot R_{Q1} \cdot I_L - \cancel{D \cdot R_L \cdot I_L} - \cancel{D \cdot V_{OUT}}$$

$$-V_{OUT} - R_{Q2} \cdot I_L - R_L \cdot I_L$$

$$+ \cancel{D \cdot V_{OUT}} + D \cdot R_{Q2} \cdot I_L + \cancel{D \cdot R_L \cdot I_L} = 0$$

$$D \cdot V_{IN} - D \cdot R_{Q1} \cdot I_L - V_{OUT} - R_{Q2} \cdot I_L - R_L \cdot I_L + D \cdot R_{Q2} \cdot I_L = 0$$

$$D \cdot V_{IN} - D \cdot R_{Q1} \cdot I_L - R_{Q2} \cdot I_L + D \cdot R_{Q2} \cdot I_L - R_L \cdot I_L - V_{OUT} = 0$$

$$D \cdot \left(V_{IN} - R_{Q1} \cdot I_L\right) - (1 - D) \cdot R_{Q2} \cdot I_L - R_L \cdot I_L - V_{OUT} = 0$$

$$V_{OUT} = D \cdot \left(V_{IN} - R_{Q1} \cdot I_L\right) - (1 - D) \cdot R_{Q2} \cdot I_L - R_L \cdot I_L$$

$$V_{OUT} = D \cdot V_{IN} - D \cdot R_{Q1} \cdot I_L - (1 - D) \cdot R_{Q2} \cdot I_L - R_L \cdot I_L$$

Conversion Ratio Derivation

Output Voltage Of An Ideal Buck Converter

These Terms Show The Effect Of The Non-Ideal Factors.

The Losses Subtract From The Ideal Output Voltage.

To Keep The Output Voltage Constant, The Controller Must Add Some Duty Cycle To Make Up For The Losses

$$V_{OUT} = D \cdot V_{IN} - D \cdot R_{Q1} \cdot I_L - (1-D) \cdot R_{Q2} \cdot I_L - R_L \cdot I_L$$

Duty Cycle As Function Of Load

$$D \cdot V_{IN} - D \cdot R_{Q1} \cdot I_L + D \cdot R_{Q2} \cdot I_L - R_{Q2} \cdot I_L - R_L \cdot I_L - V_{OUT} = 0$$

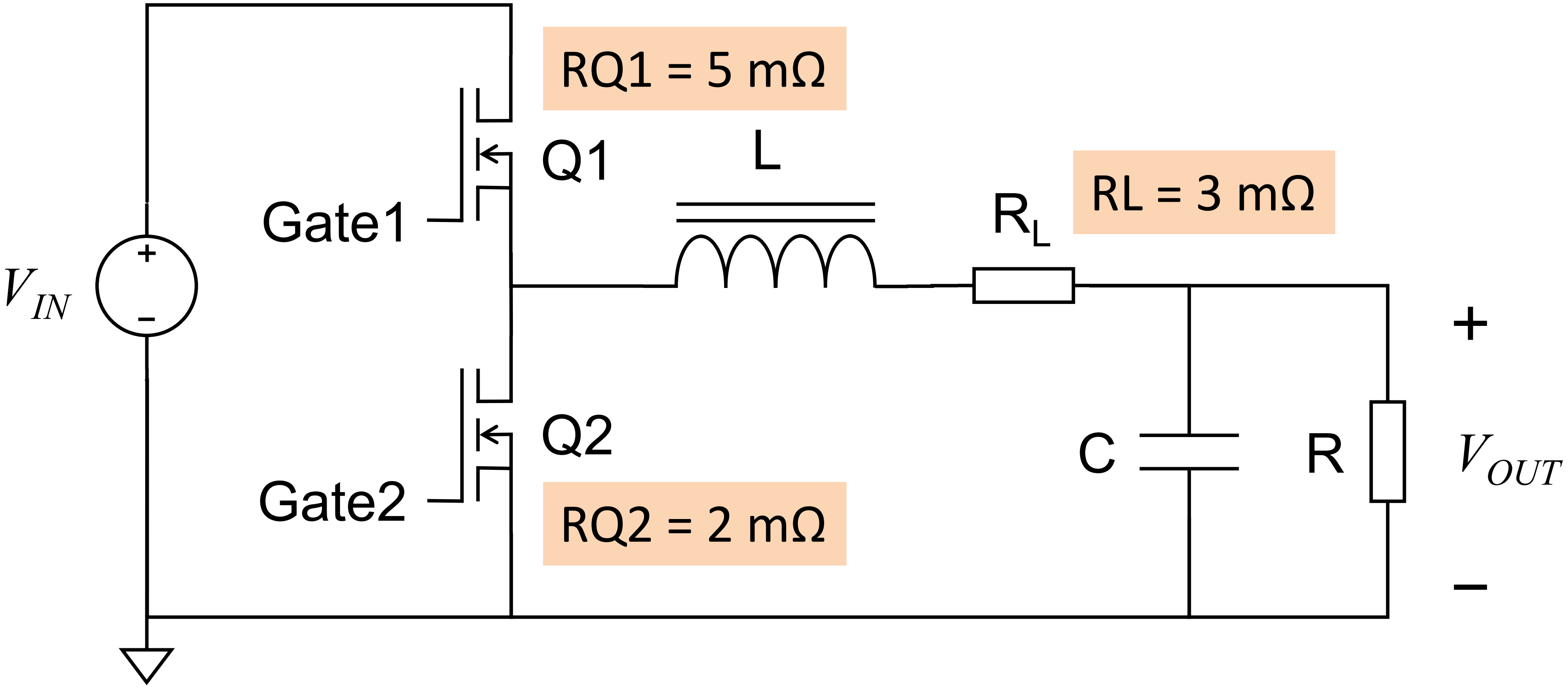
$$D \cdot V_{IN} - D \cdot R_{Q1} \cdot I_L + D \cdot R_{Q2} \cdot I_L = V_{OUT} + R_{Q2} \cdot I_L + R_L \cdot I_L$$

$$D \cdot \left(V_{IN} - R_{Q1} \cdot I_L + R_{Q2} \cdot I_L \right) = V_{OUT} + R_{Q2} \cdot I_L + R_L \cdot I_L$$

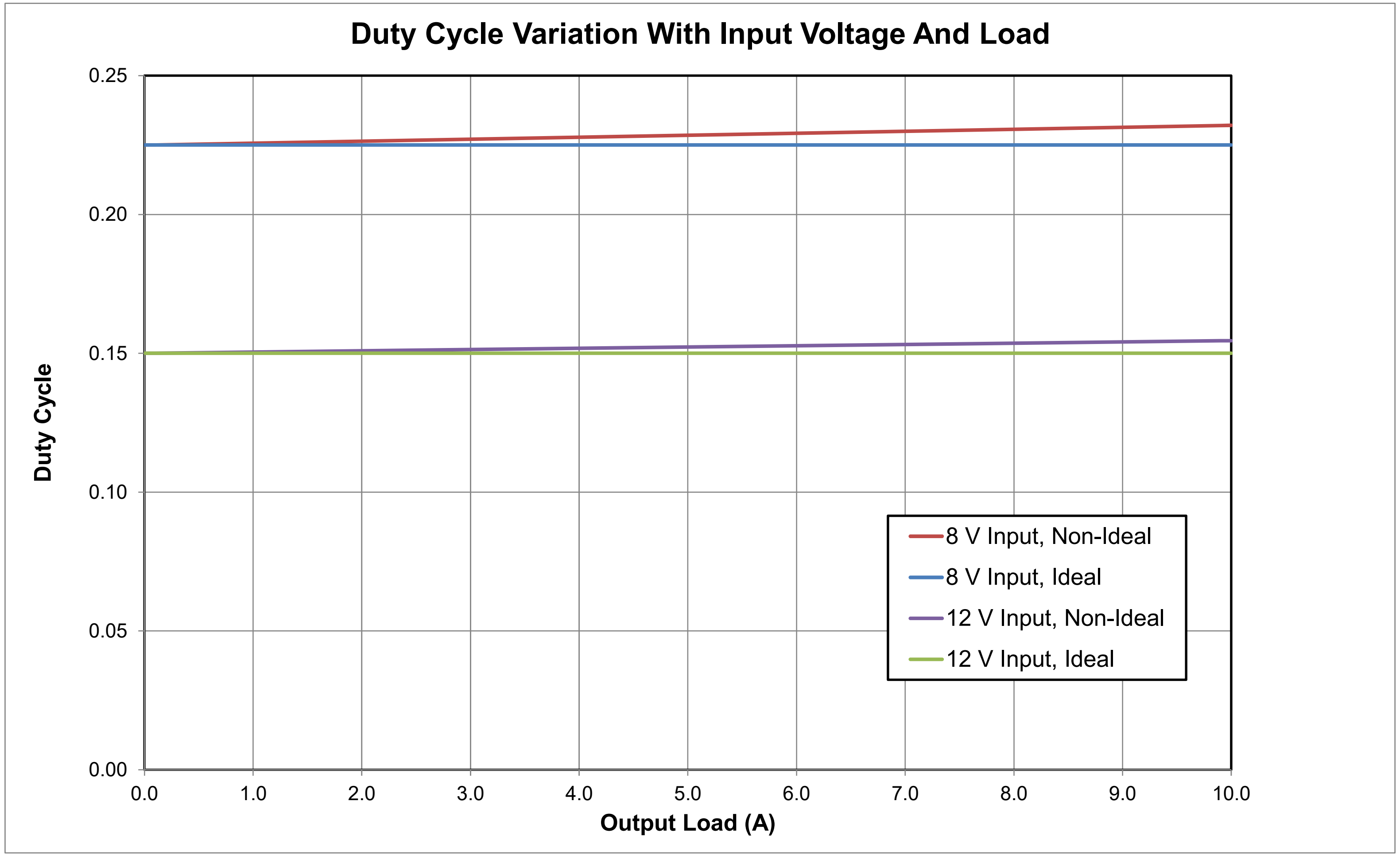
$$D = \frac{V_{OUT} + R_{Q2} \cdot I_L + R_L \cdot I_L}{V_{IN} - R_{Q1} \cdot I_L + R_{Q2} \cdot I_L}$$

Remember That The Average Inductor Current Equals The Load Current

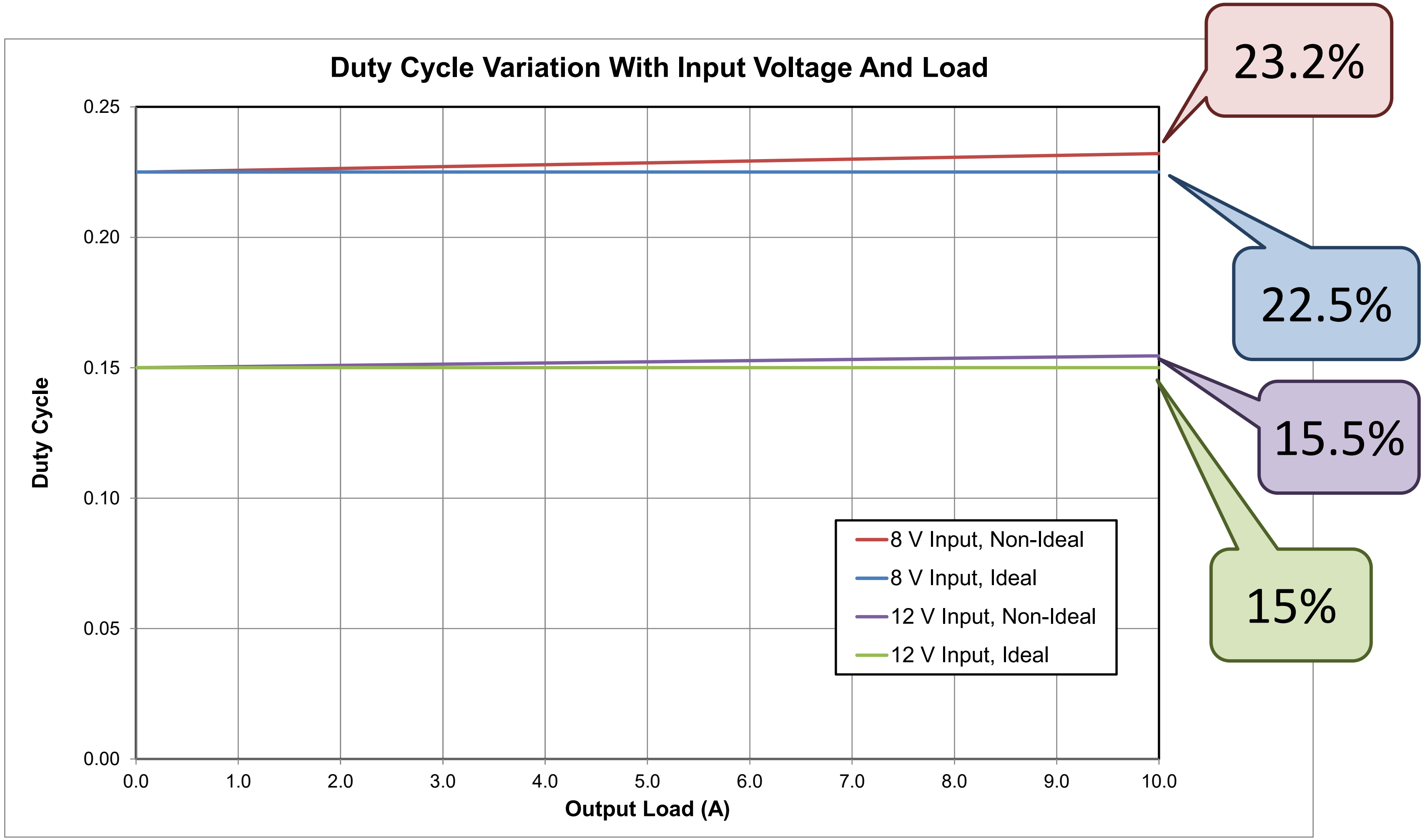
Example



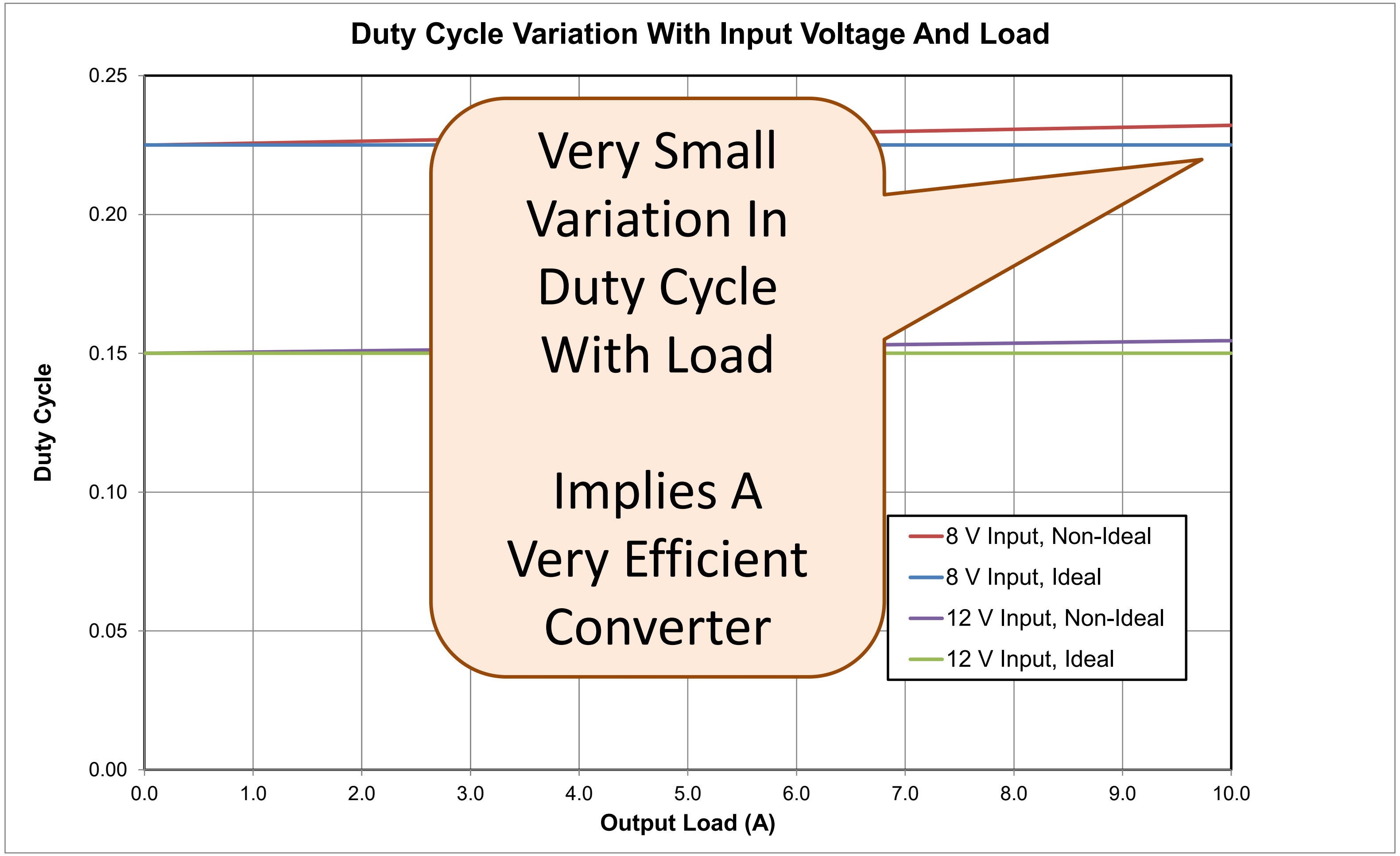
Duty Cycle Variation



Duty Cycle Variation



Duty Cycle Variation



Duty Cycle Variation

Duty Cycle Variation With Input Voltage And Load

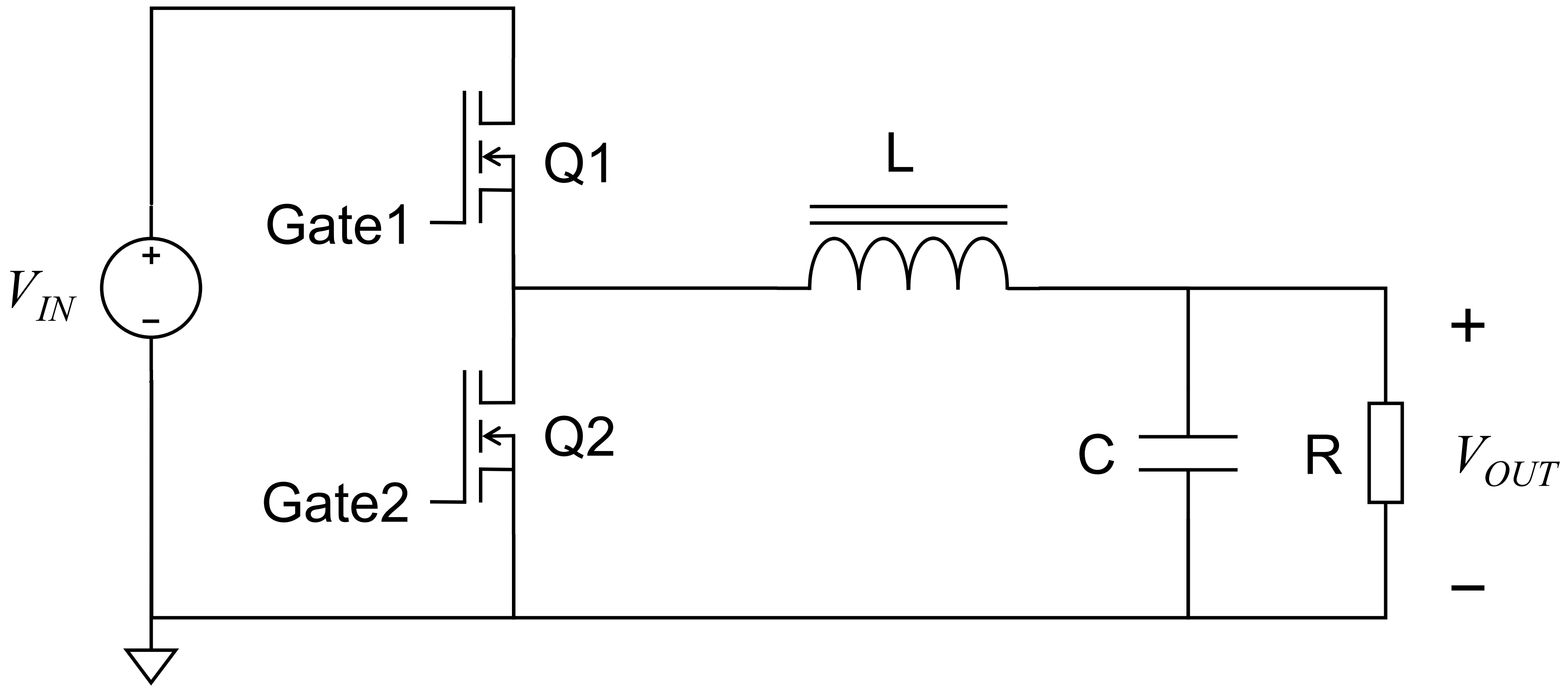
NOTE

This Is An Interesting And Informative Exercise. However, Note That Switching Losses Are Not Included And Make The Duty Cycle Needed For A Specified Output Voltage Is Larger Than Shown Here

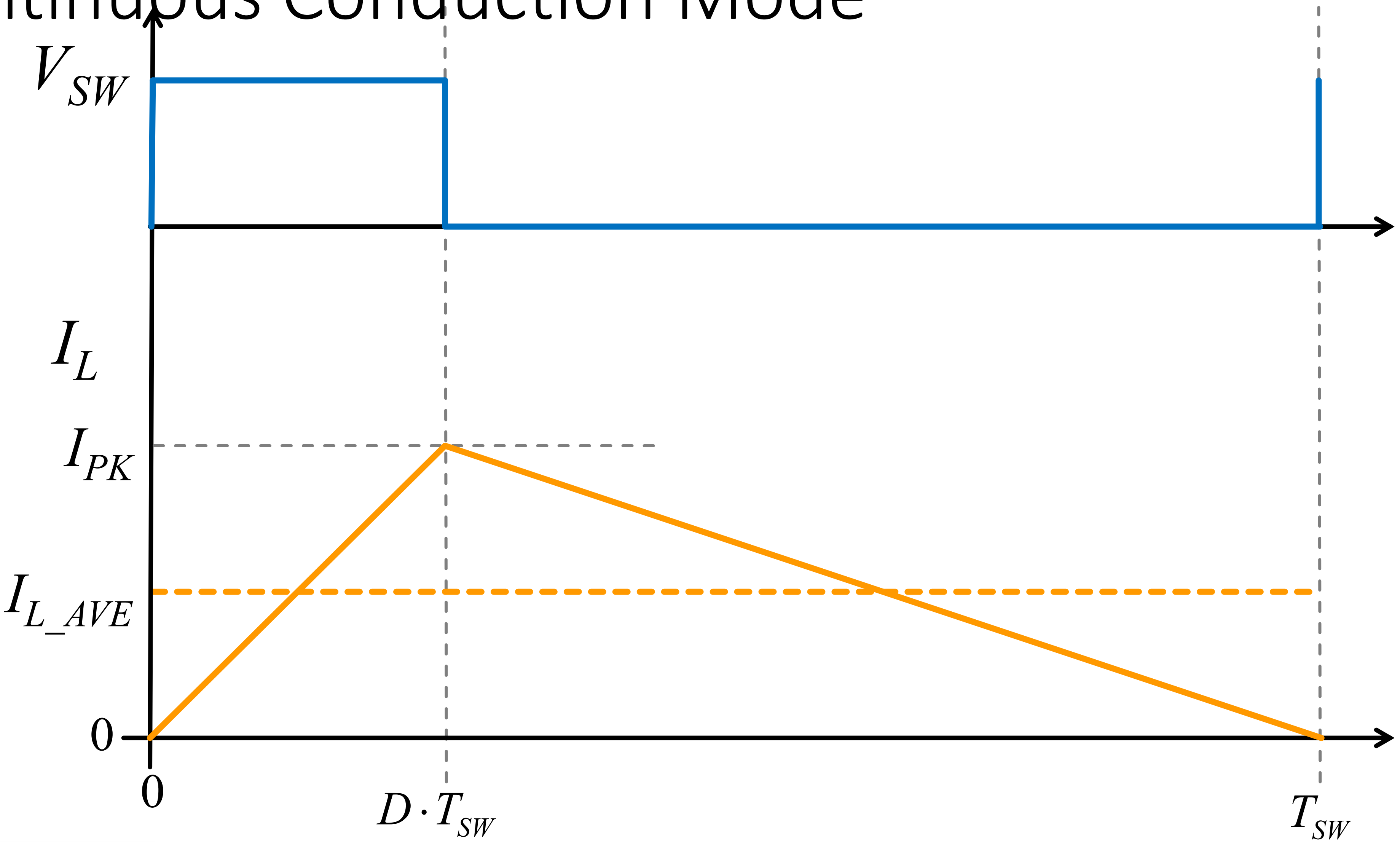
Output Load (A)

APPENDIX II.
Buck Converter
Discontinuous Conduction Mode

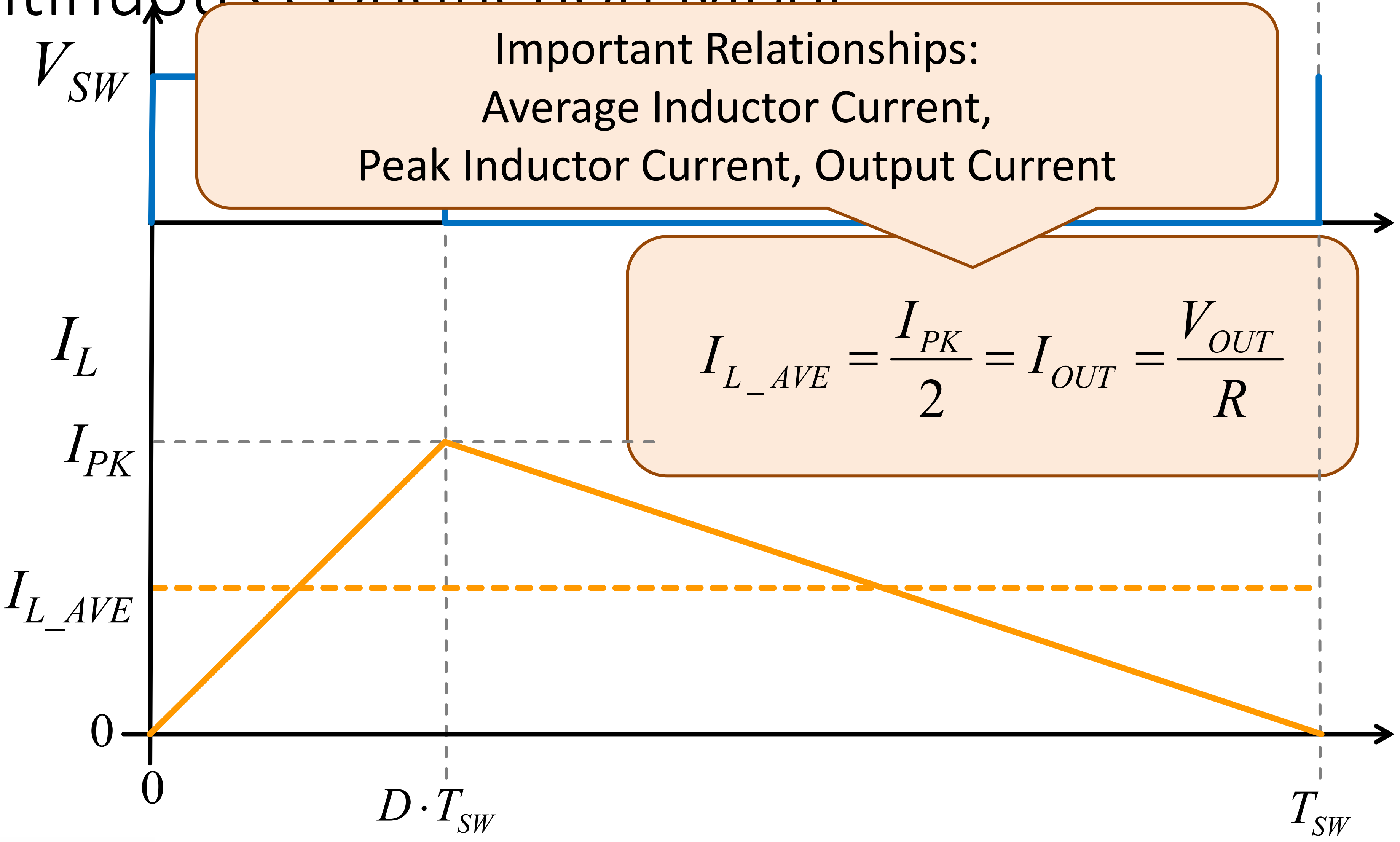
Buck Converter



Border Of Continuous And Discontinuous Conduction Mode



Border Of Continuous And Discontinuous Conduction Mode



Important Relationships:
Average Inductor Current,
Peak Inductor Current, Output Current

$$I_{L\_AVE} = \frac{I_{PK}}{2} = I_{OUT} = \frac{V_{OUT}}{R}$$

DCM And Output Inductor

Avoiding Discontinuous Mode At Light Load Often Specifies Minimum Value Of Output Inductor

$$I_{PK} = 2 \cdot I_{OUT}$$

Minimum Criteria To Avoid Discontinuous Conduction Mode

$$I_{PK} = \frac{V_{IN} - V_{OUT}}{L_{MIN}} \cdot T_{ON}$$

$$L_{MIN} = \frac{V_{IN} - V_{OUT}}{I_{PK}} \cdot T_{ON}$$

$$T_{ON} = D \cdot T_{SW} = \frac{V_{OUT}}{V_{IN}} \cdot T_{SW}$$

$$L_{MIN} = \frac{V_{IN} - V_{OUT}}{2 \cdot I_{OUT}} \cdot \frac{V_{OUT}}{V_{IN}} \cdot T_{SW}$$

Inductor Calculation Example

Circuit Parameters

$$V_{IN} = 12\,\mathrm{V}$$

$$V_{OUT} = 3.3\,\mathrm{V}$$

$$I_{OUT} = 200\,\mathrm{mA}$$

$$F_{SW} = 200\,\mathrm{kHz}$$

$$T_{SW} = 5\,\mu\mathrm{s}$$

Inductance Calculation

$$L_{MIN} = \frac{V_{IN} - V_{OUT}}{2 \cdot I_{OUT}} \cdot \frac{V_{OUT}}{V_{IN}} \cdot T_{SW}$$

$$= \frac{12\,\mathrm{V} - 3.3\,\mathrm{V}}{2 \cdot 200\,\mathrm{mA}} \cdot \frac{3.3\,\mathrm{V}}{12\,\mathrm{V}} \cdot 5\,\mu\mathrm{s}$$

$$= 29.9\,\mu\mathrm{H}$$

Standard Value Of 30 µH Is Too Close.
Better To Choose 39 µH Or 47 µH

Inductor Calculation Example

Circuit Parameters

$$V_{IN} = 12\,\text{V}$$

$$V_{OUT} = 3.3\,\text{V}$$

$$I_{OUT} = 200\,\text{mA}$$

$$R_{OUT} = \frac{3.3\,\text{V}}{200\,\text{mA}} = 16.5\,\Omega$$

$$F_{SW} = 200\,\text{kHz}$$

$$T_{SW} = 5\,\mu\text{s}$$

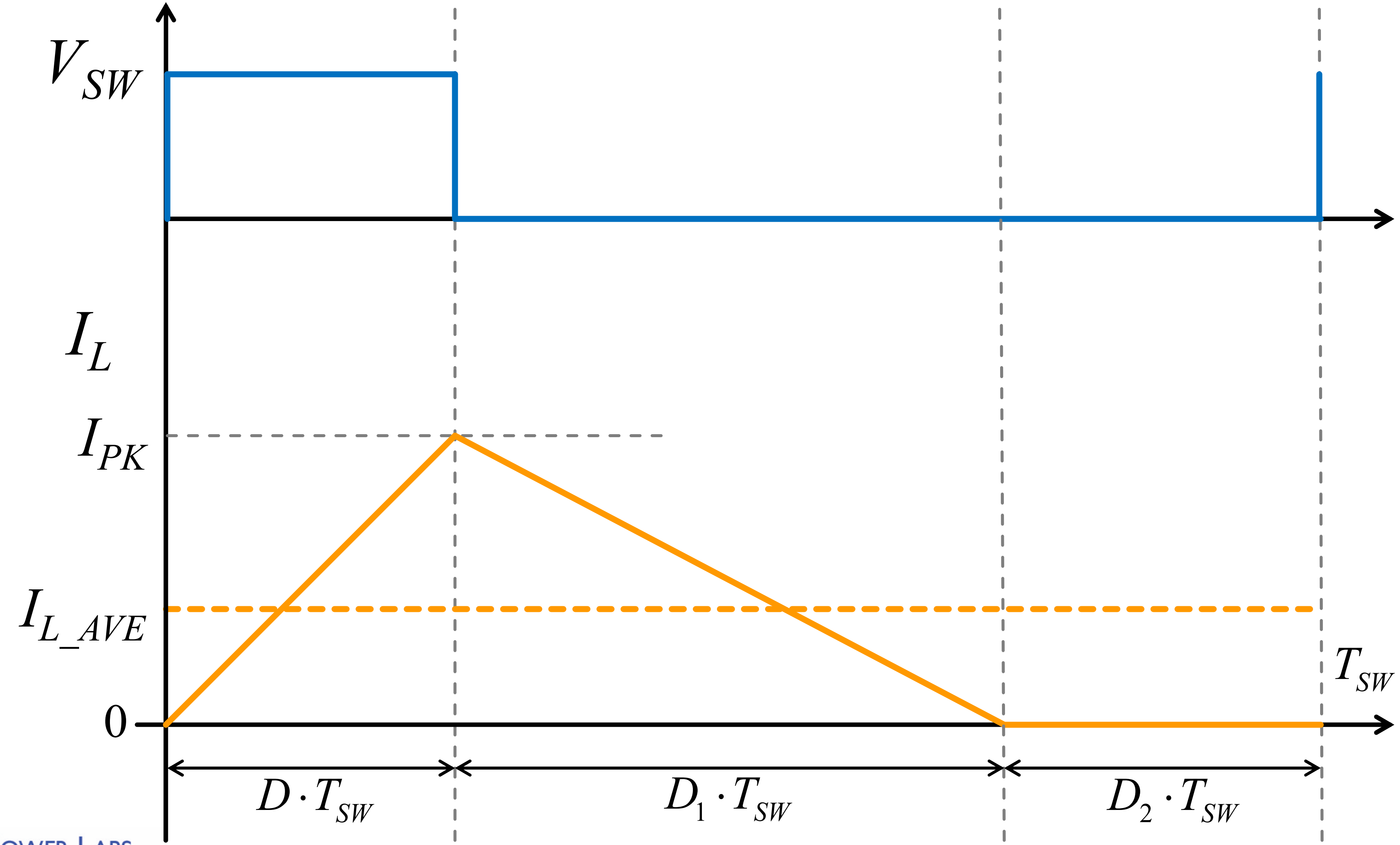
$$L = 47\,\mu\text{H}$$

How Small Can Output Current Be Before Inductor Current Becomes Discontinuous?

$$I_{PK} = \frac{V_{IN} - V_{OUT}}{L} \cdot D \cdot T_{SW} = \frac{V_{IN} - V_{OUT}}{L} \cdot \frac{V_{OUT}}{V_{IN}} \cdot T_{SW} = \frac{12\,\text{V} - 3.3\,\text{V}}{47\,\mu\text{H}} \cdot \frac{3.3\,\text{V}}{12\,\text{V}} \cdot 5\,\mu\text{s} = 254.5\,\text{mA}$$

$$I_{DCM} = \frac{I_{PK}}{2} = \frac{254.5\,\text{mA}}{2} = 127\,\text{mA}$$

Discontinuous Conduction Mode



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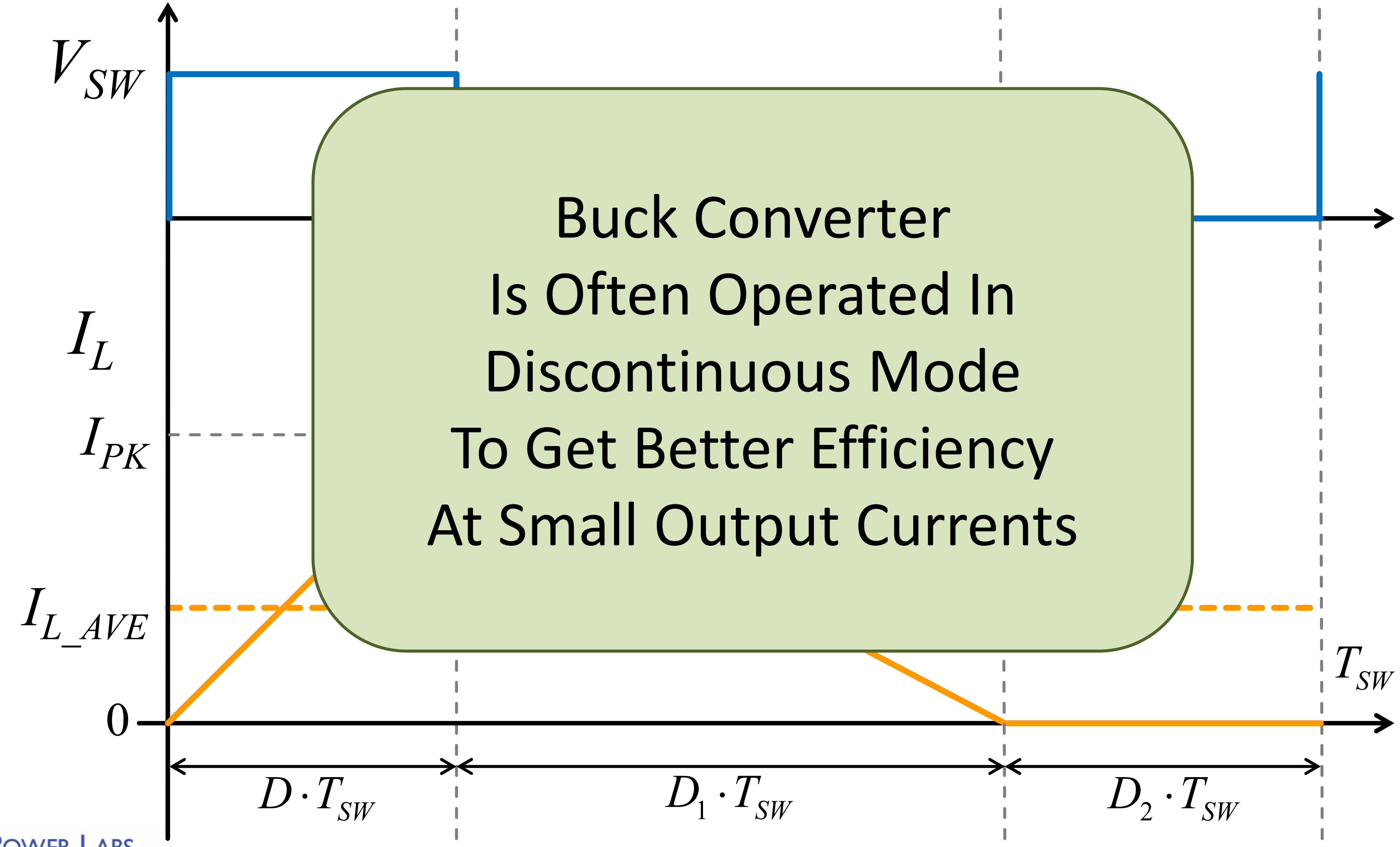
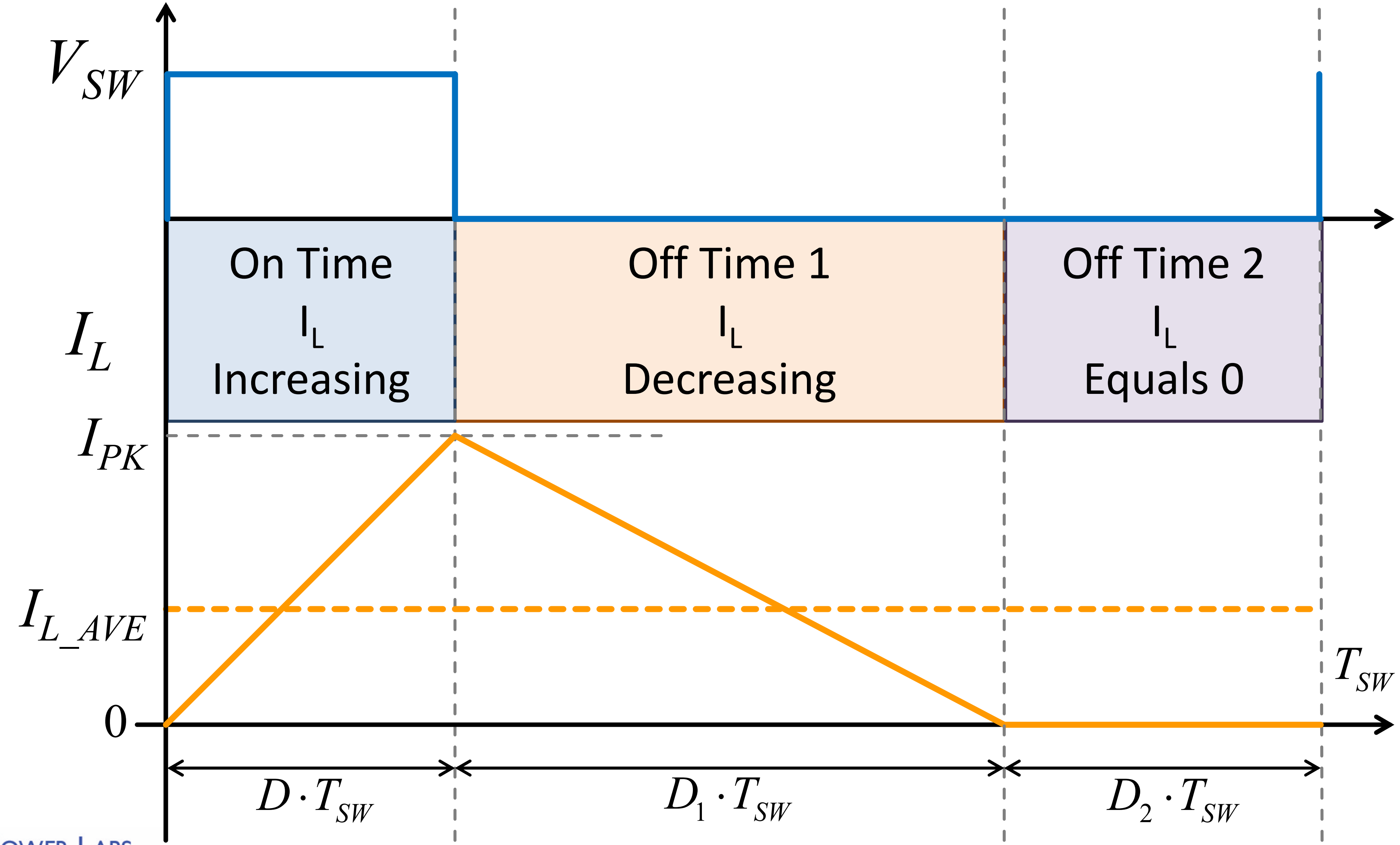
Discontinuous Conduction Mode



Buck Converter
Is Often Operated In
Discontinuous Mode
To Get Better Efficiency
At Small Output Currents

Discontinuous Conduction Mode



Conversion Ratio In DCM

Start With Inductor Voltages And Capacitor Currents In Each Of The Three Intervals

On Time

$$v_L(t) = V_{IN} - V_{OUT}$$

$$i_C(t) = i_L(t) - \frac{1}{R} \cdot V_{OUT}$$

Off Time 1

$$v_L(t) = -V_{OUT}$$

$$i_C(t) = i_L(t) - \frac{1}{R} \cdot V_{OUT}$$

Off Time 2

$$v_L(t) = 0$$

$$i_C(t) = -\frac{1}{R} \cdot V_{OUT}$$

Inductor Volt-Second Balance

$$v_L(T_{ON})\cdot T_{ON} + v_L(T_{OFF1})\cdot T_{OFF1} + v_L(T_{OFF2})\cdot T_{OFF2} = 0$$

$$(V_{IN} - V_{OUT})\cdot T_{ON} + (-V_{OUT})\cdot T_{OFF1} + 0\cdot T_{OFF2} = 0$$

$$(V_{IN} - V_{OUT})\cdot D\cdot T_{SW} + (-V_{OUT})\cdot D_1\cdot T_{SW} = 0$$

$$(V_{IN} - V_{OUT})\cdot D + (-V_{OUT})\cdot D_1 = 0$$

$$D_1\cdot V_{OUT} = (V_{IN} - V_{OUT})\cdot D$$

$$V_{OUT} = \frac{D}{D + D_1}\cdot V_{IN}$$

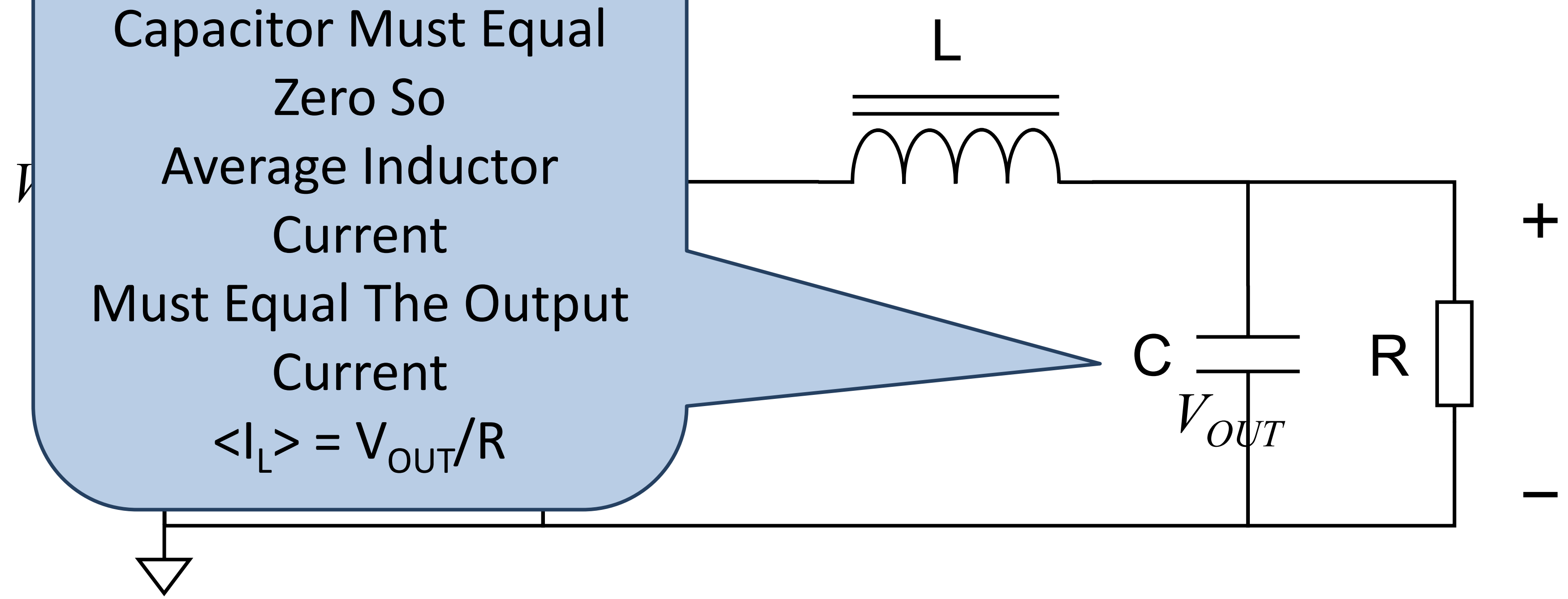
Problem: We Do Not Know D_1!

Capacitor Charge Balance

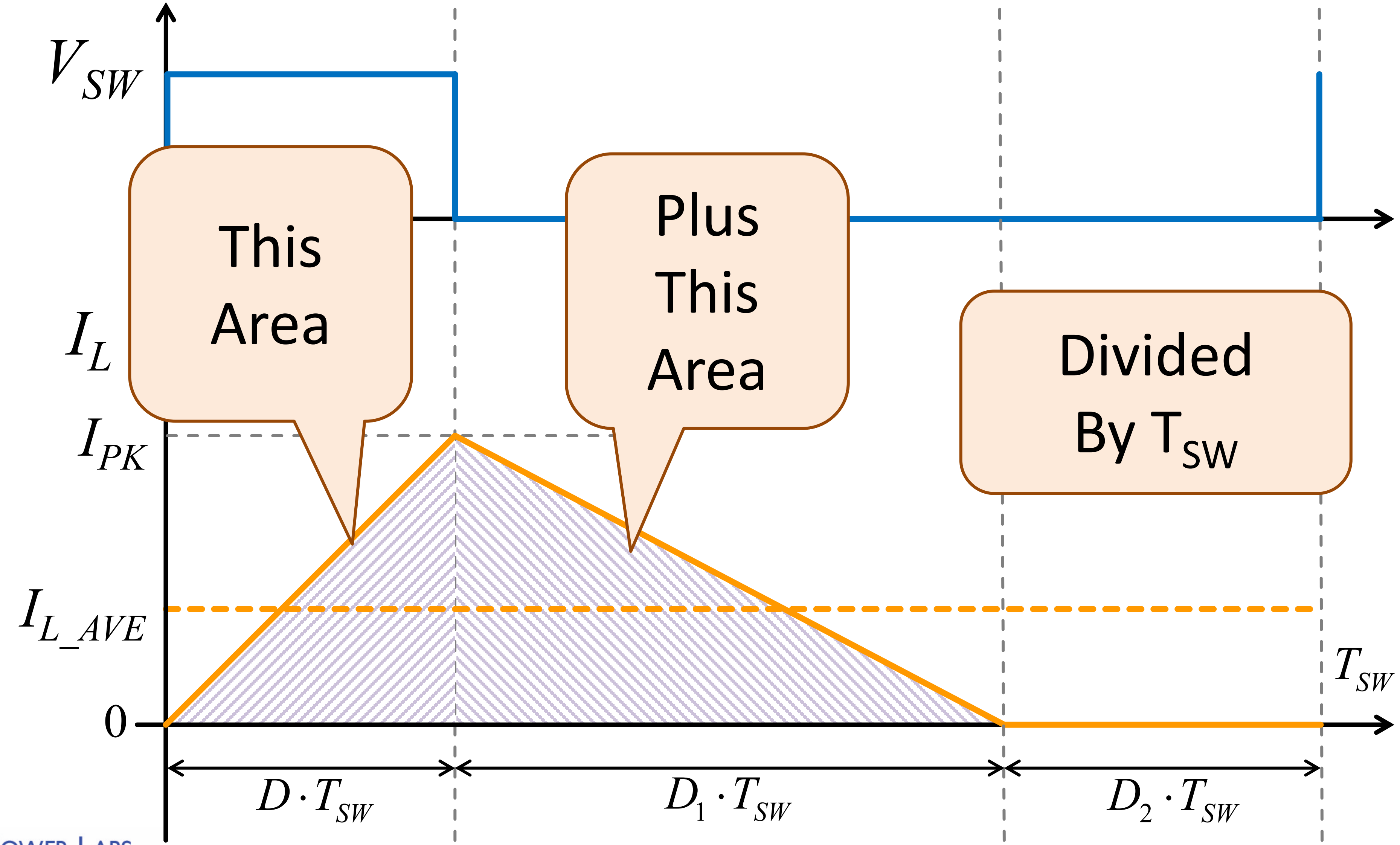
Time Consuming To Solve Analytically Because Inductor Current Is Not Constant. Use A Circuit Argument Instead.

Average Current In Capacitor Must Equal Zero So Average Inductor Current Must Equal The Output Current

$\langle I_L \rangle = V_{OUT}/R$



Average Inductor Current Equals:



Embedded Power Labs

Average Inductor Current

$$I_{L\_AVE} = \frac{\frac{1}{2} \cdot I_{PK} \cdot T_{ON} + \frac{1}{2} \cdot I_{PK} \cdot T_{OFF1}}{T_{SW}}$$

$$= \frac{1}{2} \cdot I_{PK} \cdot (D + D_1)$$

Calculating Average By Geometry

$$I_{PK} = \frac{V_{IN} - V_{OUT}}{L} \cdot T_{ON} = \frac{V_{IN} - V_{OUT}}{L} \cdot D \cdot T_{SW}$$

We Need The Peak Current

$$I_{L\_AVE} = \frac{1}{2} \cdot \left(\frac{V_{IN} - V_{OUT}}{L} \cdot D \cdot T_{SW} \right) \cdot (D + D_1)$$

$$= \frac{D \cdot T_{SW}}{2 \cdot L} \cdot (V_{IN} - V_{OUT}) \cdot (D + D_1)$$

Substituting Peak Current Expression

DCM Conversion Ratio

$$I_{L\_AVE} = \frac{1}{R} \cdot V_{OUT}$$

Result From Capacitor Charge Balance Analysis That the Average Inductor Current Must Equal The Output Current

$$\frac{1}{R} \cdot V_{OUT} = \frac{D \cdot T_{SW}}{2 \cdot L} \cdot (V_{IN} - V_{OUT}) \cdot (D + D_1)$$

Still Need D_1

$$V_{OUT} = \frac{D}{D + D_1} \cdot V_{IN}$$

Solve Expression for Output Voltage For D_1

$$D_1 = \frac{V_{IN} - V_{OUT}}{V_{OUT}} \cdot D$$

Substitute This Into The Expression Above

DCM Conversion Ratio

Now Solve This For V_{OUT}

$$\frac{1}{R}\cdot V_{OUT} = \frac{D\cdot T_{SW}}{2\cdot L}\cdot\left(V_{IN}-V_{OUT}\right)\cdot\left(D+\left(\frac{V_{IN}-V_{OUT}}{V_{OUT}}\cdot D\right)\right)$$

$$V_{OUT} = \frac{D^2\cdot R\cdot T_{SW}}{2\cdot L}\cdot\left(V_{IN}-V_{OUT}\right)\cdot\left(1+\frac{V_{IN}-V_{OUT}}{V_{OUT}}\right)$$

$$= \frac{D^2\cdot R\cdot T_{SW}}{2\cdot L}\cdot\left(V_{IN}-V_{OUT}\right)\cdot\left(\frac{V_{IN}}{V_{OUT}}\right)$$

$$V_{OUT}^2 = \frac{D^2\cdot R\cdot T_{SW}}{2\cdot L}\cdot\left(V_{IN}-V_{OUT}\right)\cdot V_{IN}$$

DCM Conversion Ratio

$$\alpha = \frac{D^2 \cdot R \cdot T_{SW}}{2 \cdot L}$$

To Make The Algebra Easier To Follow, Define Alpha

$$V_{OUT}^2 = \alpha \cdot V_{IN}^2 - \alpha \cdot V_{IN} \cdot V_{OUT}$$

Substituting Alpha

$$V_{OUT}^2 + \alpha \cdot V_{IN} \cdot V_{OUT} - \alpha \cdot V_{IN}^2 = 0$$

$$V_{OUT} = \frac{-\alpha \cdot V_{IN} \pm \sqrt{\left(\alpha \cdot V_{IN}\right)^2 - 4 \cdot \left(-\alpha \cdot V_{IN}^2\right)}}{2}$$

Applying The Formula For The Solution To A Quadratic Equation

DCM Conversion Ratio

$$V_{OUT} = \frac{-\alpha \cdot V_{IN} \pm \sqrt{(\alpha \cdot V_{IN})^2 + 4 \cdot \frac{(\alpha^2 \cdot V_{IN}^2)}{\alpha}}}{2}$$

$$= \frac{-\alpha \cdot V_{IN} \pm \alpha \cdot V_{IN} \cdot \sqrt{1 + \frac{4}{\alpha}}}{2}$$

$$\frac{V_{OUT}}{V_{IN}} = \alpha \cdot \frac{-1 + \sqrt{1 + \frac{4}{\alpha}}}{2} \cdot \frac{1 + \sqrt{1 + \frac{4}{\alpha}}}{1 + \sqrt{1 + \frac{4}{\alpha}}}$$

$$= \frac{\alpha}{2} \cdot \frac{-1 - \sqrt{1 + \frac{4}{\alpha}} + \sqrt{1 + \frac{4}{\alpha}} + \sqrt{1 + \frac{4}{\alpha}} \cdot \sqrt{1 + \frac{4}{\alpha}}}{1 + \sqrt{1 + \frac{4}{\alpha}}}$$

DCM Conversion Ratio

$$\frac{V_{OUT}}{V_{IN}} = \frac{\alpha}{2} \cdot \frac{-1+\left(1+\frac{4}{\alpha}\right)}{1+\sqrt{1+\frac{4}{\alpha}}} = \frac{\alpha}{2} \cdot \frac{\frac{4}{\alpha}}{1+\sqrt{1+\frac{4}{\alpha}}}$$

$$= \frac{2}{1+\sqrt{1+\frac{4}{\alpha}}}$$

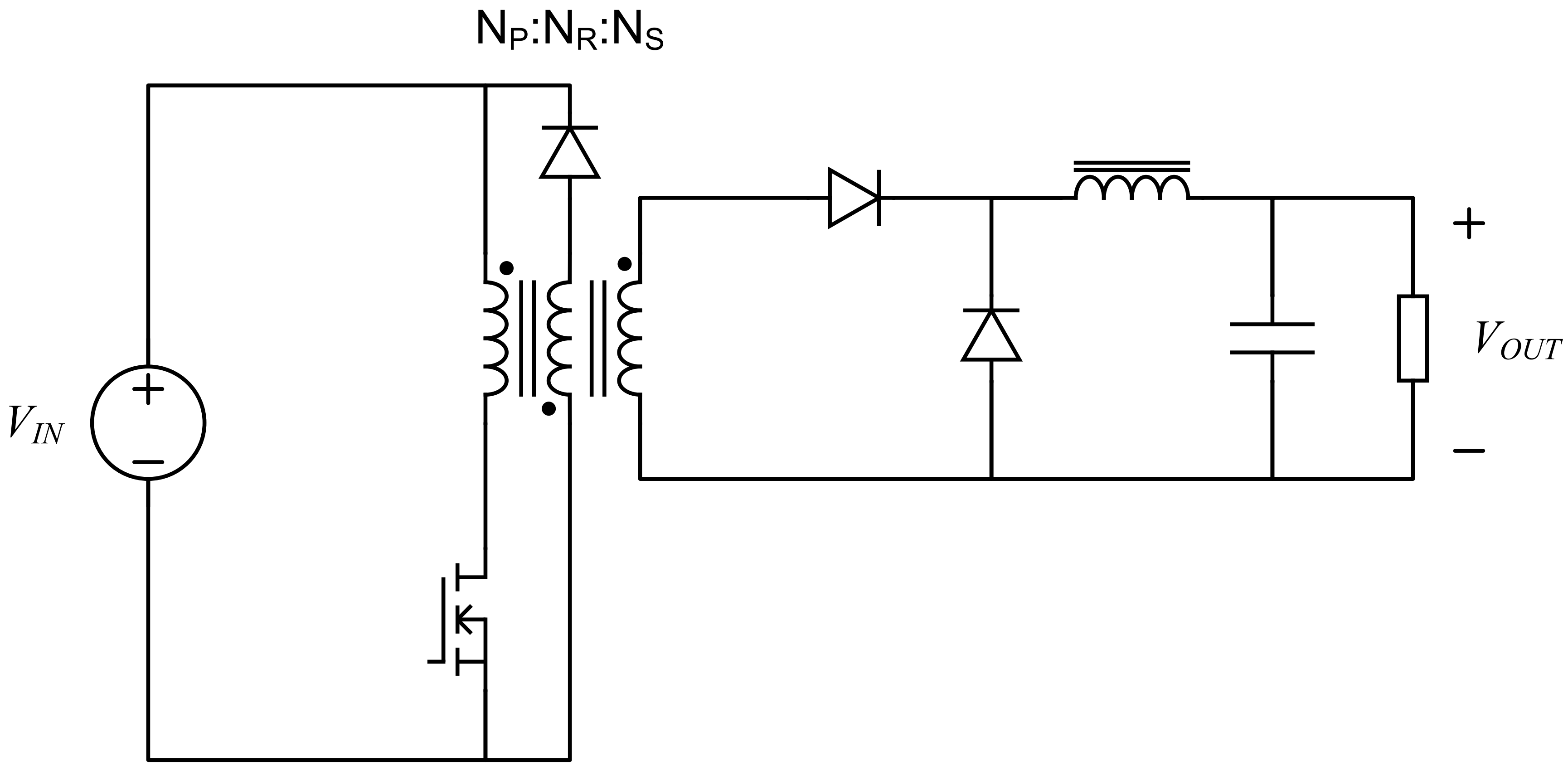
$$= \frac{2}{1+\sqrt{1+\frac{4}{\frac{D^2 \cdot R \cdot T_{SW}}{2 \cdot L}}}}$$

$$= \frac{2}{1+\sqrt{1+\frac{4}{D^2} \cdot \frac{2 \cdot L}{R \cdot T_{SW}}}}$$

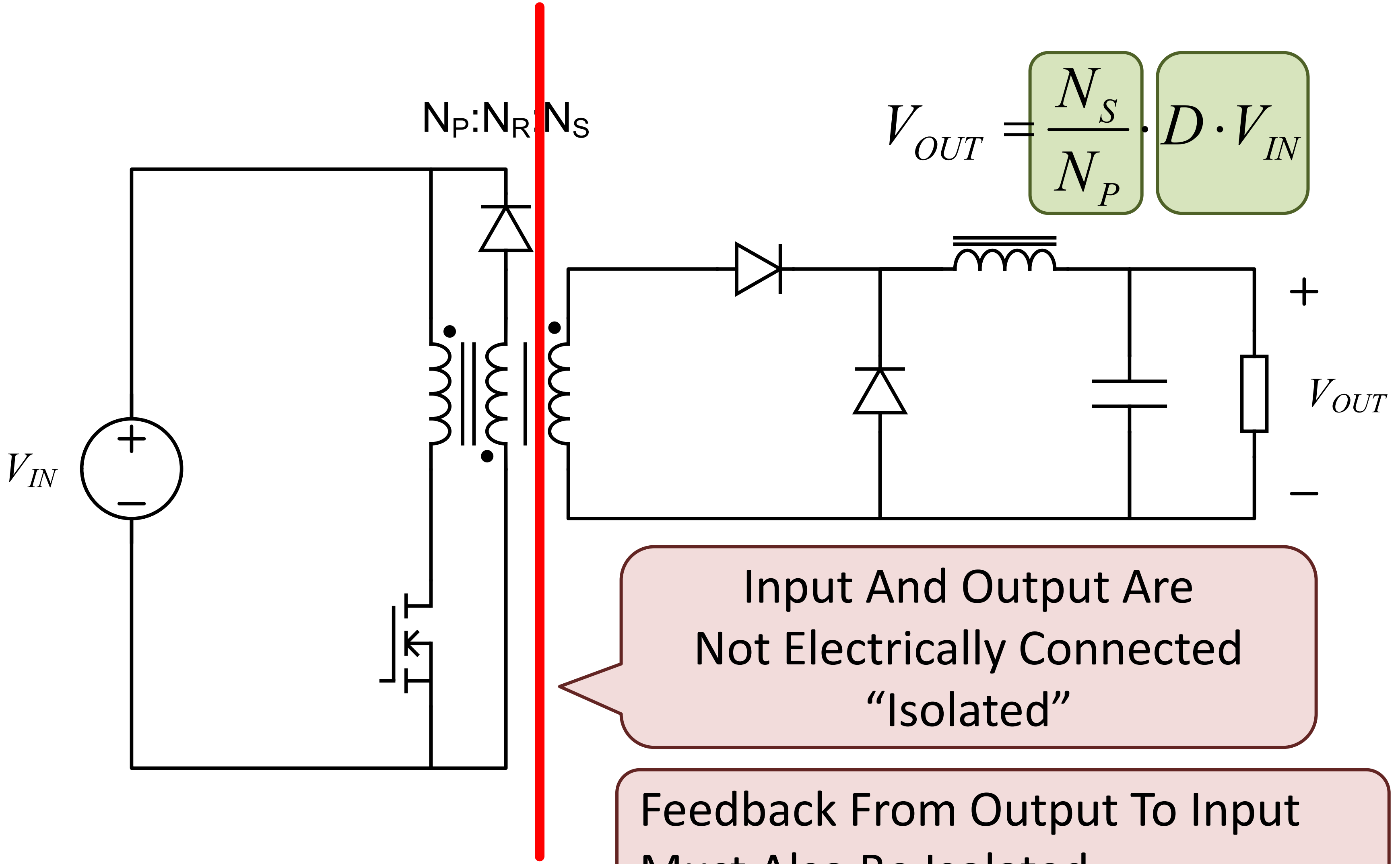
DCM
Conversion Ratio
Is Nonlinear And
Depends On
Load Resistance,
Output Inductance,
And Switching
Frequency

APPENDIX III.
More Isolated Buck Converters:
Forward Converters,
Half And Full Bridge Converters

One Transistor Forward Converter



One Transistor Forward Converter

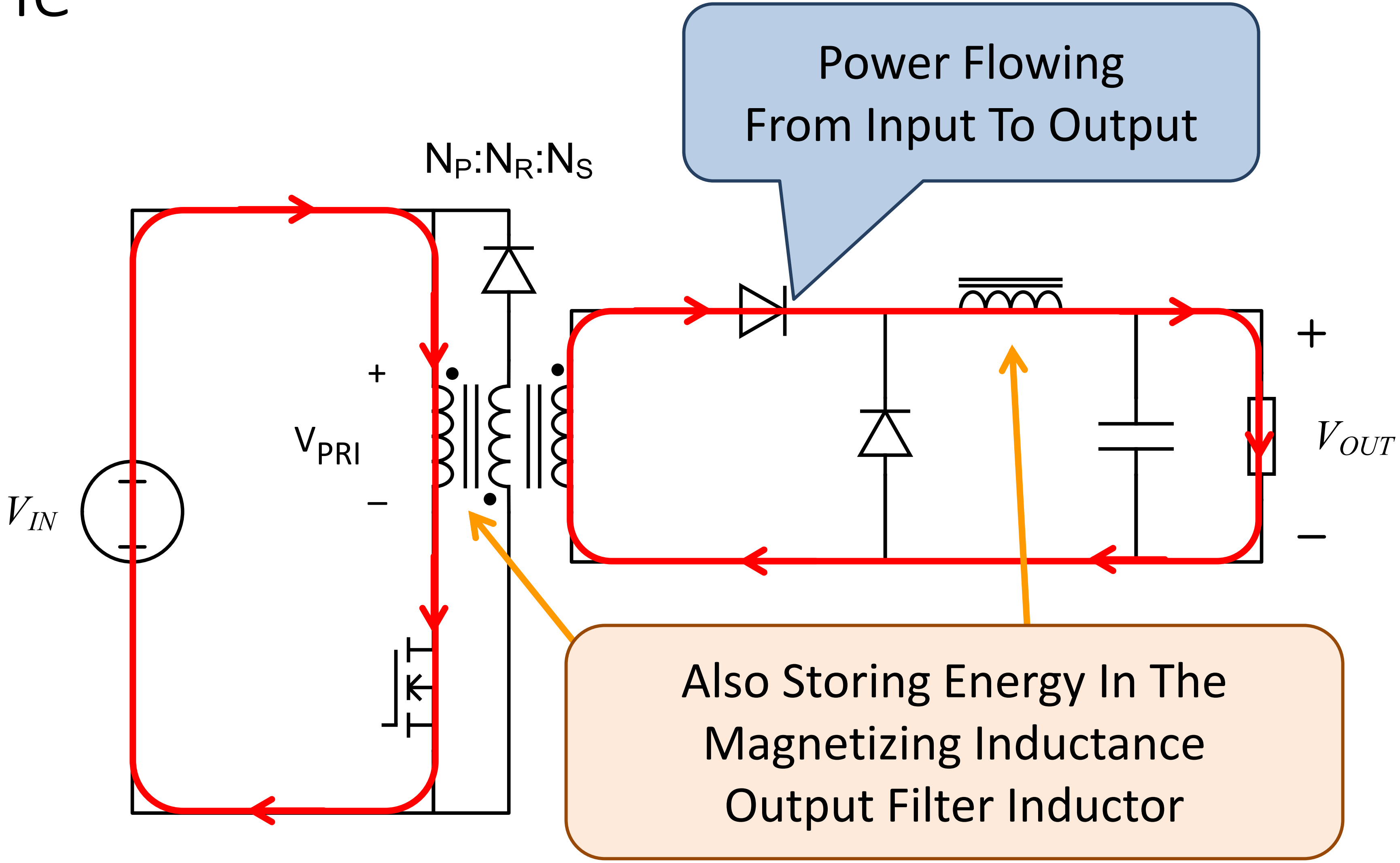


$$V_{OUT} = \frac{N_S}{N_P} \cdot D \cdot V_{IN}$$

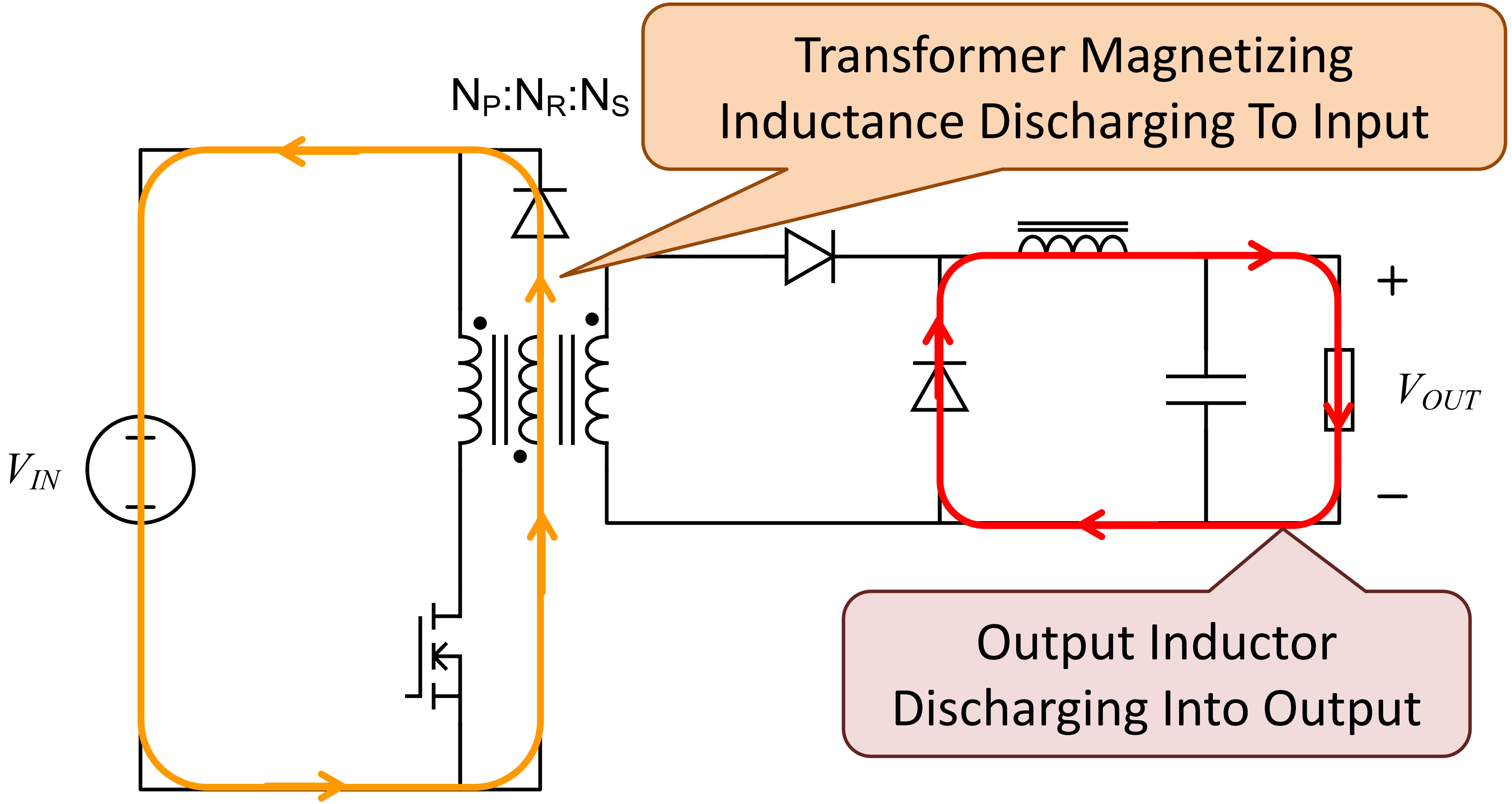
Input And Output Are
Not Electrically Connected
"Isolated"

Feedback From Output To Input
Must Also Be Isolated

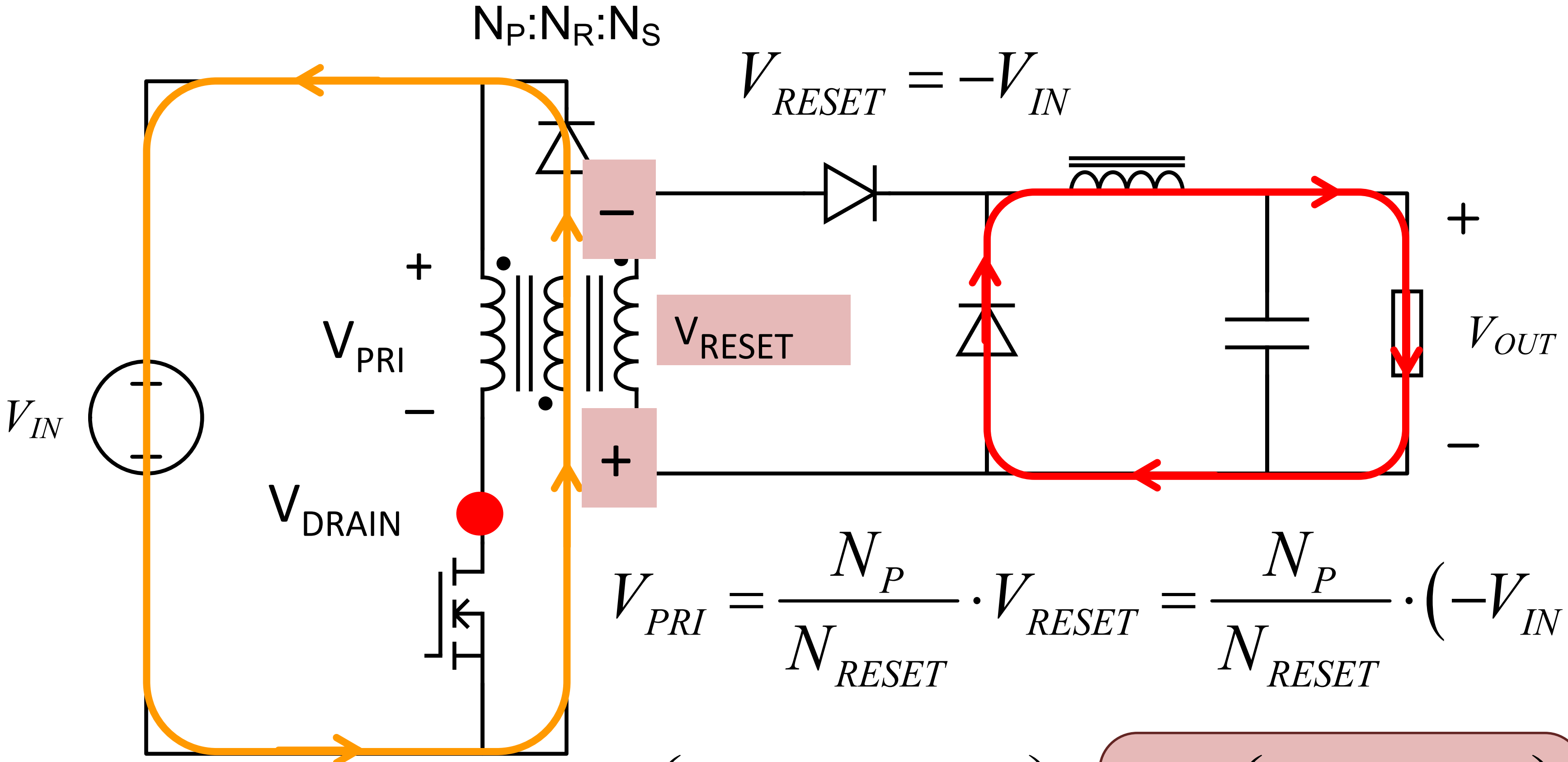
One Transistor Forward Converter
On Time



One Transistor Forward Converter Reset Time



One Transistor Forward Converter Reset Time

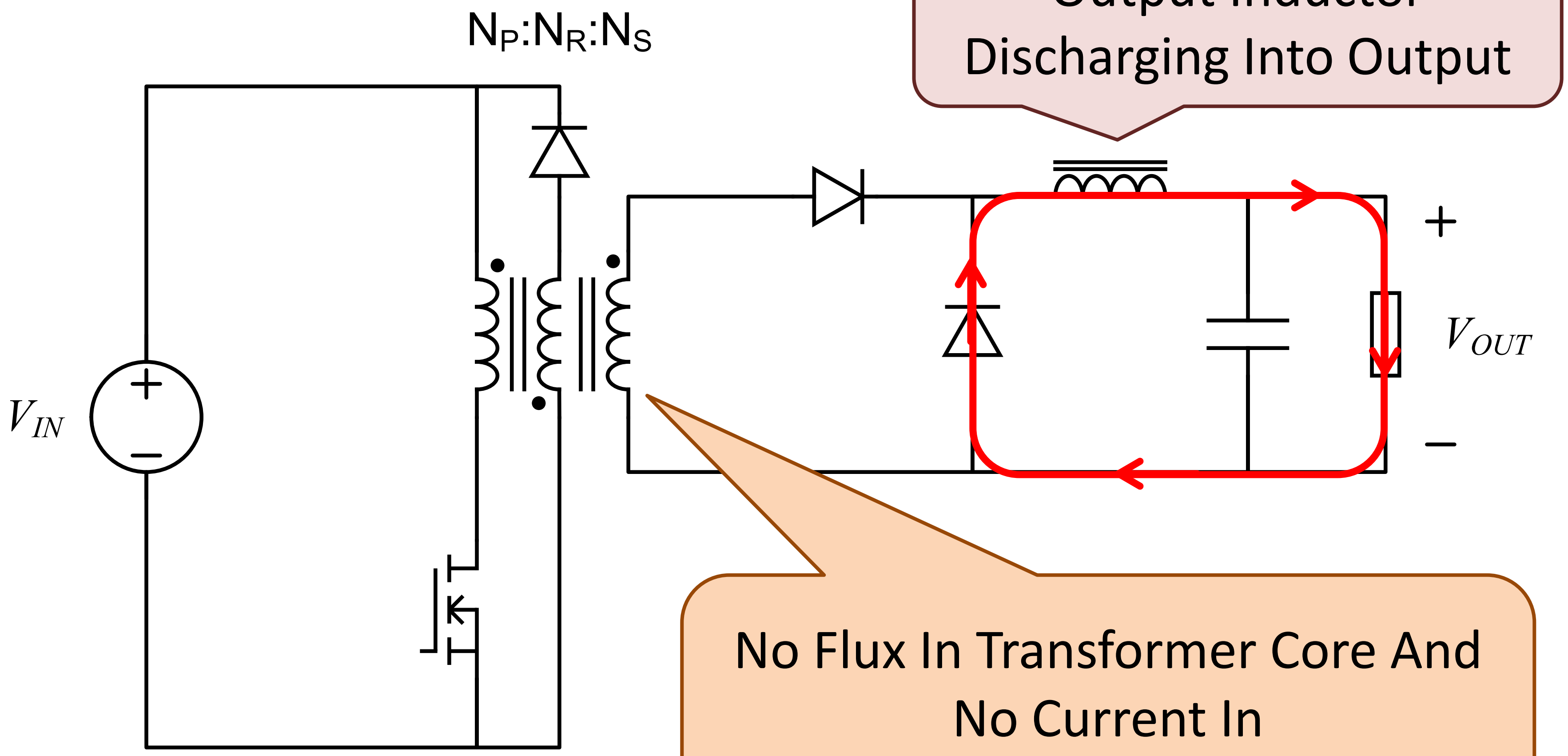


$$V_{RESET} = -V_{IN}$$

$$V_{PRI} = \frac{N_P}{N_{RESET}} \cdot V_{RESET} = \frac{N_P}{N_{RESET}} \cdot (-V_{IN})$$

$$V_{DRAIN} = V_{IN} - V_{PRI} = V_{IN} - \left(-\frac{N_P}{N_{RESET}} \cdot V_{IN} \right) = V_{IN} \cdot \left(1 + \frac{N_P}{N_{RESET}} \right)$$

One Transistor Forward Converter
Idle Time



Output Inductor Discharging Into Output

No Flux In Transformer Core And No Current In Transformer Windings

One Transistor Forward Converter Conversion Ratio

$$V_L(T_{ON})\cdot T_{ON}+V_L(T_{RESET})\cdot T_{RESET}+V_L(T_{IDLE})\cdot T_{IDLE}=0$$

Inductor Volt-Second Balance

$$\left(\frac{N_S}{N_P}\cdot V_{IN}-V_{OUT}\right)\cdot T_{ON}+(-V_{OUT})\cdot T_{RESET}+(-V_{OUT})\cdot T_{IDLE}=0$$

During On Time Voltage At Input To Inductor Is the Input Voltage Multiplied By the Transformer Turns Ratio

$$\left(\frac{N_S}{N_P}\cdot V_{IN}-V_{OUT}\right)\cdot T_{ON}+(-V_{OUT})\cdot(T_{RESET}+T_{IDLE})=0$$

$$\left(\frac{N_S}{N_P}\cdot V_{IN}-V_{OUT}\right)\cdot T_{ON}+(-V_{OUT})\cdot T_{OFF}=0$$

$$\left(\frac{N_S}{N_P}\cdot V_{IN}-V_{OUT}\right)\cdot D\cdot T_{SW}+(-V_{OUT})\cdot(1-D)\cdot T_{SW}=0$$

$$\left(\frac{N_S}{N_P}\cdot V_{IN}-V_{OUT}\right)\cdot D+(-V_{OUT})\cdot(1-D)=0$$

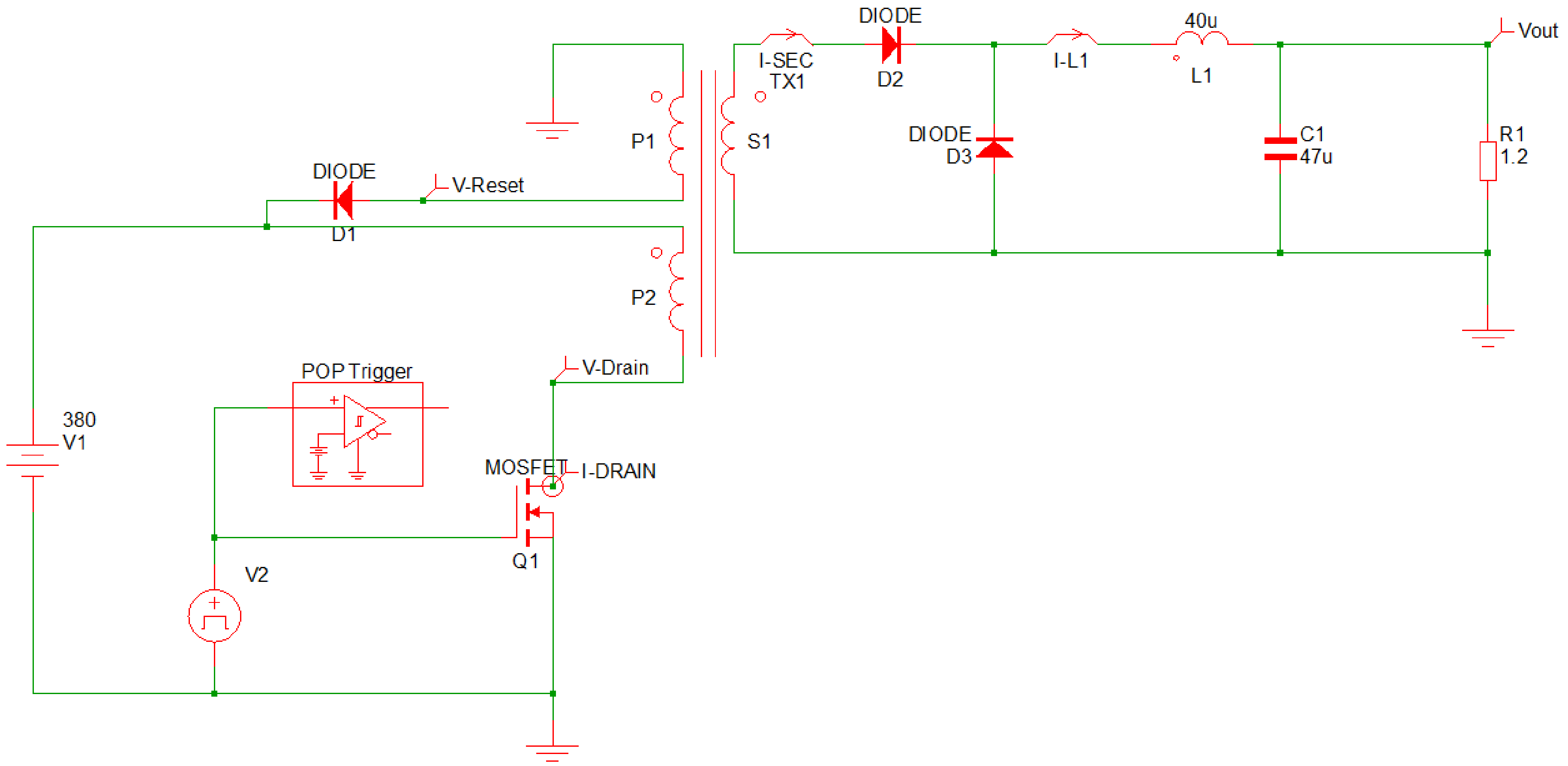
$$D\cdot\frac{N_S}{N_P}\cdot V_{IN}-D\cdot V_{OUT}-V_{OUT}+D\cdot V_{OUT}=0$$

$$D\cdot\frac{N_S}{N_P}\cdot V_{IN}-V_{OUT}=0$$

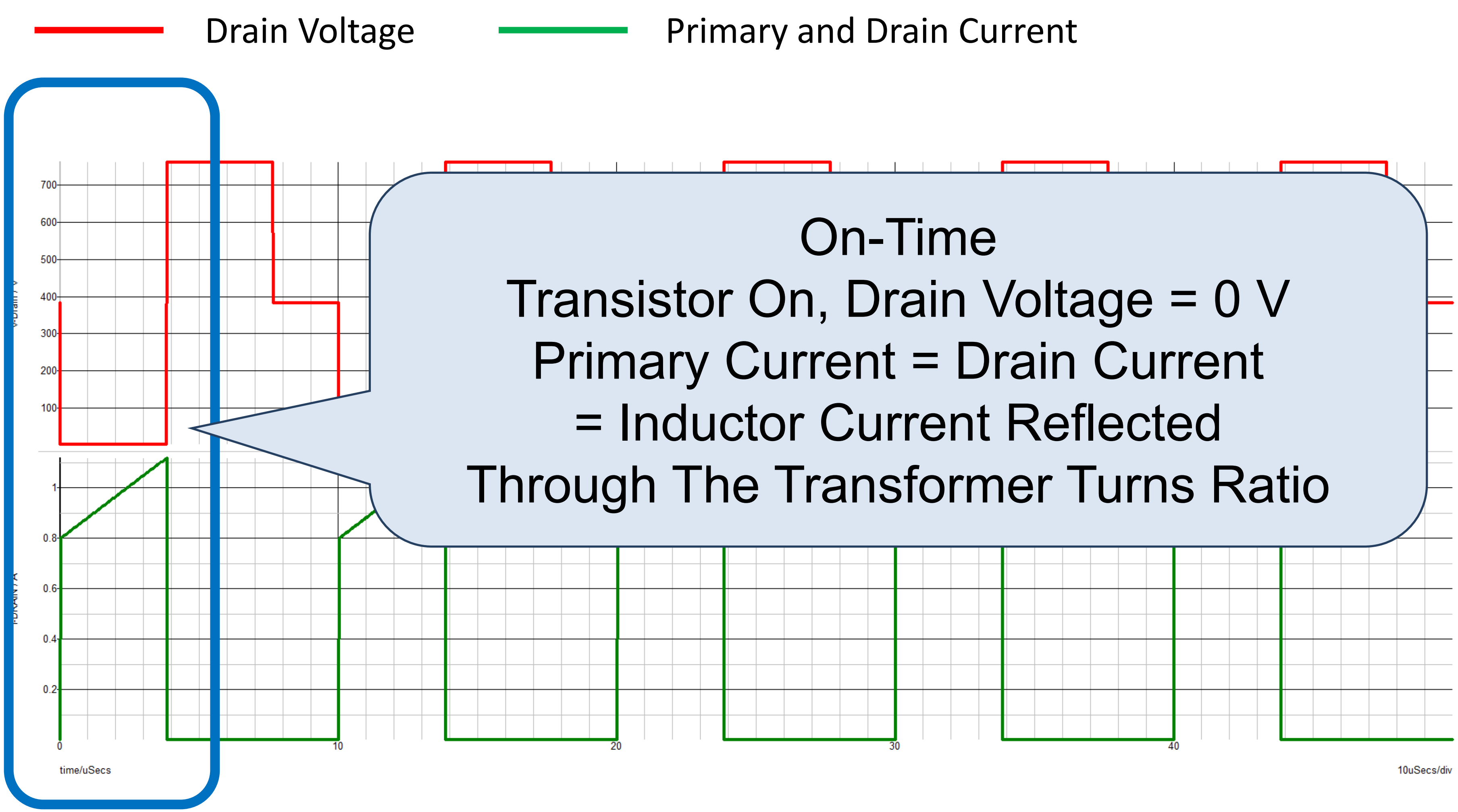
$$V_{OUT}=D\cdot\frac{N_S}{N_P}\cdot V_{IN}$$

Same As The Buck Converter Except For The Transformer Turns Ratio

Forward Converter Simulation Schematic



Forward Converter Simulation
Primary Waveforms



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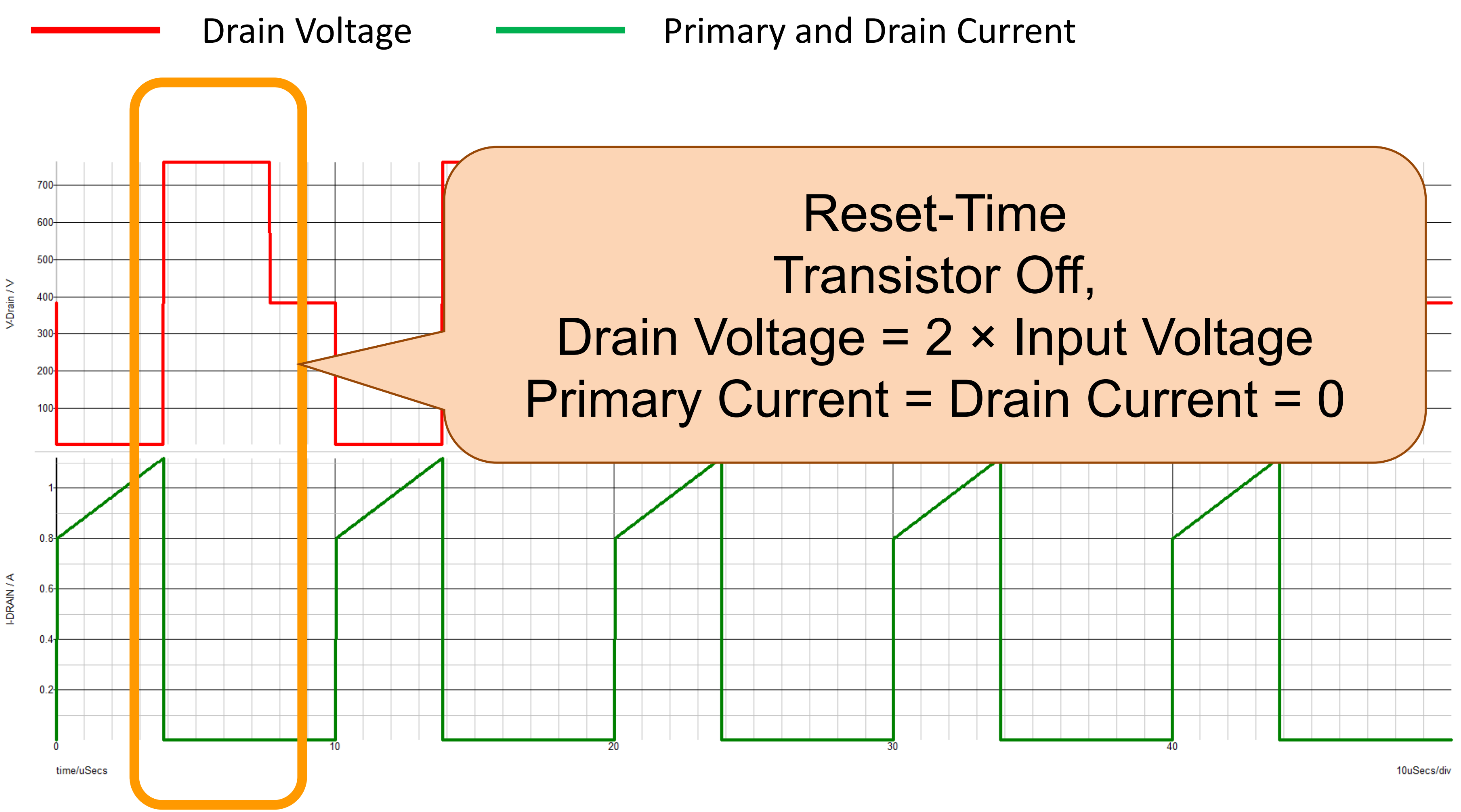
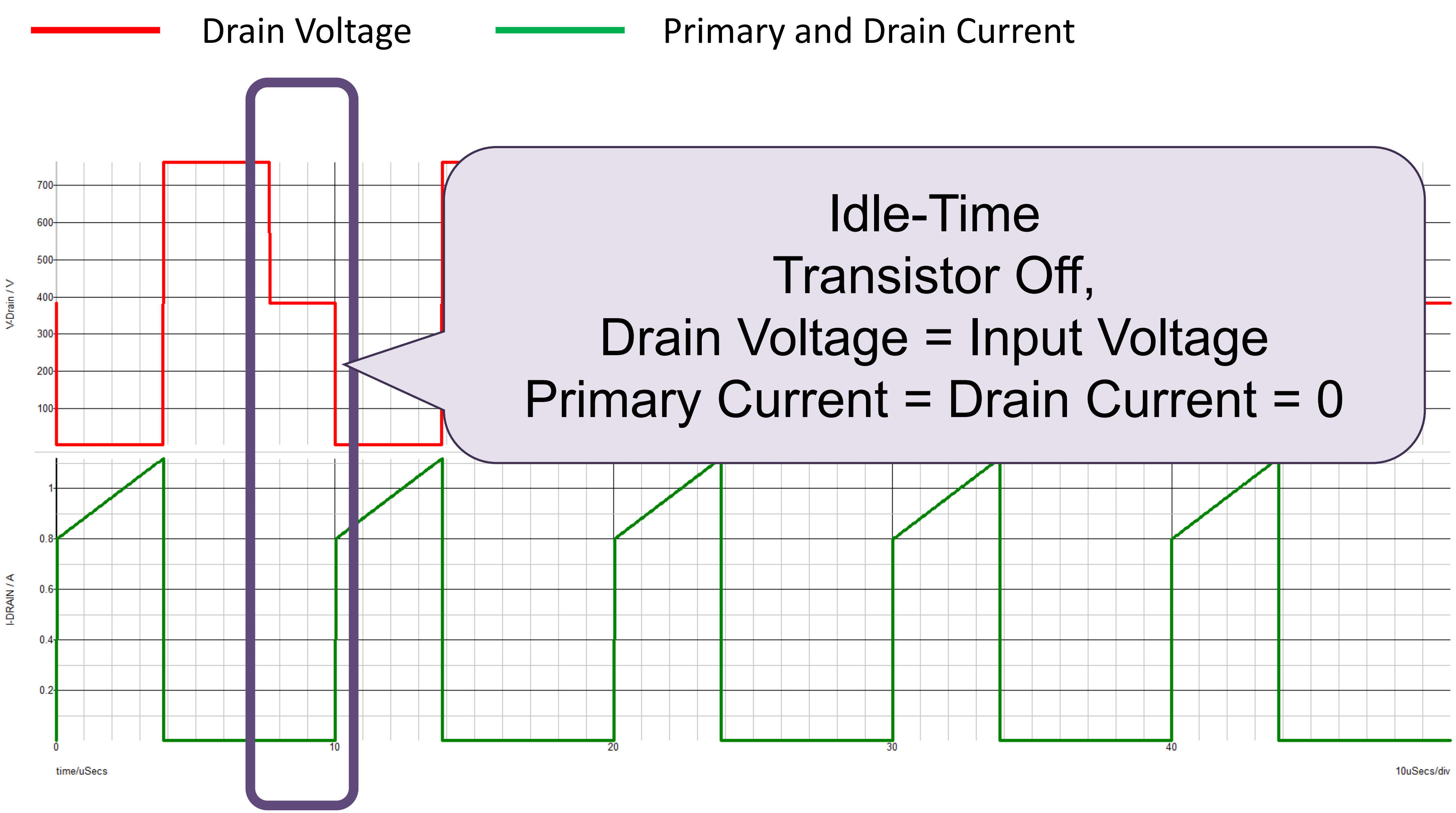
Forward Converter Simulation
Primary Waveforms

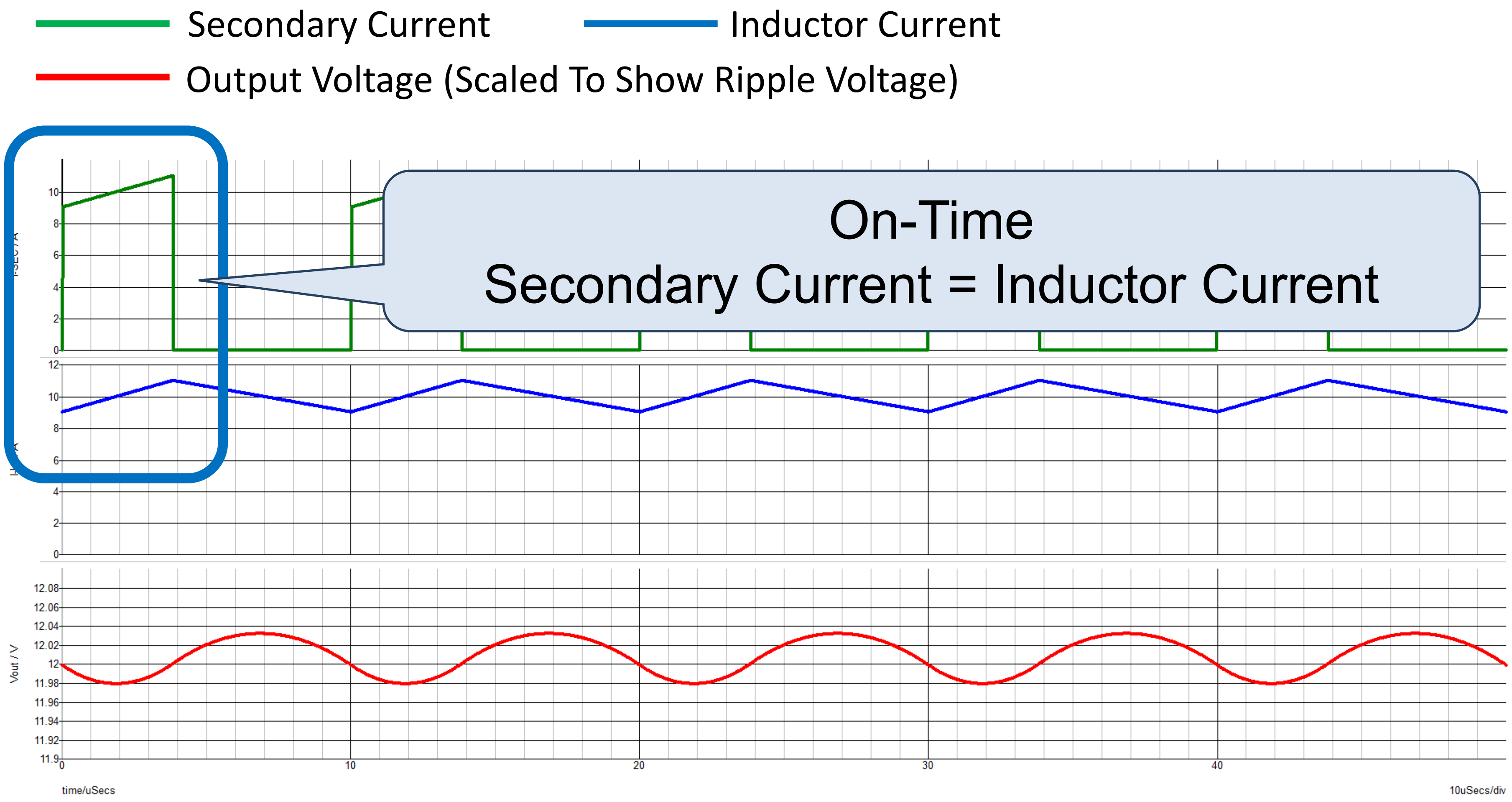


EMBEDDED POWER LABS

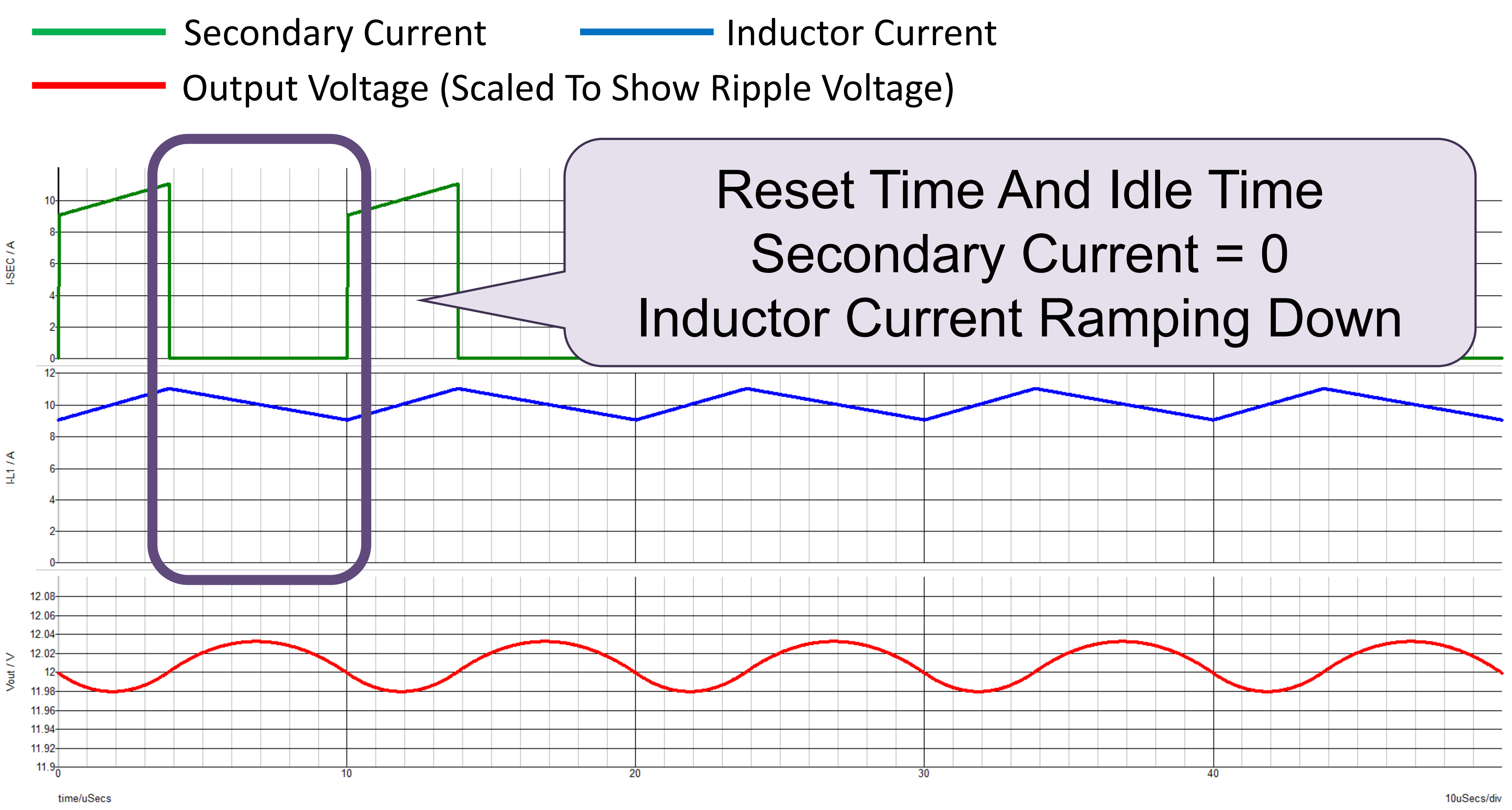
Forward Converter Simulation
Primary Waveforms



Forward Converter Simulation Secondary Waveforms

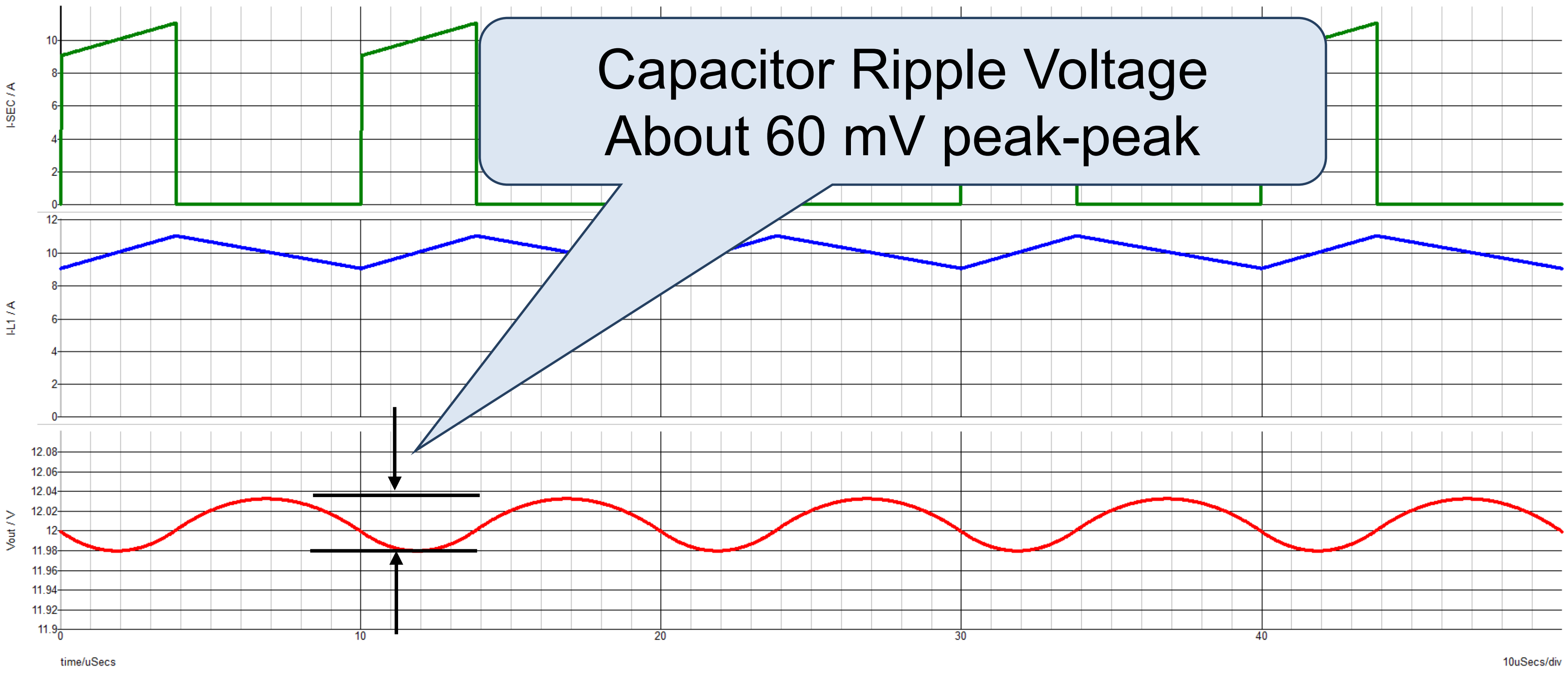


Forward Converter Simulation Secondary Waveforms



Forward Converter Simulation
Secondary Waveforms

Secondary Current
Inductor Current
Output Voltage (Scaled To Show Ripple Voltage)



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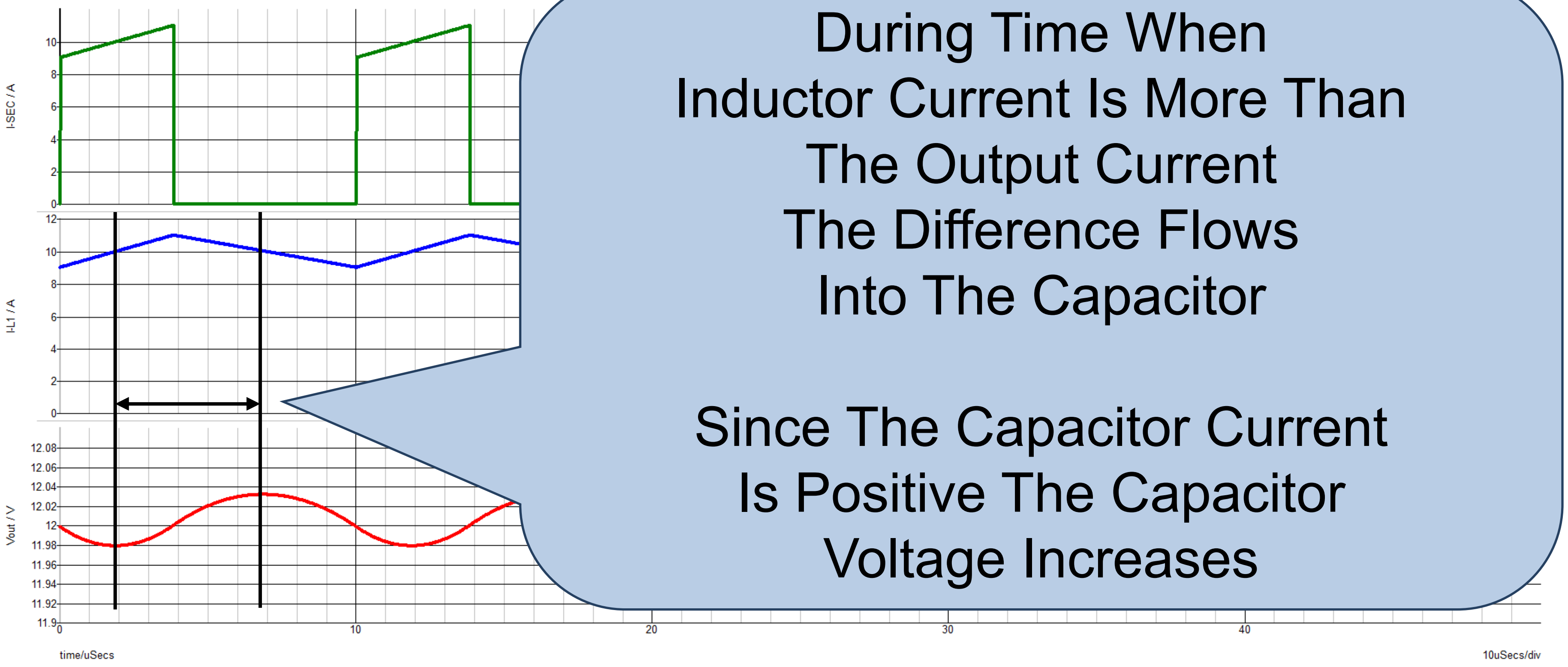
Forward Converter Simulation
Secondary Waveforms

Secondary Current — Inductor Current

Output Voltage (Scaled To Show Ripple Voltage)



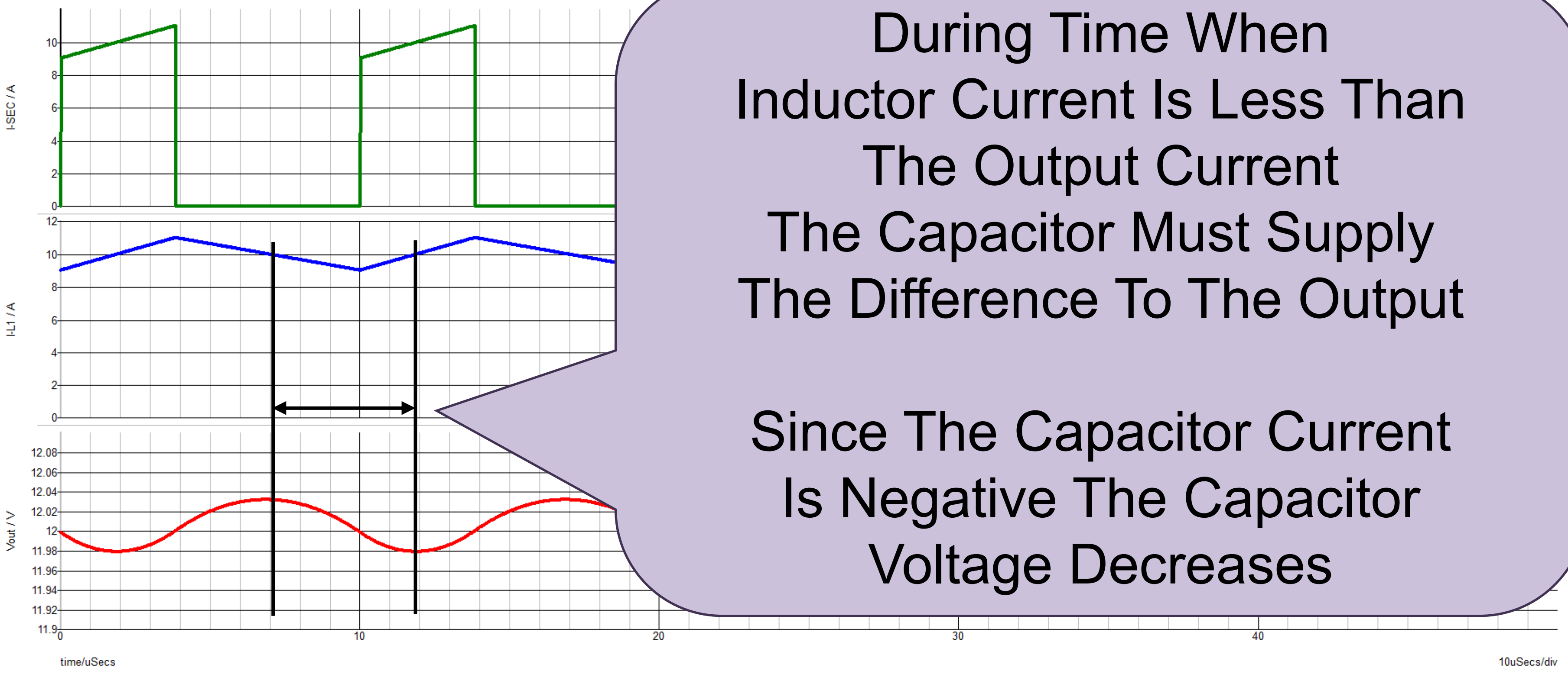
During Time When
Inductor Current Is More Than
The Output Current
The Difference Flows
Into The Capacitor

Since The Capacitor Current
Is Positive The Capacitor
Voltage Increases

Forward Converter Simulation
Secondary Waveforms

Secondary Current — Inductor Current

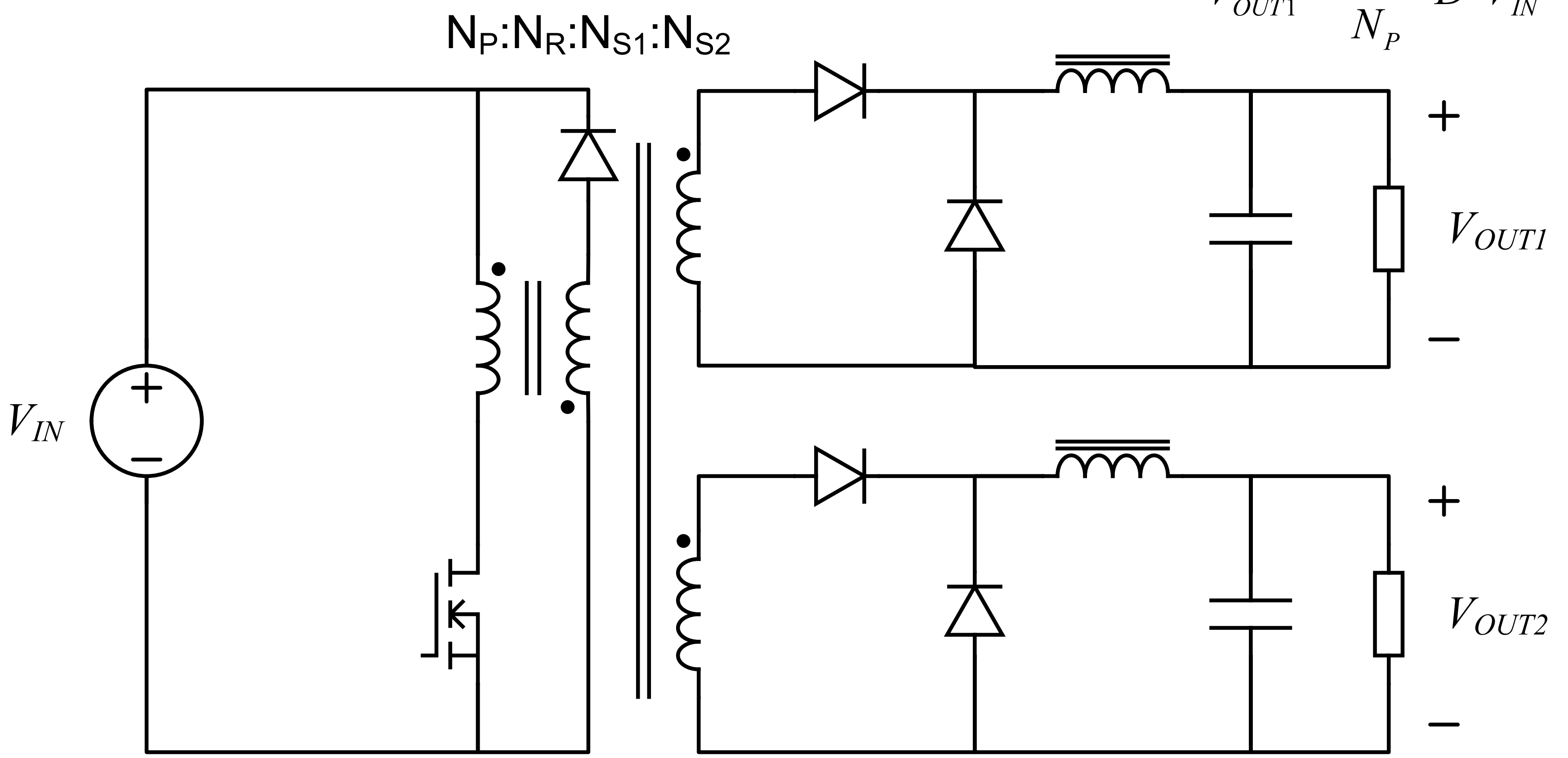
Output Voltage (Scaled To Show Ripple Voltage)



During Time When
Inductor Current Is Less Than
The Output Current
The Capacitor Must Supply
The Difference To The Output

Since The Capacitor Current
Is Negative The Capacitor
Voltage Decreases

Multiple Output Forward Converter



$$V_{OUT1} = \frac{N_{S1}}{N_P} \cdot D \cdot V_{IN}$$

$$\frac{V_{OUT2}}{V_{OUT1}} = \frac{\frac{N_{S2}}{N_P} \cdot D \cdot V_{IN}}{\frac{N_{S1}}{N_P} \cdot D \cdot V_{IN}} = \frac{N_{S2}}{N_{S1}}$$

$$V_{OUT2} = \frac{N_{S2}}{N_P} \cdot D \cdot V_{IN}$$

Forward Converter

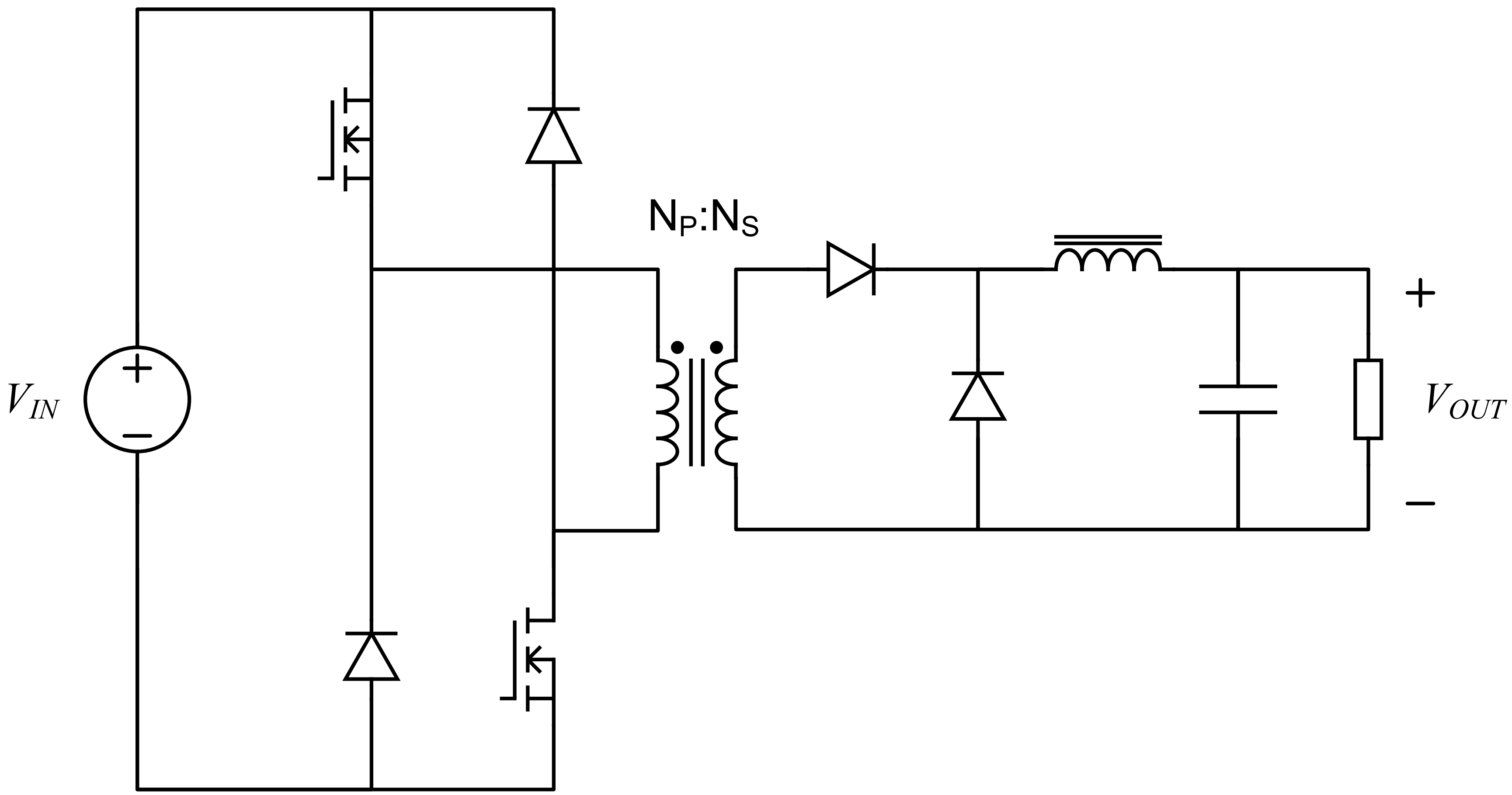
Advantages

- Isolated
- Good For 50 W To 500 W
- Multiple Isolated Outputs Possible
- Easy To Control

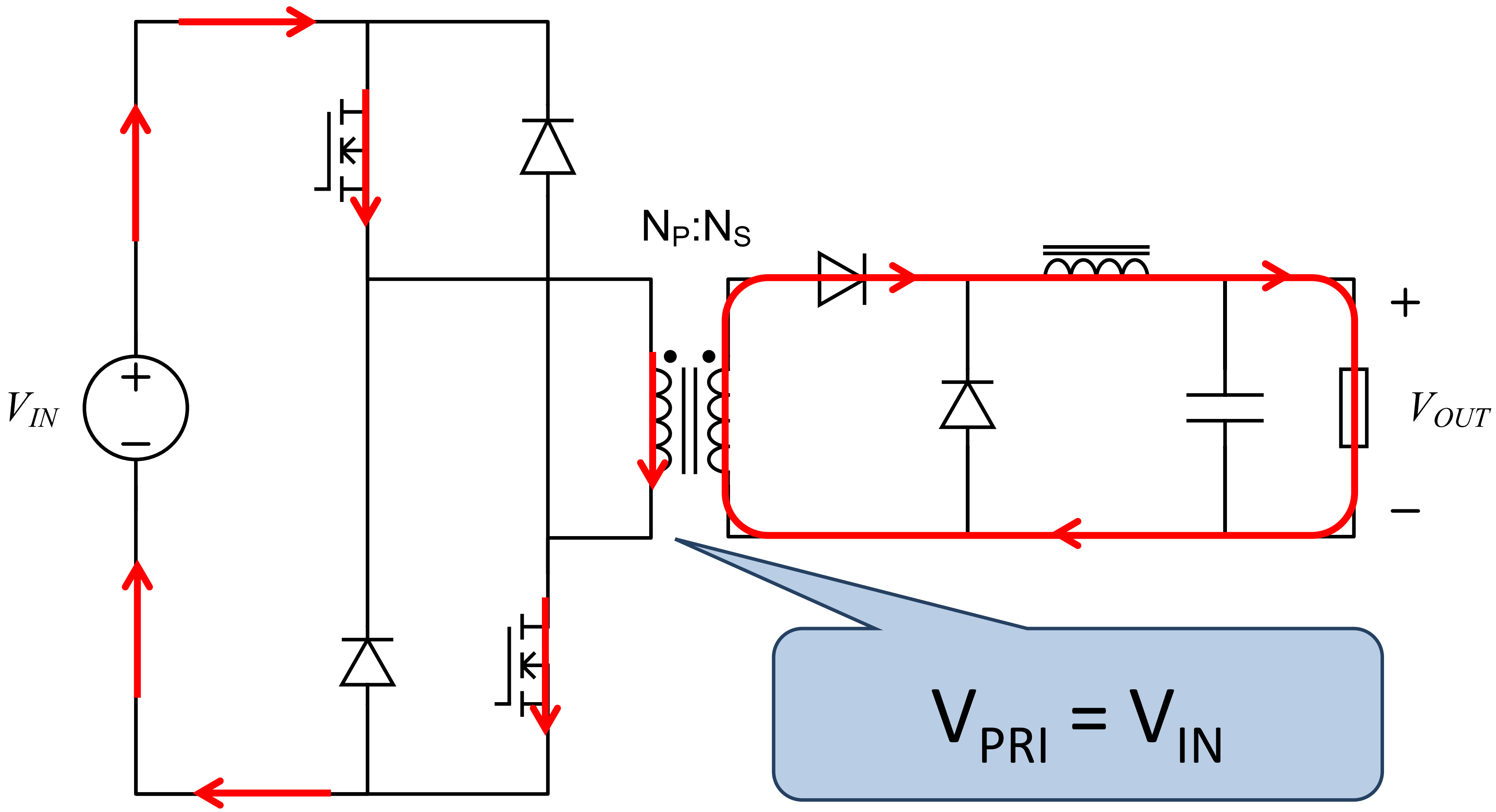
Disadvantages

- Transformer Needed
- High Voltage Stress On Transistor And Diode

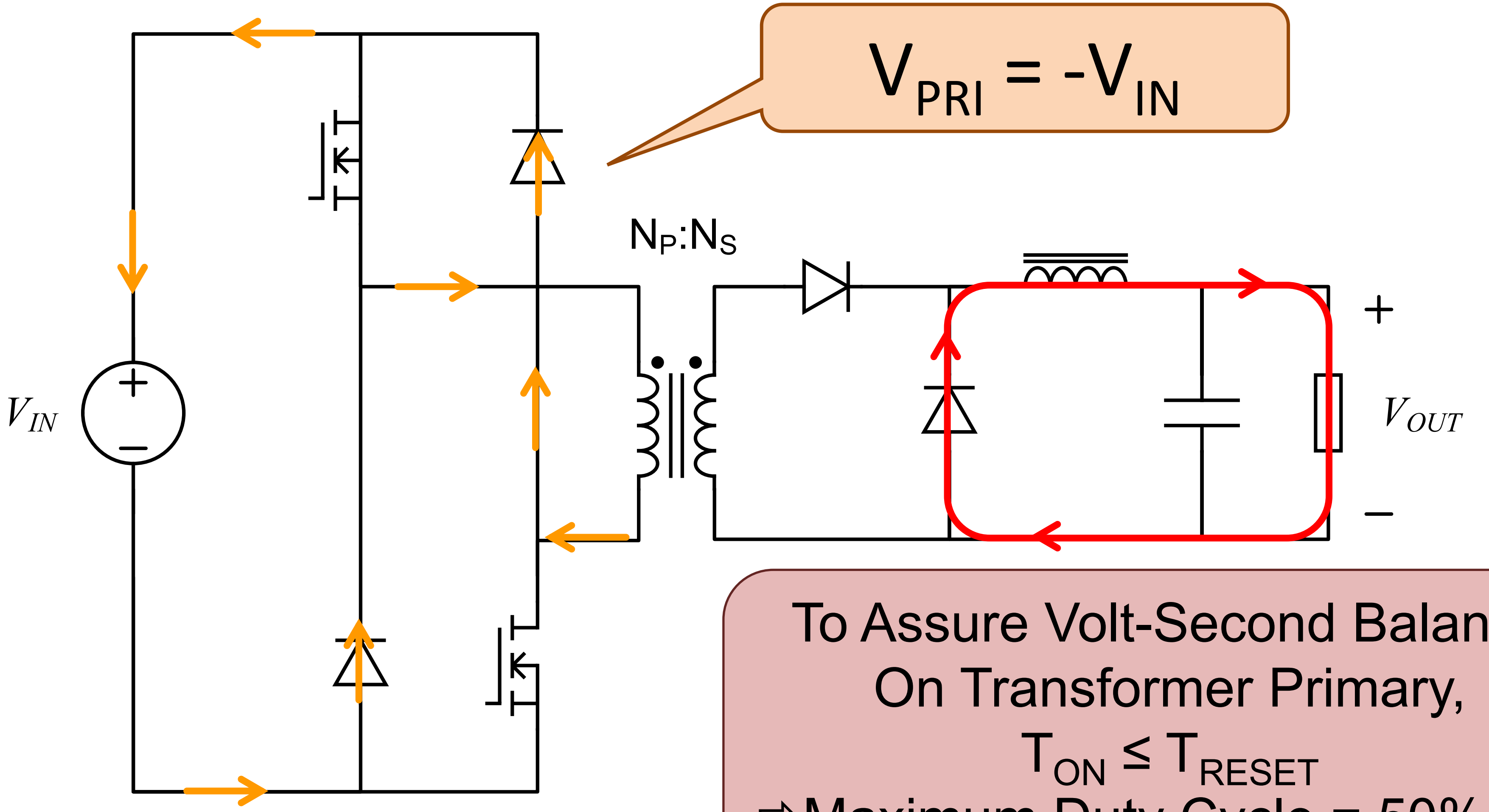
Two Transistor Forward Converter



Two Transistor Forward Converter
On-Time

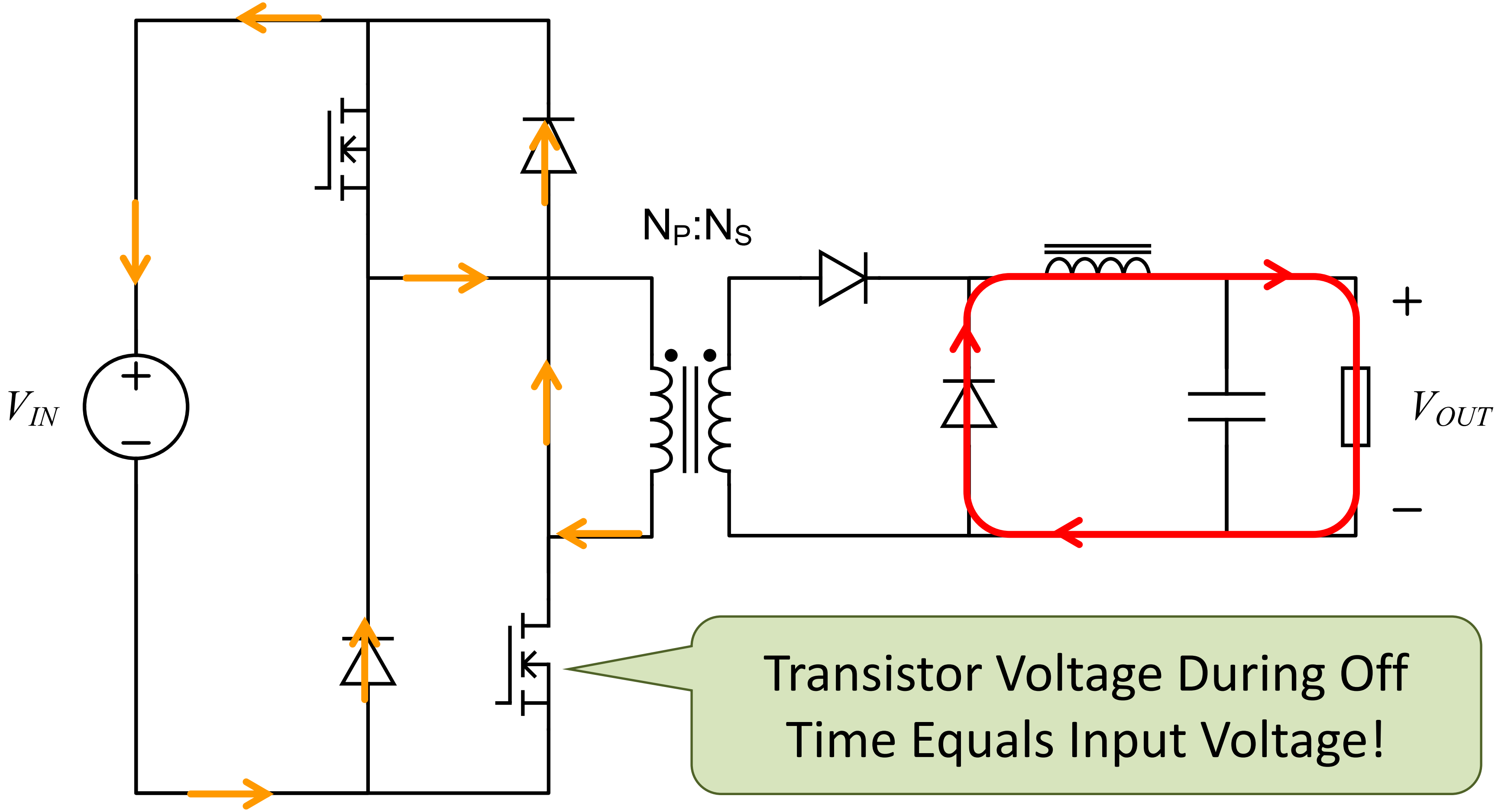


Two Transistor Forward Converter
Reset-Time



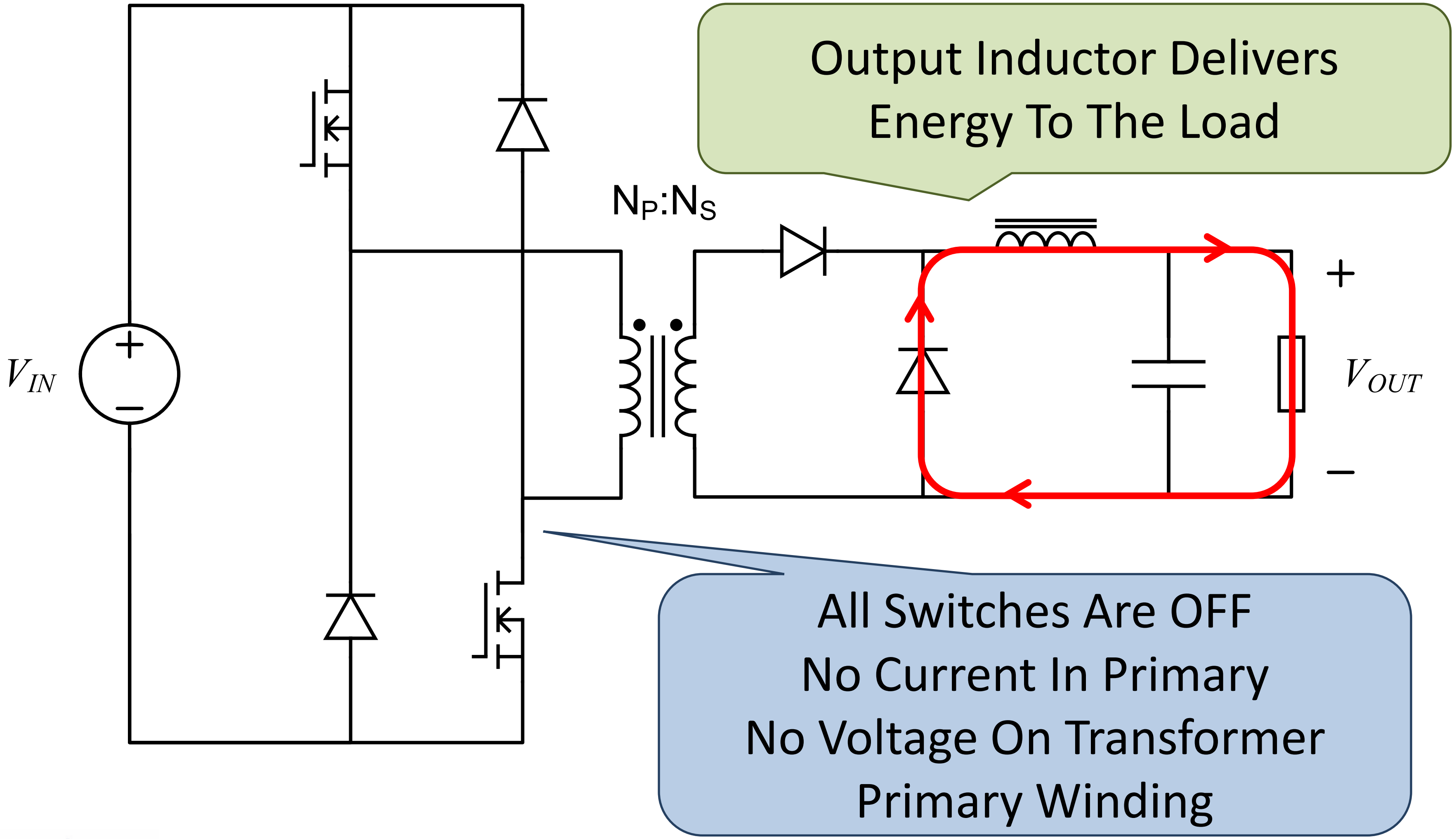
To Assure Volt-Second Balance
On Transformer Primary,
$T_{ON} \leq T_{RESET}$
⇨Maximum Duty Cycle = 50%

Two Transistor Forward Converter Reset-Time



Transistor Voltage During Off Time Equals Input Voltage!

Two Transistor Forward Converter
Idle-Time



Two Transistor Forward Converter Conversion Ratio

$$V_L(T_{ON})\cdot T_{ON}+V_L(T_{RESET})\cdot T_{RESET}+V_L(T_{IDLE})\cdot T_{IDLE}=0$$

Inductor Volt-Second Balance

$$\left(\frac{N_S}{N_P}\cdot V_{IN}-V_{OUT}\right)\cdot T_{ON}+(-V_{OUT})\cdot T_{RESET}+(-V_{OUT})\cdot T_{IDLE}=0$$

During On Time Voltage At Input To Inductor Is the Input Voltage Multiplied By the Transformer Turns Ratio

$$\left(\frac{N_S}{N_P}\cdot V_{IN}-V_{OUT}\right)\cdot T_{ON}+(-V_{OUT})\cdot(T_{RESET}+T_{IDLE})=0$$

$$\left(\frac{N_S}{N_P}\cdot V_{IN}-V_{OUT}\right)\cdot T_{ON}+(-V_{OUT})\cdot T_{OFF}=0$$

$$\left(\frac{N_S}{N_P}\cdot V_{IN}-V_{OUT}\right)\cdot D\cdot T_{SW}+(-V_{OUT})\cdot(1-D)\cdot T_{SW}=0$$

$$\left(\frac{N_S}{N_P}\cdot V_{IN}-V_{OUT}\right)\cdot D+(-V_{OUT})\cdot(1-D)=0$$

$$D\cdot\frac{N_S}{N_P}\cdot V_{IN}-D\cdot V_{OUT}-V_{OUT}+D\cdot V_{OUT}=0$$

$$D\cdot\frac{N_S}{N_P}\cdot V_{IN}-V_{OUT}=0$$

$$V_{OUT}=D\cdot\frac{N_S}{N_P}\cdot V_{IN}$$

Same At The Buck Converter Except For The Transformer Turns Ratio

Two Transistor Forward Converter

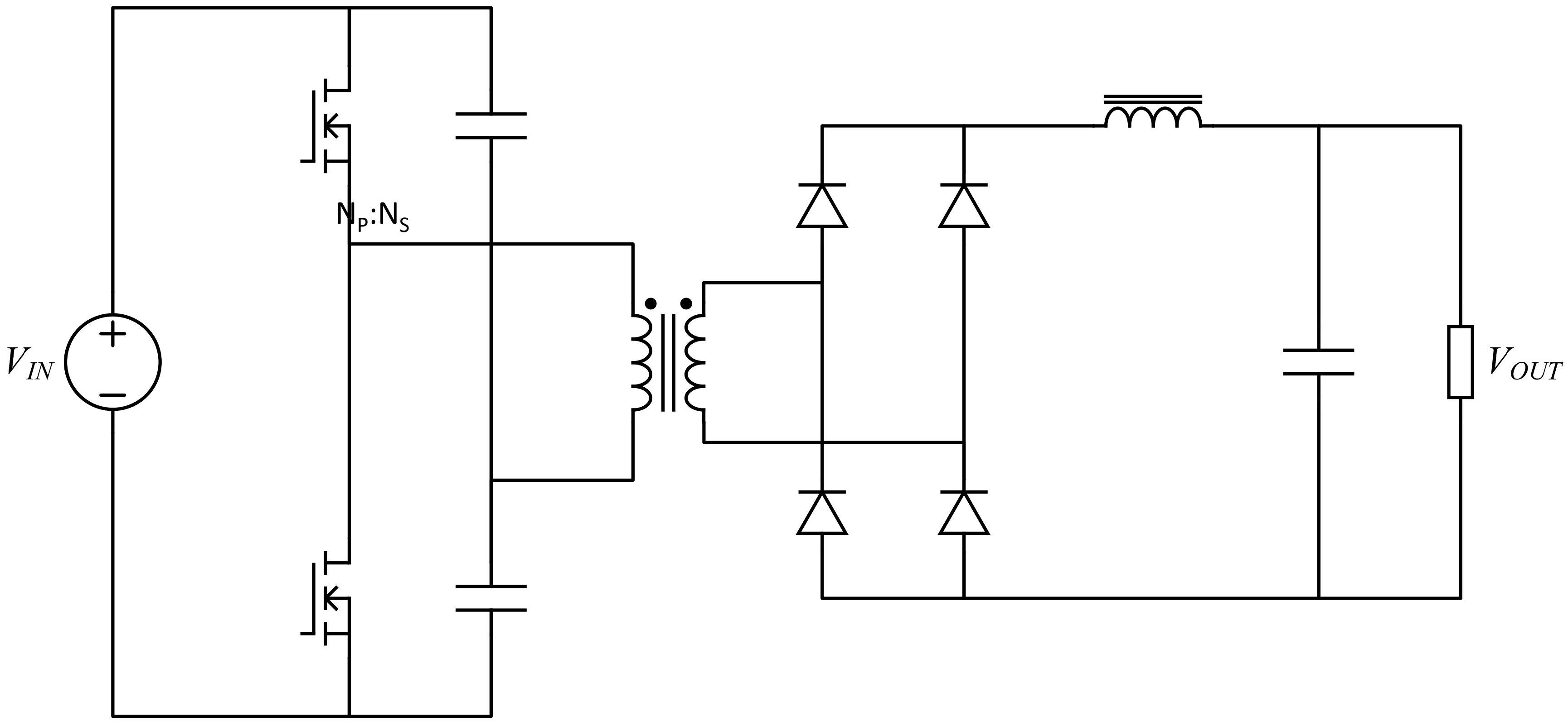
Advantages

- Isolated
- Good For
  50 W To 500 W
- Multiple Isolated Outputs Possible
- Easy To Control
- Low Voltage Stress On Switching Transistors

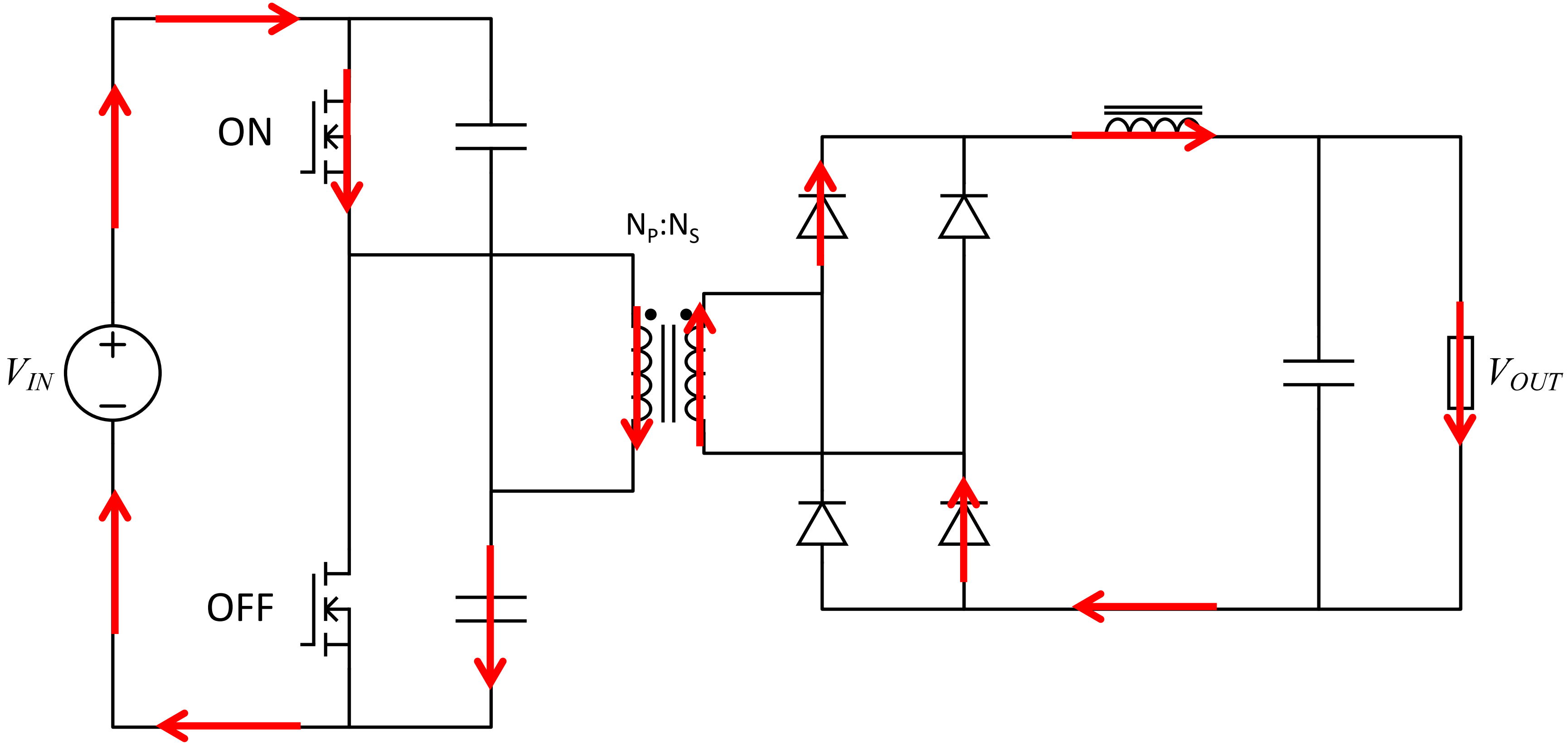
Disadvantages

- Transformer Needed
- Maximum Duty Cycle Limited To 50%
- Gate Drive Transformer Or IC Needed
- High Voltage Stress On Output Diodes

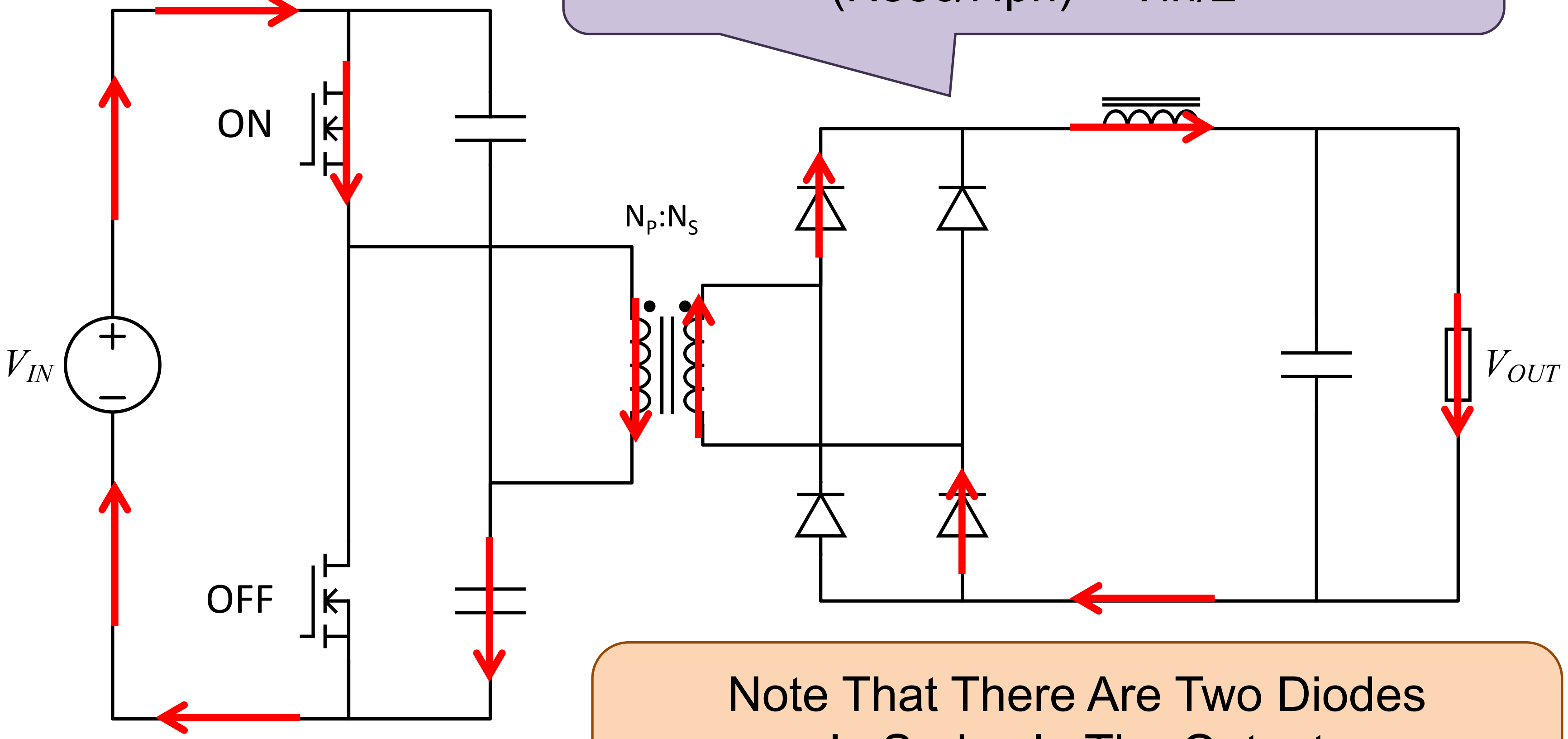
Half-Bridge Converter w/Full Wave Rectifier



Half-Bridge Converter w/FW Rectifier On Time 1



Half-Bridge Converter w/FW Rectifier On Time 1

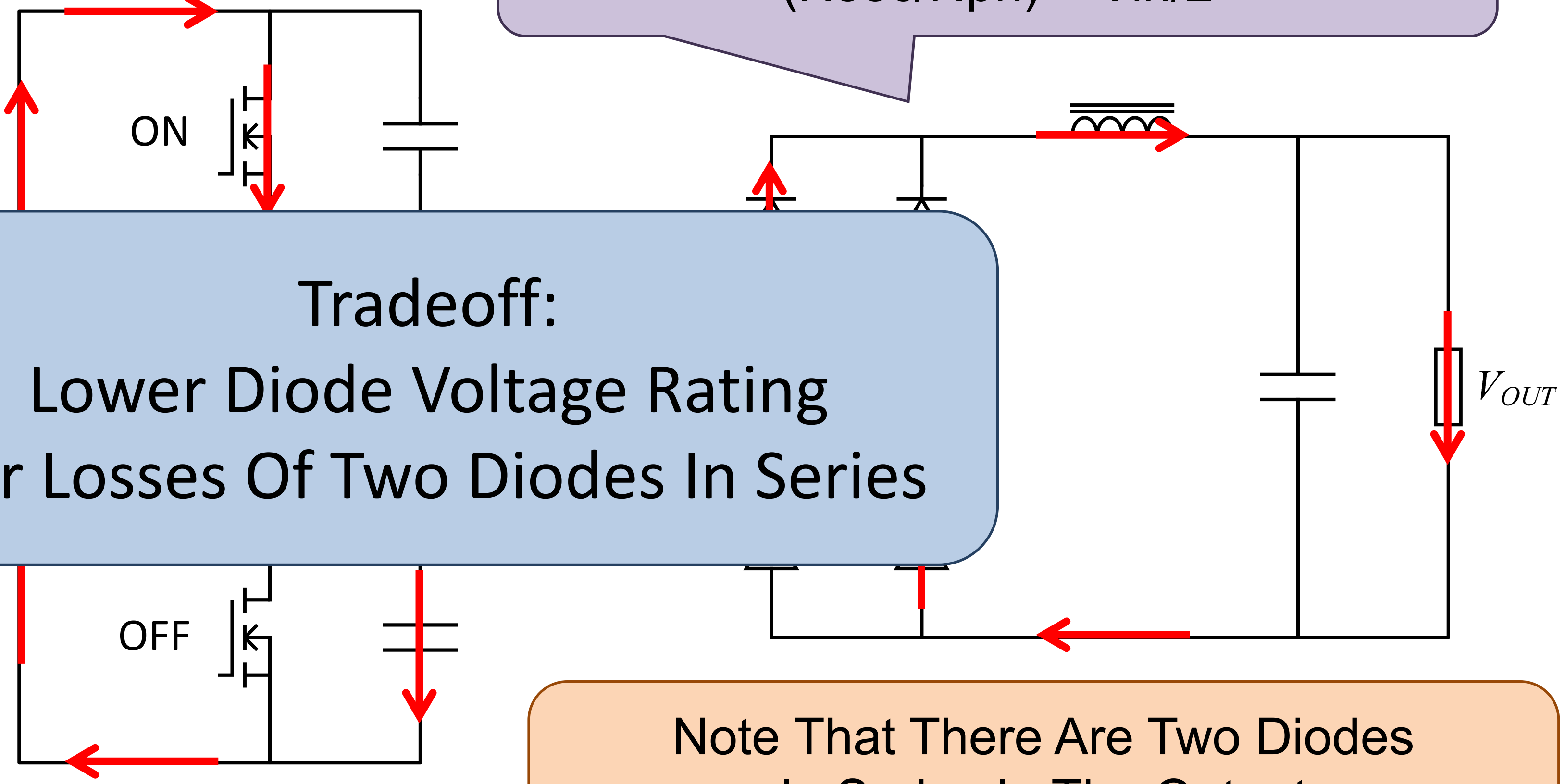


Non-Conducting Diode Blocks
(Nsec/Npri) × Vin/2

Note That There Are Two Diodes
In Series In The Output
This Causes Additional Power Losses

Half-Bridge Converter w/FW Rectifier On Time 1

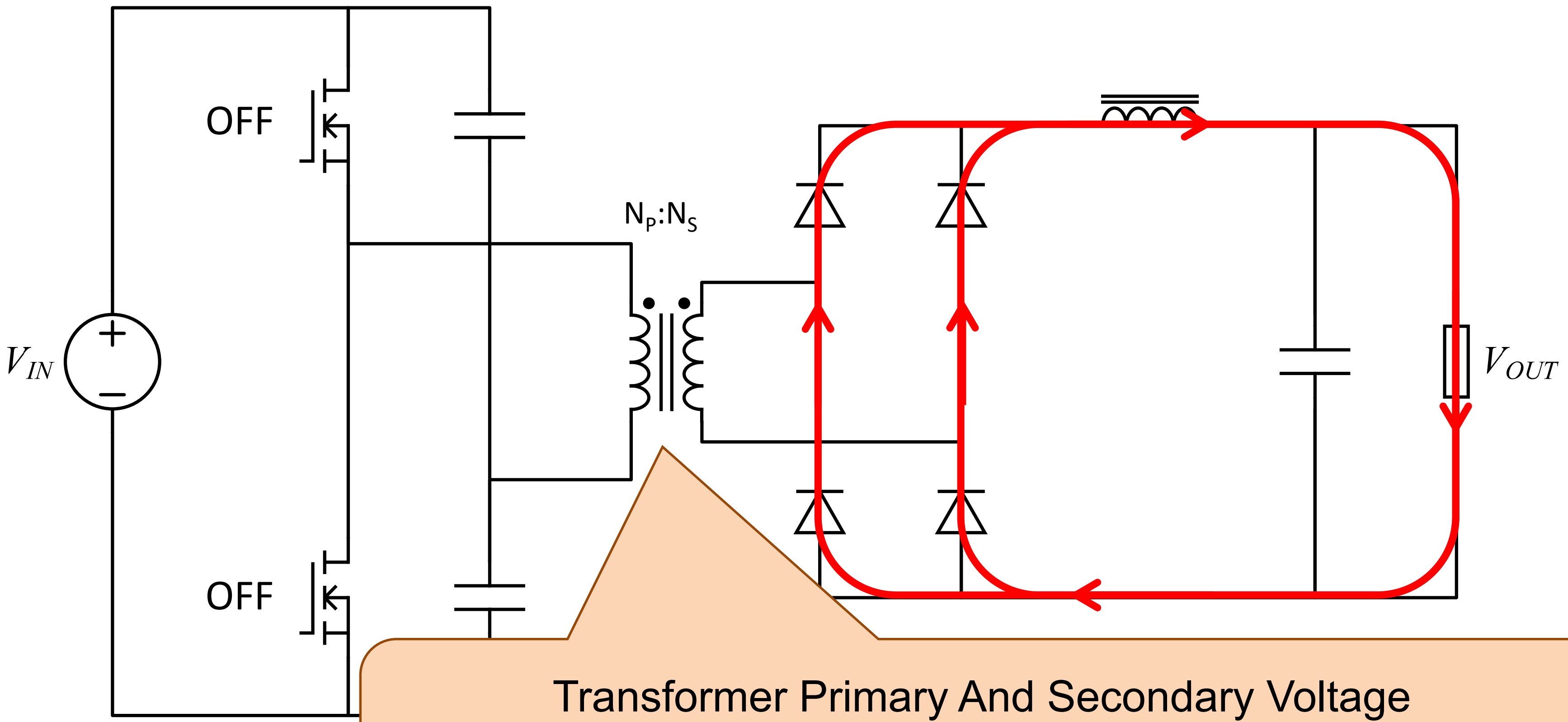
Non-Conducting Diode Blocks
(Nsec/Npri) × Vin/2



Tradeoff:
Lower Diode Voltage Rating
For Losses Of Two Diodes In Series

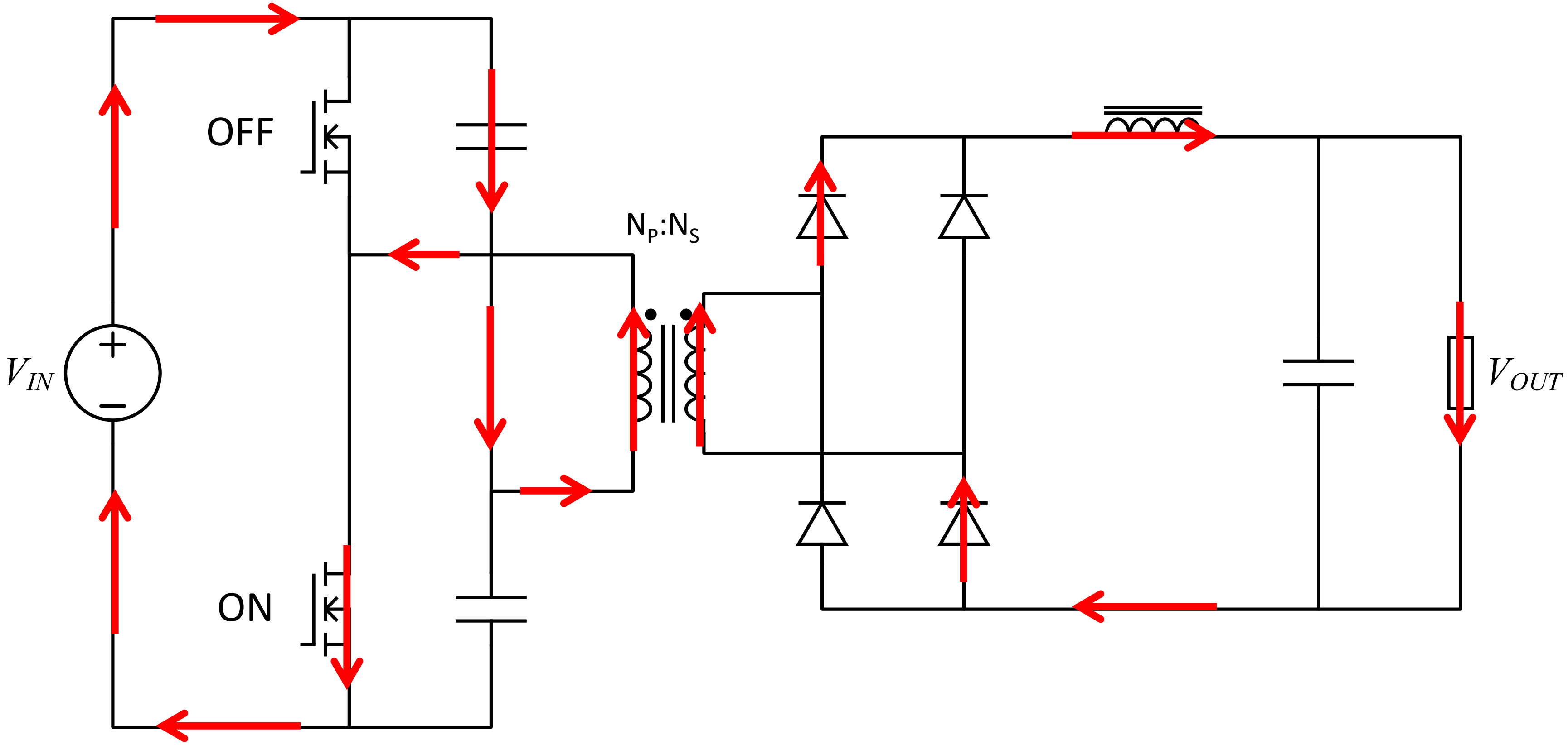
Note That There Are Two Diodes
In Series In The Output
This Causes Additional Power Losses

Half-Bridge Converter w/FW Rectifier Off Time 1

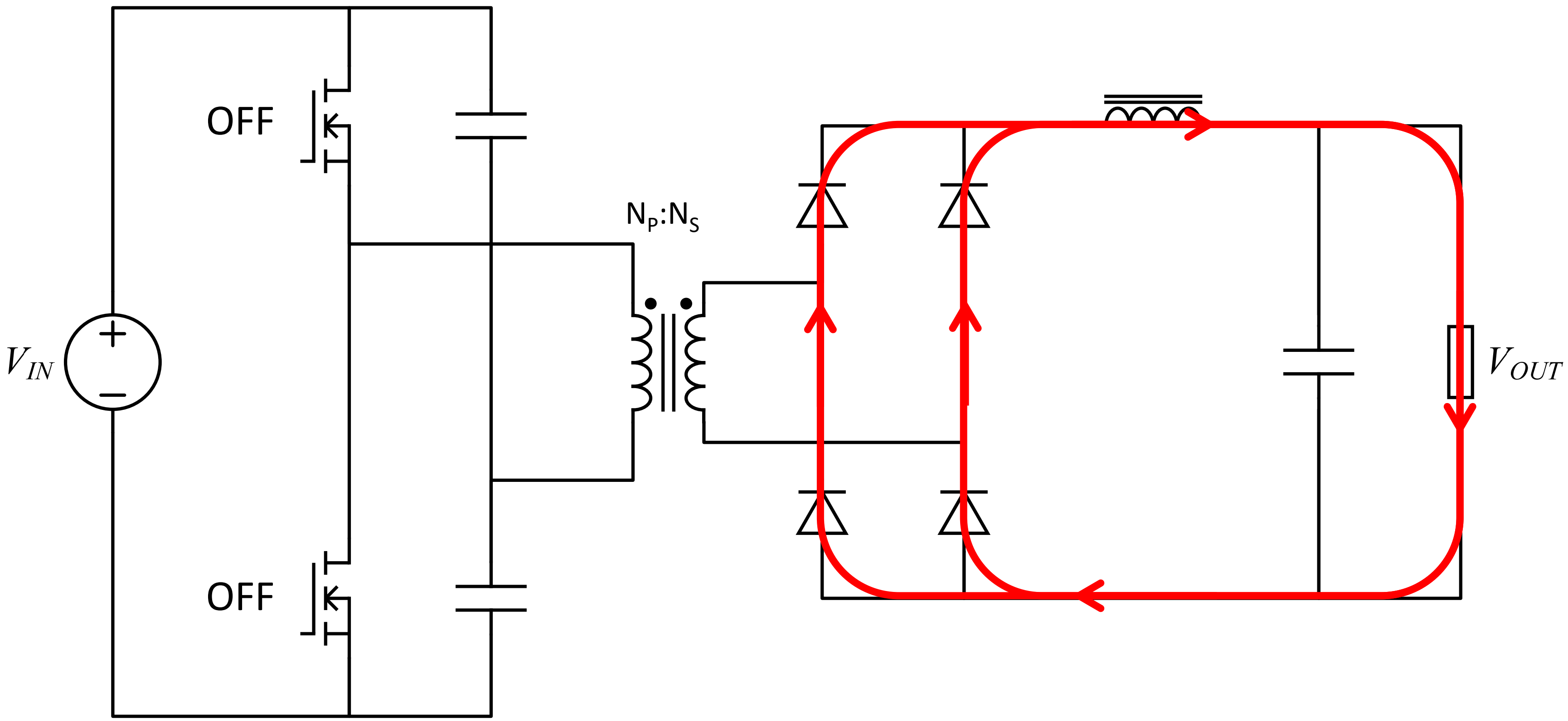


Transformer Primary And Secondary Voltage
Are 0 V During Off Time Because The
Secondary Winding Is Shorted By The Conducting Diodes

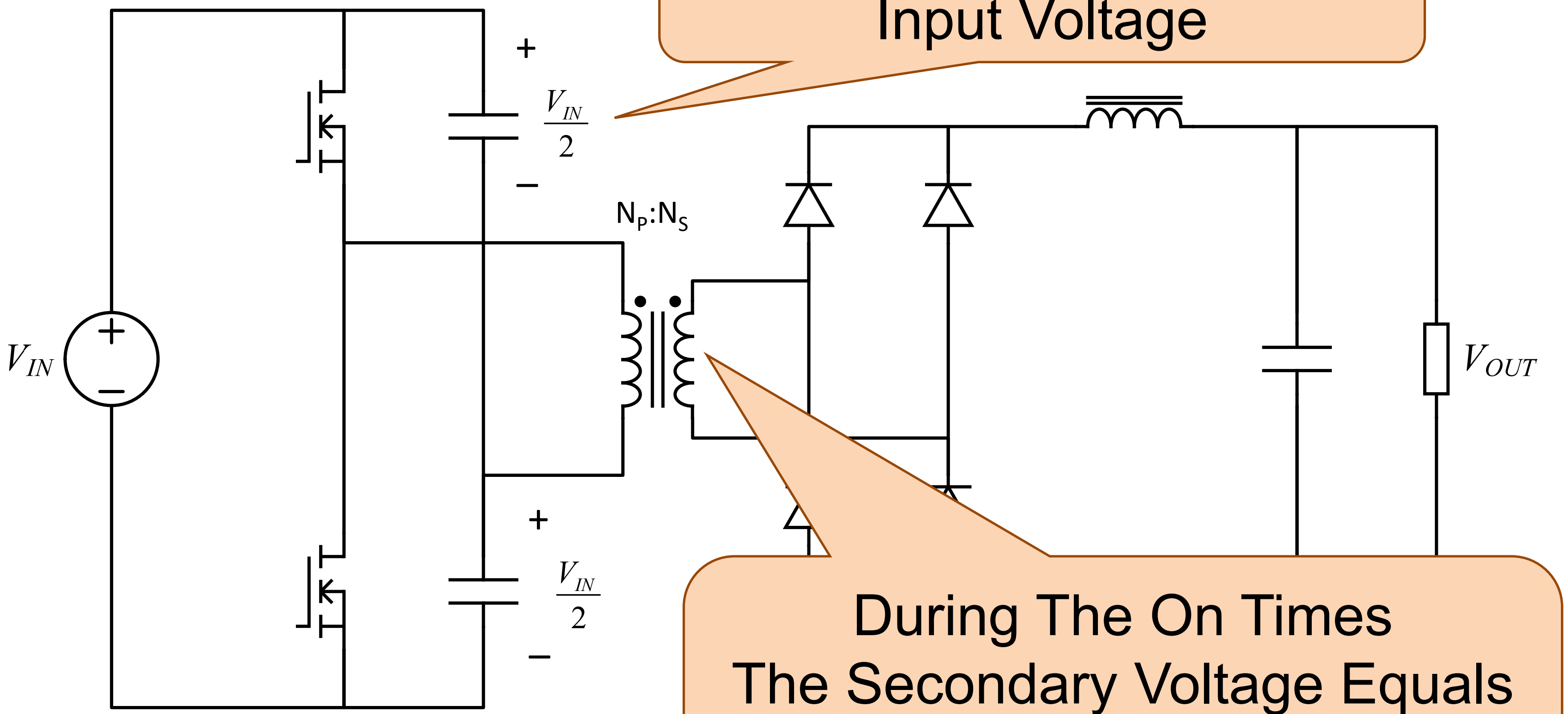
Half-Bridge Converter w/FW Rectifier On Time 2



Half-Bridge Converter w/FW Rectifier Off Time 2



Half-Bridge Converter w/FW Rectifier Conversion Ratio

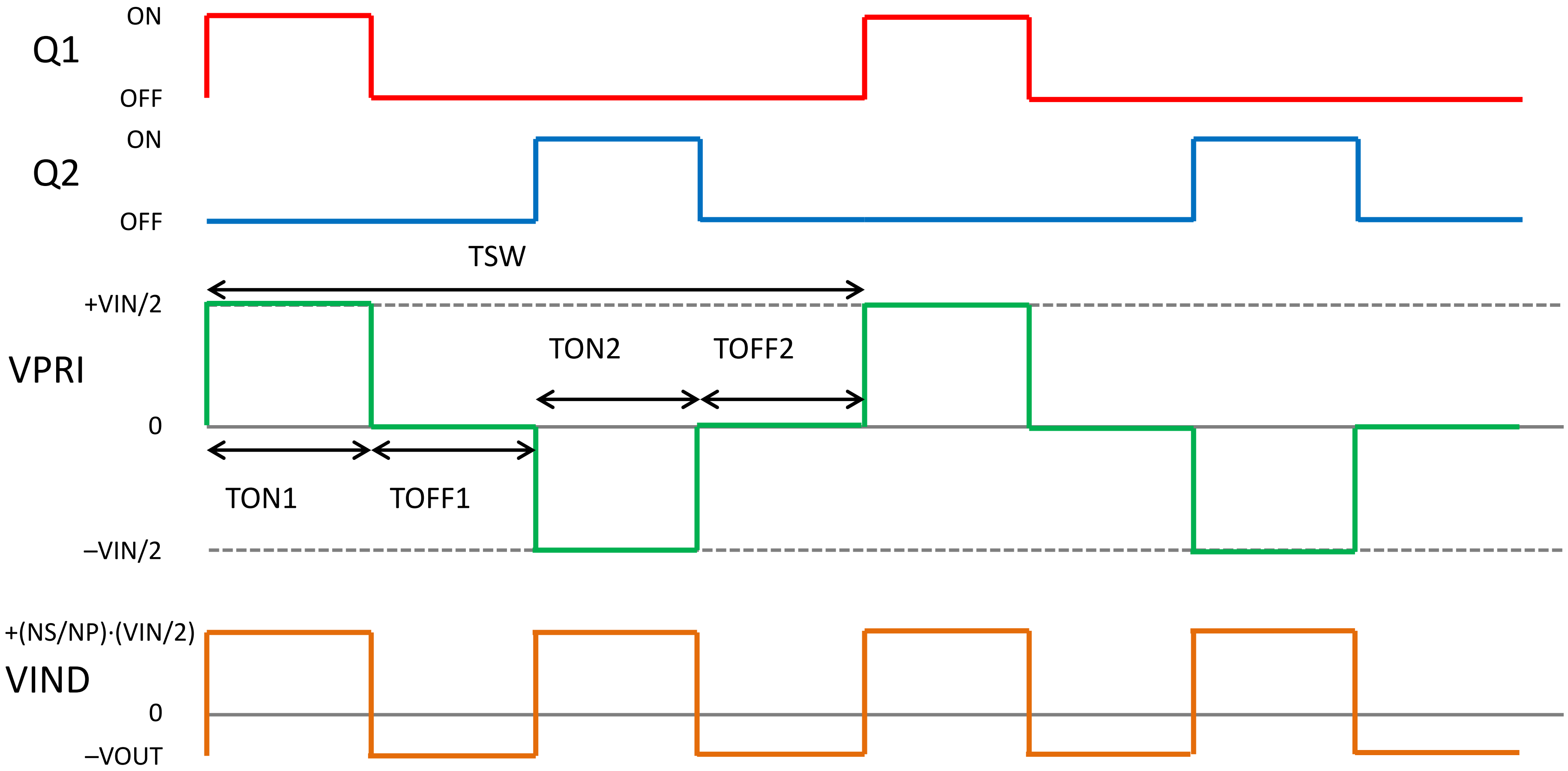


Capacitors Split The Input Voltage

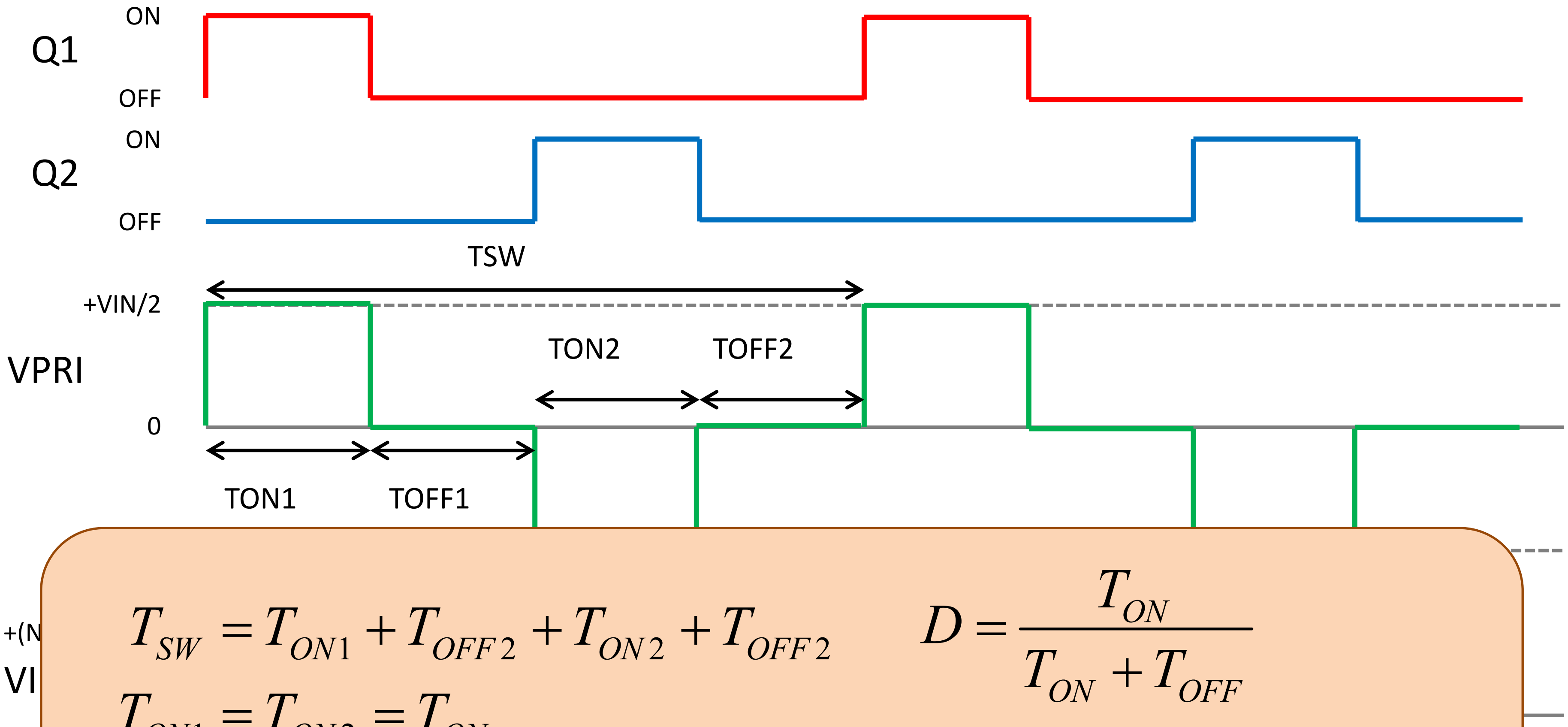
During The On Times The Secondary Voltage Equals

$$V_{SEC} = \pm \frac{N_S}{N_P} \cdot \frac{V_{IN}}{2}$$

Half-Bridge Converter w/FW Rectifier Conversion Ratio



Half-Bridge Converter w/FW Rectifier Conversion Ratio



$$T_{SW} = T_{ON1} + T_{OFF2} + T_{ON2} + T_{OFF2}$$

$$T_{ON1} = T_{ON2} = T_{ON}$$

$$T_{OFF1} = T_{OFF2} = T_{OFF}$$

$$D = \frac{T_{ON}}{T_{ON} + T_{OFF}}$$

$$= \frac{T_{ON1} + T_{ON2}}{T_{SW}} = \frac{2 \cdot T_{ON}}{T_{SW}}$$

Half-Bridge Converter w/FW Rectifier Conversion Ratio

$$V_L(T_{ON1})\cdot T_{ON1}+V_L(T_{OFF1})\cdot T_{OFF1}+V_L(T_{ON2})\cdot T_{ON2}+V_L(T_{OFF2})\cdot T_{OFF2}=0$$

$$V_L(T_{ON})\cdot T_{ON}+V_L(T_{OFF})\cdot T_{OFF}+V_L(T_{ON})\cdot T_{ON}+V_L(T_{OFF})\cdot T_{OFF}=0$$

$$V_L(T_{ON})\cdot 2\cdot T_{ON}+V_L(T_{OFF})\cdot 2\cdot T_{OFF}=0$$

$$V_L(T_{ON})\cdot\frac{2\cdot T_{ON}}{T_{SW}}+V_L(T_{OFF})\cdot\frac{T_{SW}-2\cdot T_{ON}}{T_{SW}}=0$$

$$V_L(T_{ON})\cdot D+V_L(T_{OFF})\cdot(1-D)=0$$

$$\left(\frac{N_S}{N_P}\cdot\frac{V_{IN}}{2}-V_{OUT}\right)\cdot D+(-V_{OUT})\cdot(1-D)=0$$

$$\frac{N_S}{N_P}\cdot\frac{V_{IN}}{2}\cdot D-V_{OUT}\cdot D-V_{OUT}+V_{OUT}\cdot D=0$$

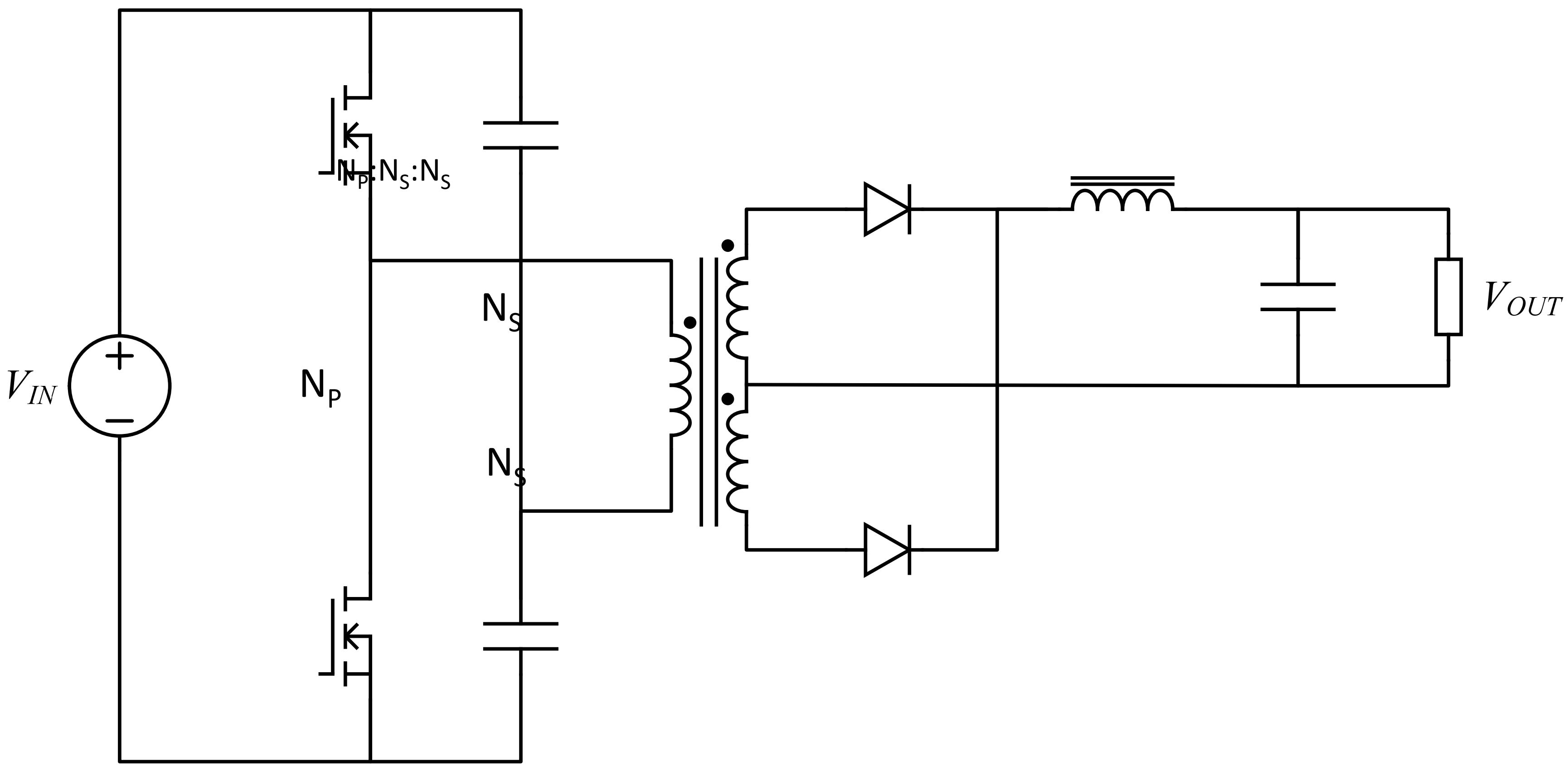
$$\frac{N_S}{N_P}\cdot\frac{V_{IN}}{2}\cdot D=V_{OUT}$$

$$V_{OUT}=D\cdot\frac{N_S}{N_P}\cdot\frac{V_{IN}}{2}$$

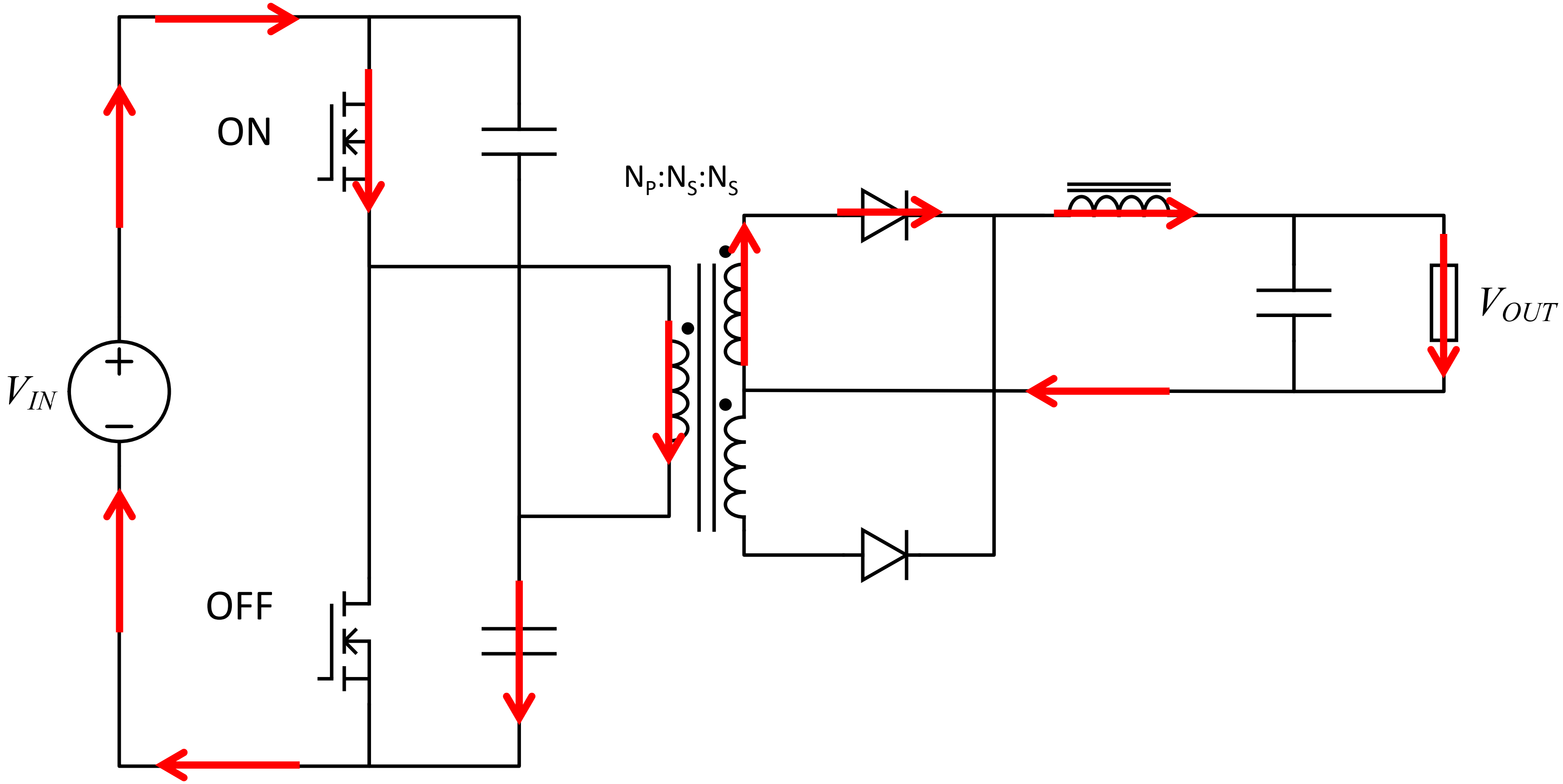
Inductor Volt-Second Balance

Same As Buck Converter Except:
- Transformer Turns Ratio
- Capacitor Split Input Voltage

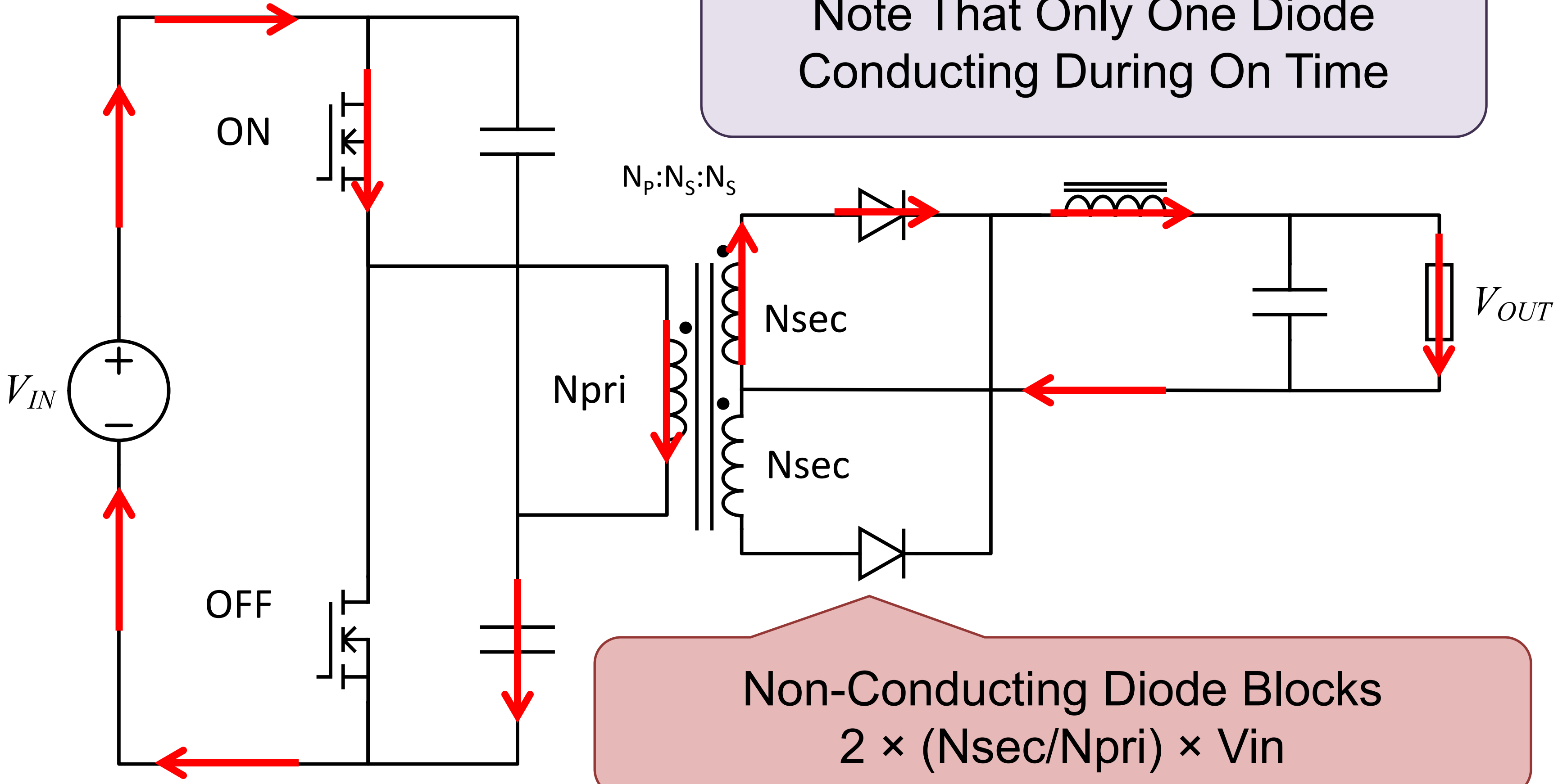
Half-Bridge Converter w/Center Tapped Rectifier



Half-Bridge Converter w/CT Rectifier
On Time 1



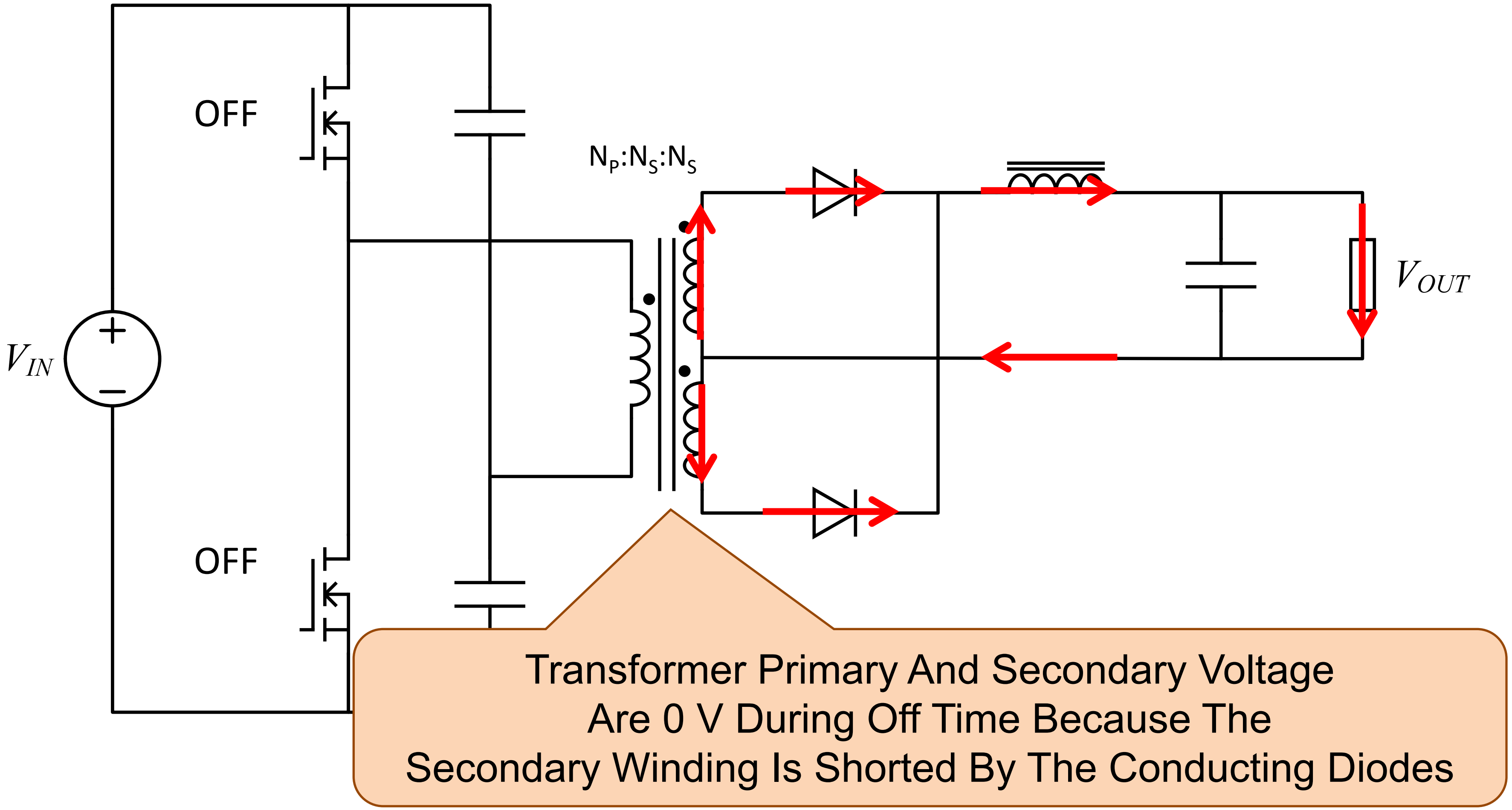
Half-Bridge Converter w/CT Rectifier
On Time 1



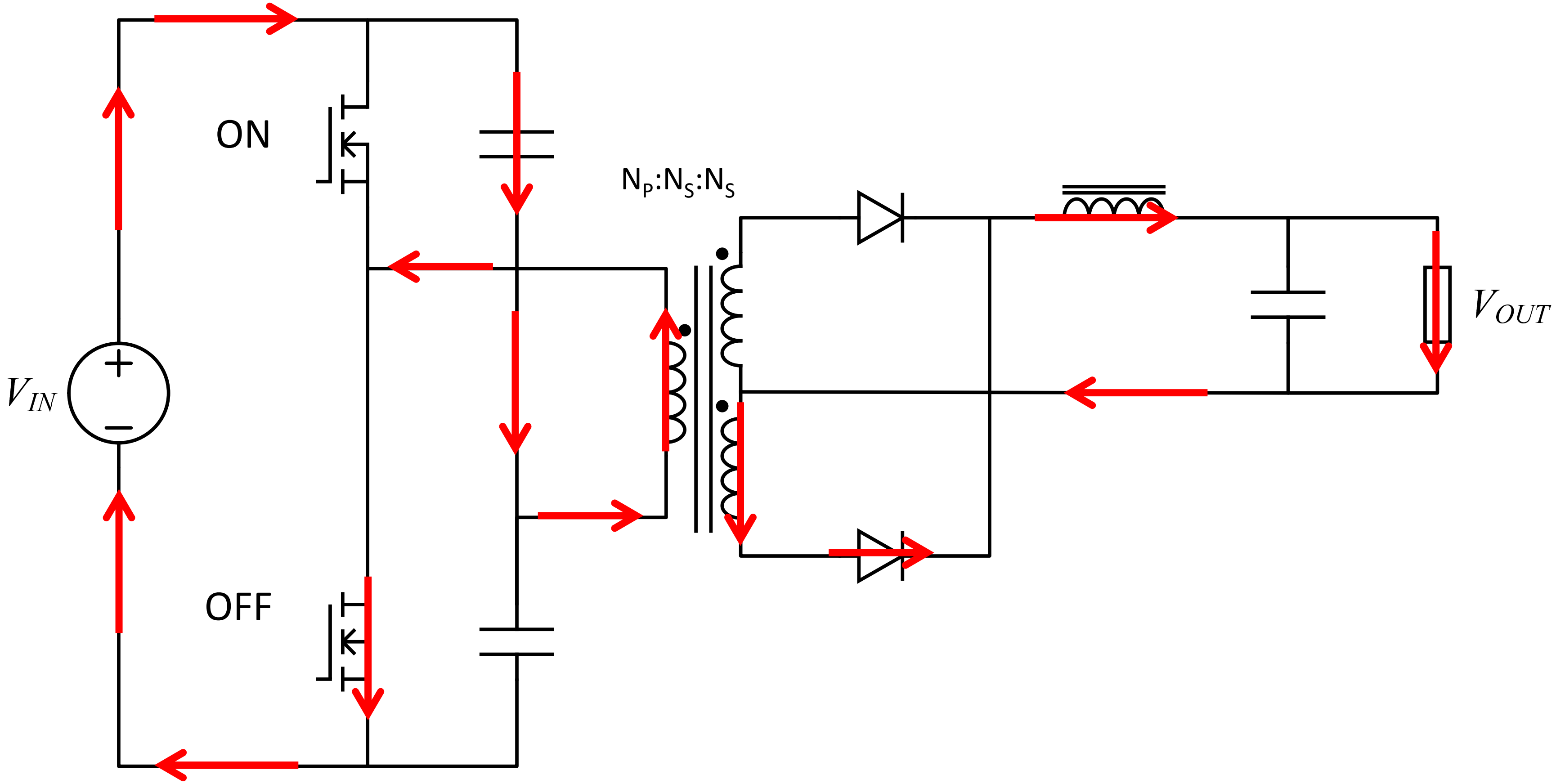
Note That Only One Diode Conducting During On Time

Non-Conducting Diode Blocks
2 × (Nsec/Npri) × Vin

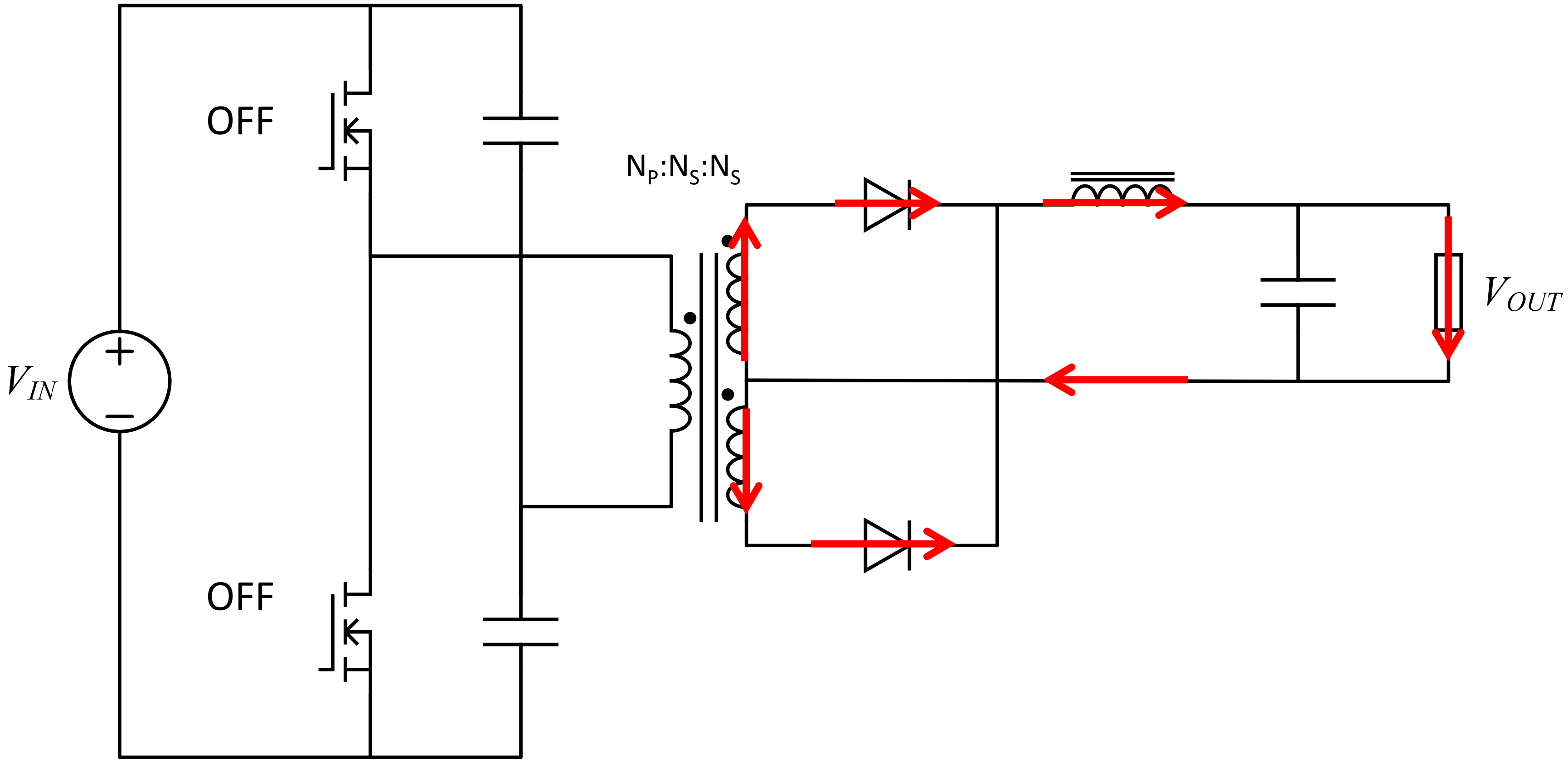
Half-Bridge Converter w/CT Rectifier
Off Time 1



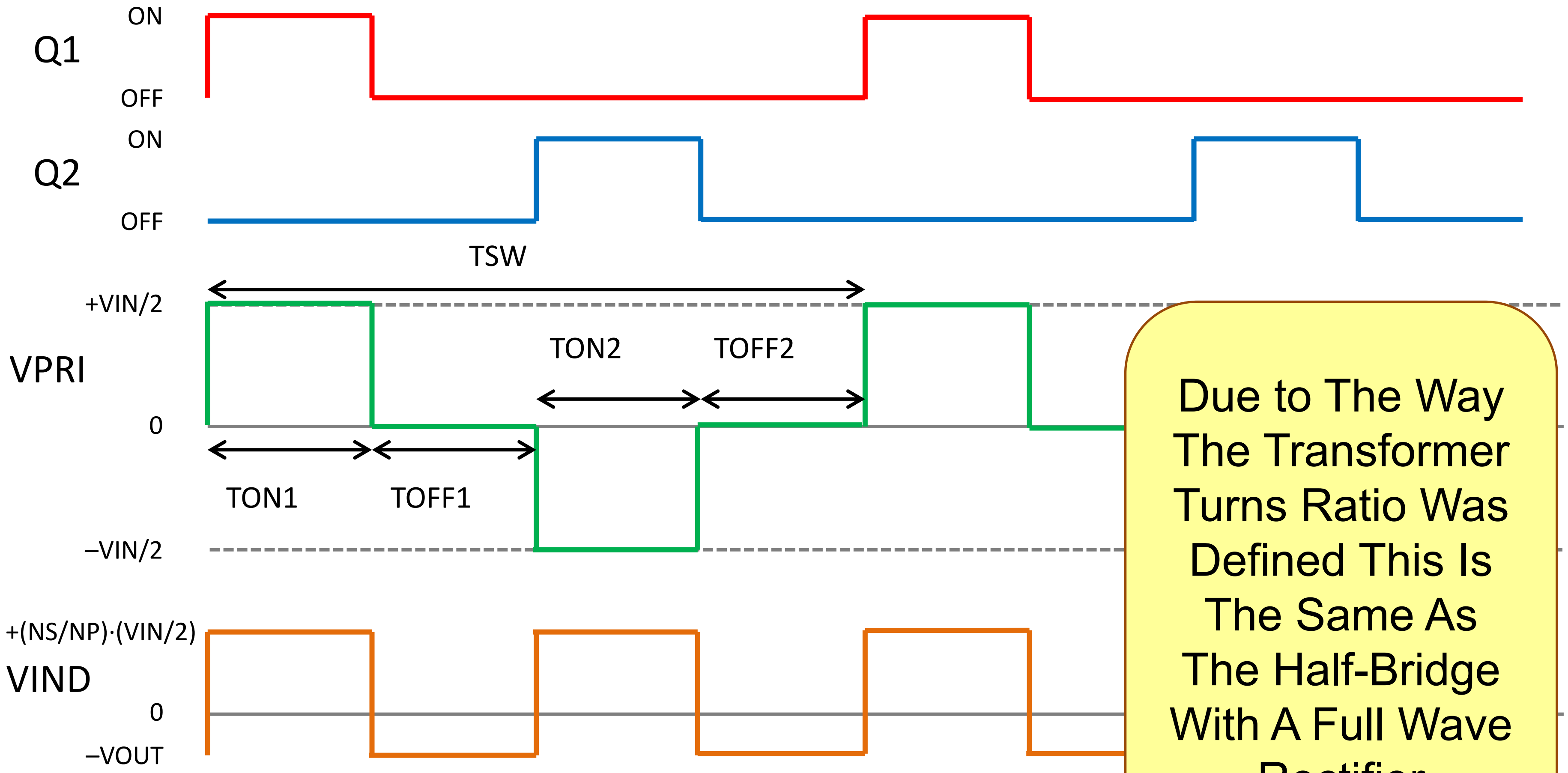
Half-Bridge Converter w/CT Rectifier
On Time 2



Half-Bridge Converter w/CT Rectifier
Off Time 2

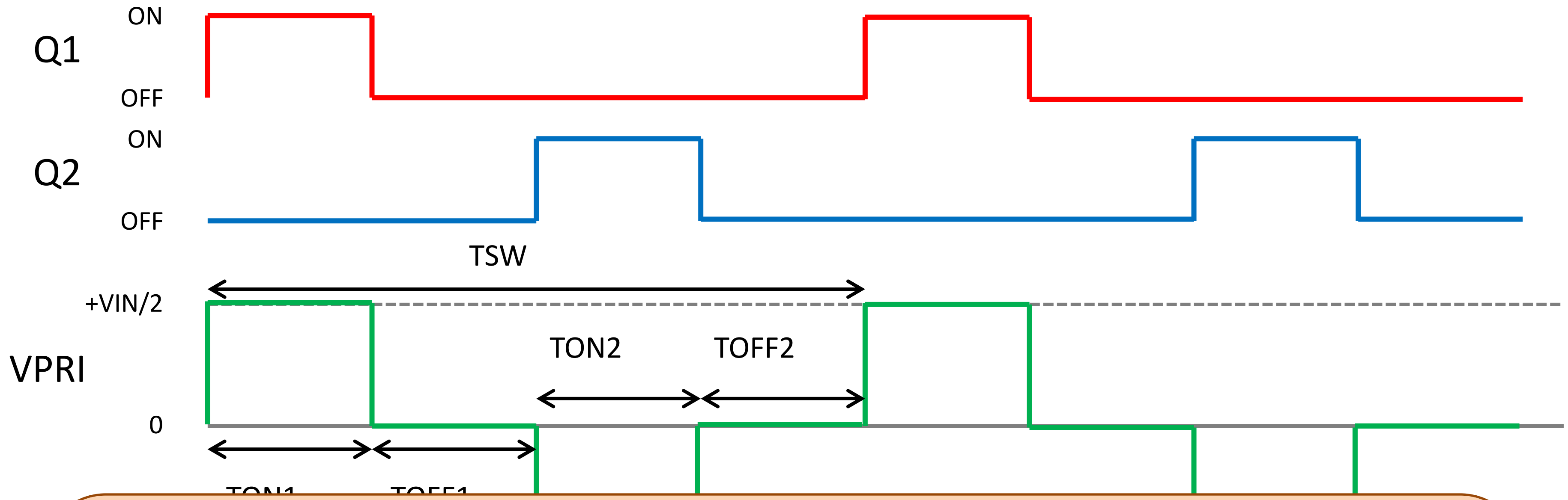


Half-Bridge Converter w/CT Rectifier Conversion Ratio



Due to The Way The Transformer Turns Ratio Was Defined This Is The Same As The Half-Bridge With A Full Wave Rectifier

Half-Bridge Converter w/CT Rectifier
Conversion Ratio



$$T_{SW} = T_{ON1} + T_{OFF2} + T_{ON2} + T_{OFF2}$$

$$T_{ON1} = T_{ON2} = T_{ON}$$

$$T_{OFF1} = T_{OFF2} = T_{OFF}$$

$$D = \frac{T_{ON}}{T_{ON} + T_{OFF}}$$

$$= \frac{T_{ON1} + T_{ON2}}{T_{SW}} = \frac{2 \cdot T_{ON}}{T_{SW}}$$

Half-Bridge Converter w/CT Rectifier Conversion Ratio

$$V_L(T_{ON1})\cdot T_{ON1}+V_L(T_{OFF1})\cdot T_{OFF1}+V_L(T_{ON2})\cdot T_{ON2}+V_L(T_{OFF2})\cdot T_{OFF2}=0$$

$$V_L(T_{ON})\cdot T_{ON}+V_L(T_{OFF})\cdot T_{OFF}+V_L(T_{ON})\cdot T_{ON}+V_L(T_{OFF})\cdot T_{OFF}=0$$

$$V_L(T_{ON})\cdot 2\cdot T_{ON}+V_L(T_{OFF})\cdot 2\cdot T_{OFF}=0$$

$$V_L(T_{ON})\cdot\frac{2\cdot T_{ON}}{T_{SW}}+V_L(T_{OFF})\cdot\frac{T_{SW}-2\cdot T_{ON}}{T_{SW}}=0$$

$$V_L(T_{ON})\cdot D+V_L(T_{OFF})\cdot(1-D)=0$$

$$\left(\frac{N_S}{N_P}\cdot\frac{V_{IN}}{2}-V_{OUT}\right)\cdot D+(-V_{OUT})\cdot(1-D)=0$$

$$\frac{N_S}{N_P}\cdot\frac{V_{IN}}{2}\cdot D-V_{OUT}\cdot D-V_{OUT}+V_{OUT}\cdot D=0$$

$$\frac{N_S}{N_P}\cdot\frac{V_{IN}}{2}\cdot D=V_{OUT}$$

$$V_{OUT}=D\cdot\frac{N_S}{N_P}\cdot\frac{V_{IN}}{2}$$

Inductor Volt-Second Balance

Due to The Way The Transformer Turns Ratio Was Defined This Is The Same As The Half-Bridge With A Full Wave Rectifier

Same As Buck Converter Except:
- Transformer Turns Ratio
- Capacitor Split Input Voltage

Half Bridge Converter

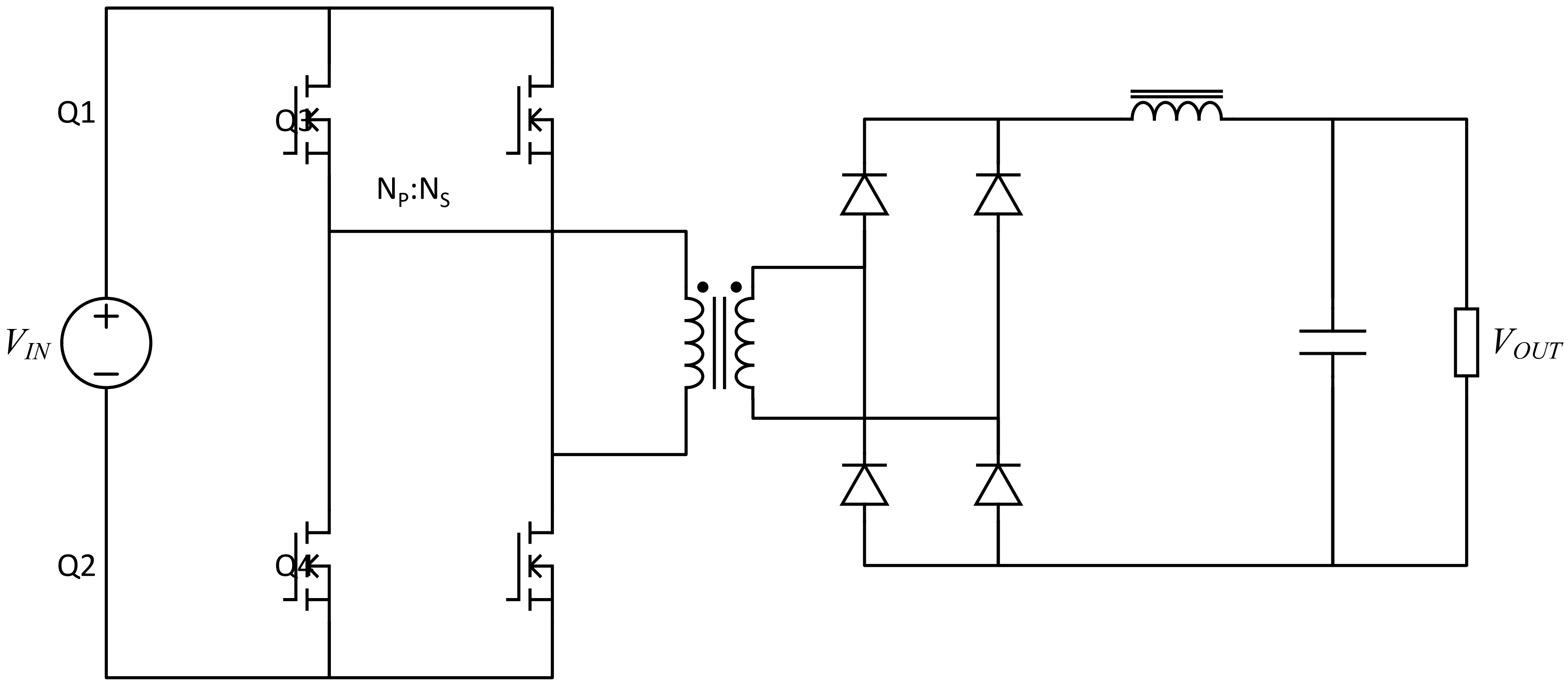
Advantages

- Isolated
- Good For
  250 W To 1000 W
- Multiple Isolated Outputs Possible
- Easy To Control
- Good Use Of Transformer Core
- Low Voltage Stress On Switching Transistors

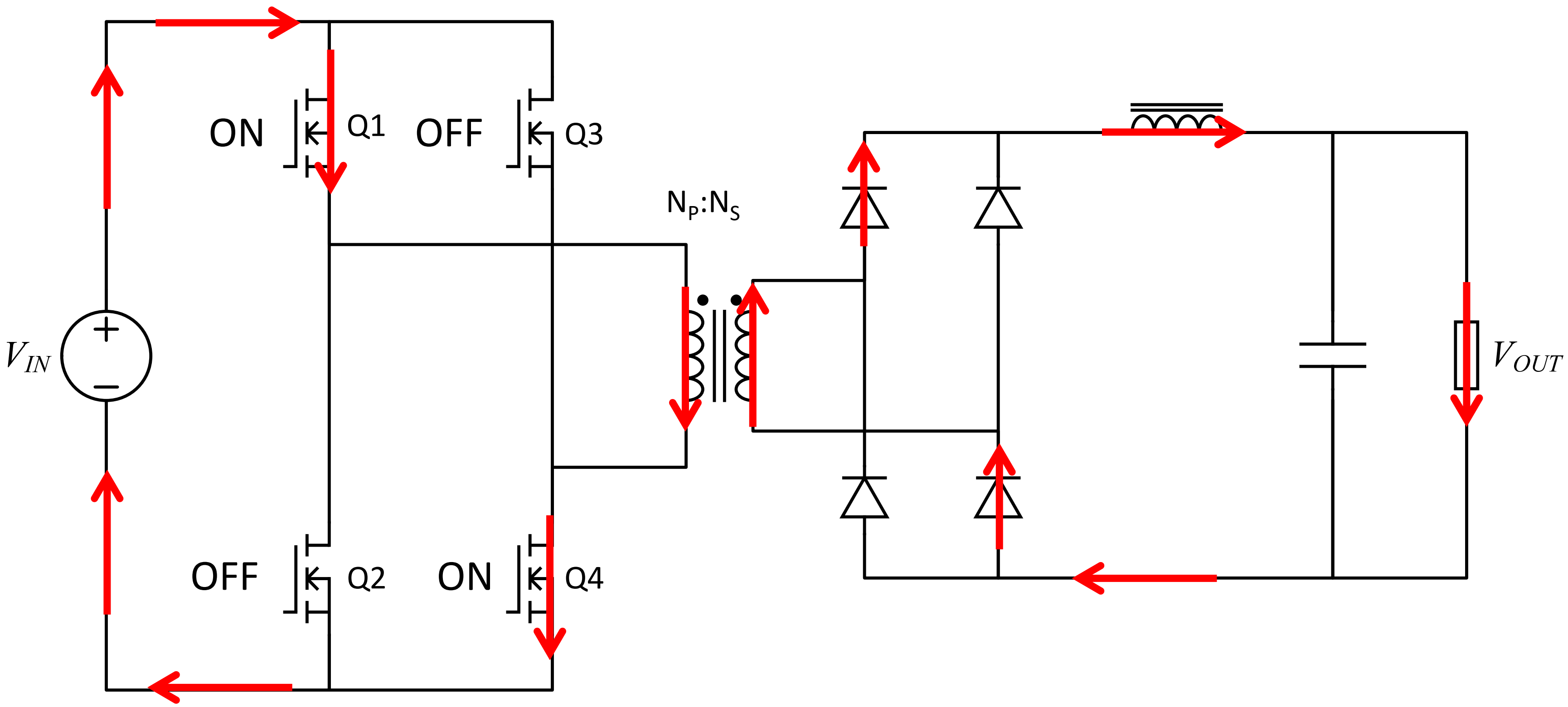
Disadvantages

- Transformer Needed
- Must Have Way TO Prevent Transformer Saturation
- Center Tap Output: High Voltage Stress On Output Diodes
- Full Wave Output:
  More Diodes And Diode Loss

Full-Bridge Converter w/Full Wave Rectifier



Full-Bridge Converter w/FW Rectifier
On Time 1



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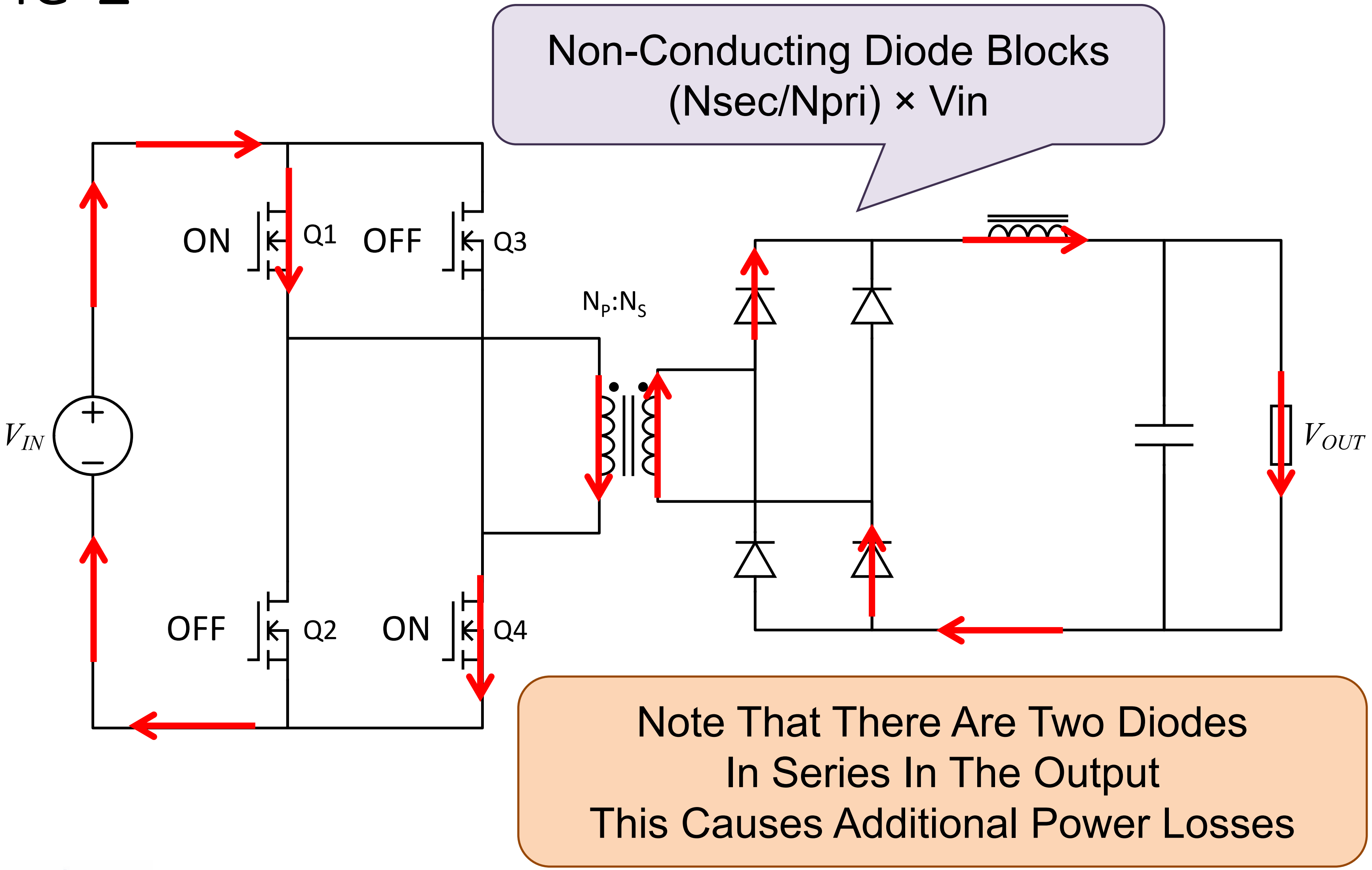
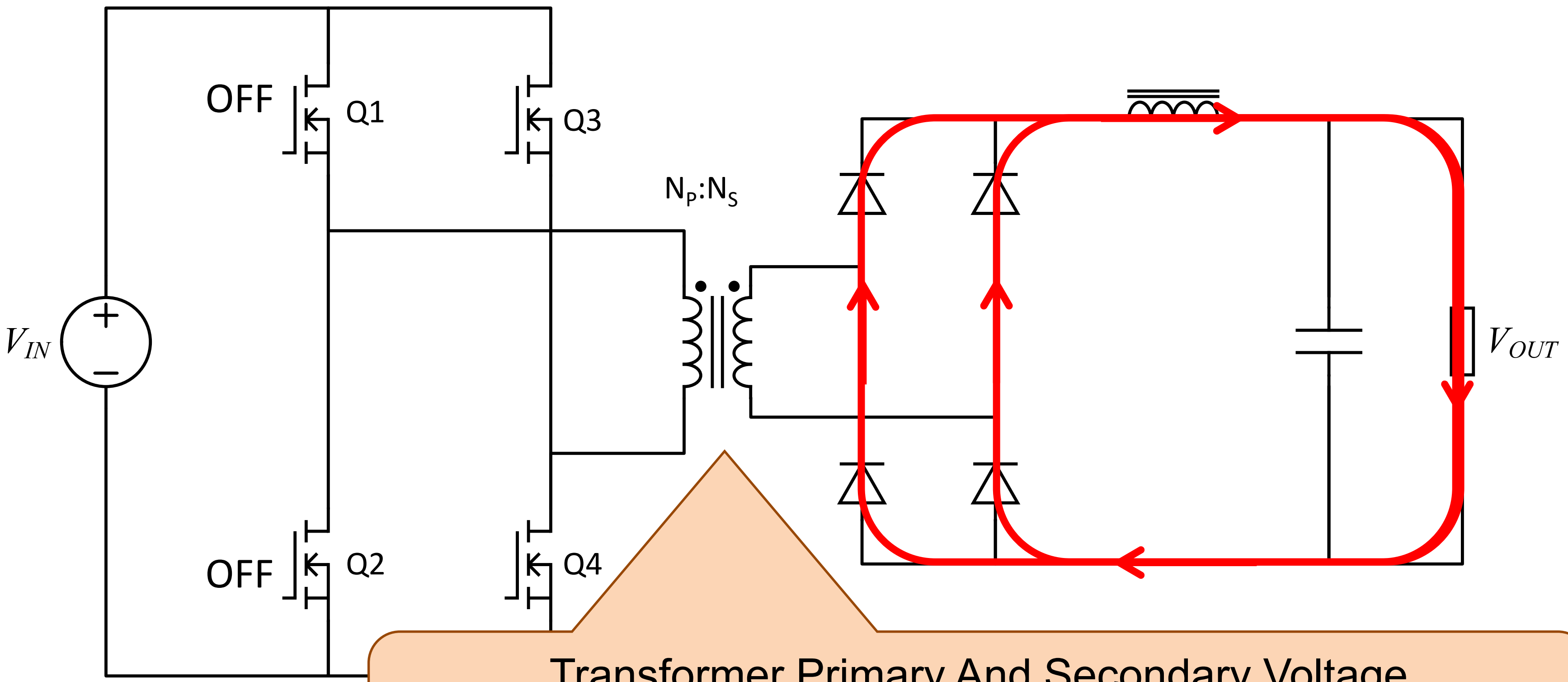
Full-Bridge Converter w/FW Rectifier
On Time 1

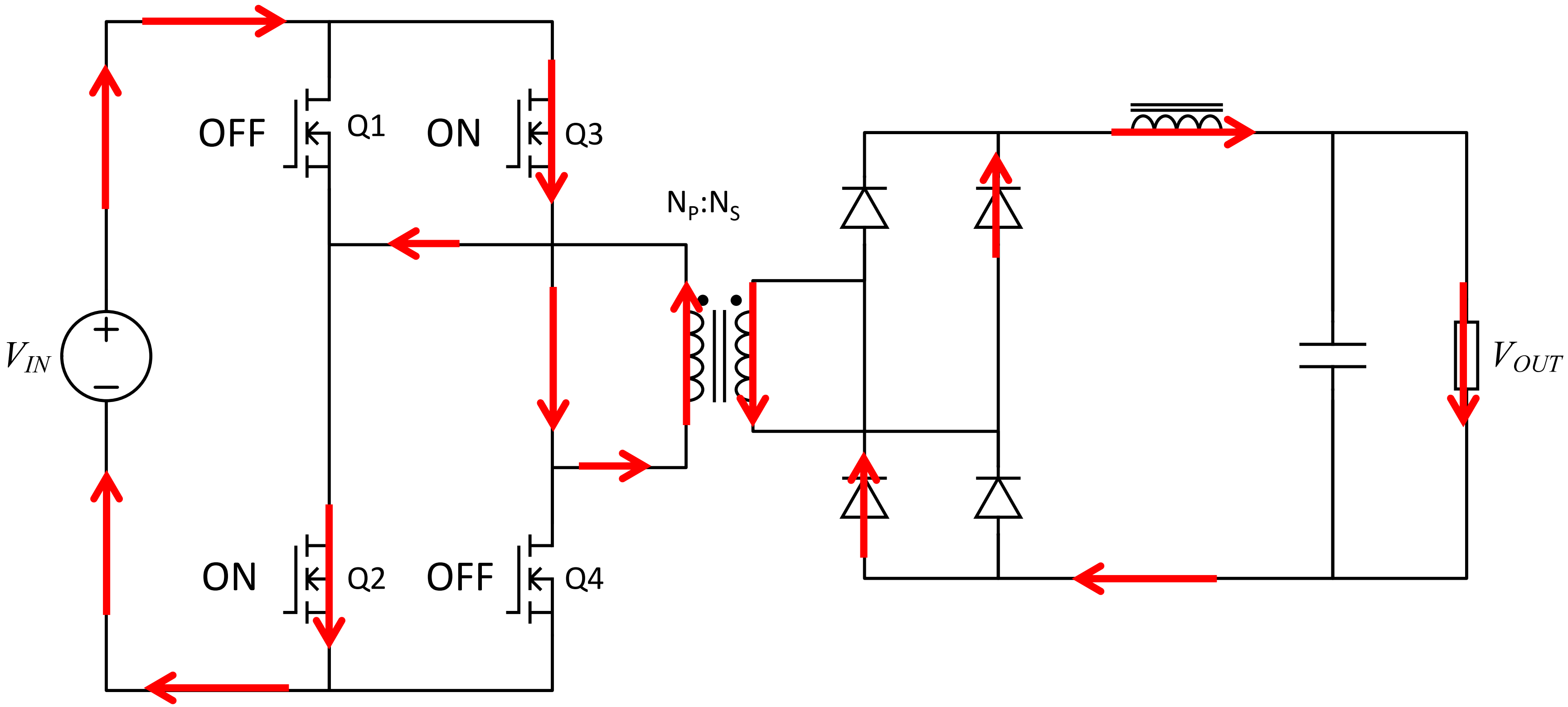


Full-Bridge Converter w/FW Rectifier
Off Time 1



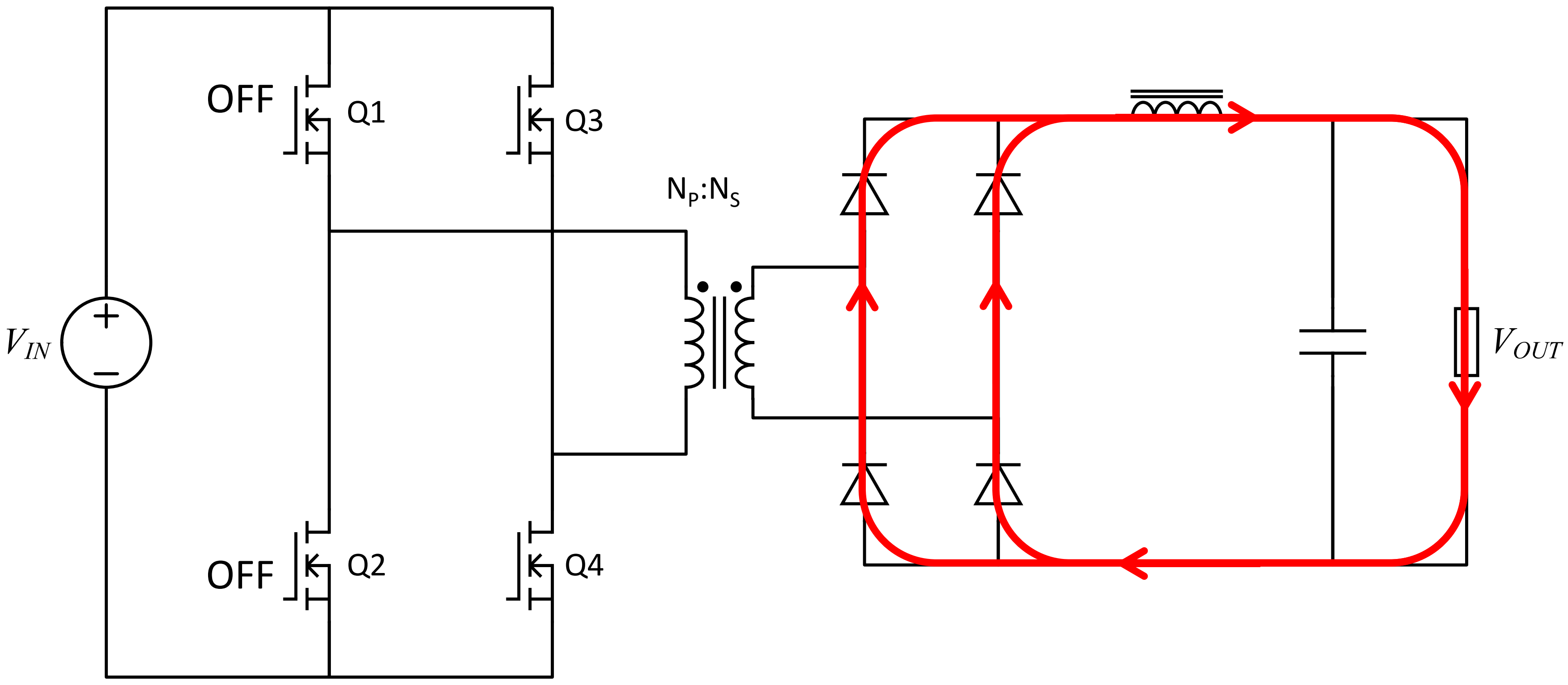
Transformer Primary And Secondary Voltage
Are 0 V During Off Time Because The
Secondary Winding Is Shorted By The Conducting Diodes

Full-Bridge Converter w/FW Rectifier
On Time 2

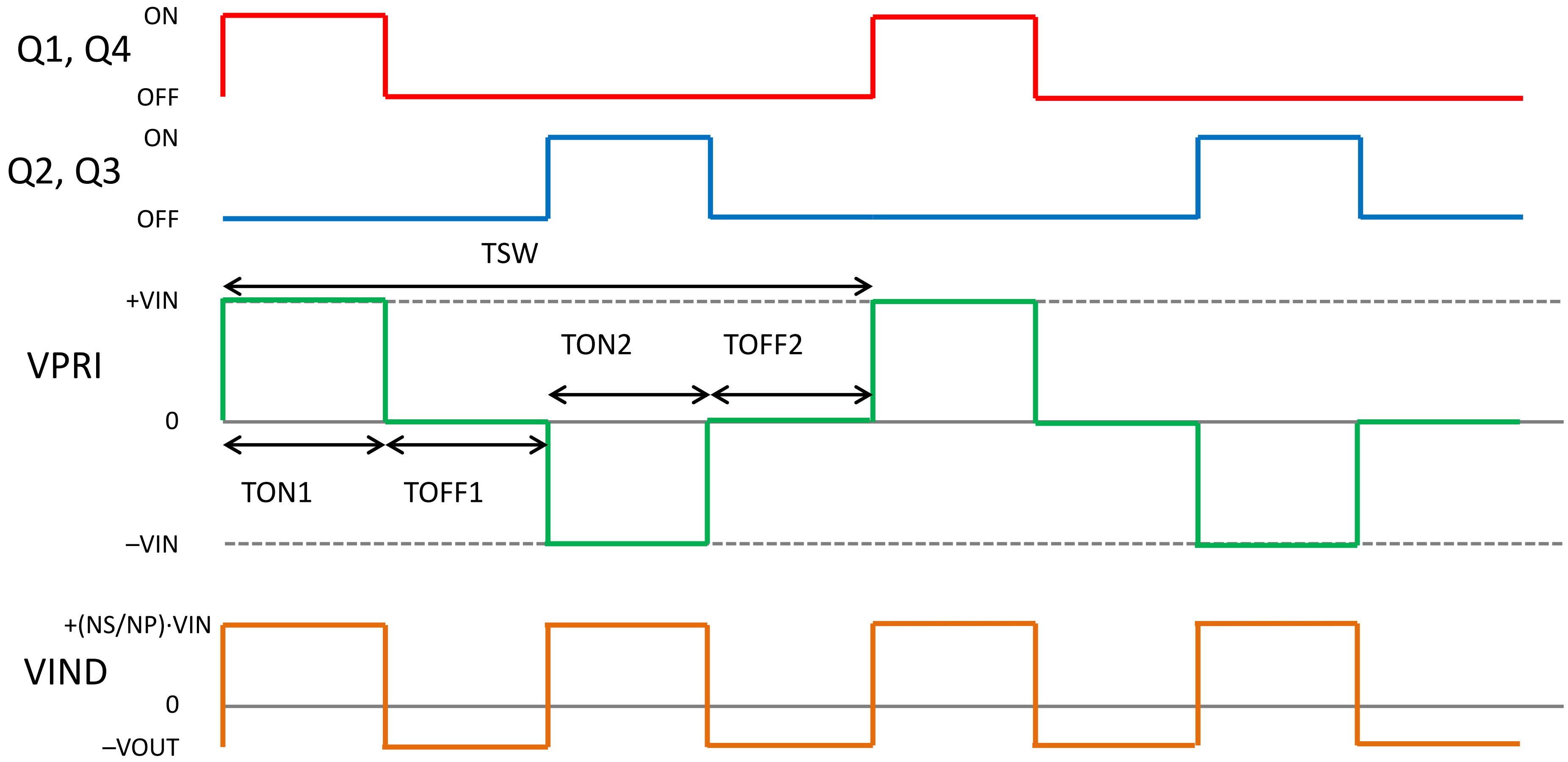


EMBEDDED POWER LABS

Full-Bridge Converter w/FW Rectifier
Off Time 2

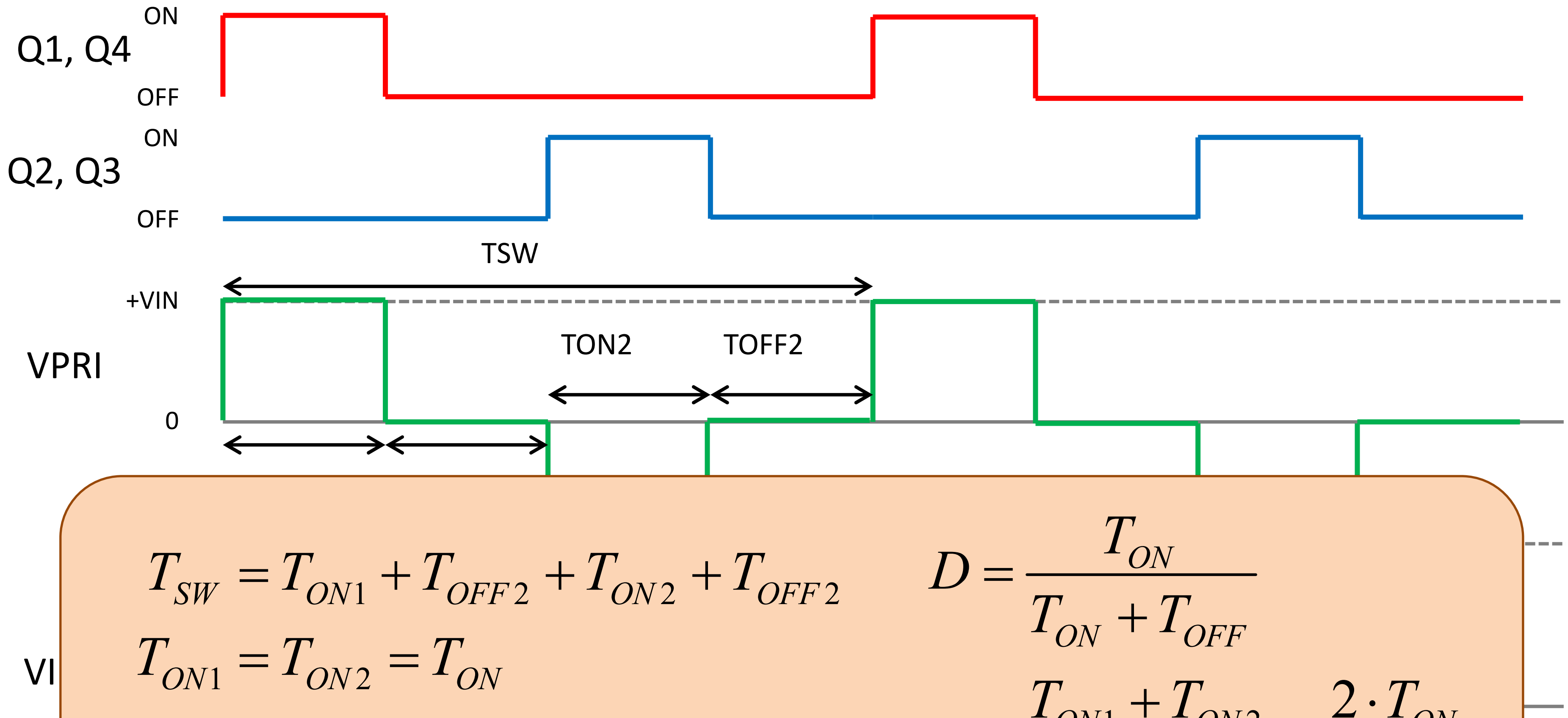


Full-Bridge Converter w/FW Rectifier Conversion Ratio



EMBEDDED POWER LABS

Full-Bridge Converter w/FW Rectifier Conversion Ratio



$$T_{SW} = T_{ON1} + T_{OFF2} + T_{ON2} + T_{OFF2}$$

$$T_{ON1} = T_{ON2} = T_{ON}$$

$$T_{OFF1} = T_{OFF2} = T_{OFF}$$

$$D = \frac{T_{ON}}{T_{ON} + T_{OFF}}$$

$$= \frac{T_{ON1} + T_{ON2}}{T_{SW}} = \frac{2 \cdot T_{ON}}{T_{SW}}$$

Full-Bridge Converter w/FW Rectifier Conversion Ratio

$$V_L(T_{ON1})\cdot T_{ON1} + V_L(T_{OFF1})\cdot T_{OFF1} + V_L(T_{ON2})\cdot T_{ON2} + V_L(T_{OFF2})\cdot T_{OFF2} = 0$$

$$V_L(T_{ON})\cdot T_{ON} + V_L(T_{OFF})\cdot T_{OFF} + V_L(T_{ON})\cdot T_{ON} + V_L(T_{OFF})\cdot T_{OFF} = 0$$

$$V_L(T_{ON})\cdot 2\cdot T_{ON} + V_L(T_{OFF})\cdot 2\cdot T_{OFF} = 0$$

$$V_L(T_{ON})\cdot \frac{2\cdot T_{ON}}{T_{SW}} + V_L(T_{OFF})\cdot \frac{T_{SW} - 2\cdot T_{ON}}{T_{SW}} = 0$$

$$V_L(T_{ON})\cdot D + V_L(T_{OFF})\cdot (1-D) = 0$$

$$\left(\frac{N_S}{N_P}\cdot V_{IN} - V_{OUT}\right)\cdot D + (-V_{OUT})\cdot(1-D) = 0$$

$$\frac{N_S}{N_P}\cdot V_{IN}\cdot D - V_{OUT}\cdot D - V_{OUT} + V_{OUT}\cdot D = 0$$

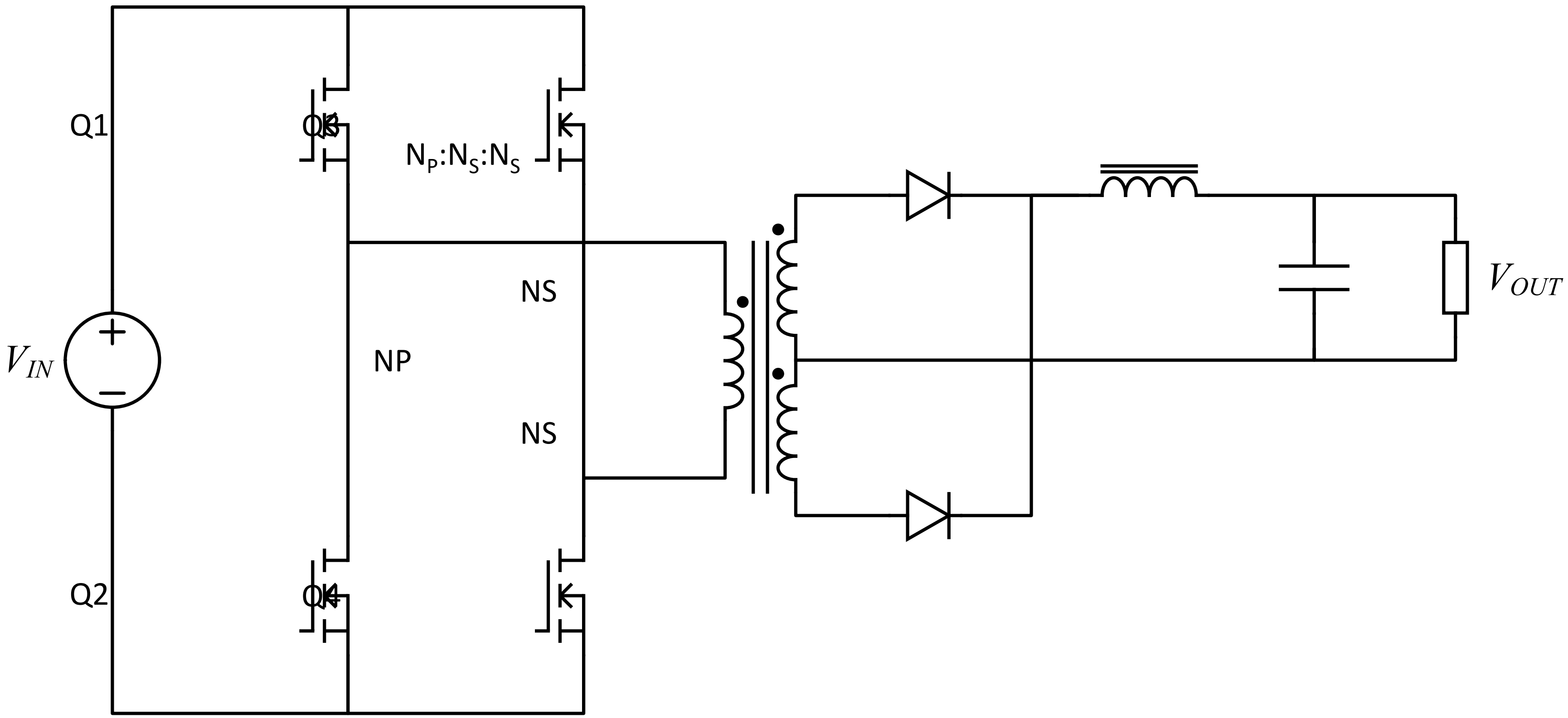
$$\frac{N_S}{N_P}\cdot V_{IN}\cdot D = V_{OUT}$$

$$V_{OUT} = D\cdot\frac{N_S}{N_P}\cdot V_{IN}$$

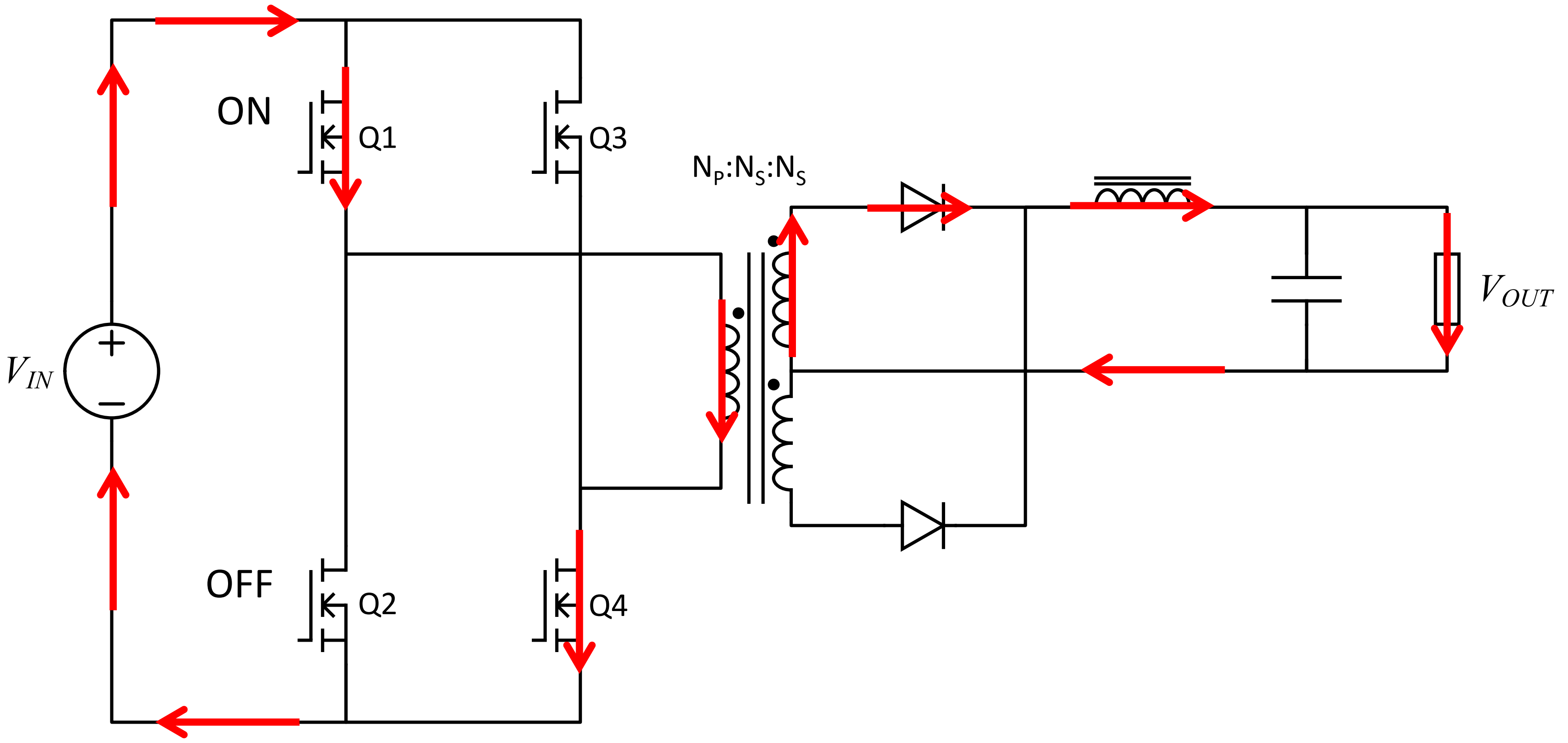
Inductor Volt-Second Balance

Same As Buck Converter Except:
- Transformer Turns Ratio

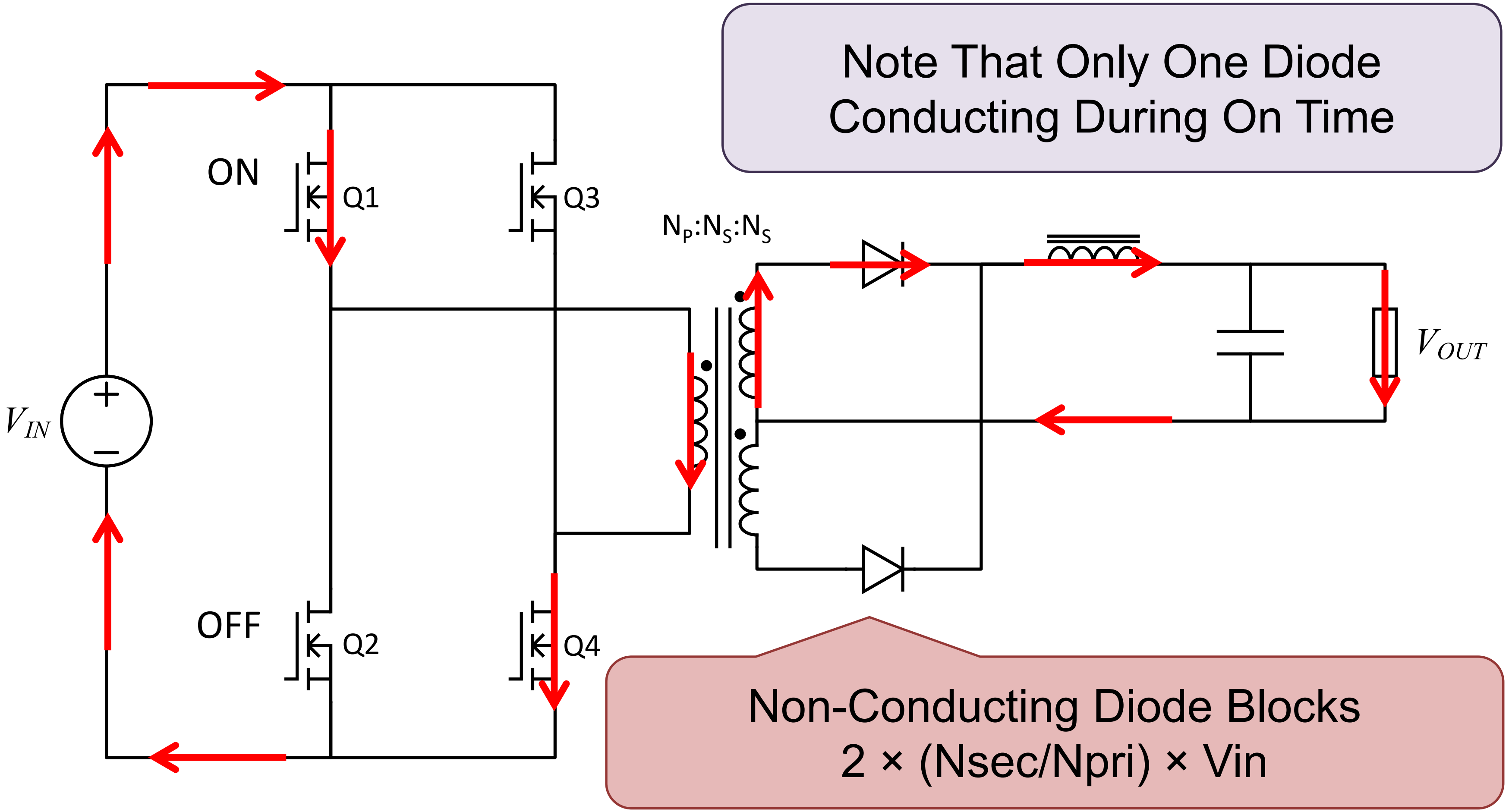
Full-Bridge Converter w/Center Tapped Rectifier



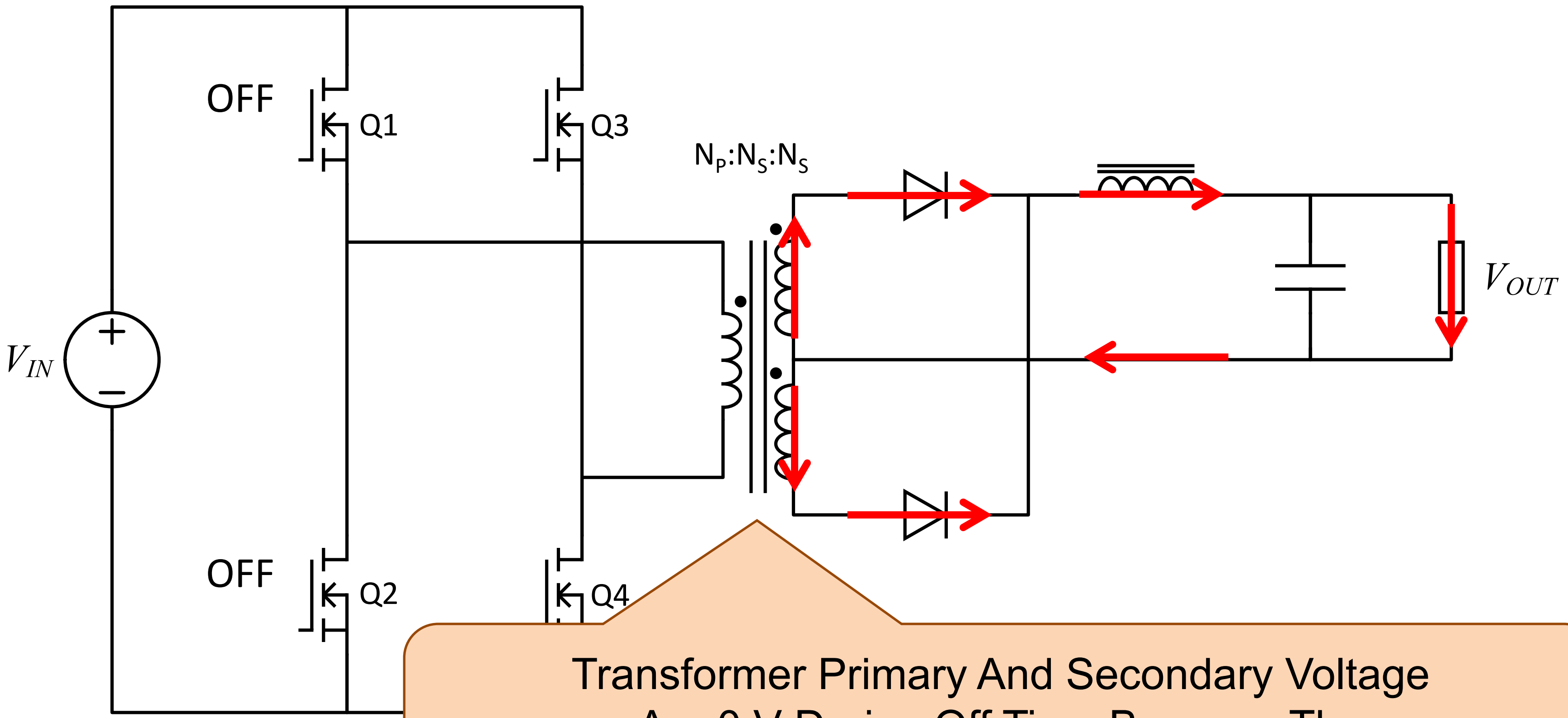
Full-Bridge Converter w/CT Rectifier
On Time 1



Full-Bridge Converter w/CT Rectifier
On Time 1

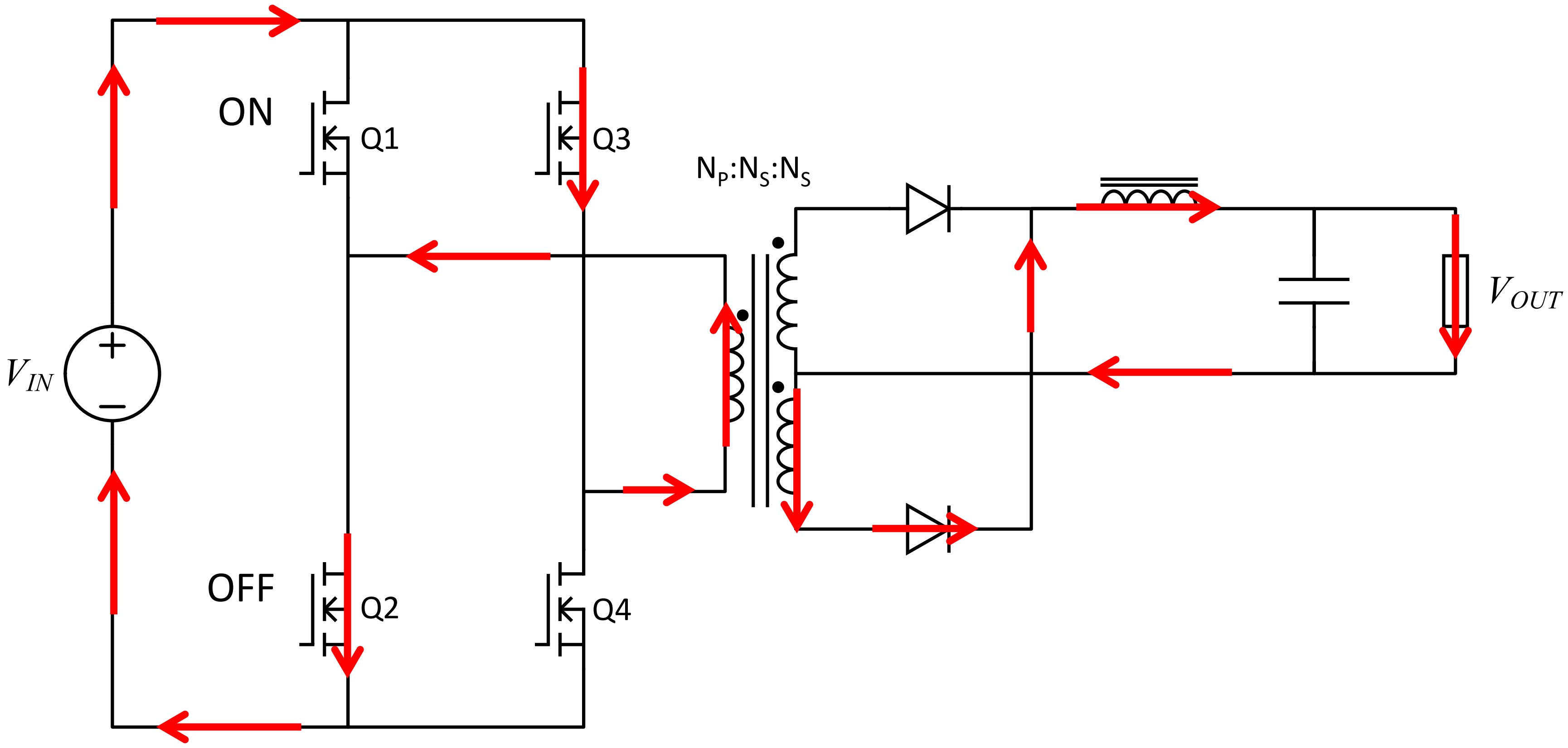


Full-Bridge Converter w/CT Rectifier
Off Time 1

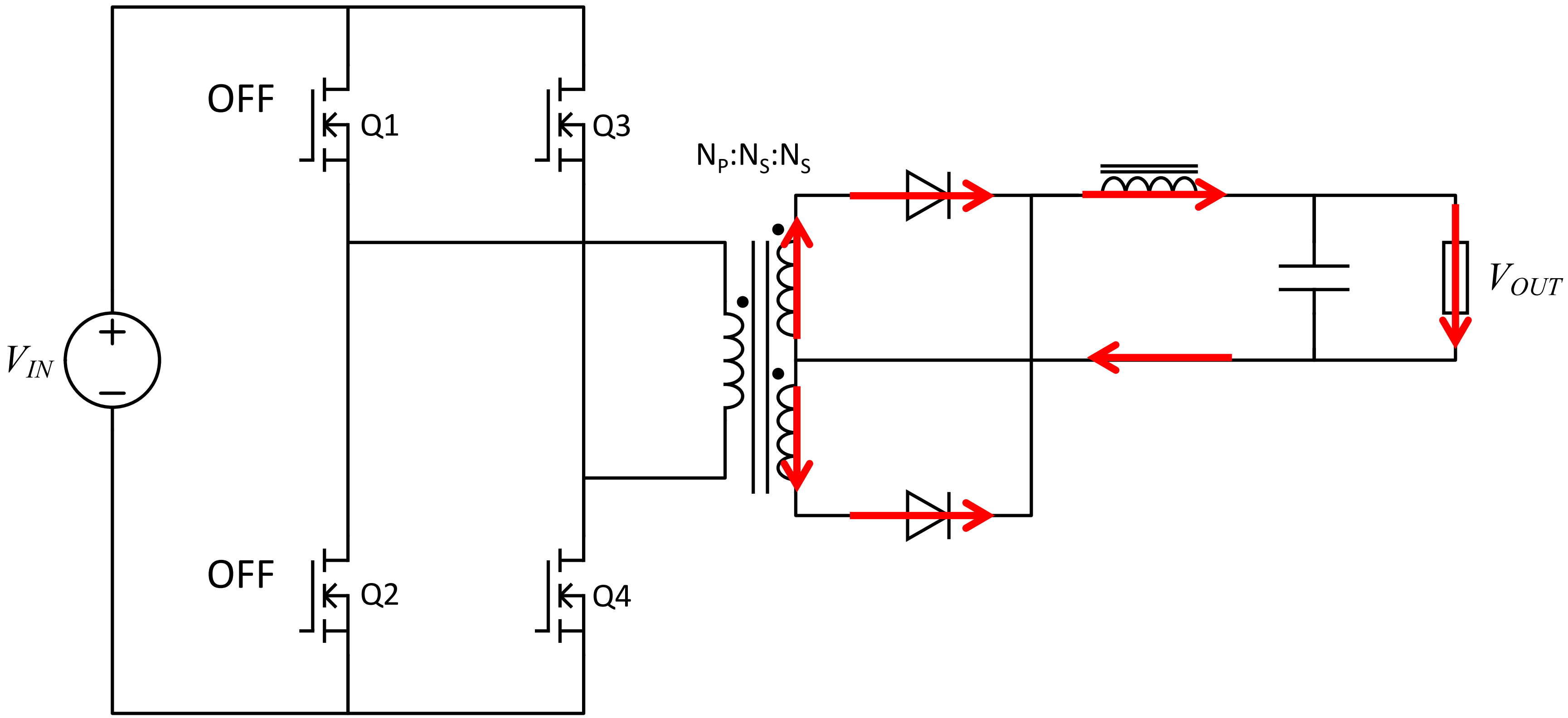


Transformer Primary And Secondary Voltage
Are 0 V During Off Time Because The
Secondary Winding Is Shorted By The Conducting Diodes

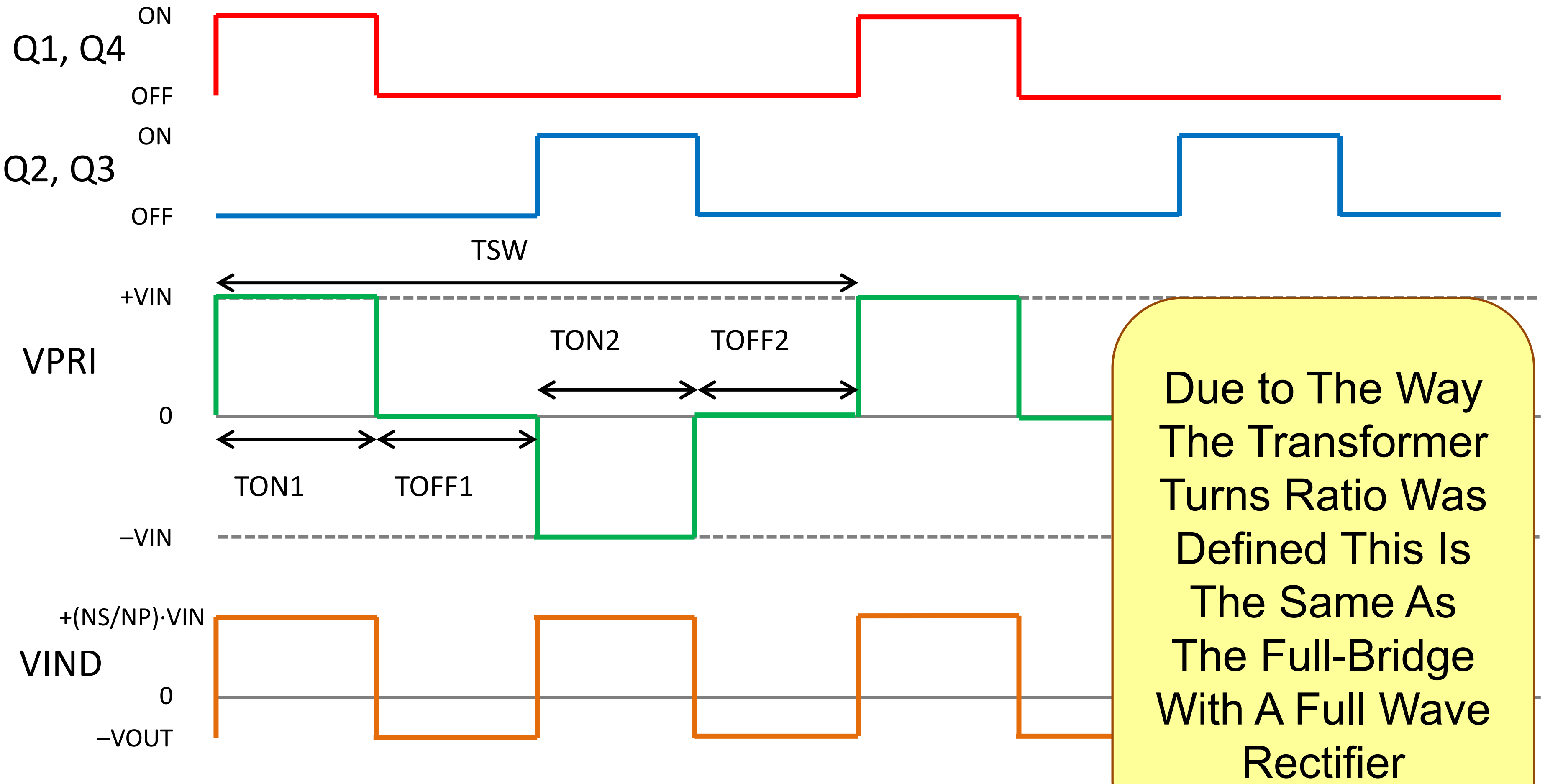
Full-Bridge Converter w/CT Rectifier
On Time 2



Full-Bridge Converter w/CT Rectifier
Off Time 2



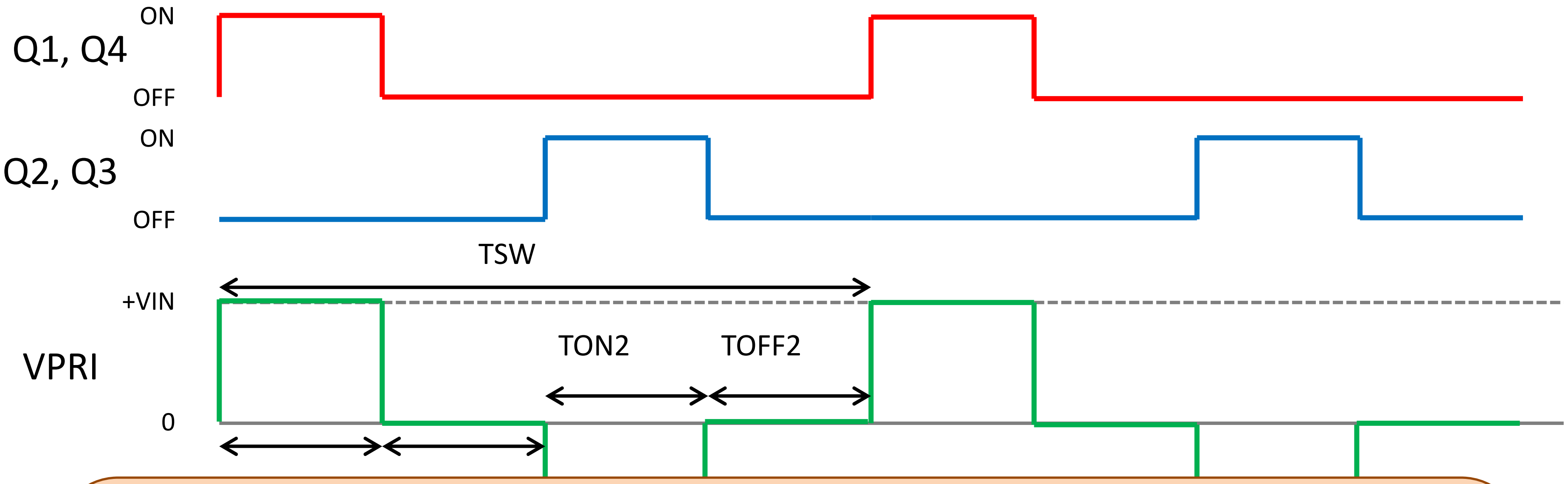
Full-Bridge Converter w/CT Rectifier Conversion Ratio



Due to The Way The Transformer Turns Ratio Was Defined This Is The Same As The Full-Bridge With A Full Wave Rectifier

EMBEDDED POWER LABS

Full-Bridge Converter w/CT Rectifier Conversion Ratio



$$T_{SW} = T_{ON1} + T_{OFF2} + T_{ON2} + T_{OFF2}$$

$$T_{ON1} = T_{ON2} = T_{ON}$$

$$T_{OFF1} = T_{OFF2} = T_{OFF}$$

$$D = \frac{T_{ON}}{T_{ON} + T_{OFF}}$$

$$= \frac{T_{ON1} + T_{ON2}}{T_{SW}} = \frac{2 \cdot T_{ON}}{T_{SW}}$$

Full-Bridge Converter w/CT Rectifier Conversion Ratio

$$V_L(T_{ON1}) \cdot T_{ON1} + V_L(T_{OFF1}) \cdot T_{OFF1} + V_L(T_{ON2}) \cdot T_{ON2} + V_L(T_{OFF2}) \cdot T_{OFF2} = 0$$

$$V_L(T_{ON}) \cdot T_{ON} + V_L(T_{OFF}) \cdot T_{OFF} + V_L(T_{ON}) \cdot T_{ON} + V_L(T_{OFF}) \cdot T_{OFF} = 0$$

$$V_L(T_{ON}) \cdot 2 \cdot T_{ON} + V_L(T_{OFF}) \cdot 2 \cdot T_{OFF} = 0$$

$$V_L(T_{ON}) \cdot \frac{2 \cdot T_{ON}}{T_{SW}} + V_L(T_{OFF}) \cdot \frac{T_{SW} - 2 \cdot T_{ON}}{T_{SW}} = 0$$

$$V_L(T_{ON}) \cdot D + V_L(T_{OFF}) \cdot (1 - D) = 0$$

$$\left(\frac{N_S}{N_P} \cdot V_{IN} - V_{OUT} \right) \cdot D + (-V_{OUT}) \cdot (1 - D) = 0$$

$$\frac{N_S}{N_P} \cdot V_{IN} \cdot D - V_{OUT} \cdot D - V_{OUT} + V_{OUT} \cdot D = 0$$

$$\frac{N_S}{N_P} \cdot V_{IN} \cdot D = V_{OUT}$$

$$V_{OUT} = D \cdot \frac{N_S}{N_P} \cdot V_{IN}$$

Inductor Volt-Second Balance

Due to The Way The Transformer Turns Ratio Was Defined This Is The Same As The Full-Bridge With A Full Wave Rectifier

Same As Buck Converter Except:
- Transformer Turns Ratio

Full Bridge Converter

Advantages

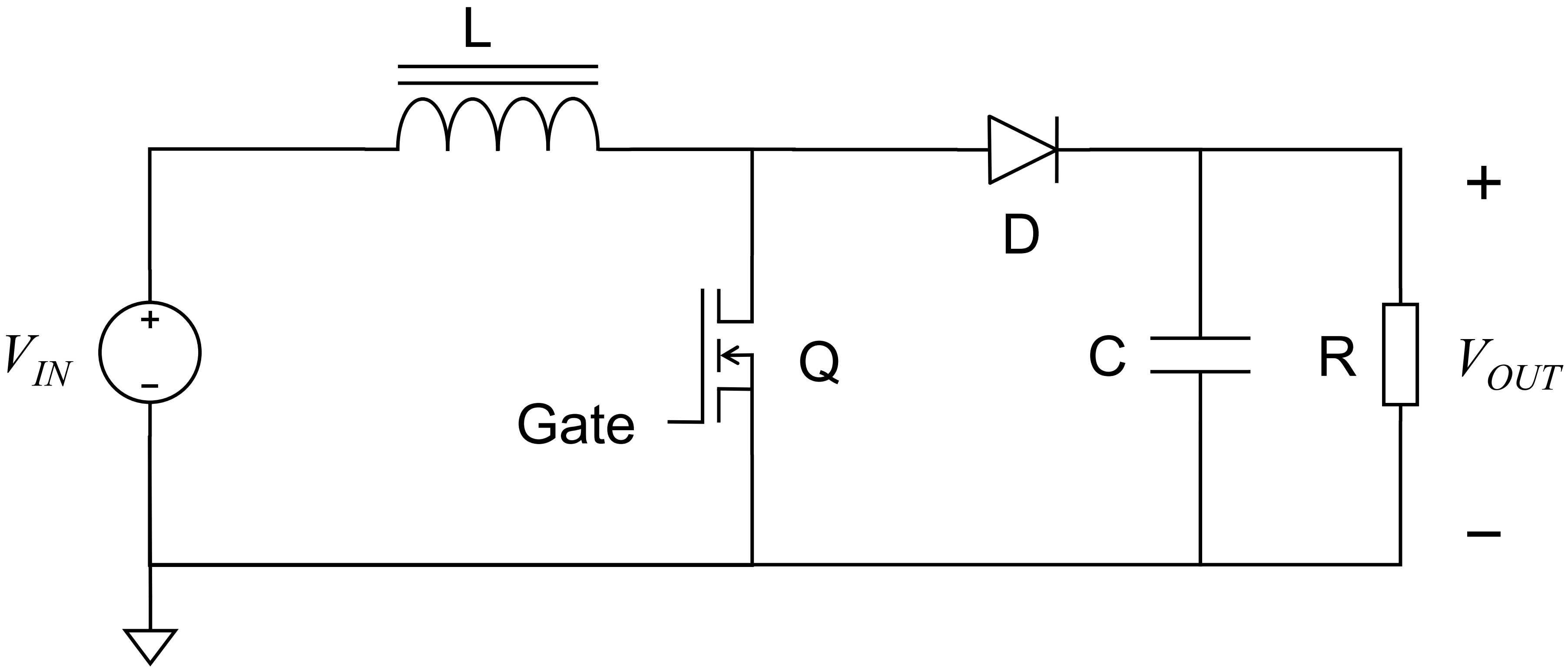
- Isolated
- Good For 500 W And Up
- Multiple Isolated Outputs Possible
- Easy To Control
- Good Use Of Transformer Core
- Low Voltage Stress On Switching Transistors

Disadvantages

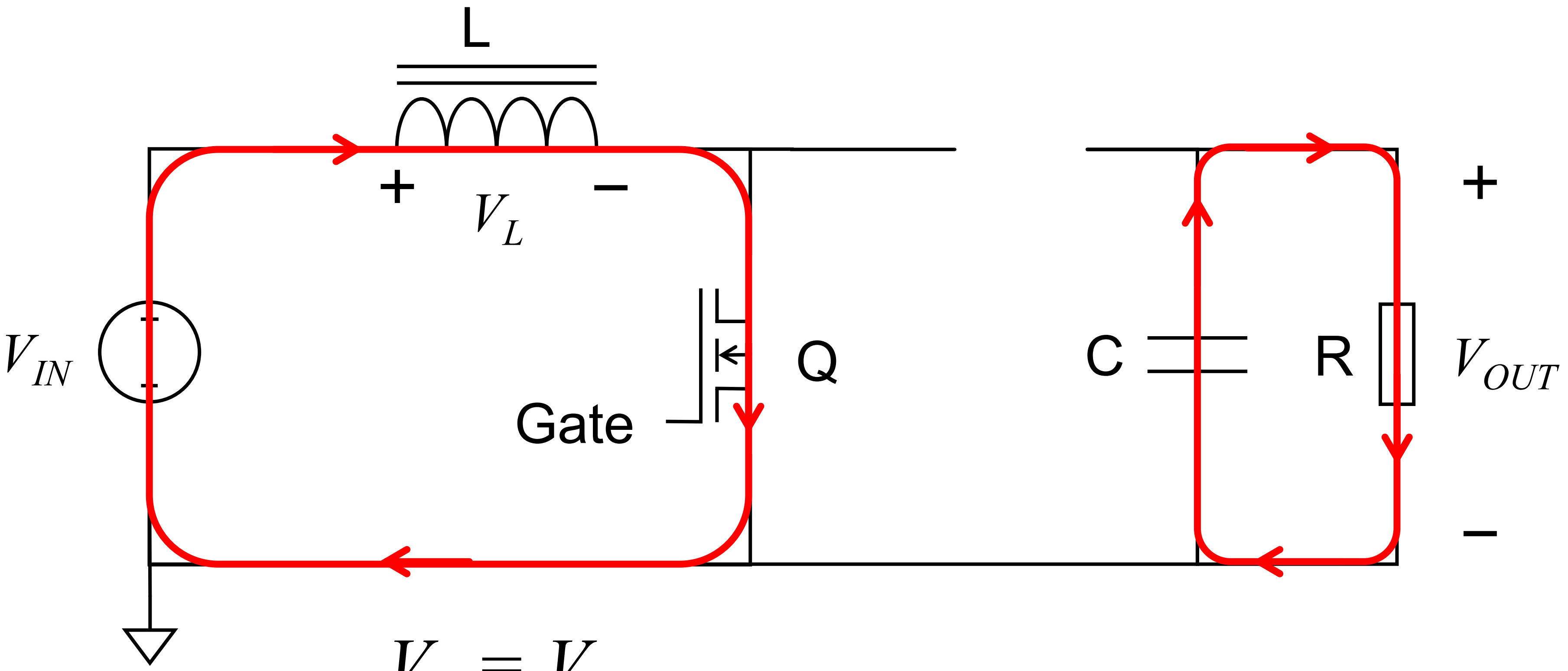
- Transformer Needed
- Four Switches And Associated Drive Circuits
- Must Have Way To Prevent Transformer Saturation
- Center Tap Output: High Voltage Stress On Output Diodes
- Full Wave Output: More Diodes And Diode Loss

APPENDIX IV.
Boost Converter
Discontinuous Conduction
And Critical Conduction Modes

Boost Converter

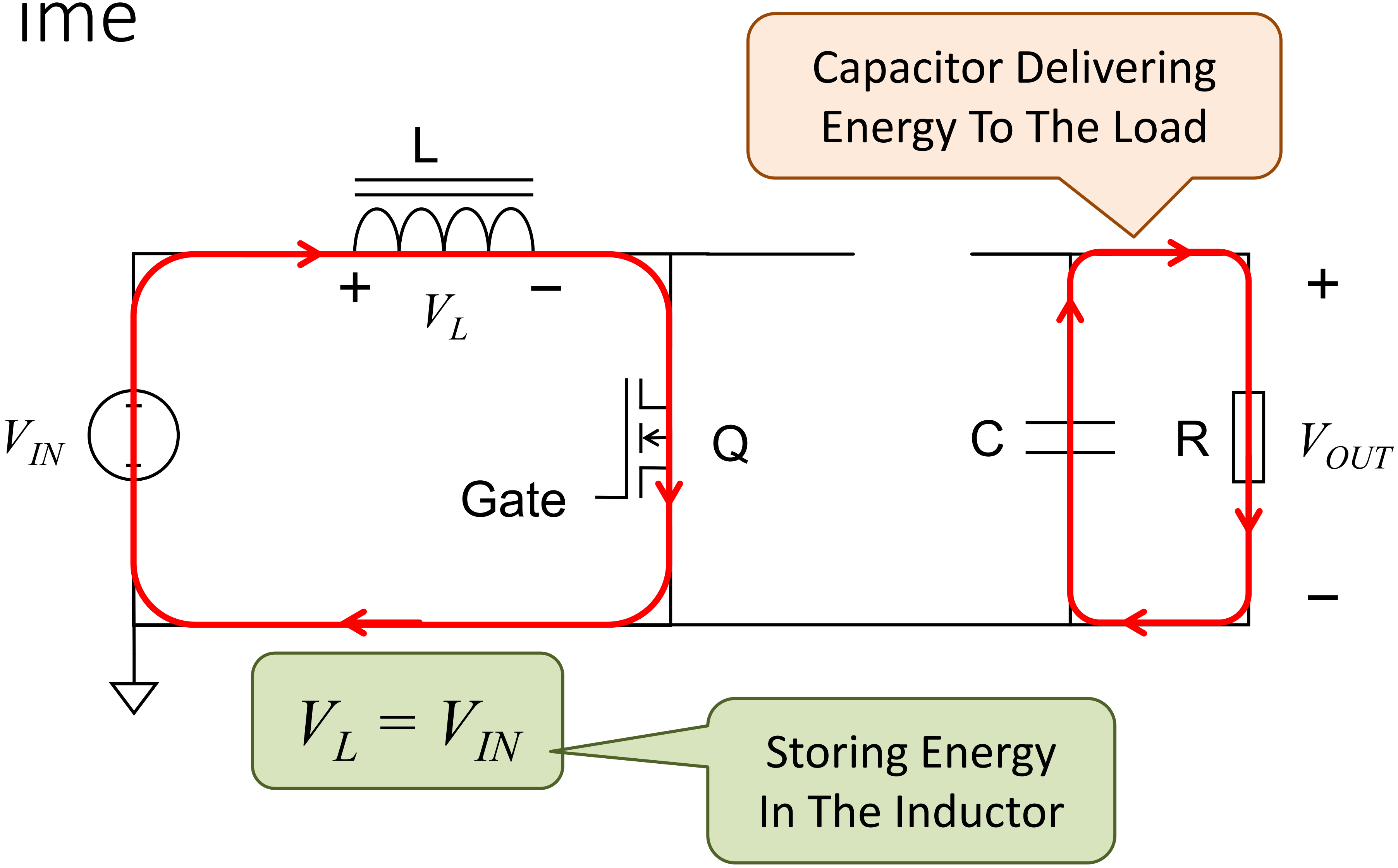


Boost Converter Discontinuous Conduction Mode: On Time

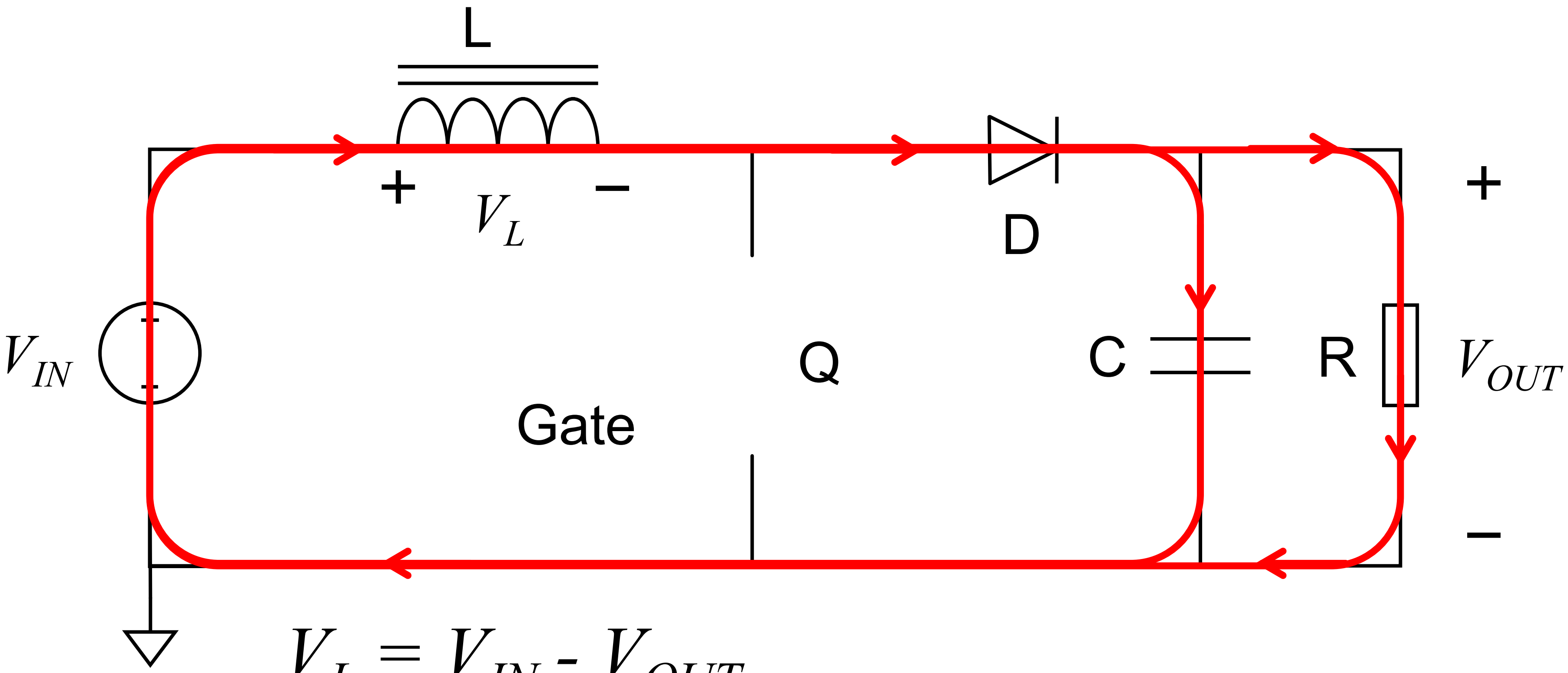


$$V_L = V_{IN}$$

Boost Converter Discontinuous Conduction Mode: On Time



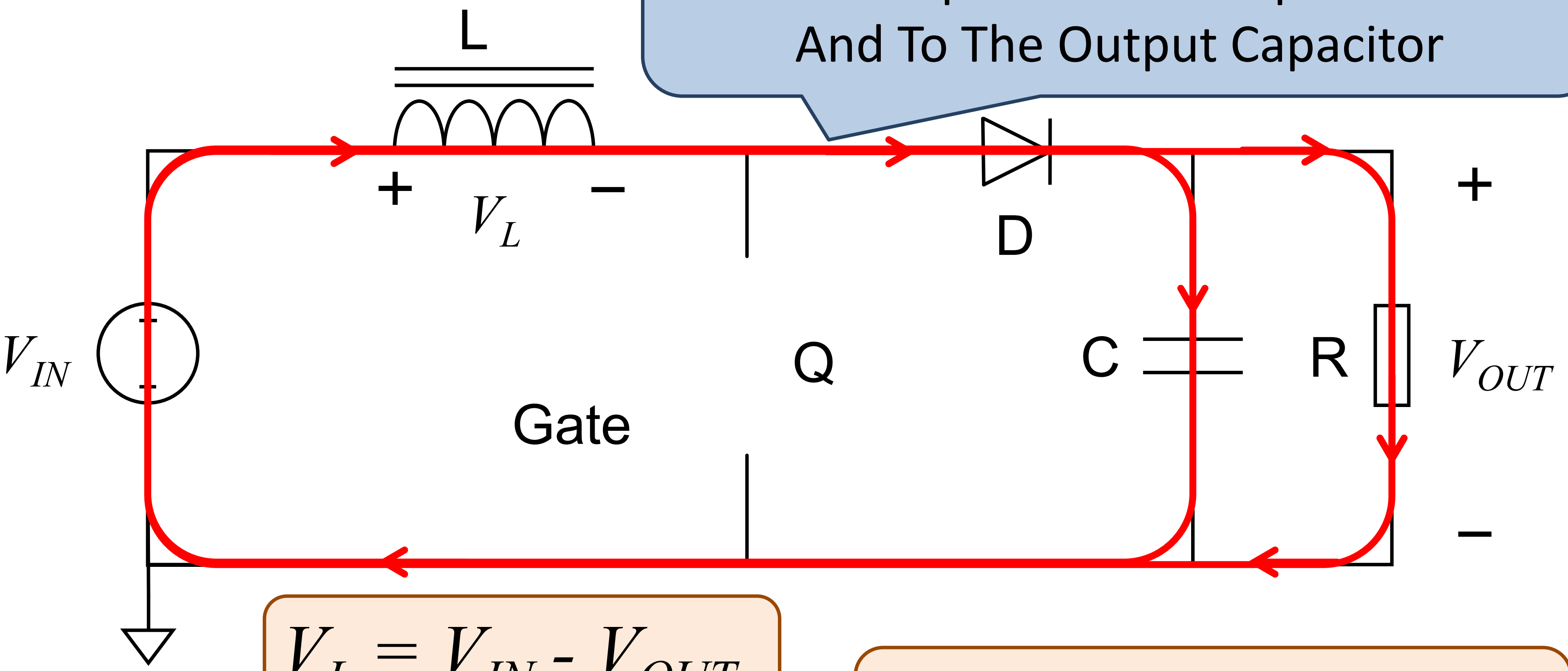
Boost Converter Discontinuous Conduction Mode: Off Time 1



$$V_L = V_{IN} - V_{OUT}$$

Boost Converter Discontinuous Conduction Mode: Off Time 1

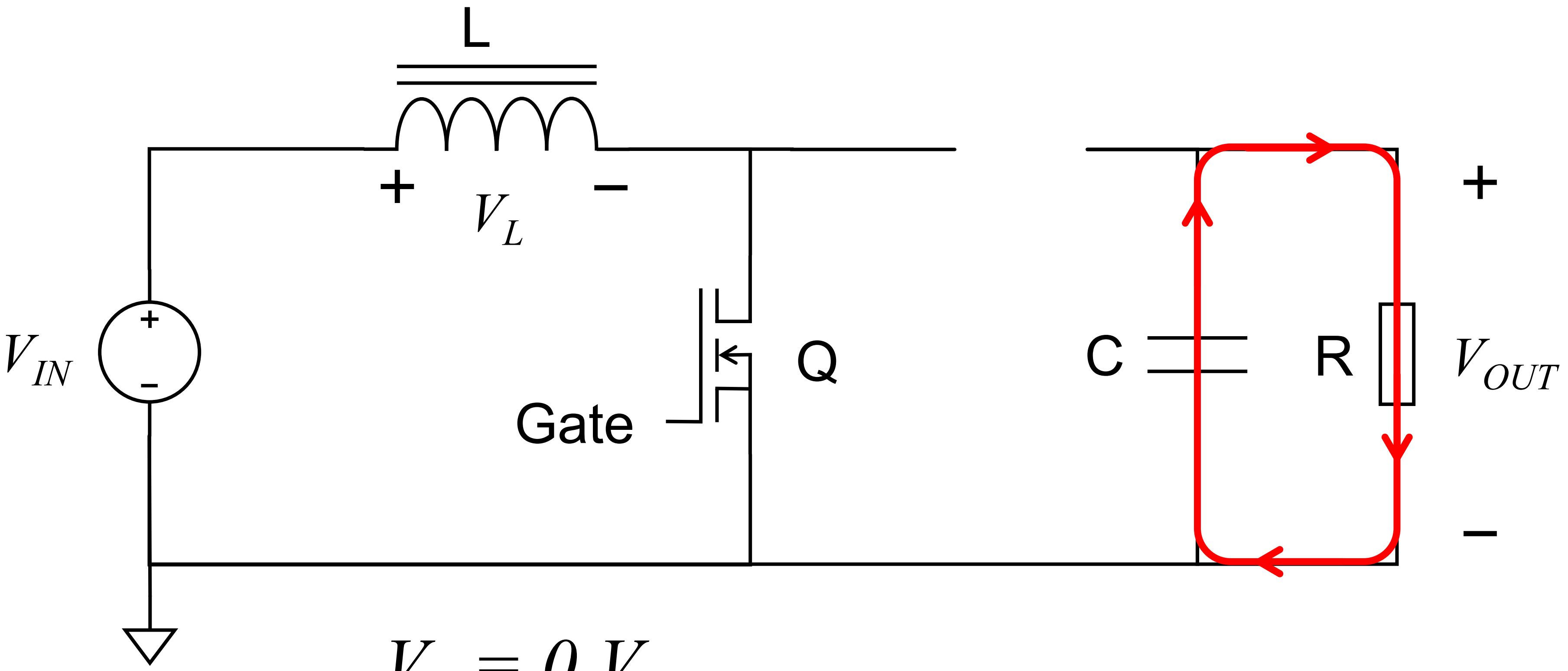
Power Flowing
From Input To The Output Load
And To The Output Capacitor



$$V_L = V_{IN} - V_{OUT}$$

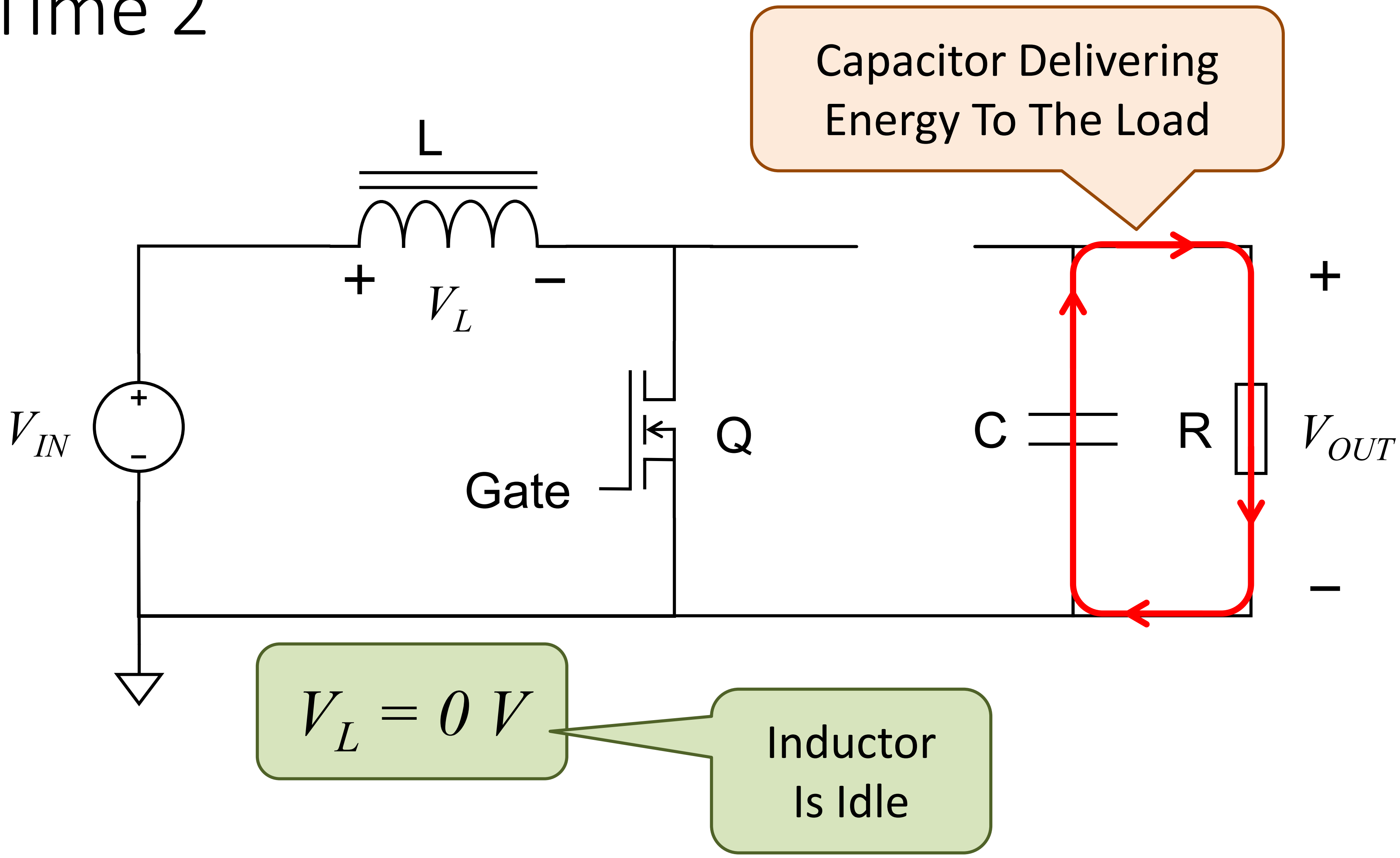
Also Releasing Stored Energy
In The Inductor

Boost Converter Discontinuous Conduction Mode: Off Time 2

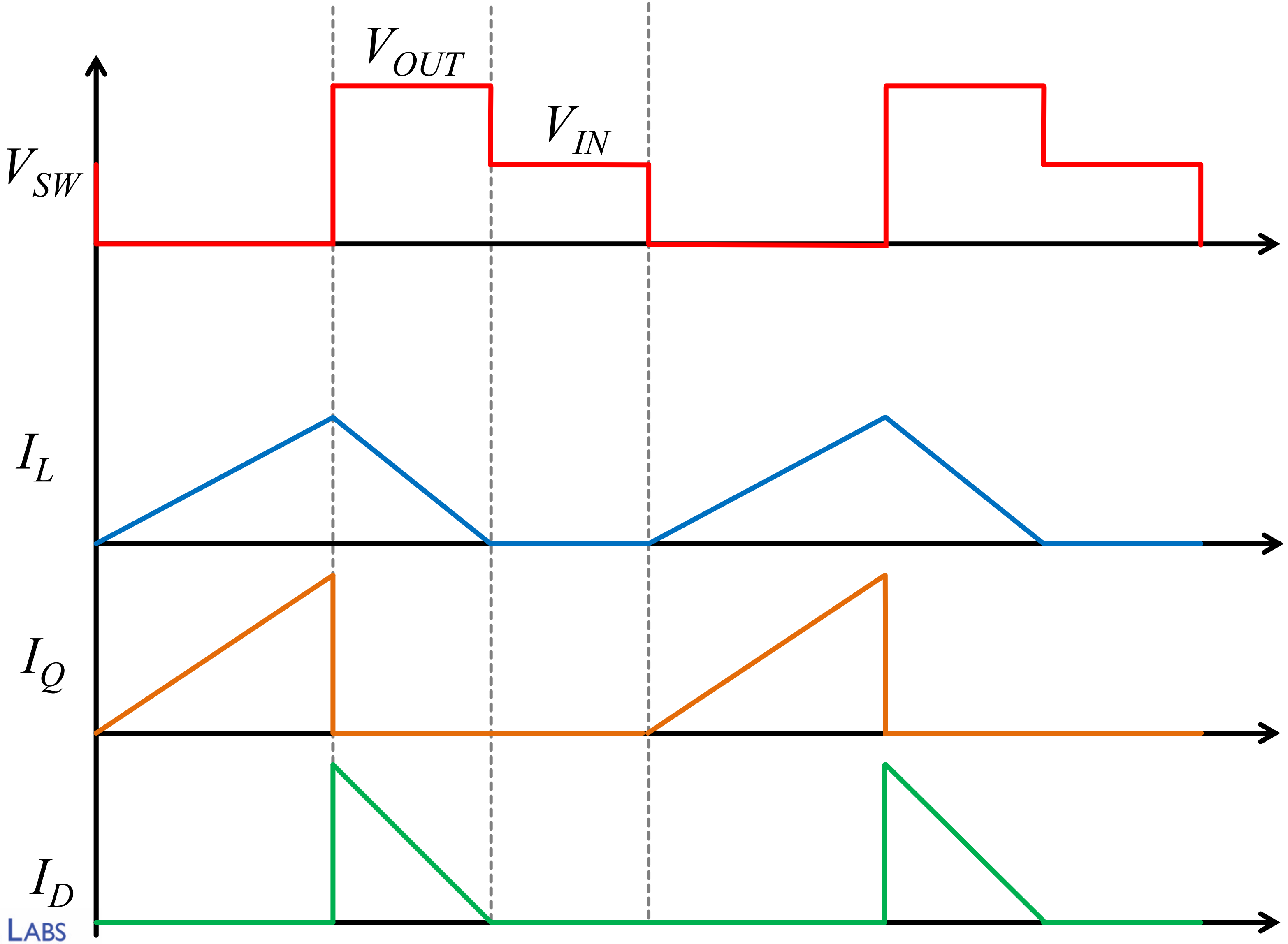


$$V_L = 0\ V$$

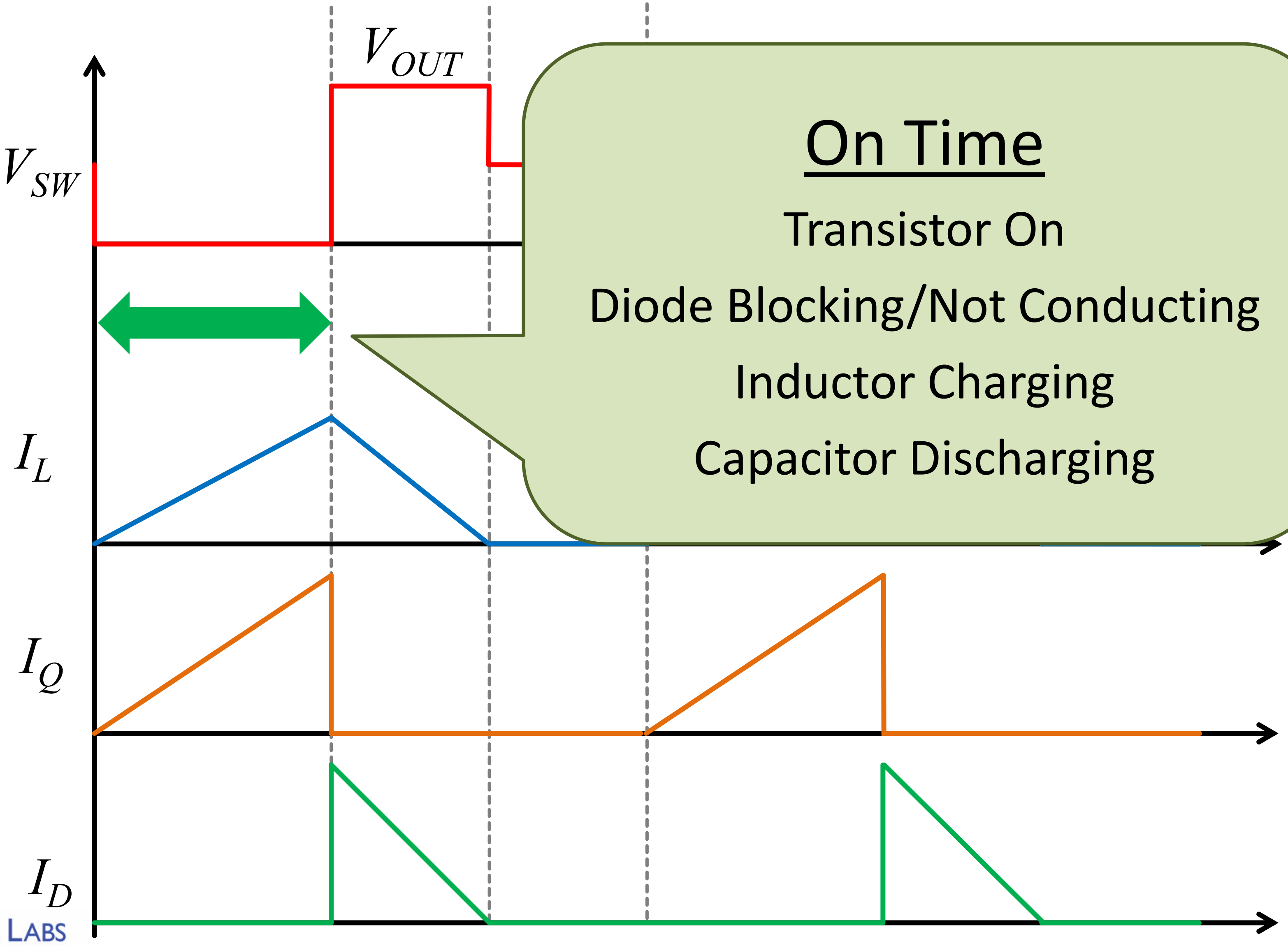
Boost Converter Discontinuous Conduction Mode: Off Time 2



Discontinuous Conduction Mode



Discontinuous Conduction Mode



On Time

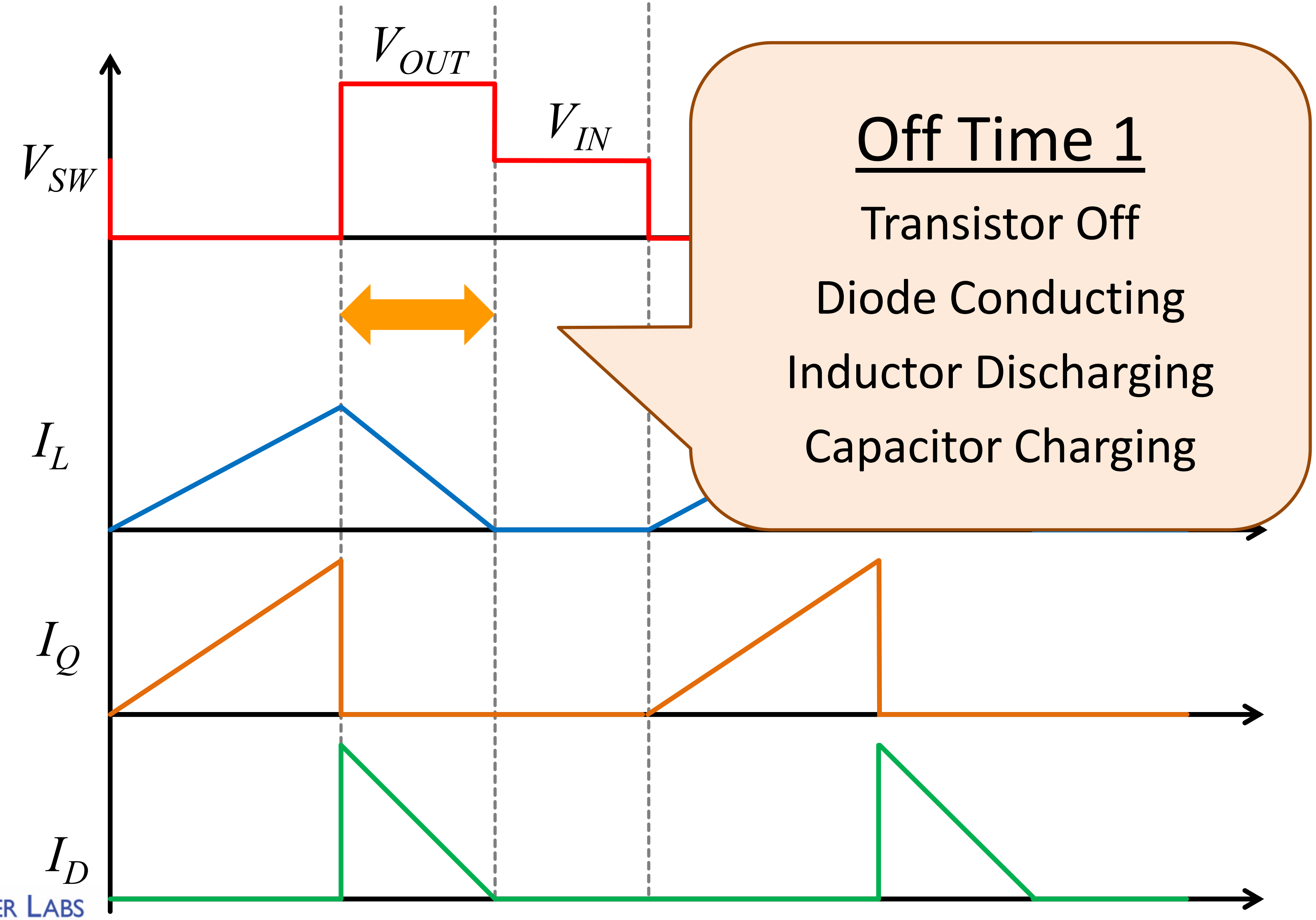
Transistor On

Diode Blocking/Not Conducting

Inductor Charging

Capacitor Discharging

Discontinuous Conduction Mode



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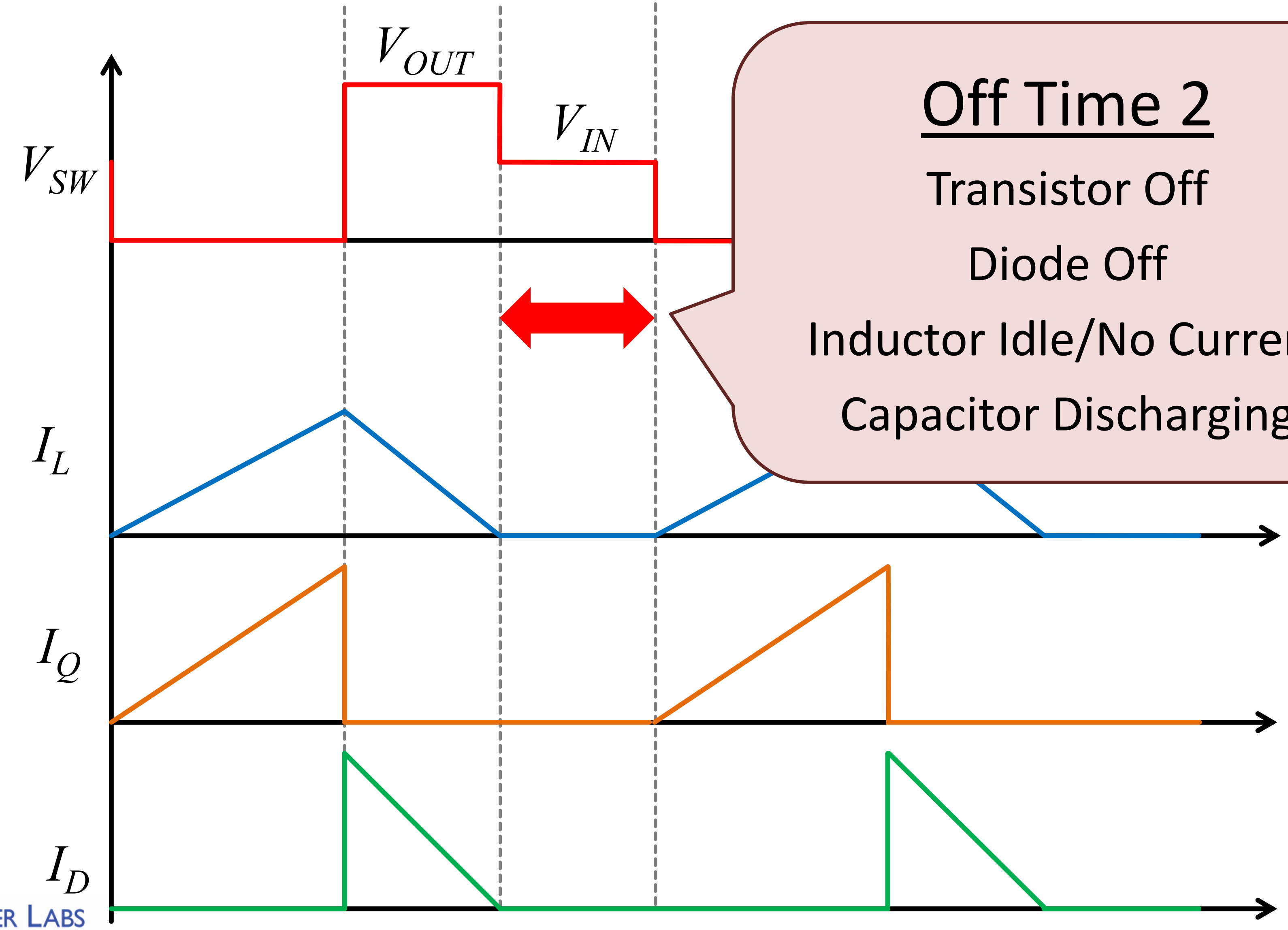
Discontinuous Conduction Mode



Off Time 2

Transistor Off

Diode Off

Inductor Idle/No Current

Capacitor Discharging

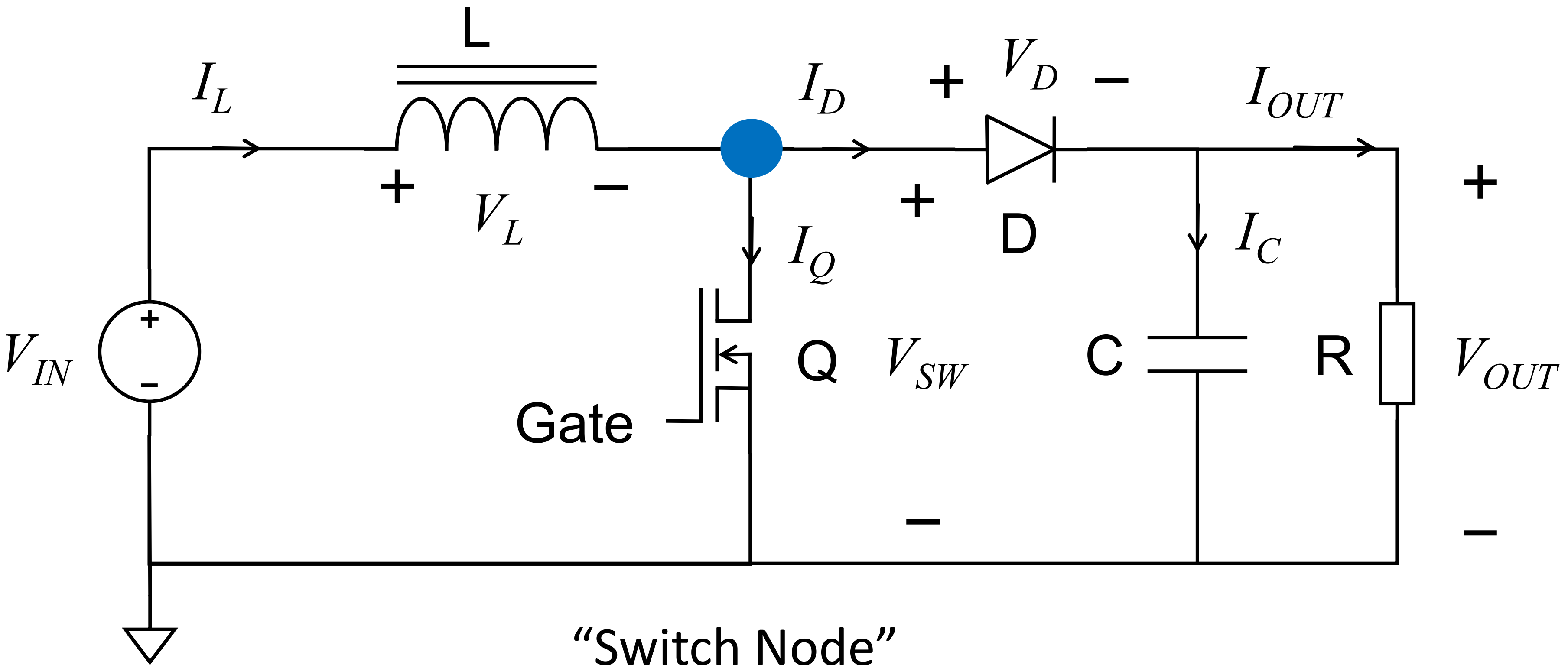
DCM Conversion Ratio

$$\frac{V_{OUT}}{V_{IN}} = \frac{1 + \sqrt{1 + 4 \cdot D^2 \cdot \dfrac{R \cdot T_{SW}}{2 \cdot L}}}{2}$$

- Nonlinear
- Depends On Load Resistance, Inductor, Switching Frequency
- Becomes Continuous Conduction Mode When:

$$\frac{2 \cdot L}{R \cdot T_{SW}} \geq D \cdot (1 - D)^2$$

Video Lab 11
Boost Converter In Discontinuous Mode



"Switch Node"

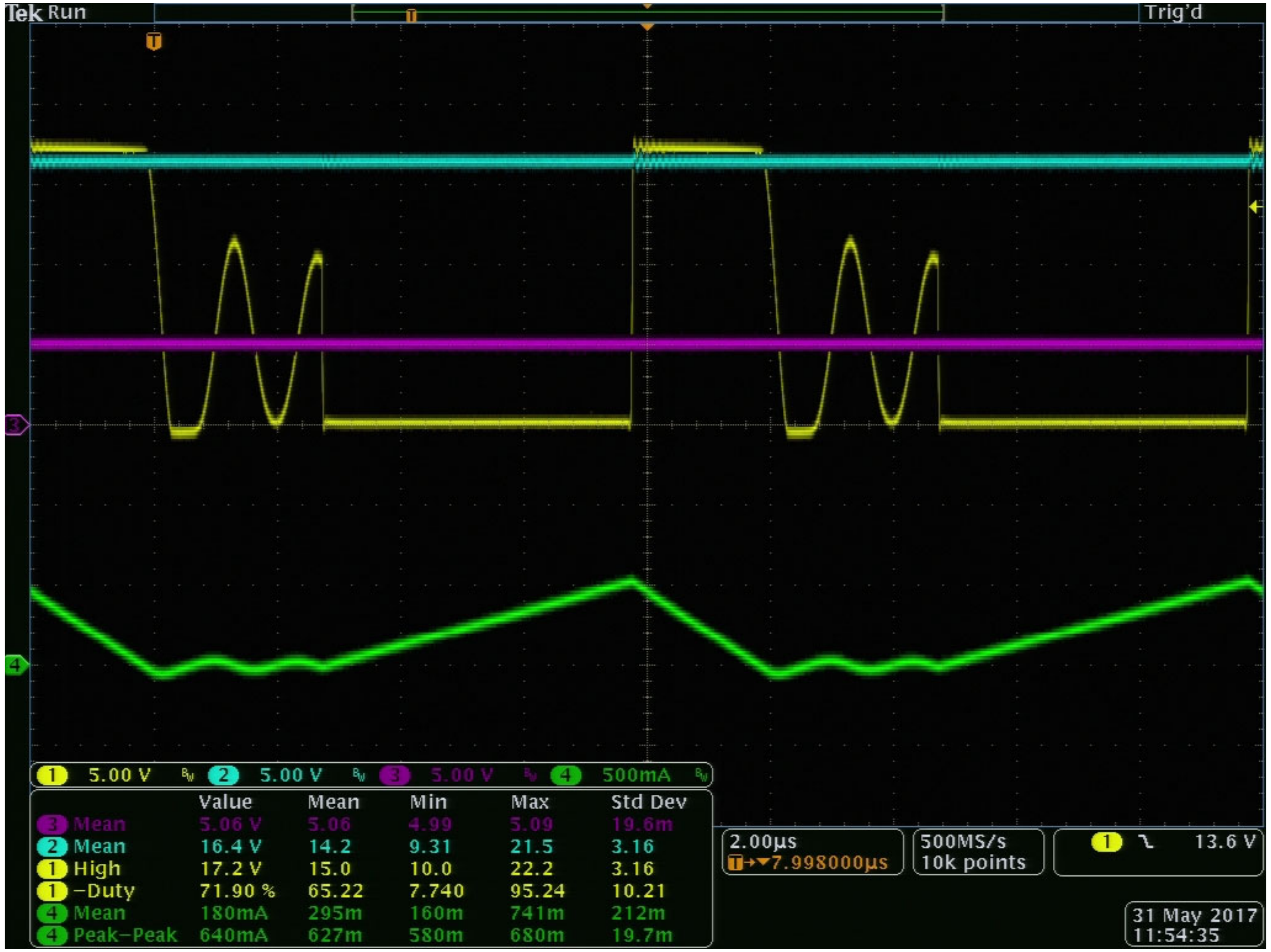
Video Lab 11
Boost Converter In Discontinuous Mode

Switch Node

Output Voltage

Input Voltage

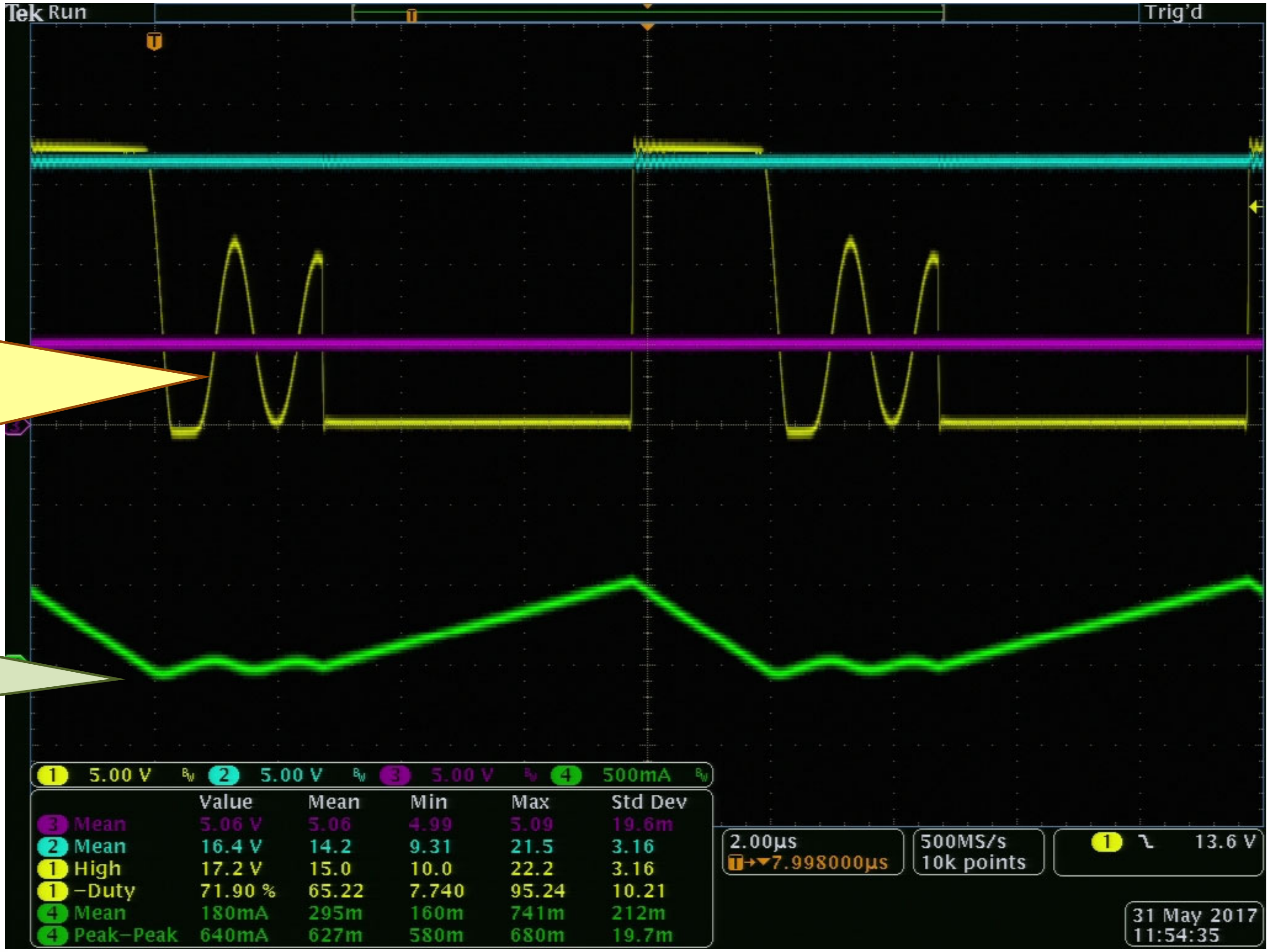
Inductor Current



Video Lab 11
Boost Converter In Discontinuous Mode

We See Here The Switch Node Voltage Oscillating As The Inductor Rings With the Output Capacitance Of The MOSFET

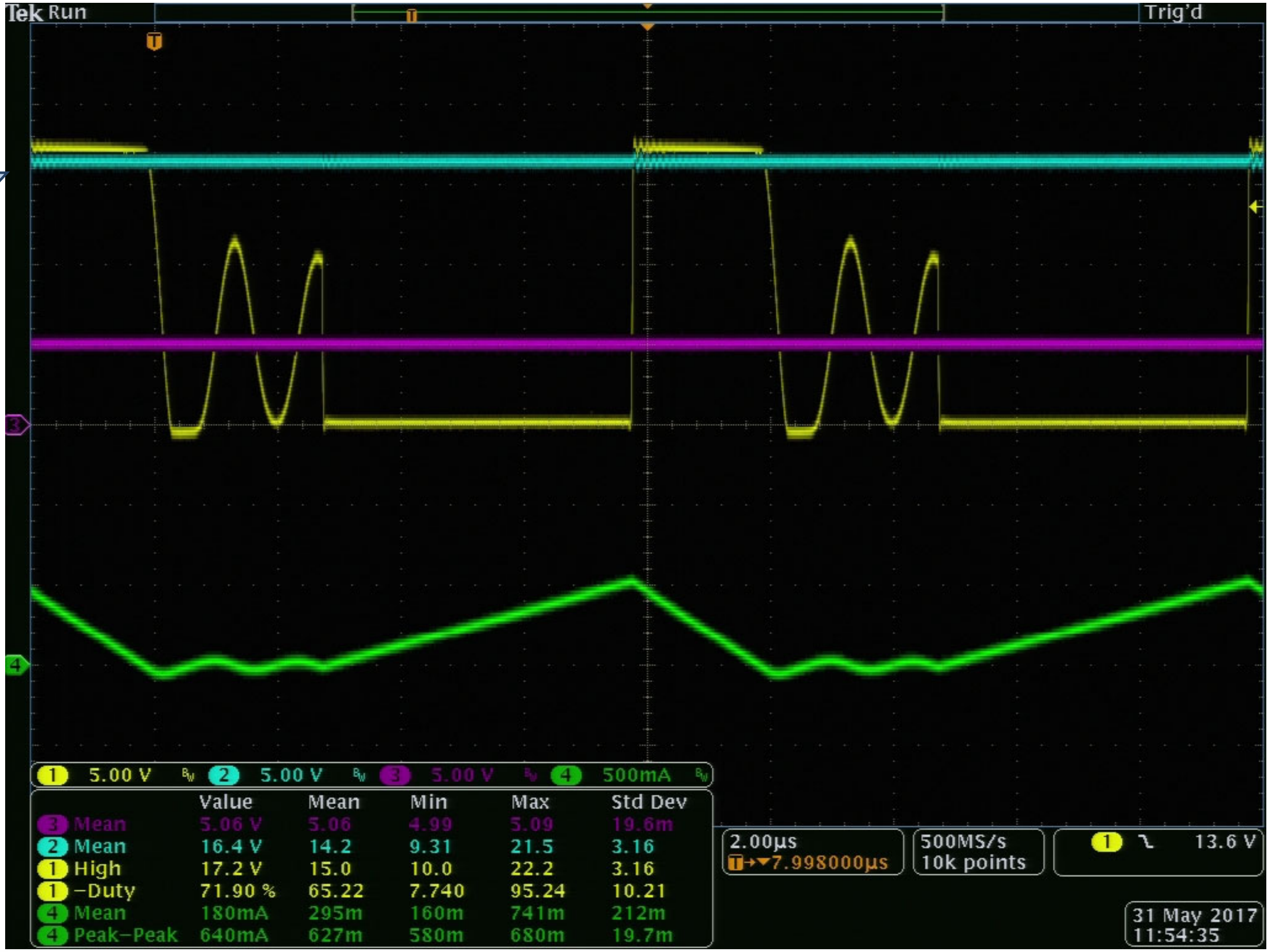
Can Also See The Current In The Inductor Oscillating/ Ringing



Video Lab 11
Boost Converter In Discontinuous Mode

In The Video, Note That The Output Voltage *Increases* As The Load Current Is *Decreased*.

Why Do You Think that Happens?



Boost Discontinuous Mode

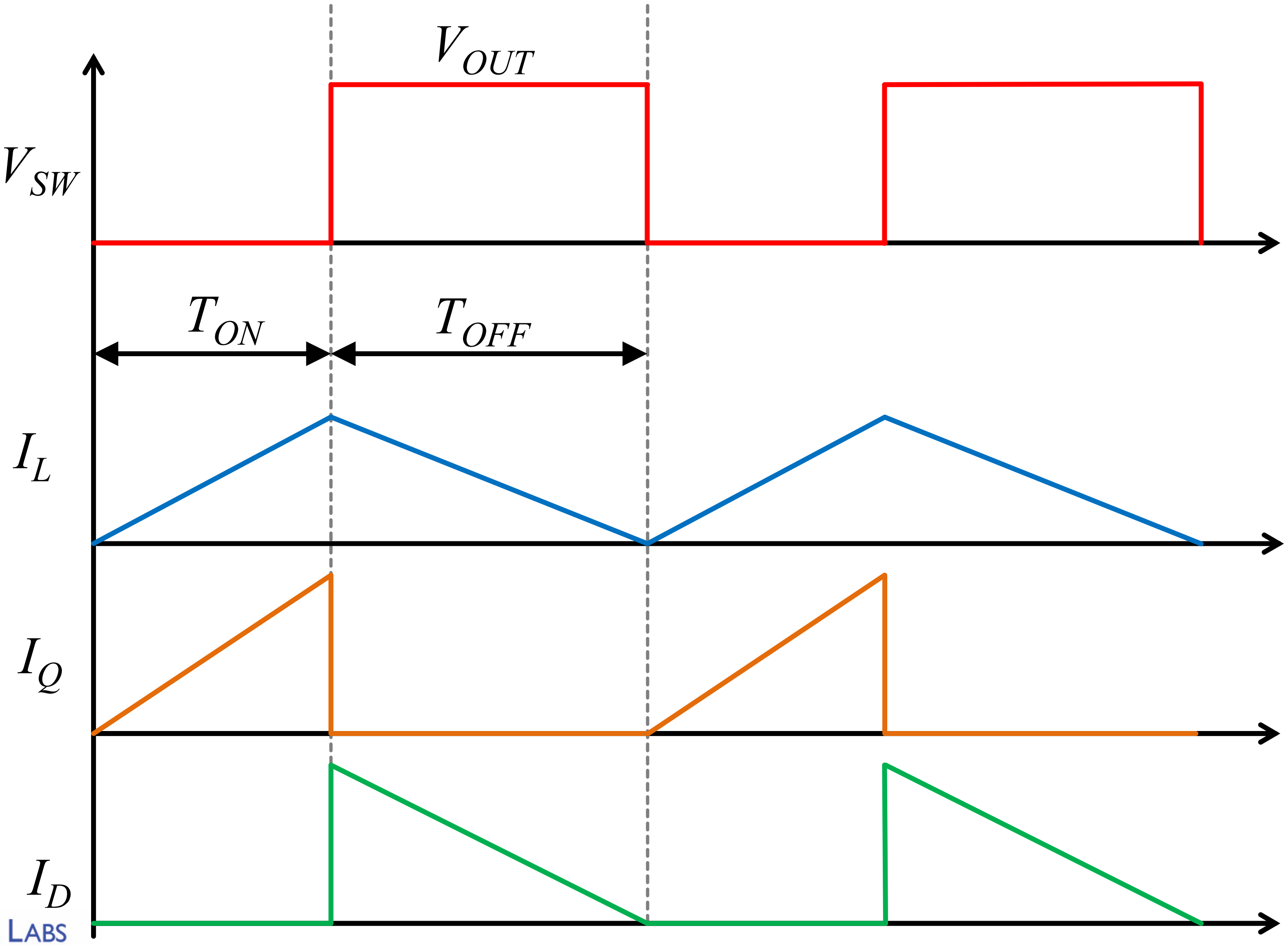
Advantages

- Easy To Design The Controller
- No Losses From Diode Reverse Recovery

Disadvantages

- High Peak Currents In Transistor, Diode, Inductor Means Significant I^2R Losses
- Large Output Capacitor

Critical Conduction Mode



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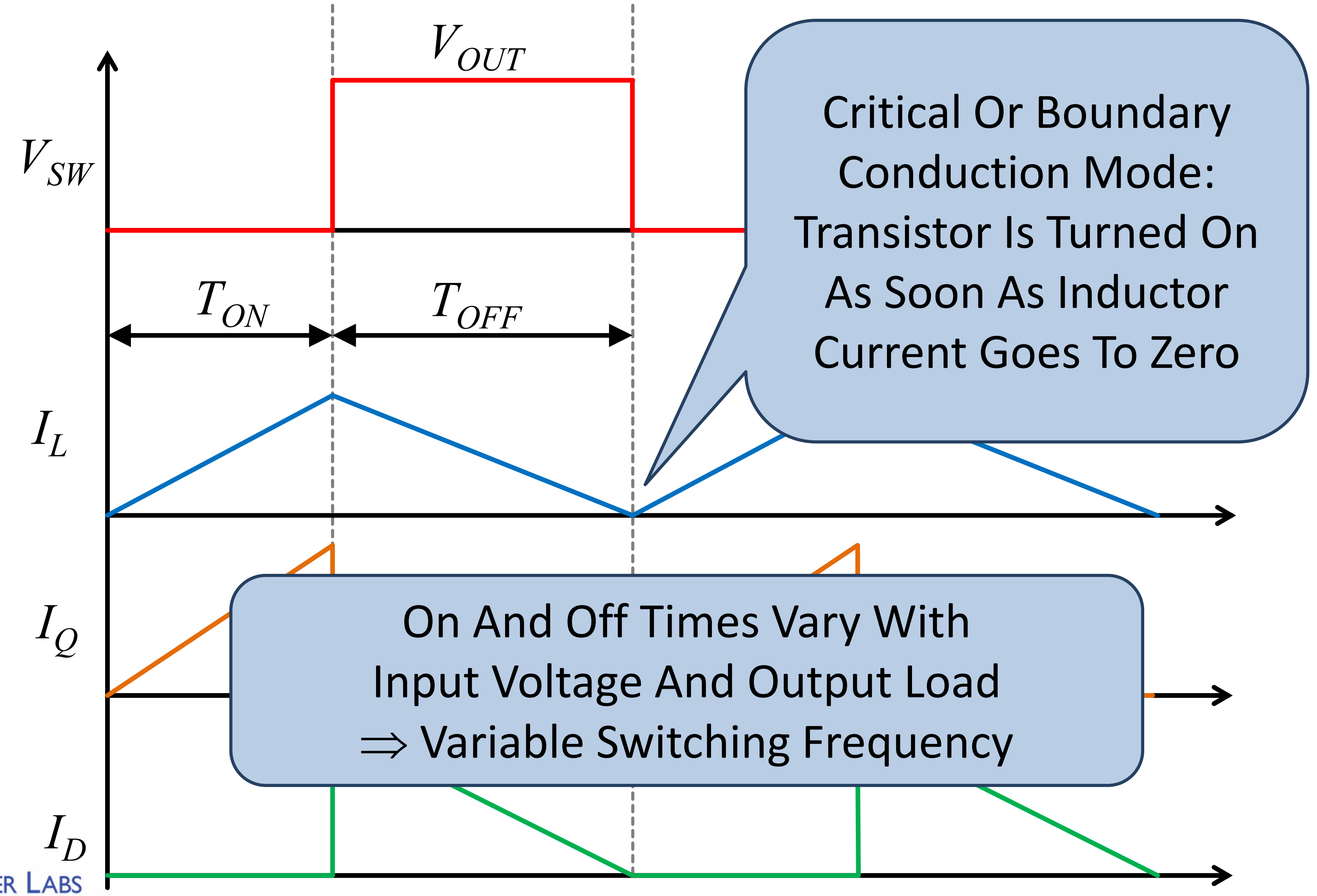
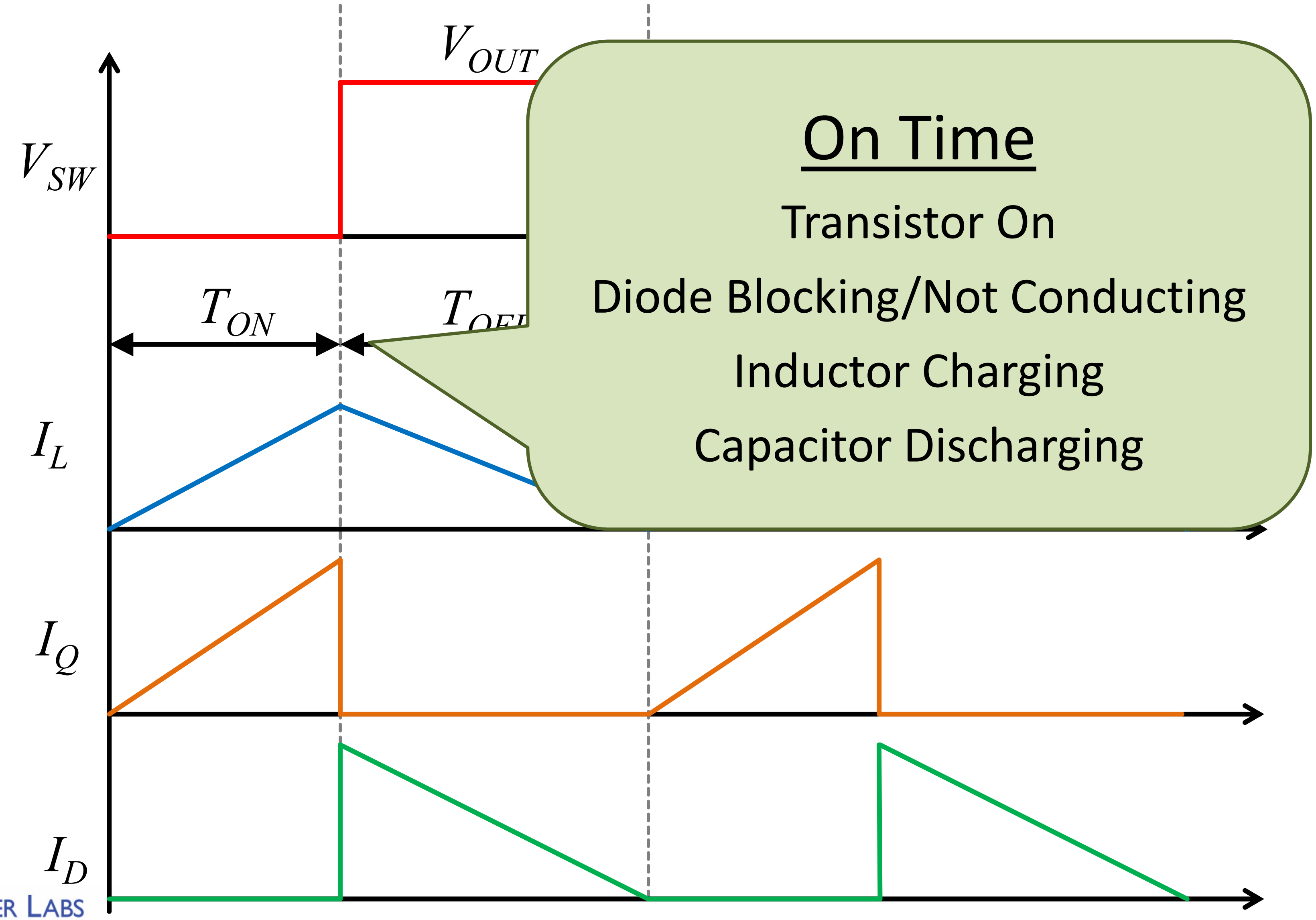
Critical Conduction Mode



EMBEDDED POWER LABS

Critical Conduction Mode



On Time

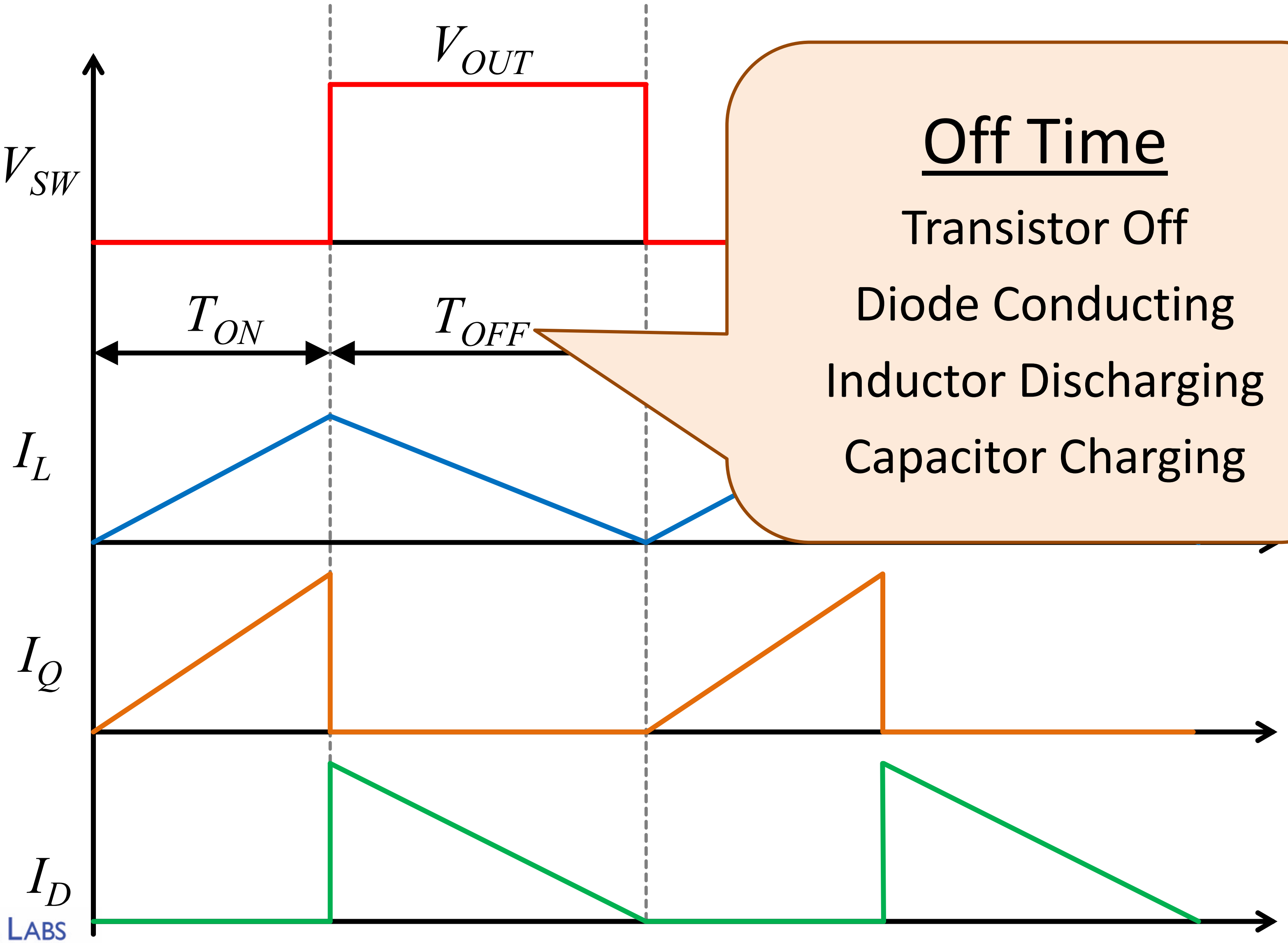
Transistor On

Diode Blocking/Not Conducting

Inductor Charging

Capacitor Discharging

Critical Conduction Mode



Off Time

Transistor Off

Diode Conducting

Inductor Discharging

Capacitor Charging

EMBEDDED POWER LABS

Boost Critical Conduction Mode

Advantages

- No Losses From Diode Reverse Recovery
- Smaller Peak Currents Than Discontinuous Mode

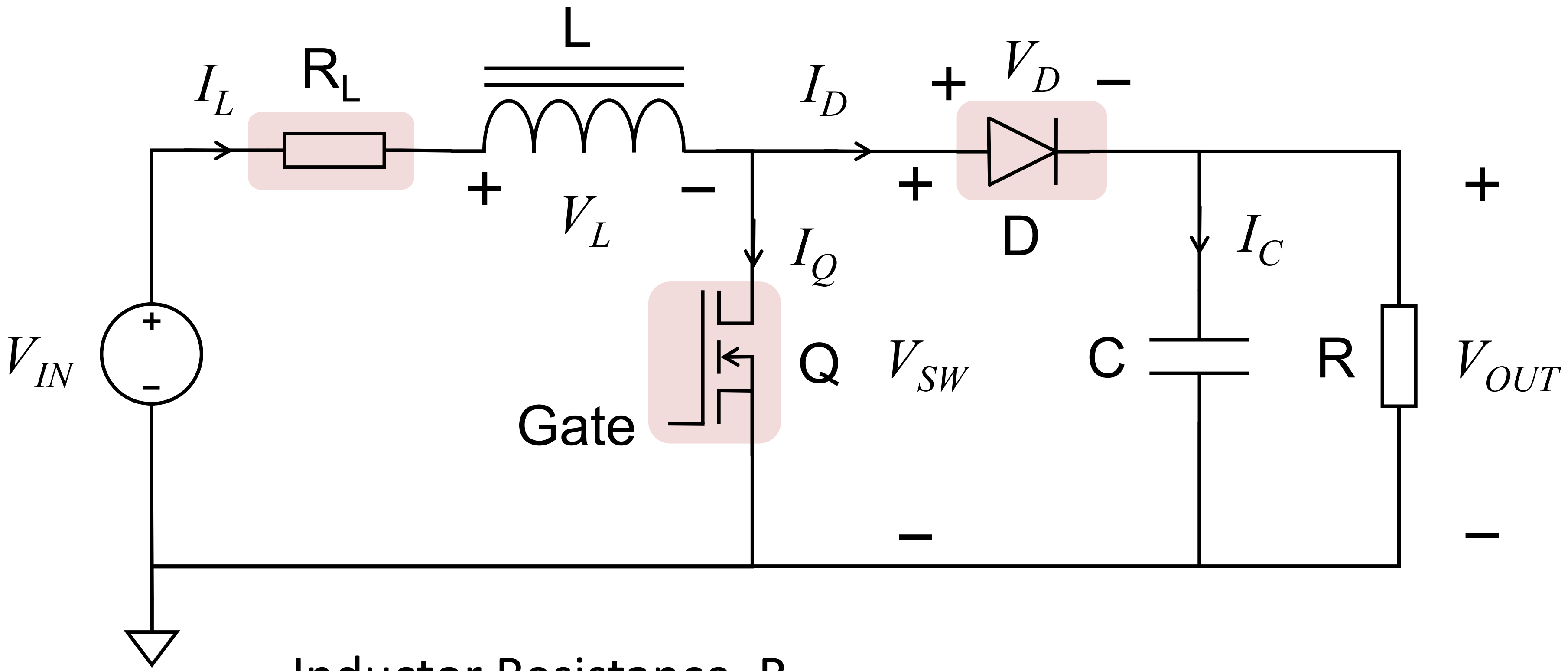
Disadvantages

- Specialized Controller
- Variable Frequency Operation
- Large Output Capacitor

This Mode Of Operation Is Popular
When A Boost Converter Is Used To Make
An Active Rectifier (Power Factor Correction, PFC)
In An AC-DC Power Supply Because The
Lack Of Diode Reverse Recovery Improves Efficiency

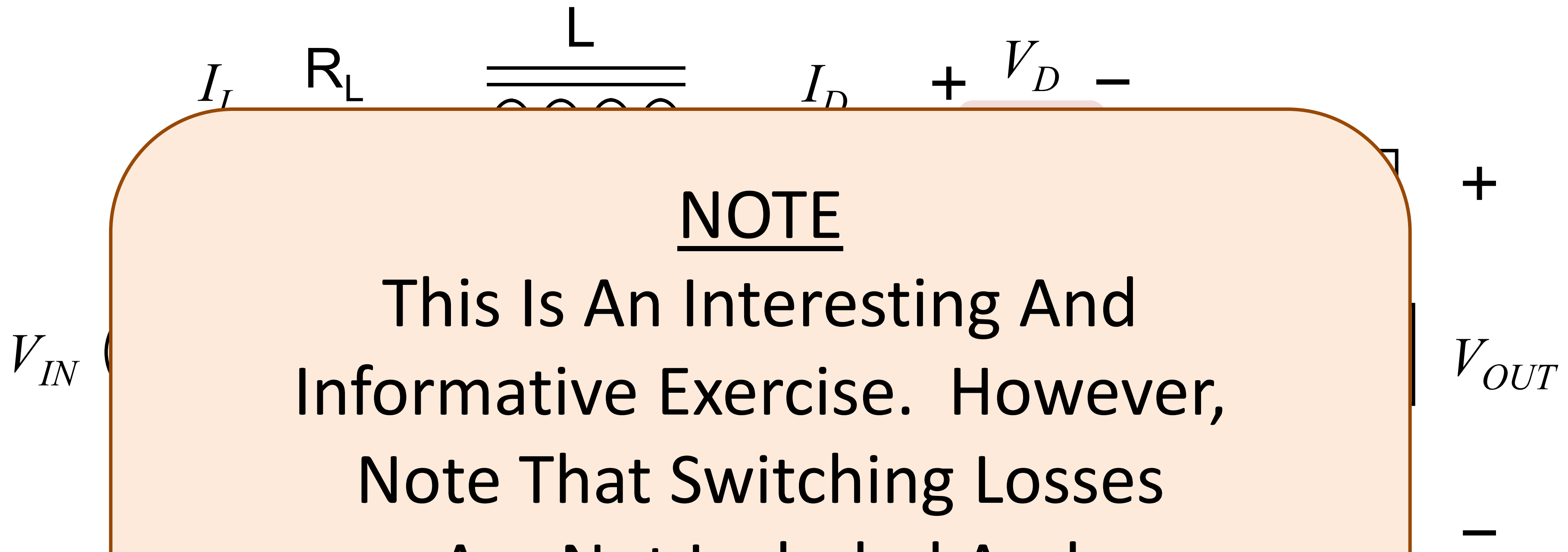
APPENDIX V.
Non-Ideal Boost Converter

Non-Ideal Boost Converter



Inductor Resistance, R_L
Transistor On-Resistance, R_Q
Diode Forward Voltage, V_D

Non-Ideal Boost Converter



NOTE

This Is An Interesting And Informative Exercise. However, Note That Switching Losses Are Not Included And Reduce The Ideal Conversion Ratio Even More Than Shown Here.

Boost Converter Input Current

$$I_C(T_{ON}) \cdot T_{ON} + I_C(T_{OFF}) \cdot T_{OFF} = 0$$

$$I_C(T_{ON}) = -\frac{V_{OUT}}{R}$$

$$I_C(T_{OFF}) = I_L - \frac{V_{OUT}}{R}$$

$$\left(-\frac{V_{OUT}}{R}\right) \cdot T_{ON} + \left(I_L - \frac{V_{OUT}}{R}\right) \cdot T_{OFF} = 0$$

$$\left(-\frac{V_{OUT}}{R}\right) \cdot D \cdot T_{SW} + \left(I_L - \frac{V_{OUT}}{R}\right) \cdot (1-D) \cdot T_{SW} = 0$$

$$\left(-\frac{V_{OUT}}{R}\right) \cdot D + \left(I_L - \frac{V_{OUT}}{R}\right) \cdot (1-D) = 0$$

$$-D \cdot \frac{V_{OUT}}{R} + I_L - D \cdot I_L - \frac{V_{OUT}}{R} + D \cdot \frac{V_{OUT}}{R} = 0$$

$$I_L - D \cdot I_L - \frac{V_{OUT}}{R} = 0$$

$$(1-D) \cdot I_L = \frac{V_{OUT}}{R}$$

$$I_L = I_{IN} = \frac{1}{1-D} \cdot \frac{V_{OUT}}{R}$$

$$D \to 1 \Rightarrow I_{IN} \to \infty$$

Inductor Voltages

$$V_L\left(T_{ON}\right)=V_{IN}-R_L\cdot I_L-R_Q\cdot I_L$$

$$V_L\left(T_{OFF}\right)=V_{IN}-R_L\cdot I_L-V_D-V_{OUT}$$

Both Expressions Use The Inductor Current. We Use The Expression For The Inductor Current We Just Derived

$$I_L=\frac{1}{D'}\cdot\frac{V_{OUT}}{R}$$

Inductor Volt-Second Balance

$$V_L\left(T_{ON}\right)=V_{IN}-R_L\cdot\frac{1}{D'}\cdot\frac{V_{OUT}}{R}-R_Q\cdot\frac{1}{D'}\cdot\frac{V_{OUT}}{R}$$
$$=V_{IN}-\frac{\left(R_L+R_Q\right)}{D'}\cdot\frac{V_{OUT}}{R}$$

$$V_L\left(T_{OFF}\right)=V_{IN}-R_L\cdot\frac{1}{D'}\cdot\frac{V_{OUT}}{R}-V_D-V_{OUT}$$
$$=V_{IN}-V_D-V_{OUT}-R_L\cdot\frac{1}{D'}\cdot\frac{V_{OUT}}{R}$$
$$=V_{IN}-V_D-V_{OUT}\cdot\left(1+R_L\cdot\frac{1}{D'}\cdot\frac{1}{R}\right)$$
$$=V_{IN}-V_D-V_{OUT}\cdot\left(1+\frac{1}{D'}\cdot\frac{R_L}{R}\right)$$
$$=V_{IN}-V_D-\frac{V_{OUT}}{R}\cdot\left(\frac{D'\cdot R+R_L}{D'}\right)$$

$$0=V_L\left(T_{ON}\right)\cdot T_{ON}+V_L\left(T_{OFF}\right)\cdot T_{OFF}$$
$$=V_L\left(T_{ON}\right)\cdot D+V_L\left(T_{OFF}\right)\cdot D'$$
$$=\left(V_{IN}-\frac{\left(R_L+R_Q\right)}{D'}\cdot\frac{V_{OUT}}{R}\right)\cdot D$$
$$+\left(V_{IN}-V_D-\frac{V_{OUT}}{R}\cdot\left(\frac{D'\cdot R+R_L}{D'}\right)\right)\cdot D'$$
$$=D\cdot V_{IN}-D\cdot\frac{\left(R_L+R_Q\right)}{D'}\cdot\frac{V_{OUT}}{R}$$
$$+D'\cdot V_{IN}-D'\cdot V_D-D'\cdot\frac{V_{OUT}}{R}\cdot\left(\frac{D'\cdot R+R_L}{D'}\right)$$

Inductor Volt-Second Balance

$$0 = D \cdot V_{IN} + D' \cdot V_{IN} - D' \cdot V_D$$
$$- D \cdot \frac{\left(R_L + R_Q\right)}{D'} \cdot \frac{V_{OUT}}{R} - D' \cdot \left(\frac{D' \cdot R + R_L}{D'}\right) \cdot \frac{V_{OUT}}{R}$$
$$= \left(D + D'\right) V_{IN} - D' \cdot V_D$$
$$- D \cdot \frac{\left(R_L + R_Q\right)}{D'} \cdot \frac{V_{OUT}}{R} - \left(D' \cdot R + R_L\right) \cdot \frac{V_{OUT}}{R}$$
$$= V_{IN} - D' \cdot V_D$$
$$- \frac{D}{D'} \cdot R_L \cdot \frac{V_{OUT}}{R} - R_L \cdot \frac{V_{OUT}}{R} - \frac{D}{D'} \cdot R_Q \cdot \frac{V_{OUT}}{R} - D' \cdot R \cdot \frac{V_{OUT}}{R}$$

$$0 = V_{IN} - D' \cdot V_D$$
$$- \left(1 + \frac{D}{D'}\right) R_L \cdot \frac{V_{OUT}}{R} - \frac{D}{D'} \cdot R_Q \cdot \frac{V_{OUT}}{R} - D' \cdot V_{OUT}$$
$$= V_{IN} - D' \cdot V_D$$
$$- \left(\frac{1 - D}{D'} + \frac{D}{D'}\right) R_L \cdot \frac{V_{OUT}}{R} - \frac{D}{D'} \cdot R_Q \cdot \frac{V_{OUT}}{R} - D' \cdot V_{OUT}$$
$$= V_{IN} - D' \cdot V_D$$
$$- \frac{1}{D'} R_L \cdot \frac{V_{OUT}}{R} - \frac{D}{D'} \cdot R_Q \cdot \frac{V_{OUT}}{R} - D' \cdot V_{OUT}$$
$$= V_{IN} - D' \cdot V_D$$
$$- \left(\frac{1}{D'} R_L \cdot \frac{1}{R} + \frac{D}{D'} \cdot R_Q \cdot \frac{1}{R} + D'\right) \cdot V_{OUT}$$

Non-Ideal Conversion Ratio

$$\left(D' + \frac{1}{D'}\frac{R_L}{R} + \frac{D}{D'}\cdot\frac{R_Q}{R}\right)\cdot V_{OUT} = V_{IN} - D'\cdot V_D$$

$$V_{OUT} = \frac{V_{IN} - D'\cdot V_D}{D' + \frac{1}{D'}\frac{R_L}{R} + \frac{D}{D'}\cdot\frac{R_Q}{R}}$$

$$= \frac{V_{IN}}{D'}\cdot\frac{1 - D'\cdot\frac{V_D}{V_{IN}}}{1 + \frac{1}{D'^2}\cdot\frac{R_L}{R} + \frac{D}{D'^2}\frac{R_Q}{R}}$$

Non-Ideal Conversion Ratio

$$V_{OUT} = \frac{V_{IN} - D' \cdot V_D}{D' + \frac{1}{D'} \cdot \frac{R_L}{R} + \frac{D}{D'} \cdot \frac{R_Q}{R}}$$

$$= \frac{V_{IN}}{D'} \cdot \frac{1 - D' \cdot \frac{V_D}{V_{IN}}}{1 + \frac{1}{D'^2} \cdot \frac{R_L}{R} + \frac{D}{D'^2} \cdot \frac{R_Q}{R}}$$

Ideal Conversion Ratio

Effect Of The Inductor Resistance

Effect Of The Diode Forward Voltage

Effect Of The Transistor Resistance

Non-Ideal Conversion Ratio

$$\left(D' + \frac{1}{D'}\frac{R_L}{R} + \frac{D}{D'}\cdot\frac{R_Q}{R}\right)\cdot V_{OUT} = V_{IN} - D'\cdot V_D$$

$$V_{OUT} = \frac{V_{IN} - D'\cdot V_D}{D' + \frac{1}{D'}\frac{R_L}{R} + \frac{D}{D'}\cdot\frac{R_Q}{R}}$$

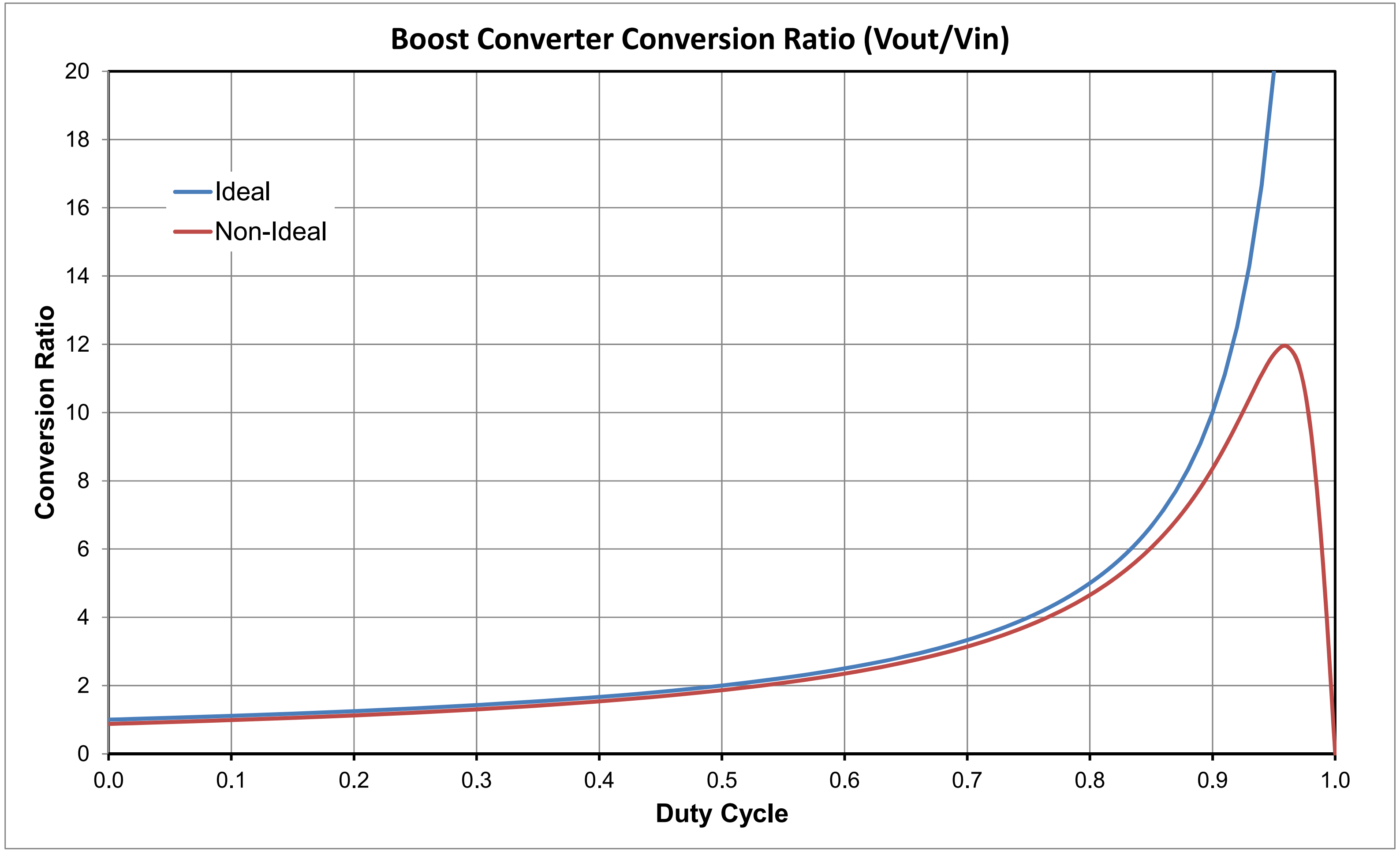
$$= \frac{V_{IN}}{D'}\cdot\frac{1 - D'\cdot\frac{V_D}{V_{IN}}}{1 + \frac{1}{D'^2}\cdot\frac{R_L}{R} + \frac{D}{D'^2}\frac{R_Q}{R}}$$

In This Form, It Is Easy To See That If:
$V_D = 0$, $R_L = 0$,
And $R_Q = 0$,
We Get The Ideal Conversion Ratio

Example Calculation

- V_{IN} = 3.3 V
- R_{OUT} = 12 Ω
  - Nominally 12 V @ 1 A
- R_L = 20 m Ω
- R_Q = 20 m Ω
- V_D = 0.4 V (Schottky)

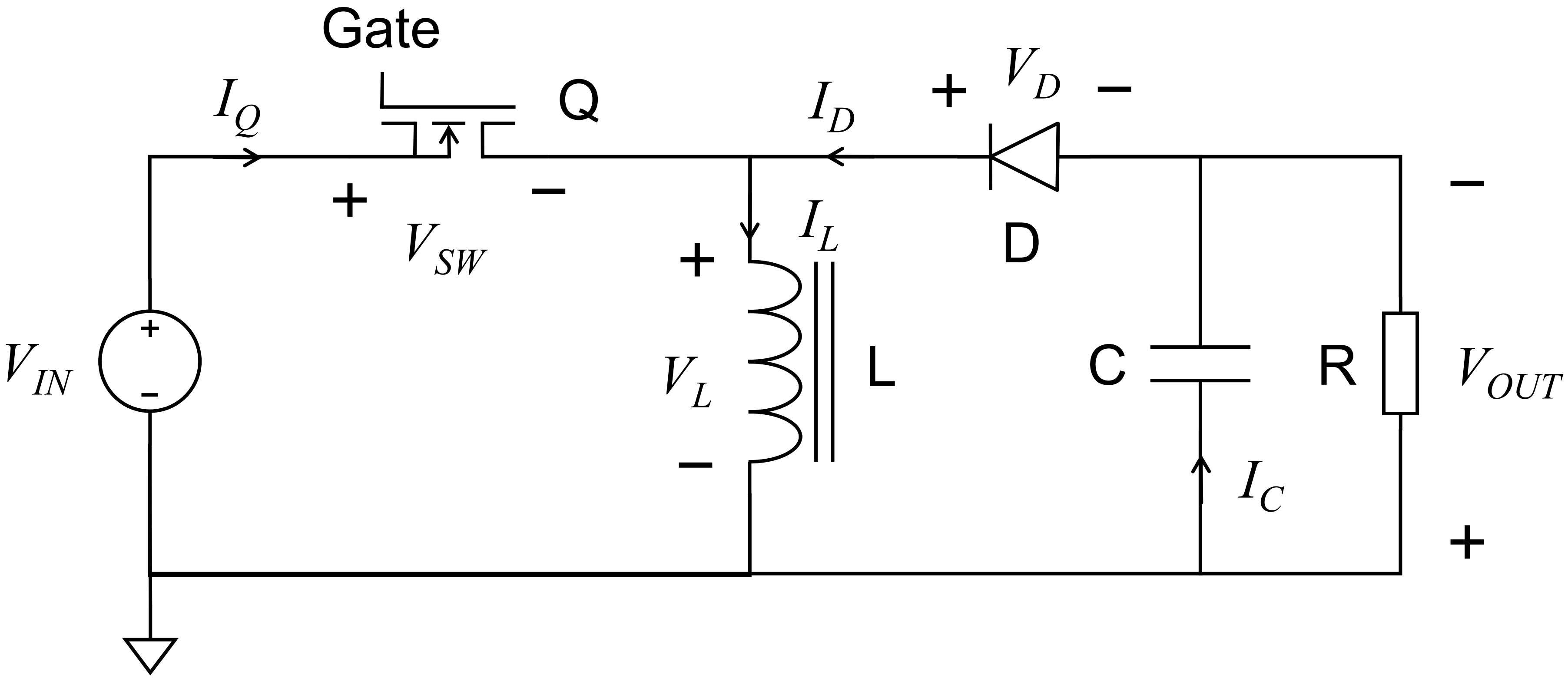
Conversion Ratio Plot



Embedded Power Labs

APPENDIX VI.
Buck-Boost Converter Analysis

Buck-Boost Converter



Conversion Ratio

$$V_L(T_{ON}) \cdot T_{ON} + V_L(T_{OFF}) \cdot T_{OFF} = 0$$

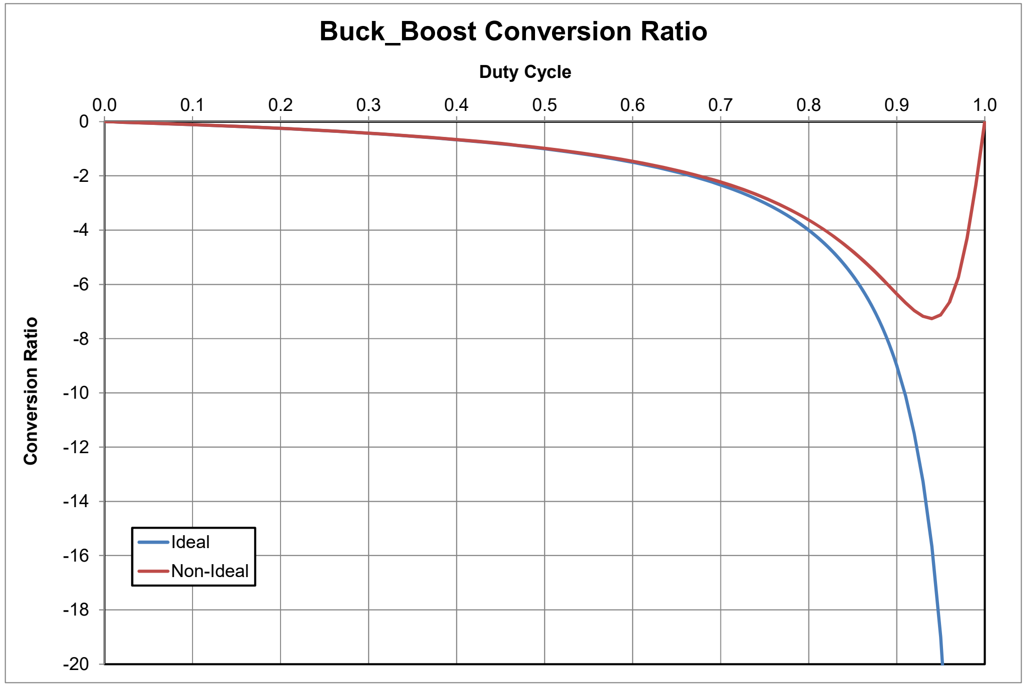
$$V_{IN} \cdot T_{ON} + V_{OUT} \cdot T_{OFF} = 0$$

$$D \cdot V_{IN} \cdot T_{SW} + V_{OUT} \cdot (1-D) \cdot T_{SW} = 0$$

$$D \cdot V_{IN} + (1-D) \cdot V_{OUT} = 0$$

$$(1-D) \cdot V_{OUT} = -D \cdot V_{IN}$$

$$V_{OUT} = -\frac{D}{1-D} \cdot V_{IN} = -\frac{D}{D'} \cdot V_{IN}$$



Conversion Ratio

Buck-Boost Has The Same Loss Limiting On The Maximum Boost Ratio As The Boost Converter

$$V_L(T_{ON})\cdot T_{ON}+V_L(T_{OFF})\cdot T_{OFF}=0$$

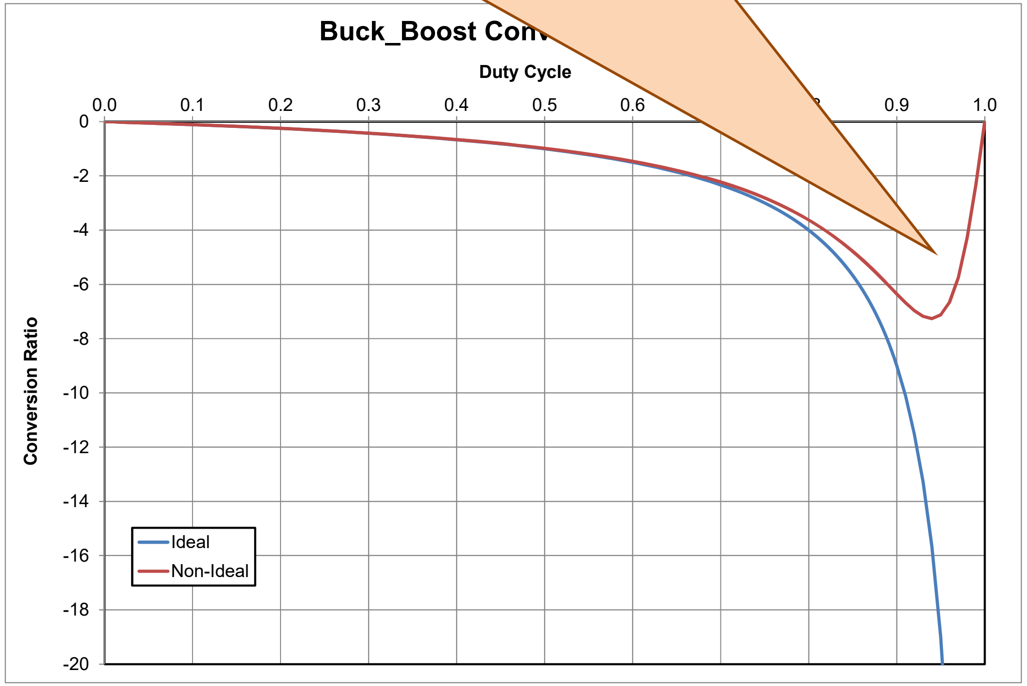
$$V_{IN}\cdot T_{ON}+V_{OUT}\cdot T_{OFF}=0$$

$$D\cdot V_{IN}\cdot T_{SW}+V_{OUT}\cdot(1-D)\cdot T_{SW}=0$$

$$D\cdot V_{IN}+(1-D)\cdot V_{OUT}=0$$

$$(1-D)\cdot V_{OUT}=-D\cdot V_{IN}$$

$$V_{OUT}=-\frac{D}{1-D}\cdot V_{IN}=-\frac{D}{D'}\cdot V_{IN}$$

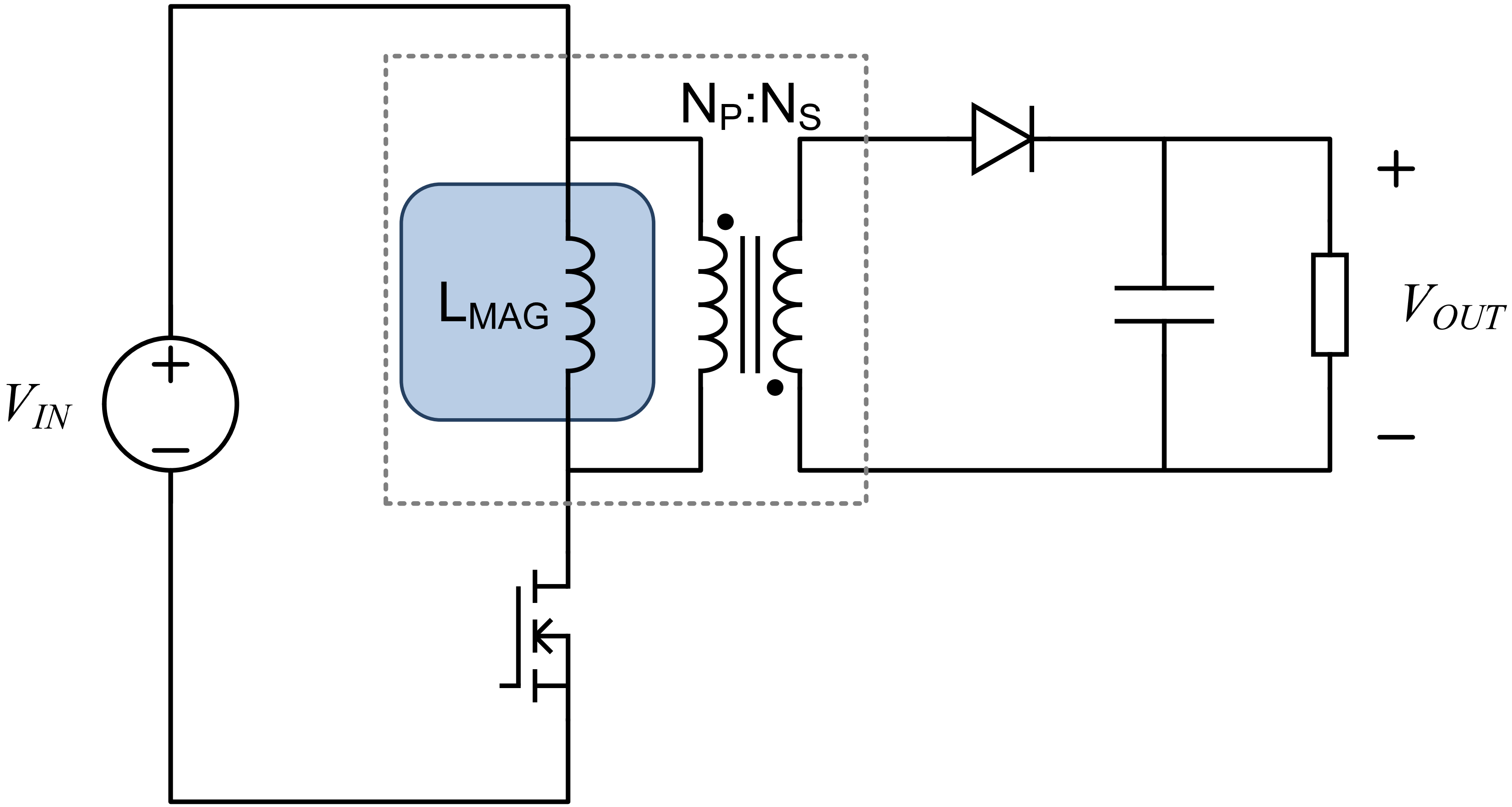


APPENDIX VII.
DCM Flyback Converter
Deeper Dive

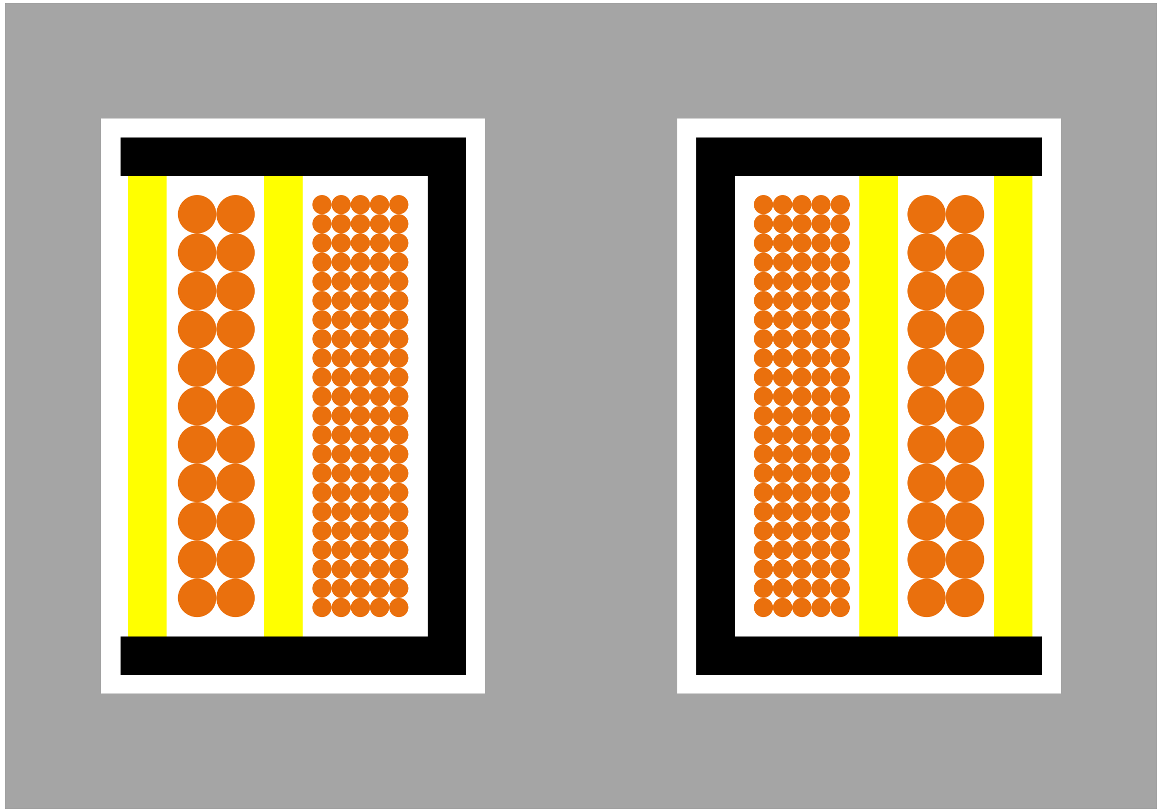
DCM Flyback Deeper Dive

- If You Are Going To Use The Flyback Converter, Important To Understand Effect Of:
  - Leakage Inductance
  - Stray Capacitances
- Leakage Inductance Effects:
  - Loss Of Efficiency
  - Needs A Clamp Circuit
  - Higher Switch Transistor Voltage Rating

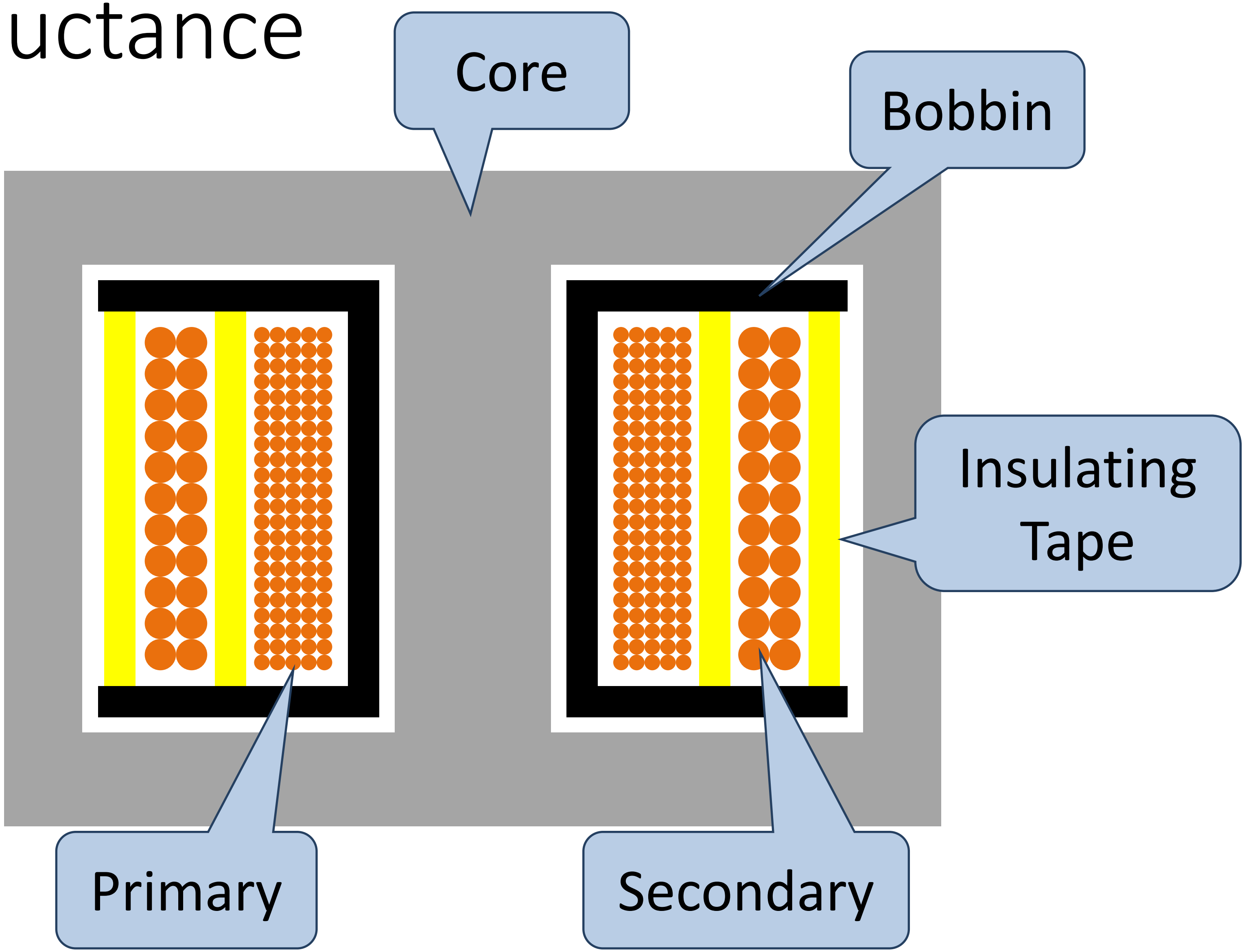
Flyback Converter



Leakage Inductance

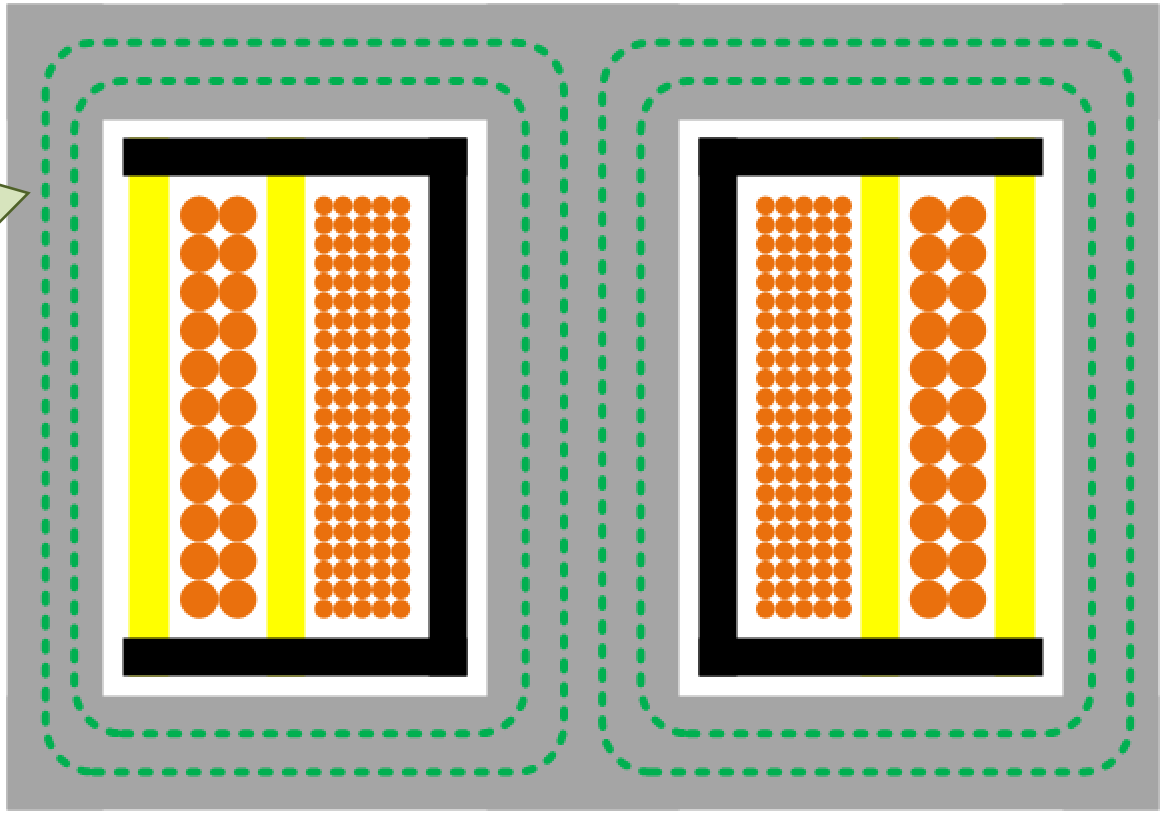


Leakage Inductance



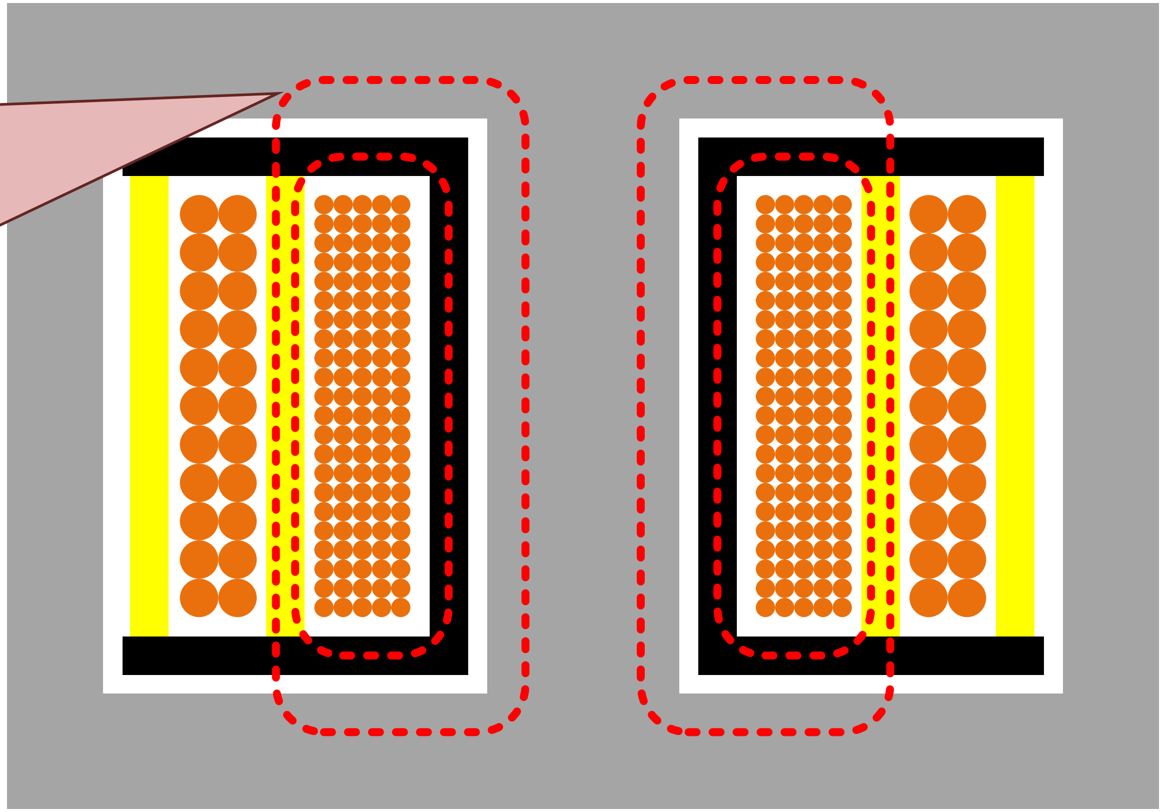
Leakage Inductance

Magnetic Flux In the Core Coupling Both Windings



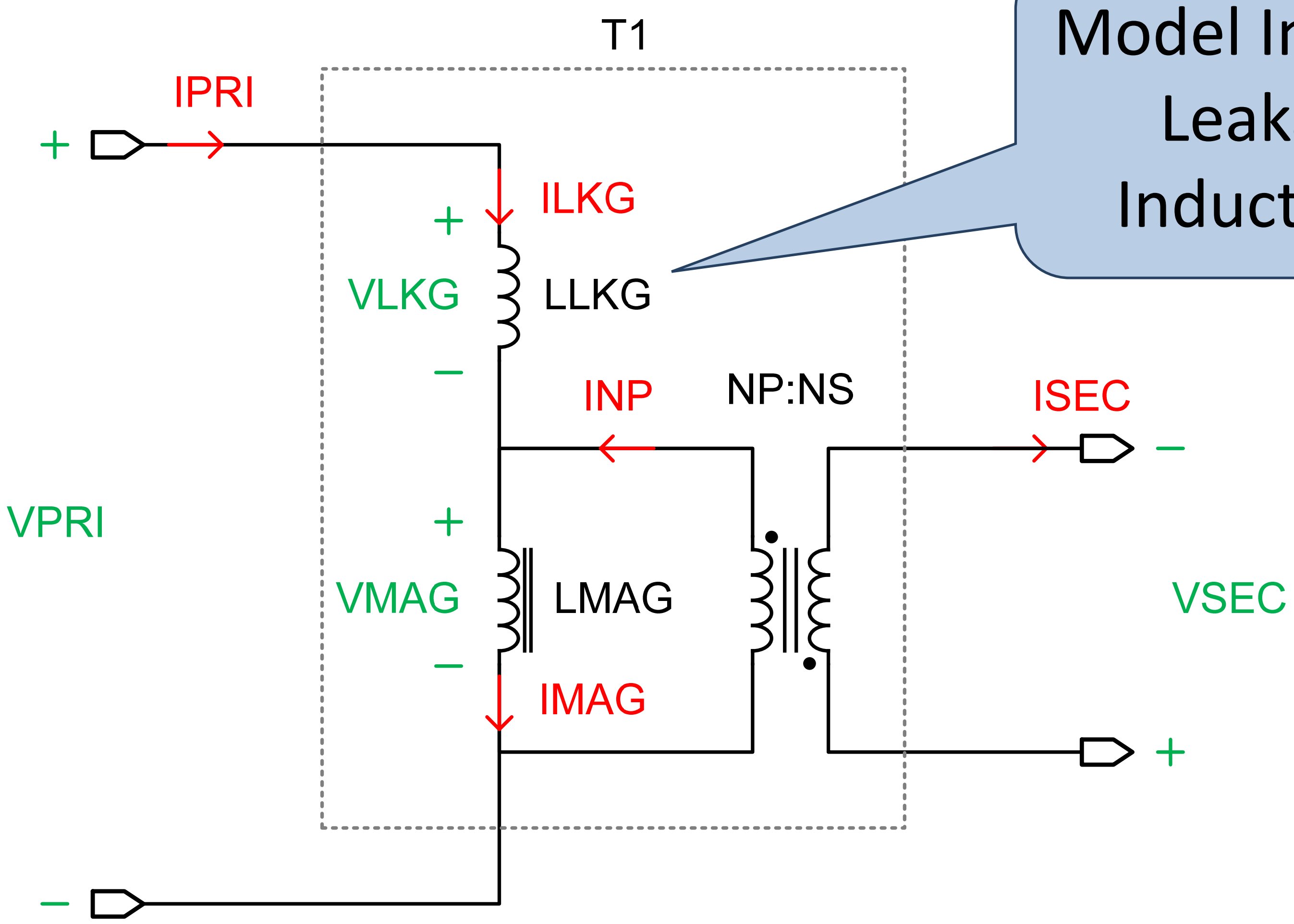
Leakage Inductance

Magnetic Flux In the Core Coupled Only To The Primary Winding



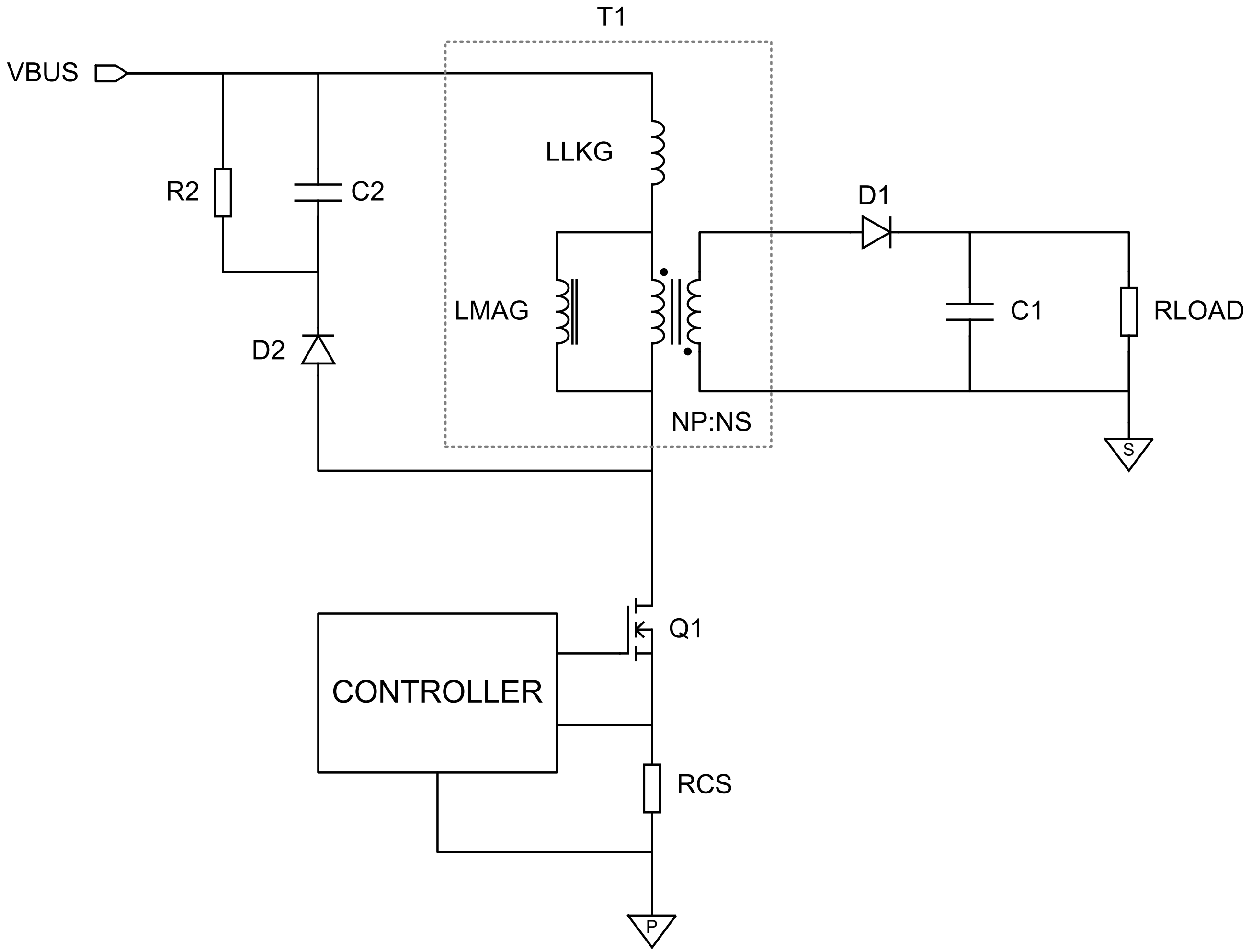
This “Leakage Flux” Results In The “Leakage Inductance”

Transformer With Leakage Inductance Model

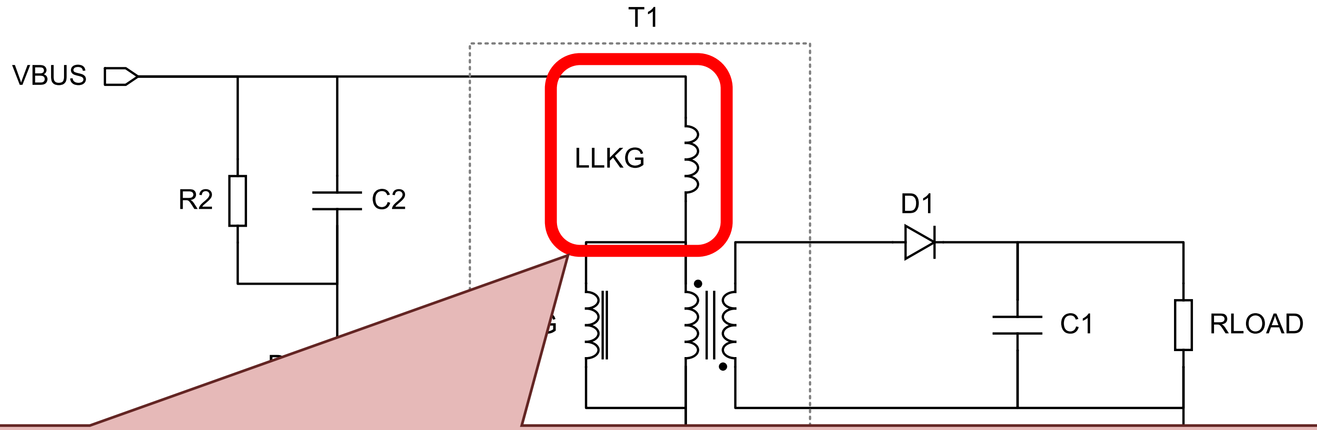


Model Includes Leakage Inductance

Flyback With Leakage Inductance



Flyback With Leakage Inductance



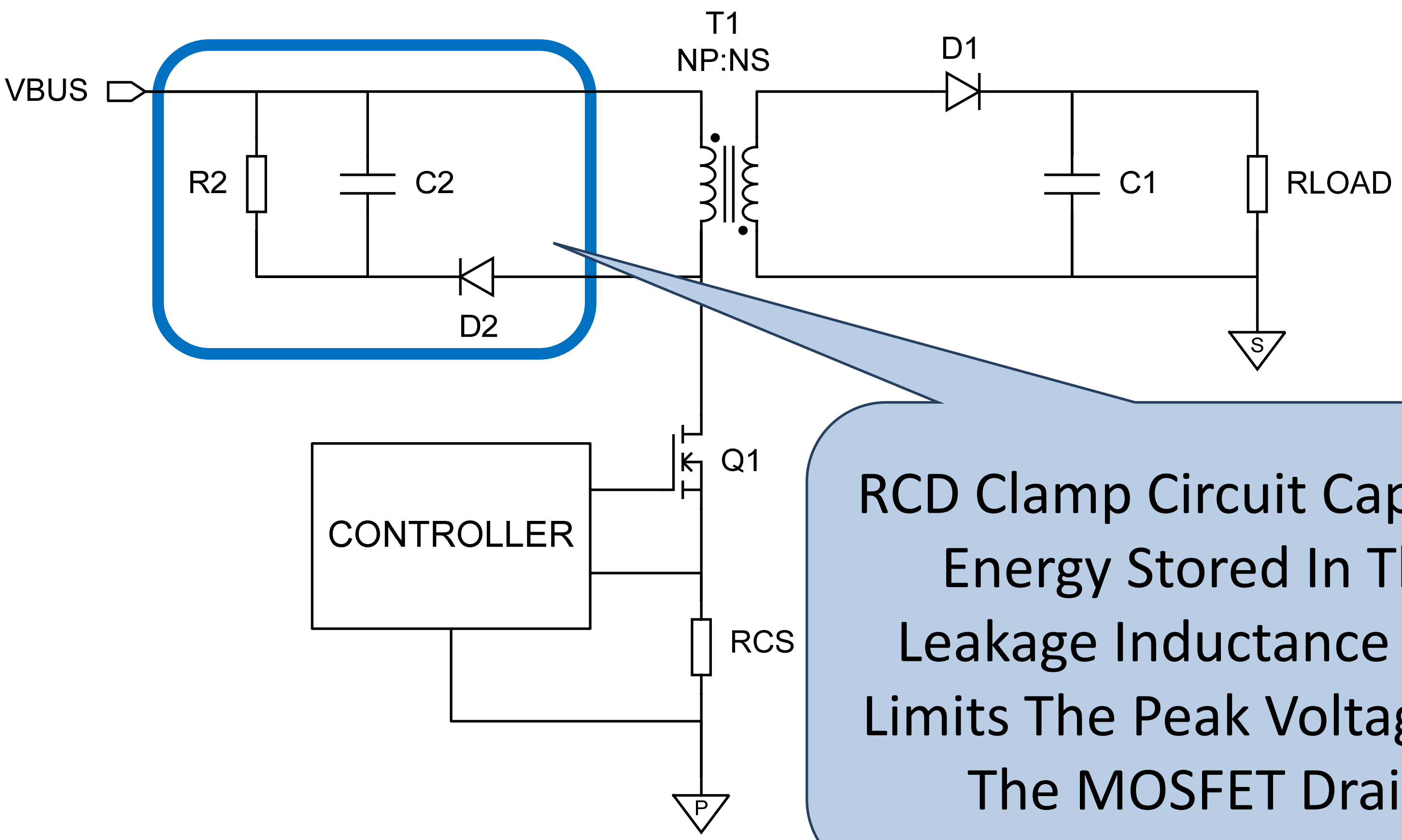
The Leakage Inductance Causes Many Problems In A Flyback Converter Including:

Significant Loss Of Power/Efficiency

Need For Clamping Circuit To Absorb The Energy Stored In The Leakage Inductance

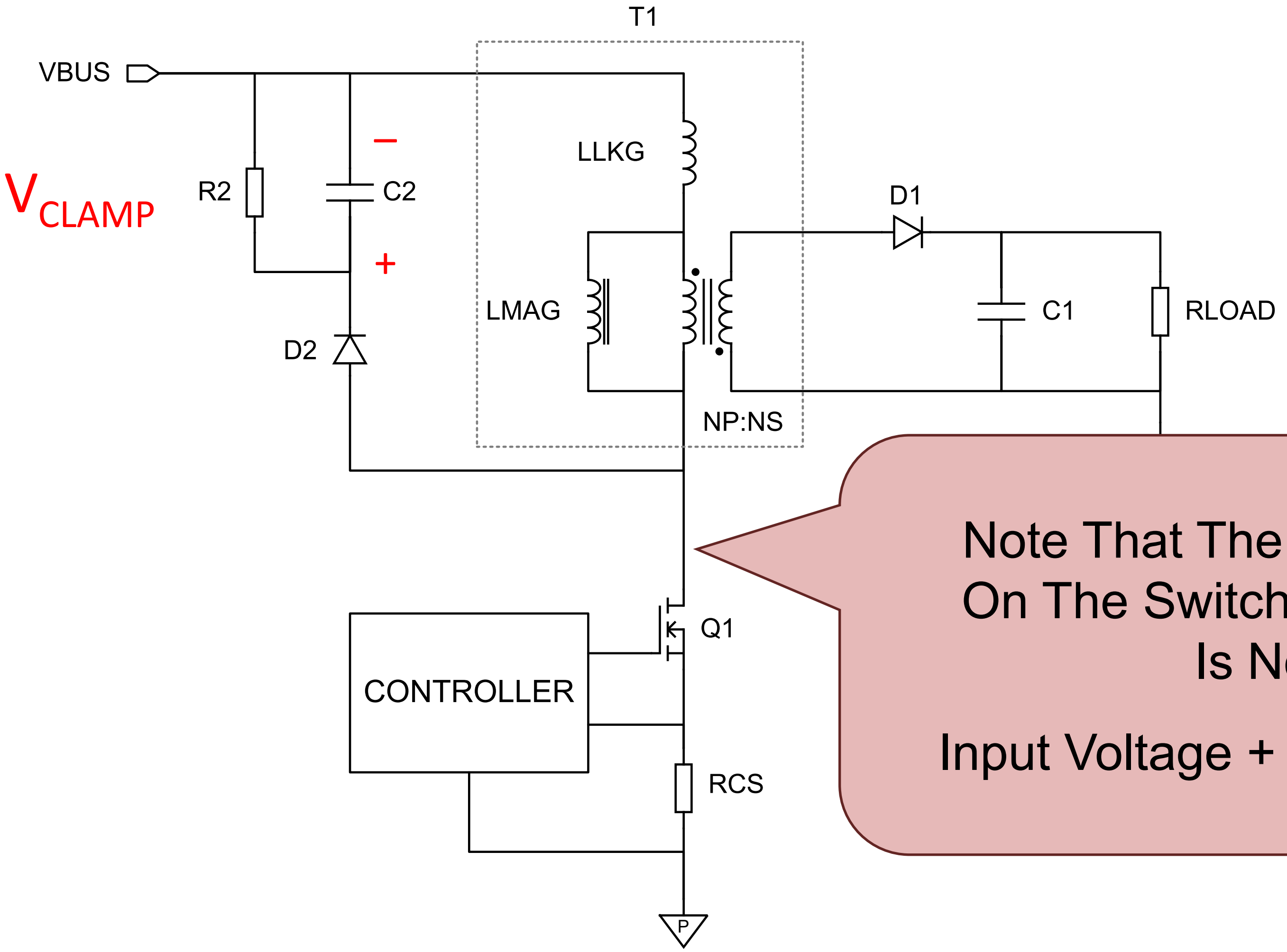
Much Higher Peak Voltage On Switching Transistor

Basic Flyback Circuit With RCD Clamp



RCD Clamp Circuit Captures Energy Stored In The Leakage Inductance And Limits The Peak Voltage On The MOSFET Drain

Flyback With Leakage Inductance

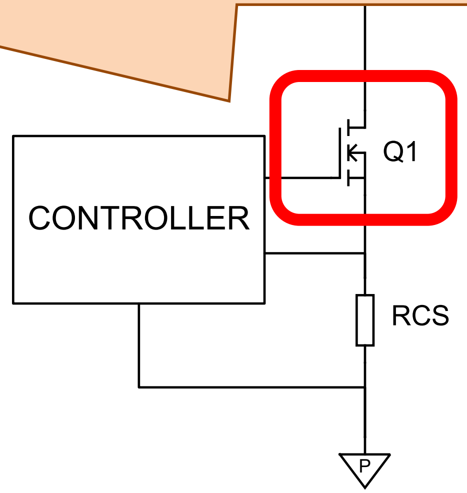


Note That The Peak Voltage On The Switching Transistor Is Now:

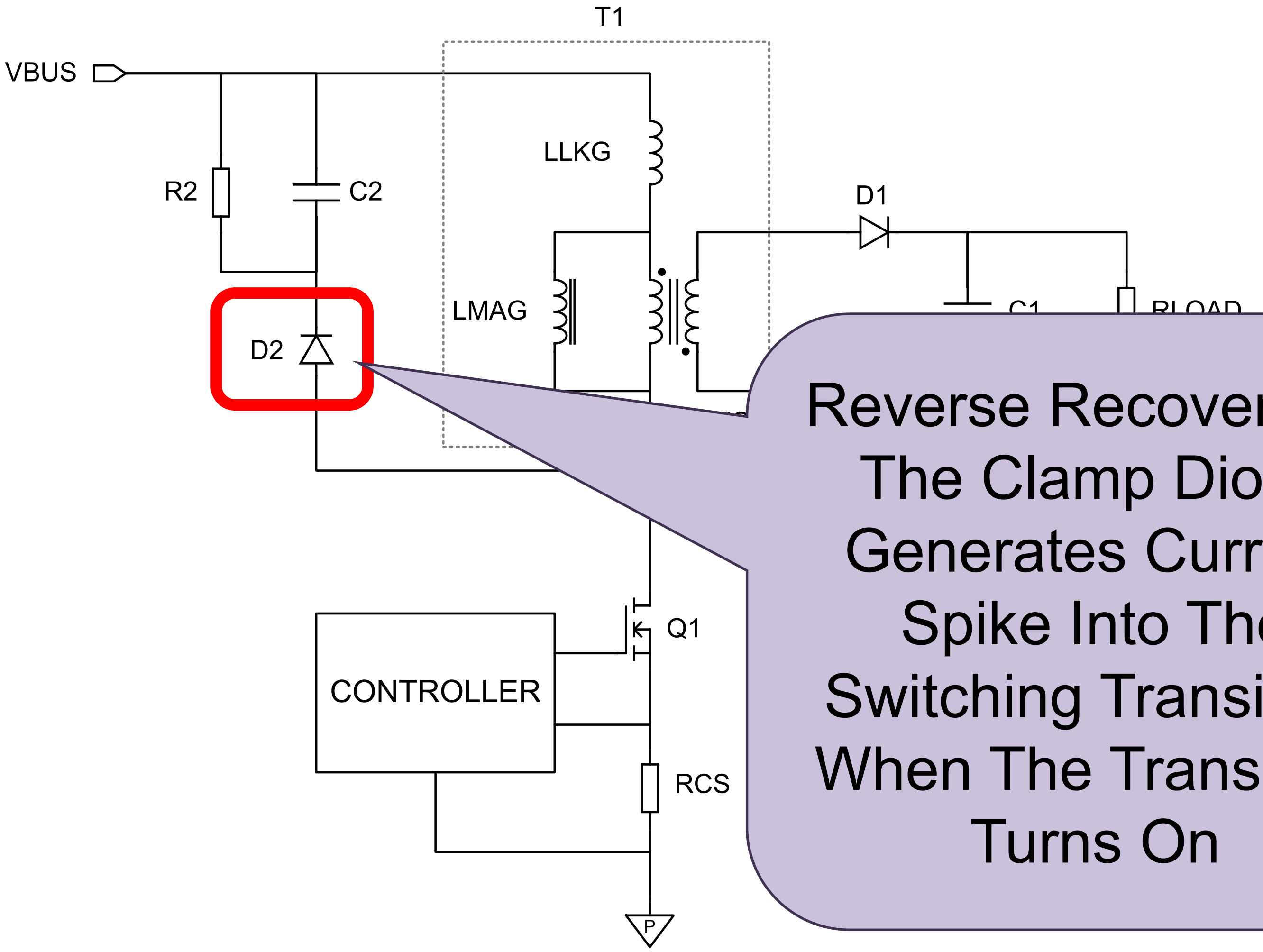
Input Voltage + Clamp Voltage

Flyback With Leakage Inductance

Semiconductor Capacitances,
Such As The Output Capacitance Of The Switching Transistor (C_{OSS}) And The Inter-Winding Capacitance Of The Flyback Inductor, Allow Significant Ringing With The Leakage And Magnetizing Inductance.
This Can Be A Major Source Of EMI.

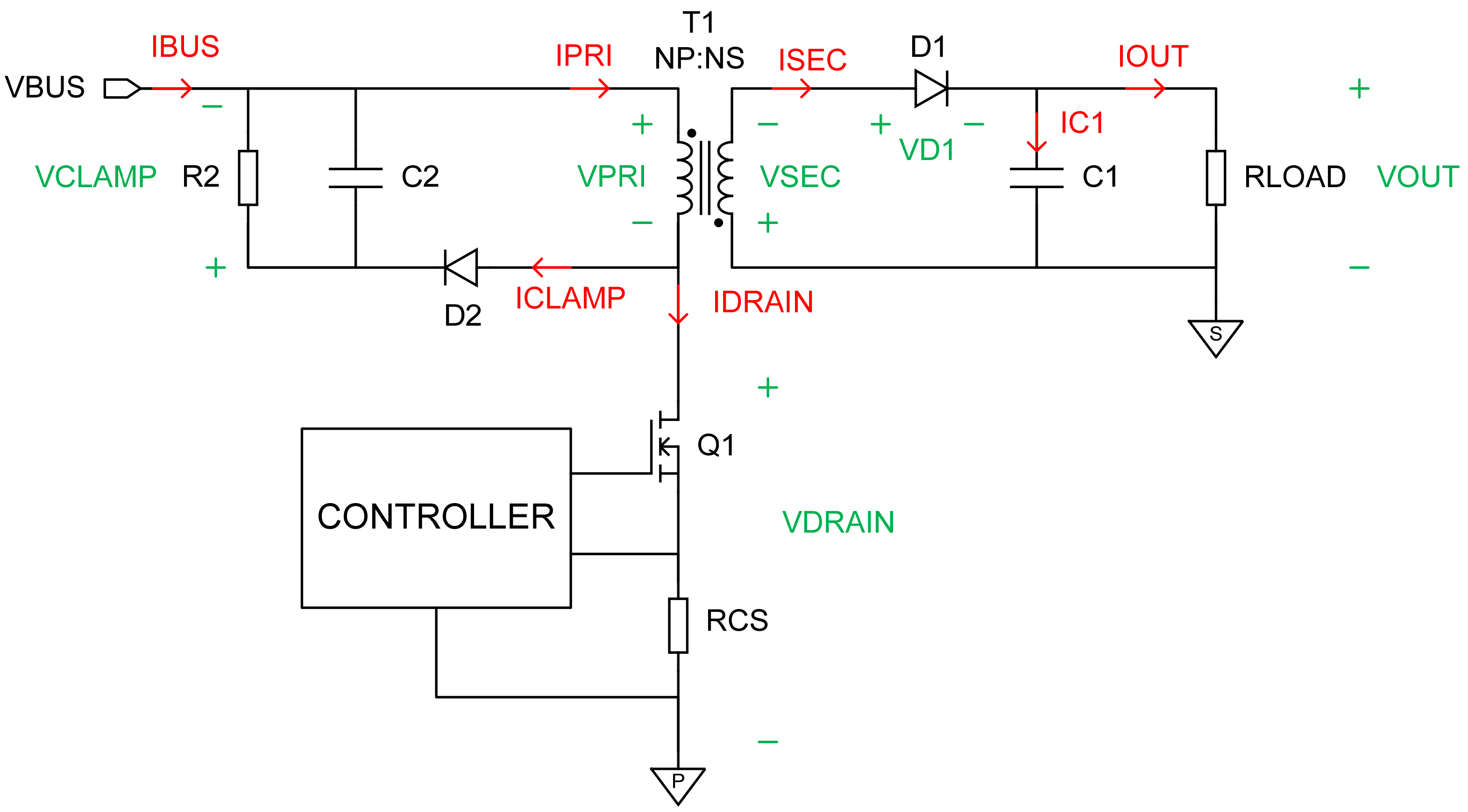


Flyback With Leakage Inductance

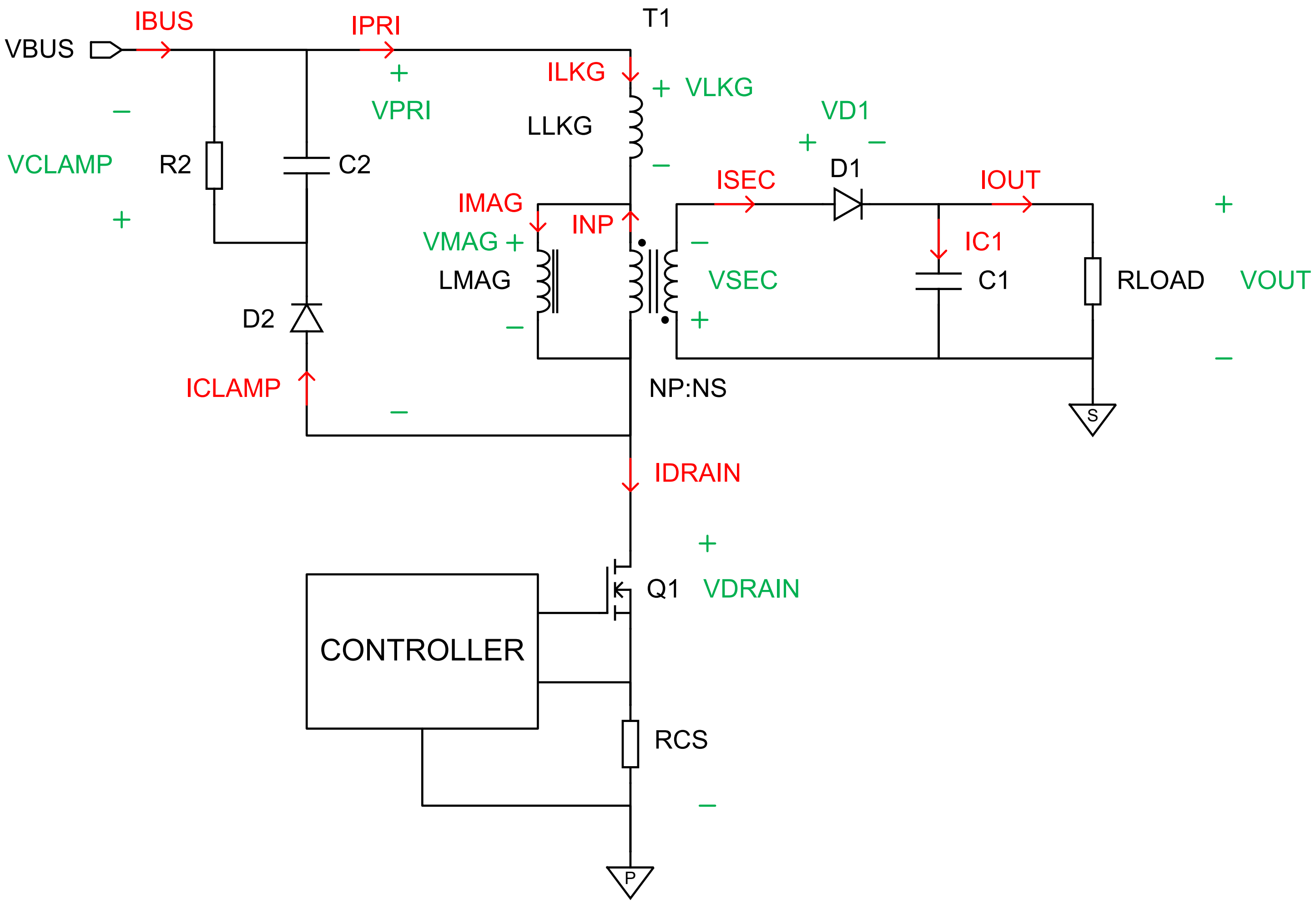


Reverse Recovery Of The Clamp Diode Generates Current Spike Into The Switching Transistor When The Transistor Turns On

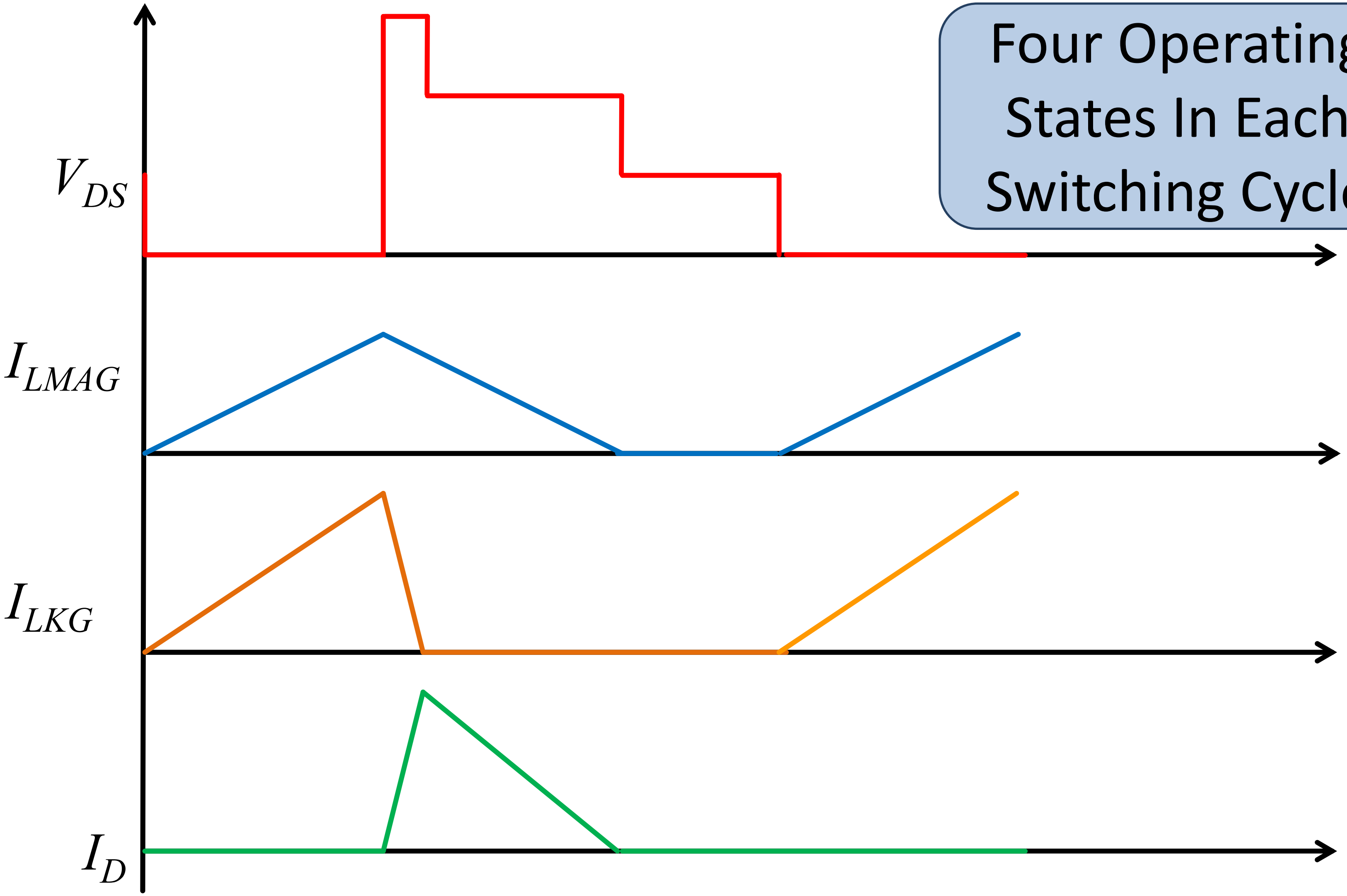
Basic Flyback Circuit With RCD Clamp



Flyback With Leakage Inductance



Discontinuous Conduction Mode



Four Operating States In Each Switching Cycle

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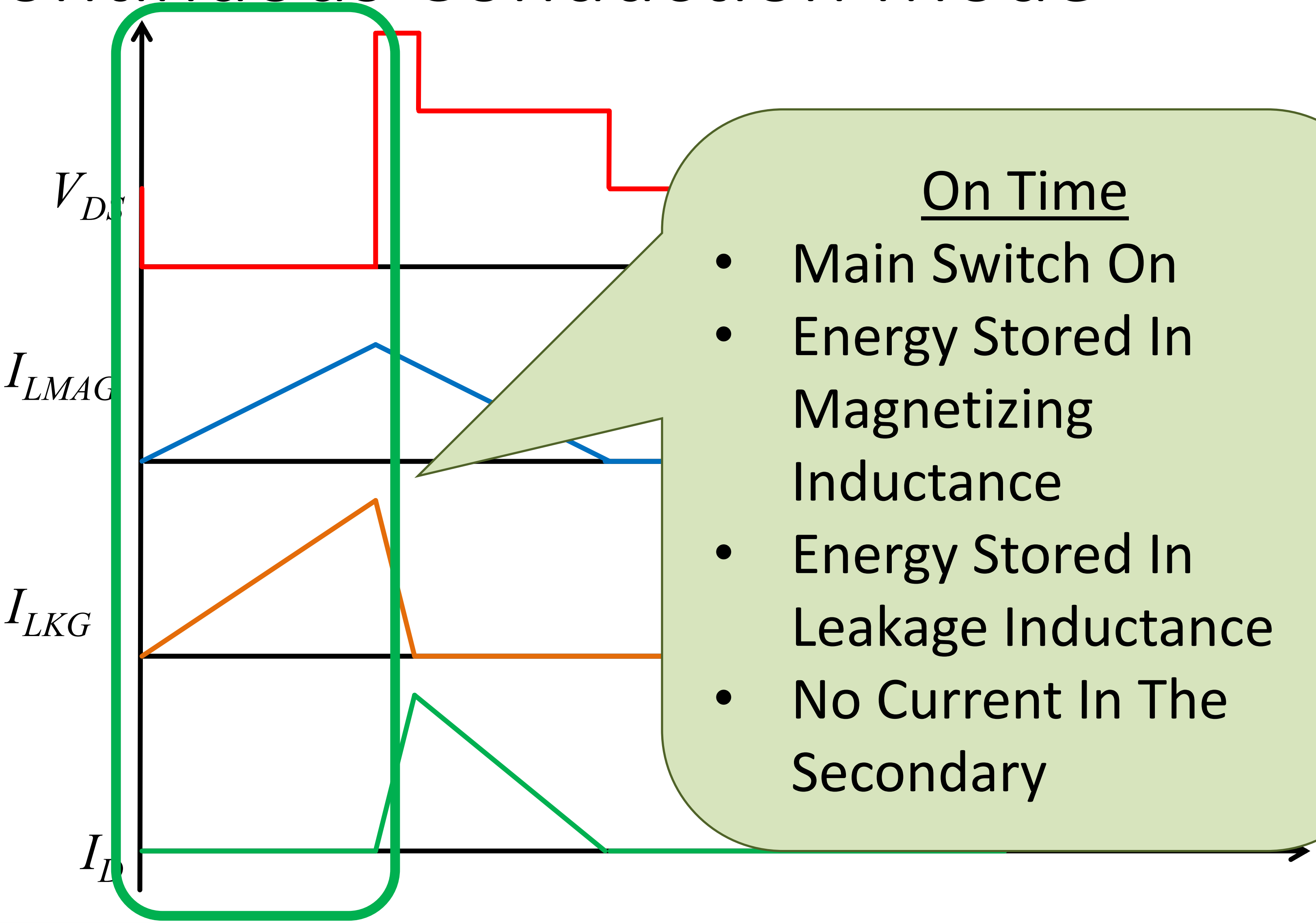
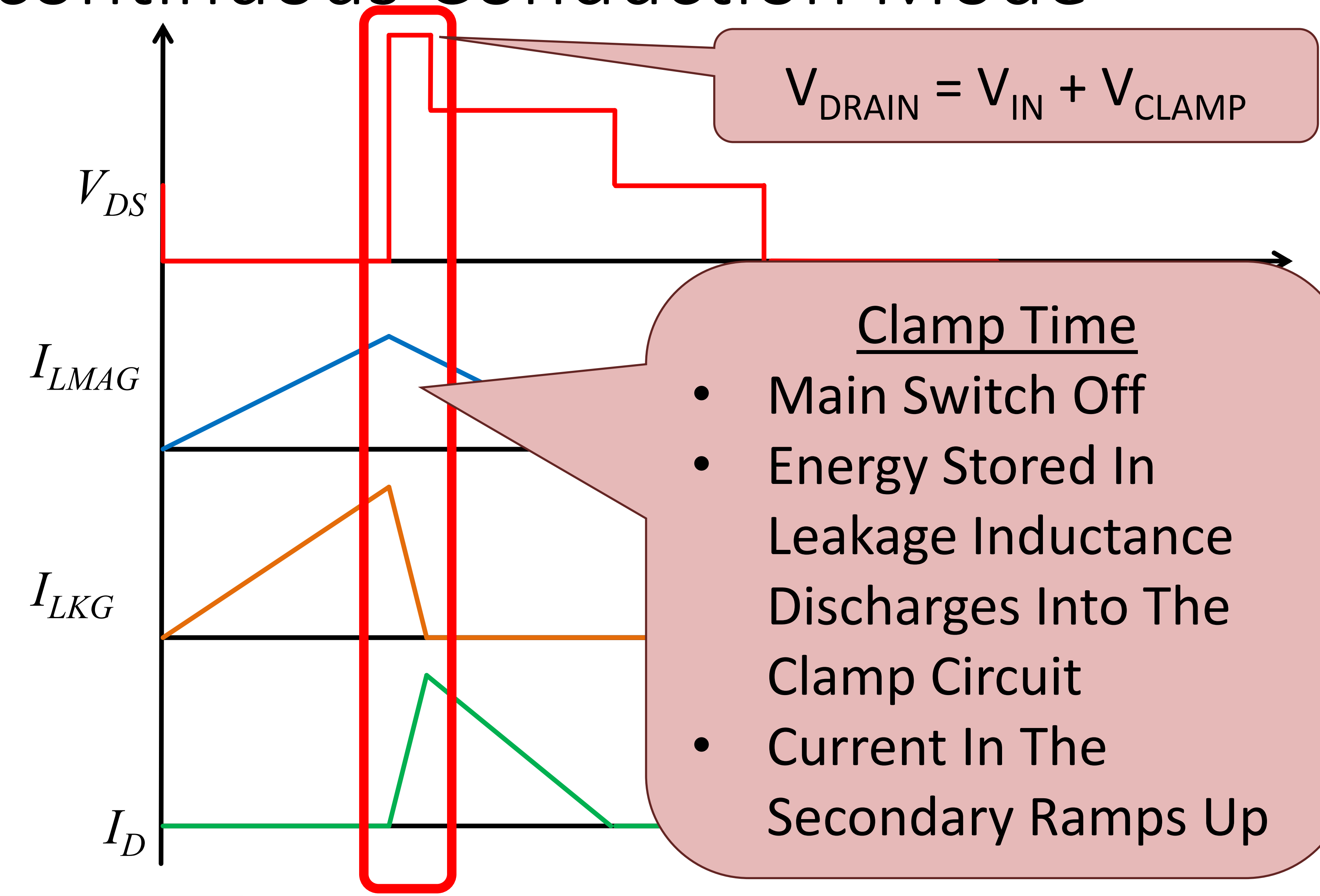
Discontinuous Conduction Mode



On Time

- Main Switch On
- Energy Stored In Magnetizing Inductance
- Energy Stored In Leakage Inductance
- No Current In The Secondary

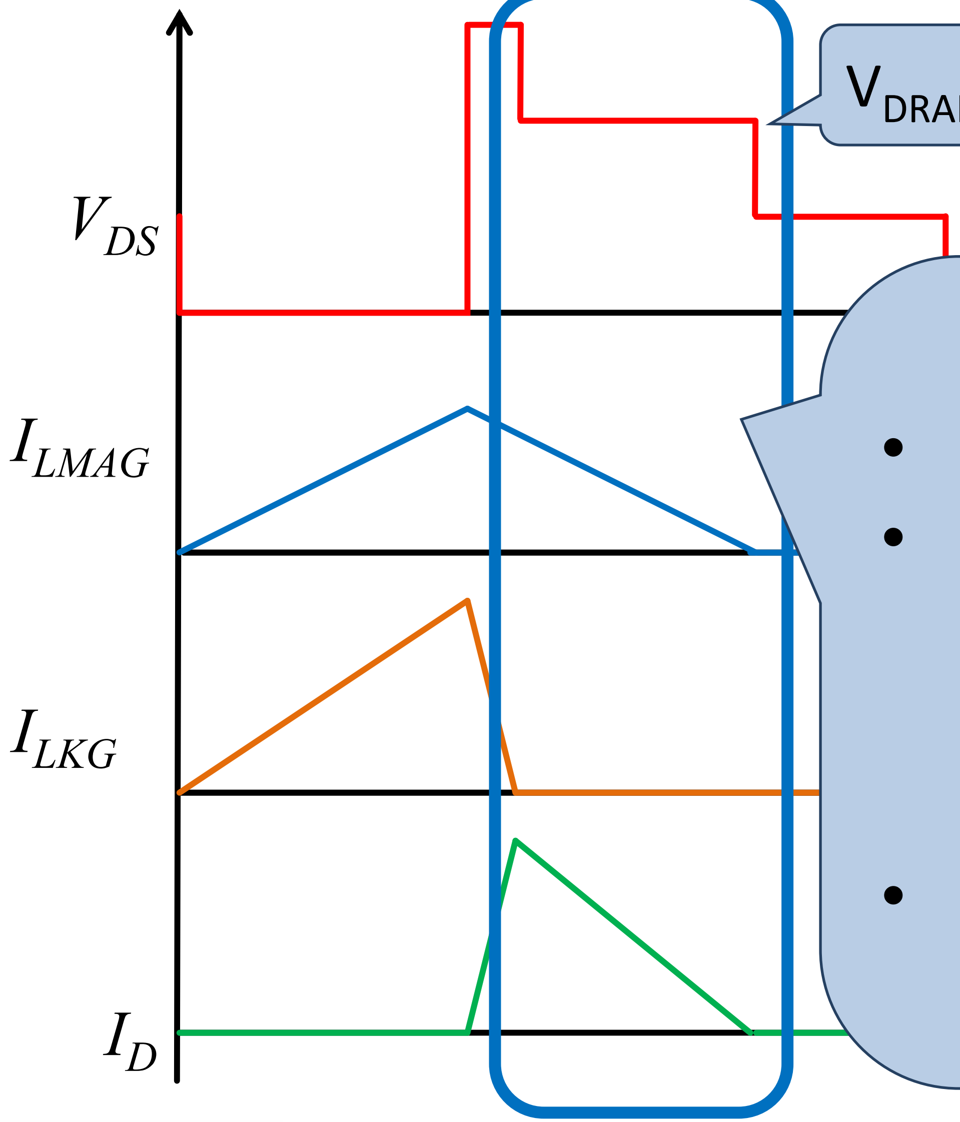
Discontinuous Conduction Mode



Clamp Time

- Main Switch Off
- Energy Stored In Leakage Inductance Discharges Into The Clamp Circuit
- Current In The Secondary Ramps Up

Discontinuous Conduction Mode

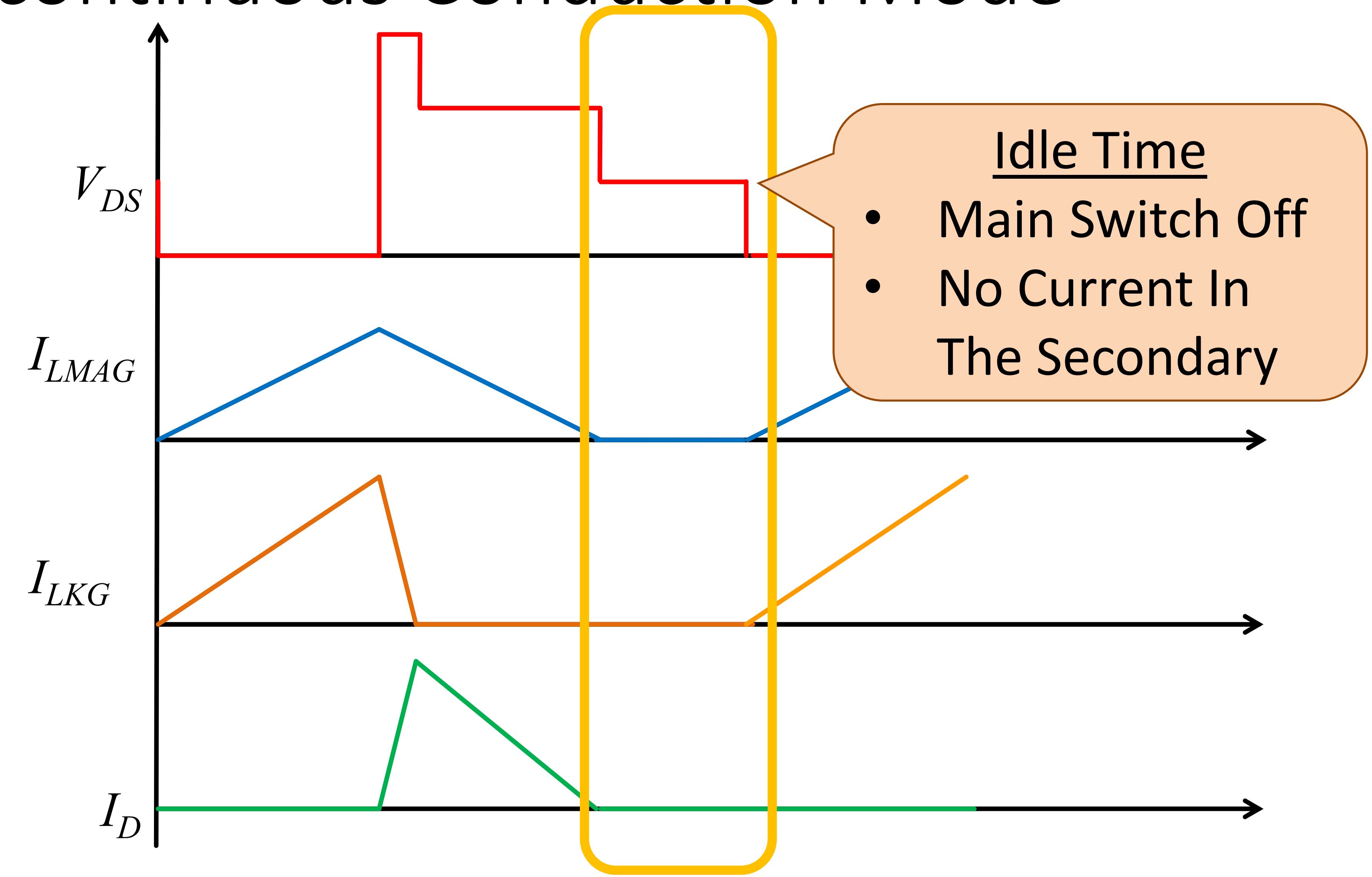


$V_{DRAIN} = V_{IN} + (NP/NS) \cdot (V_{OUT} + V_{DIODE})$

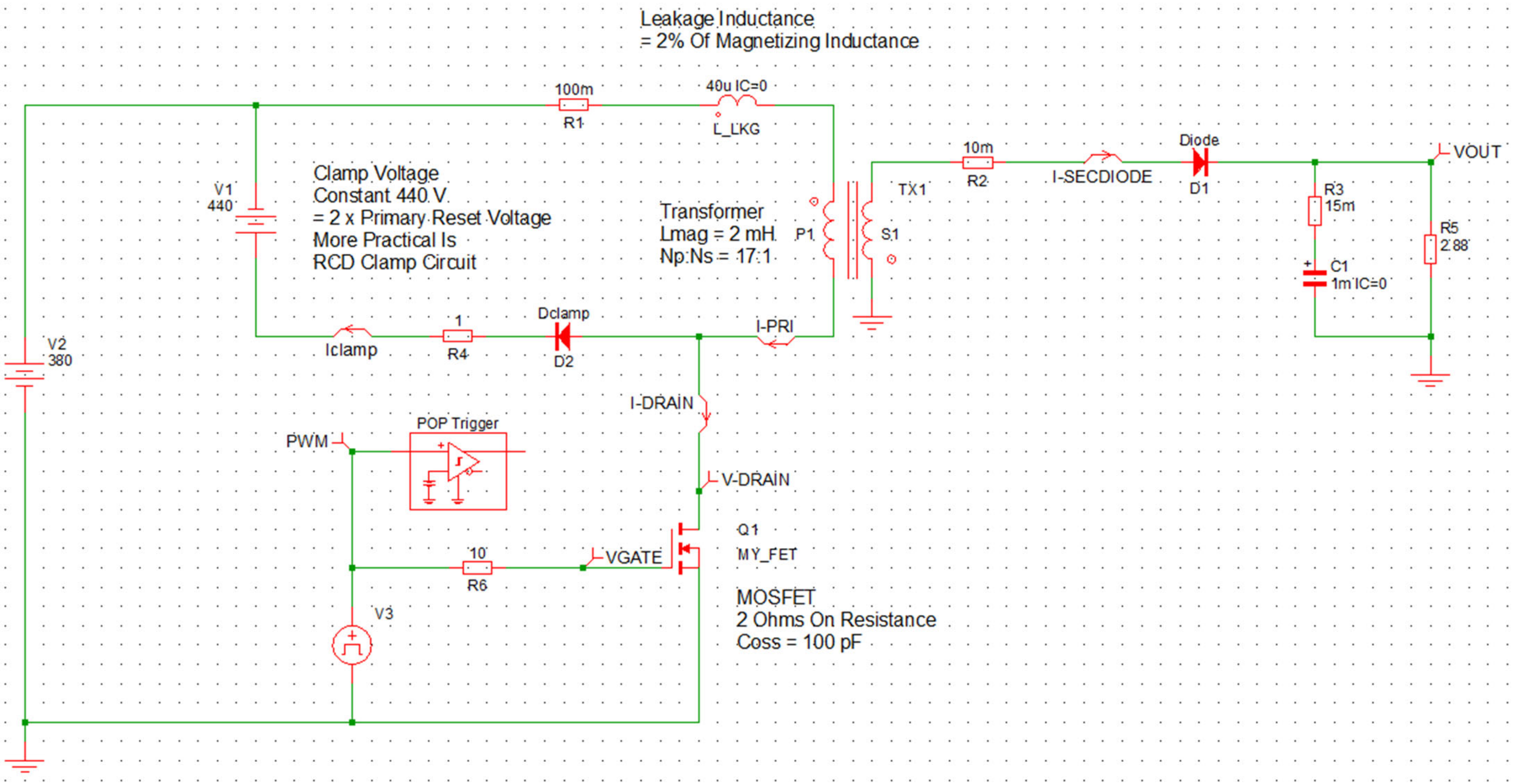
Reset Time

- Main Switch Off
- Energy Stored In Magnetizing Inductance Discharges Into The Output
- No Current In The Primary

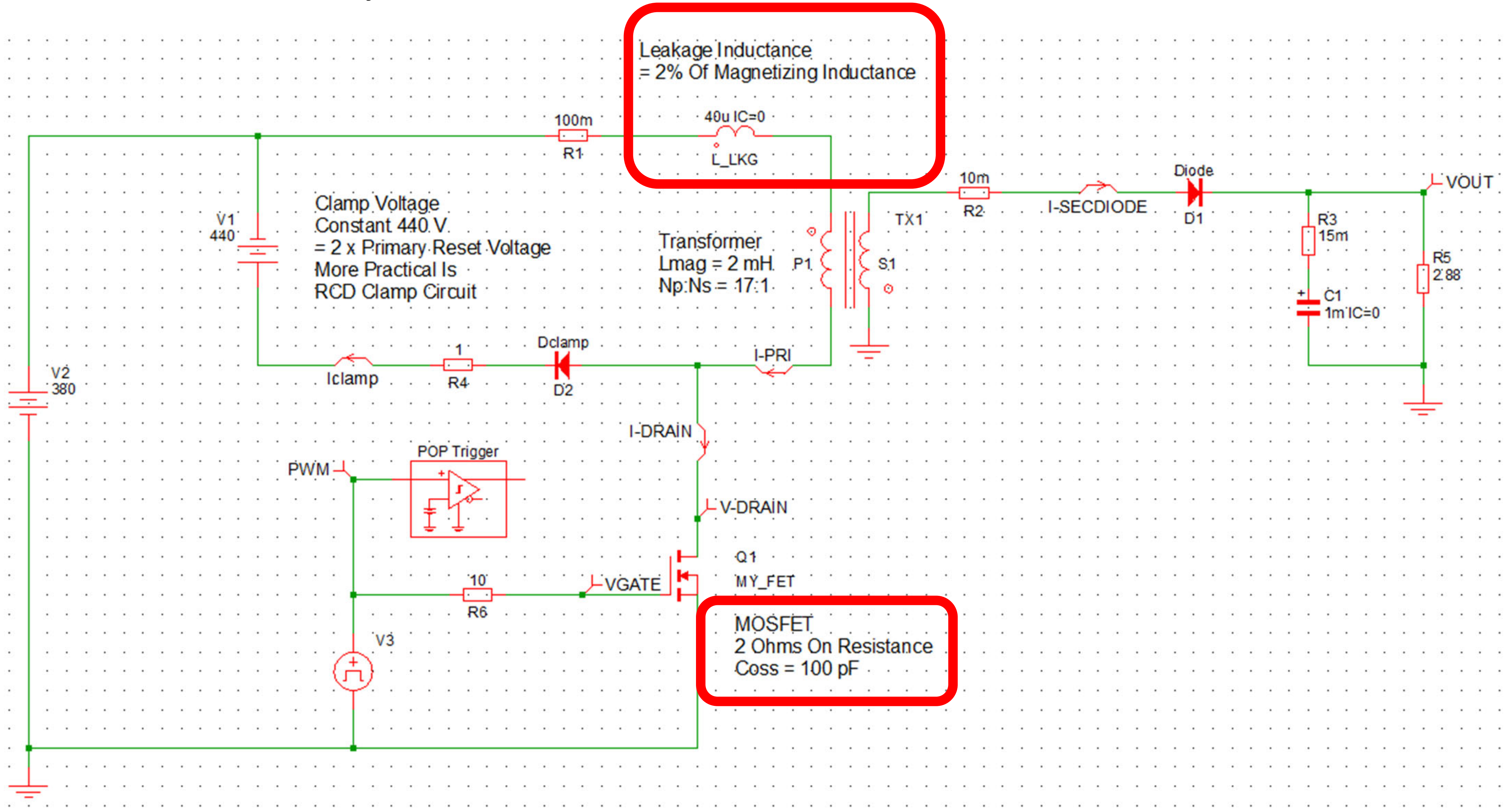
Discontinuous Conduction Mode



Discontinuous Conduction Mode
More Realistic Flyback Simulation

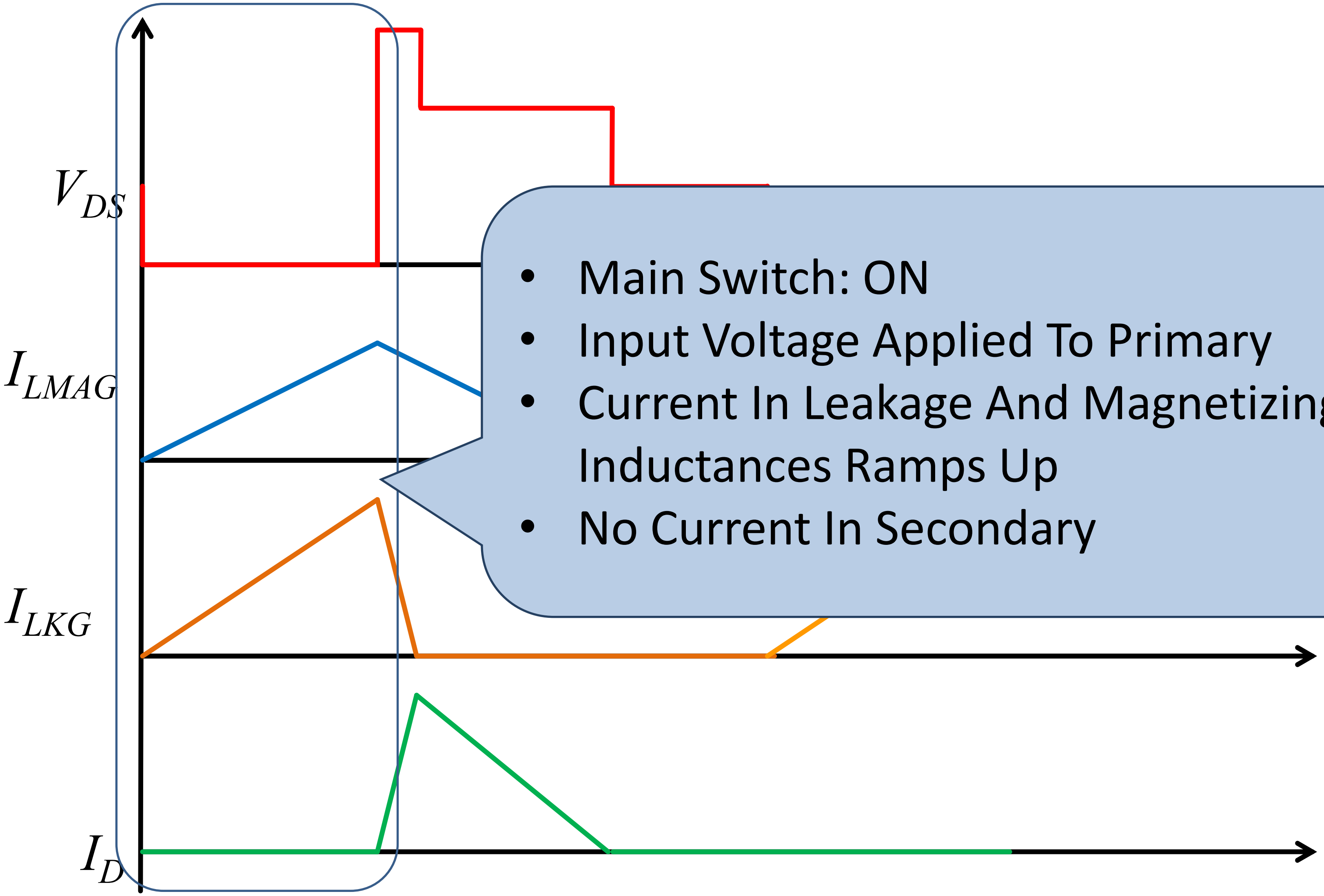


Discontinuous Conduction Mode
More Realistic Flyback Simulation



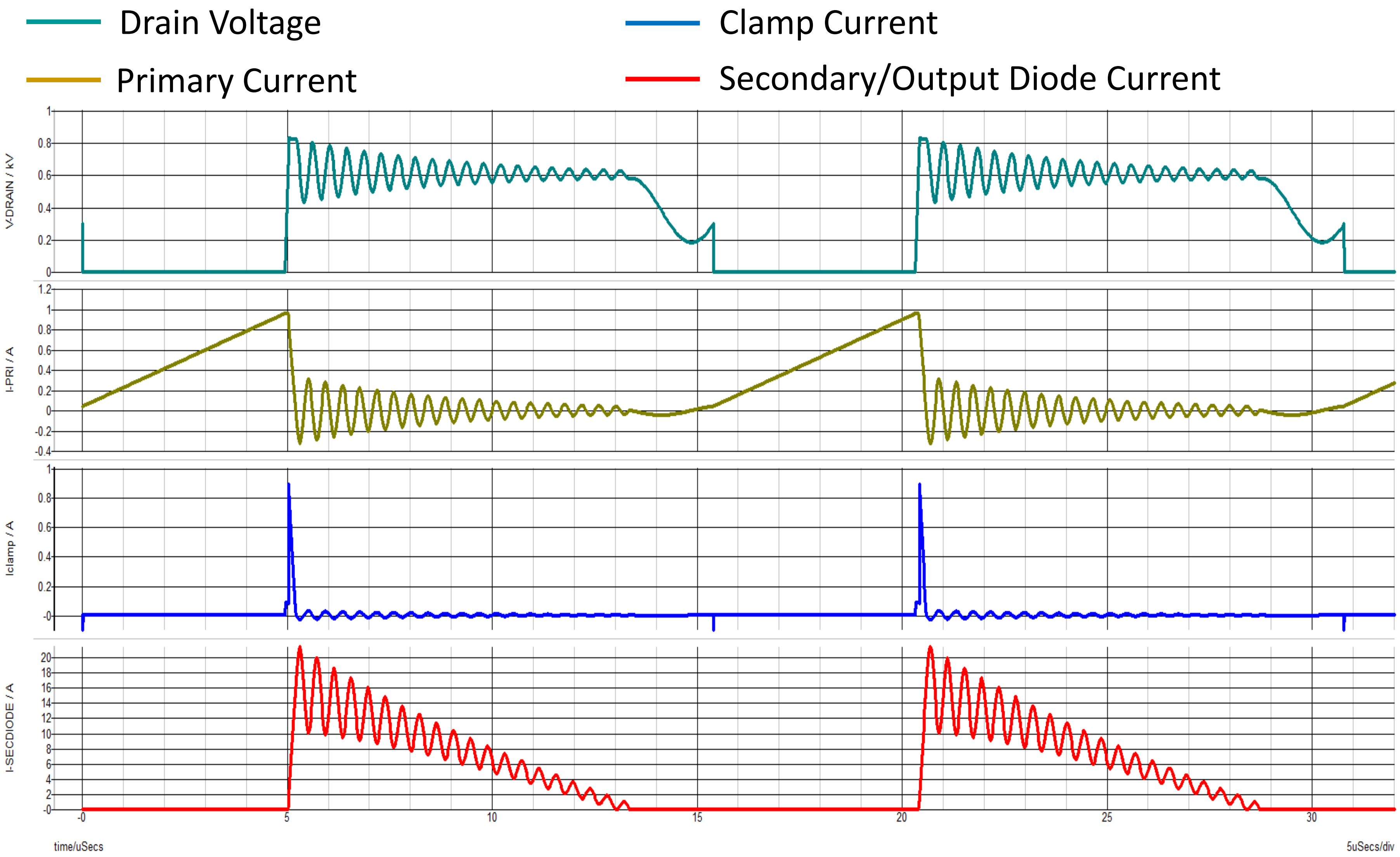
DCM On Time

DCM: On Time

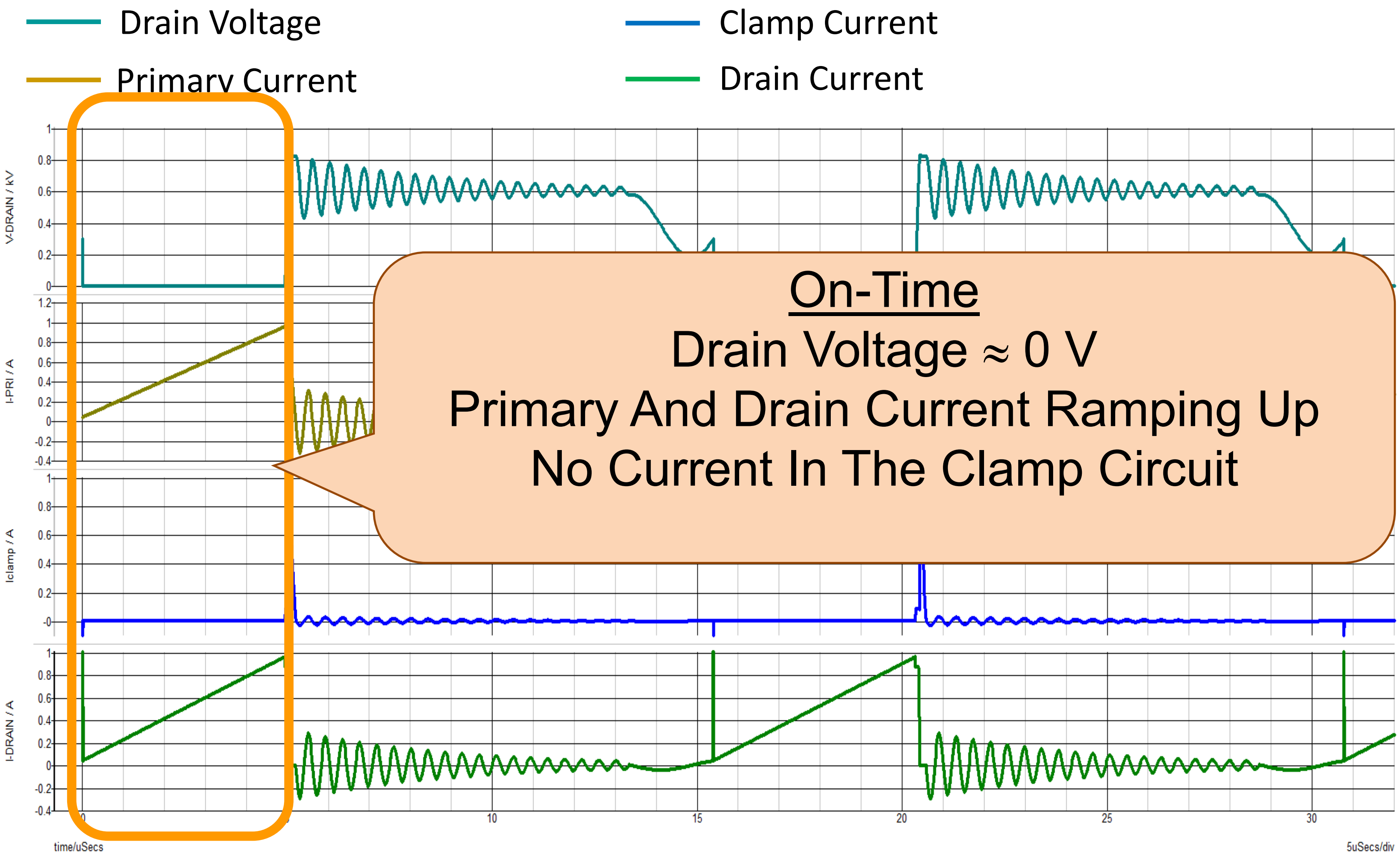


- Main Switch: ON
- Input Voltage Applied To Primary
- Current In Leakage And Magnetizing Inductances Ramps Up
- No Current In Secondary

Discontinuous Conduction Mode
More Realistic Flyback Simulation



Discontinuous Conduction Mode
More Realistic Flyback Simulation



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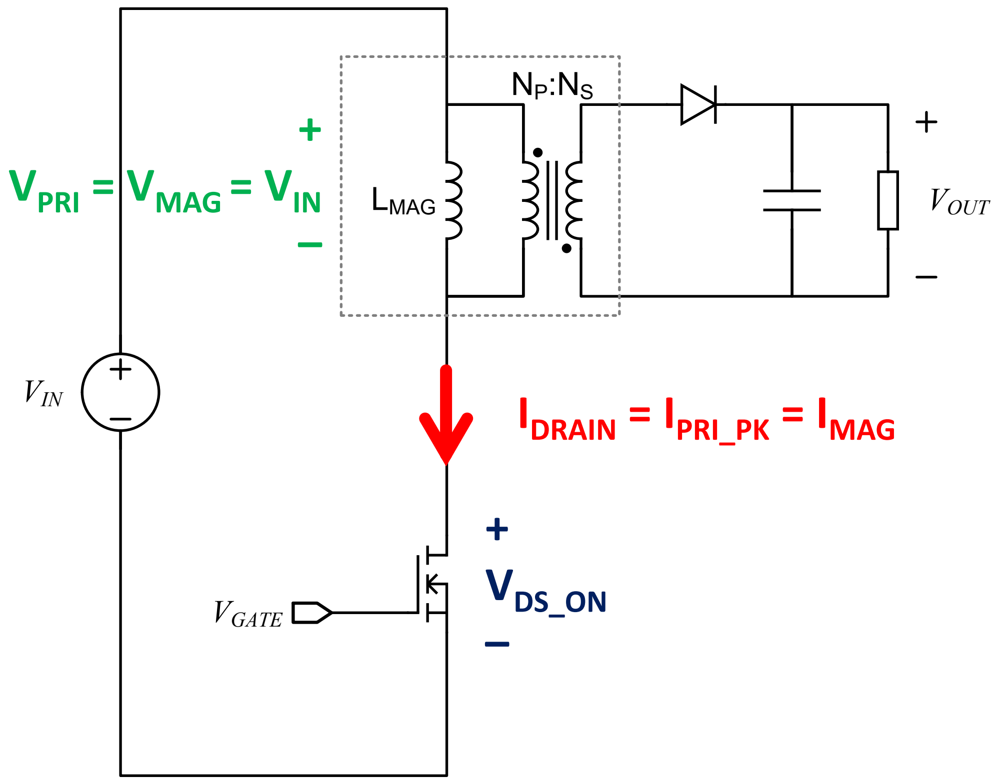
DCM Clamping Interval

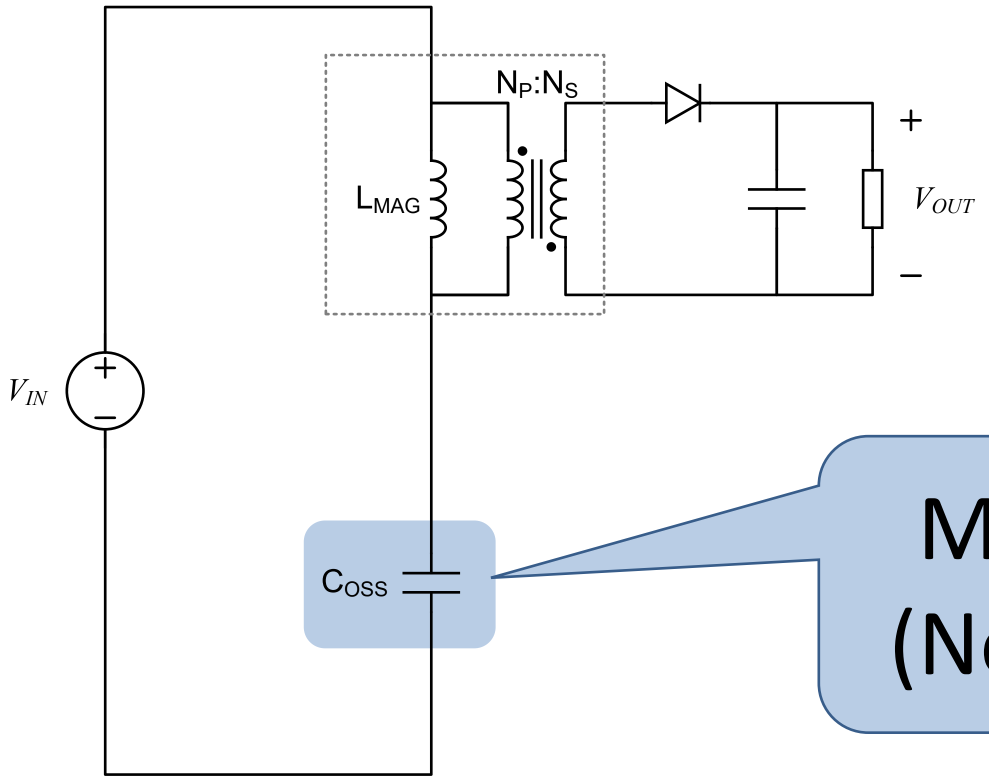
Just Before MOSFET Turns Off



How Does This Circuit Model Change When The MOSFET Is Turned Off?

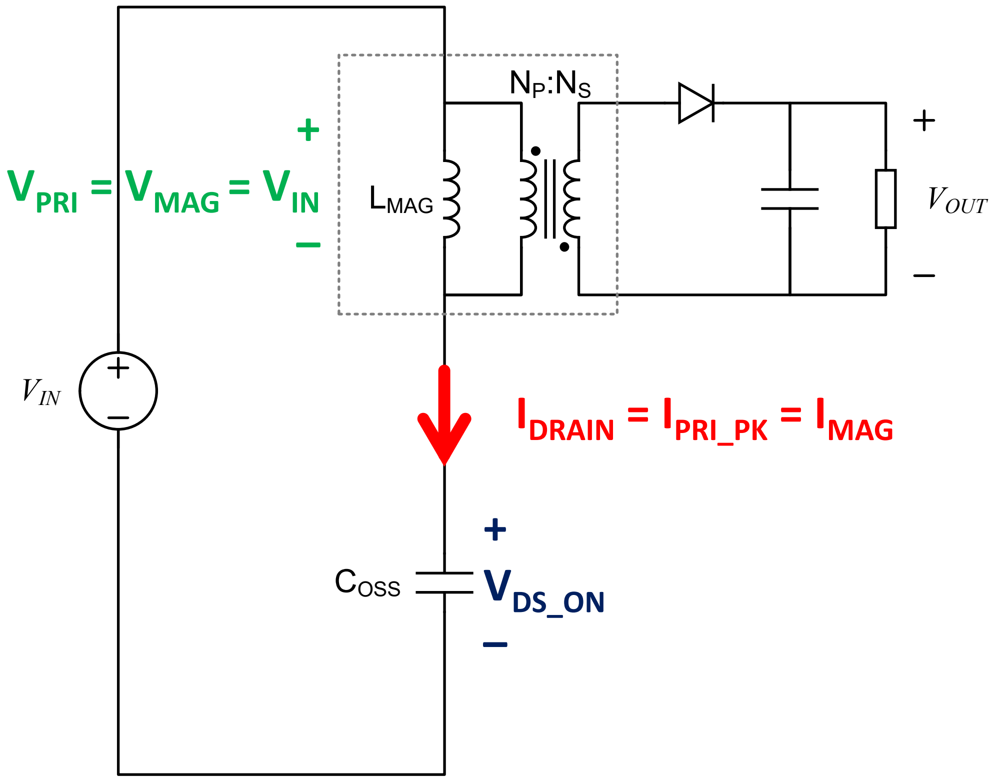
Ignoring L_{LKG} For The Moment…

Just After MOSFET Turned Off



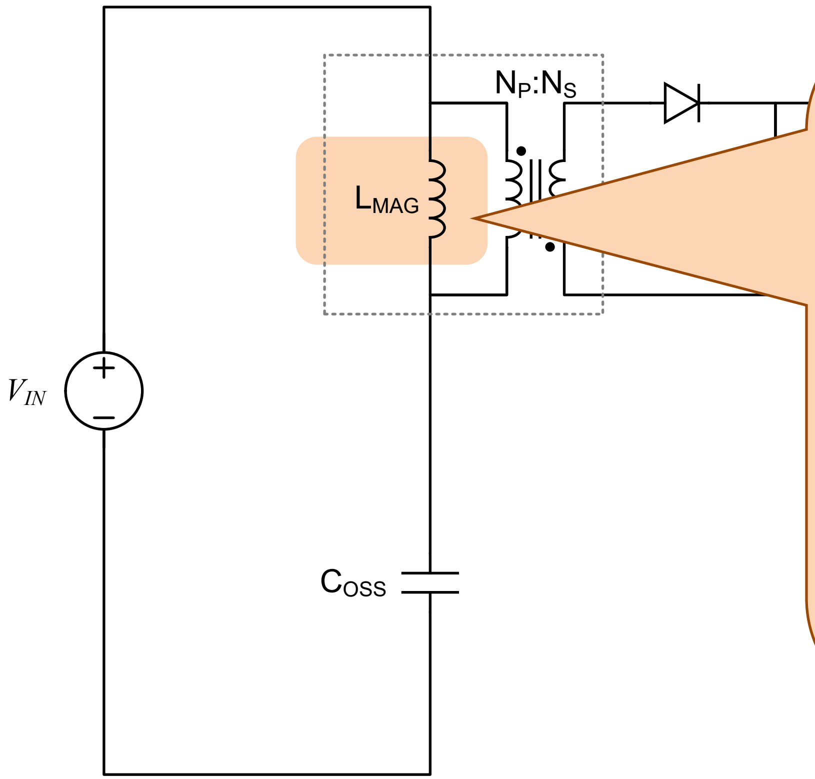
MOSFET Becomes A (Nonlinear) Capacitor

Just After MOSFET Turned Off



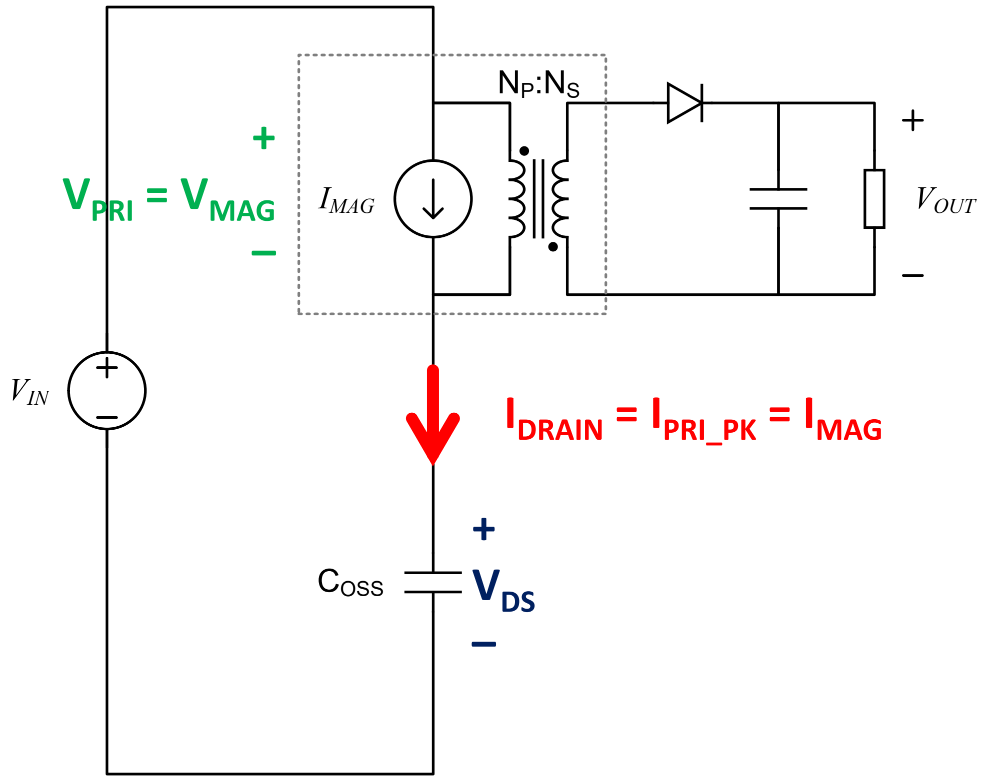
L_{MAG} Rings With C_{OSS}

Just After MOSFET Turned Off

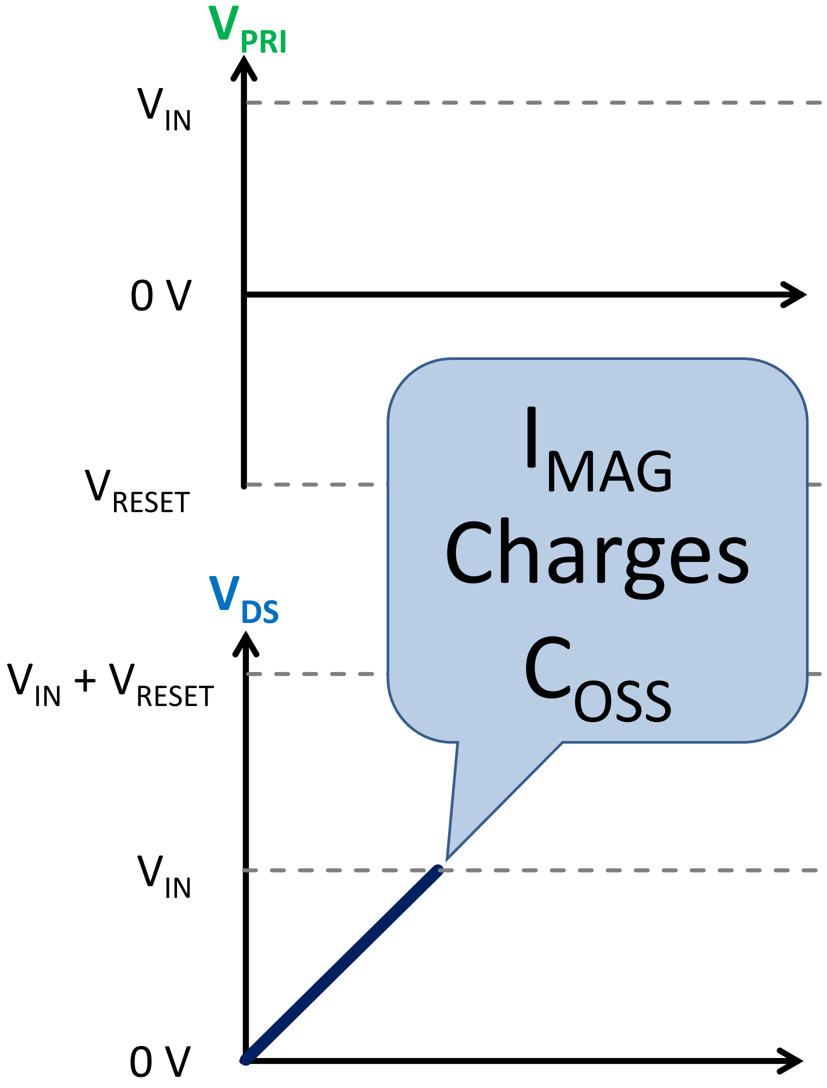


During Time Interval Of Interest I_{MAG} Does Not Change Much. We Model It As A Constant Current Source

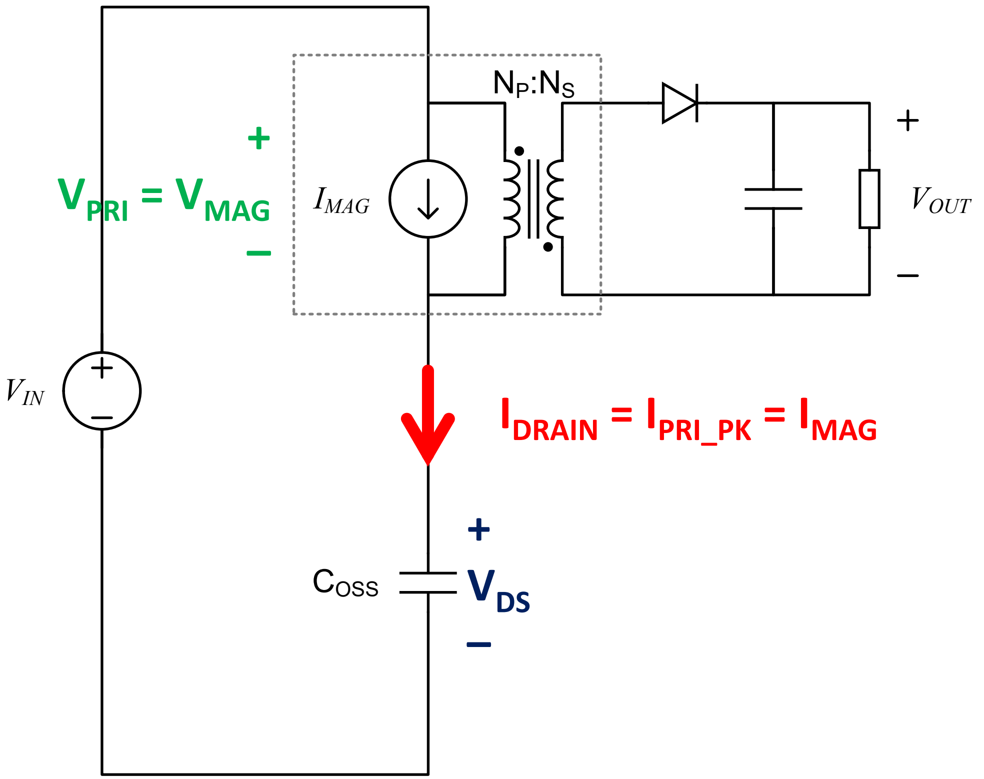
Just After MOSFET Turned Off



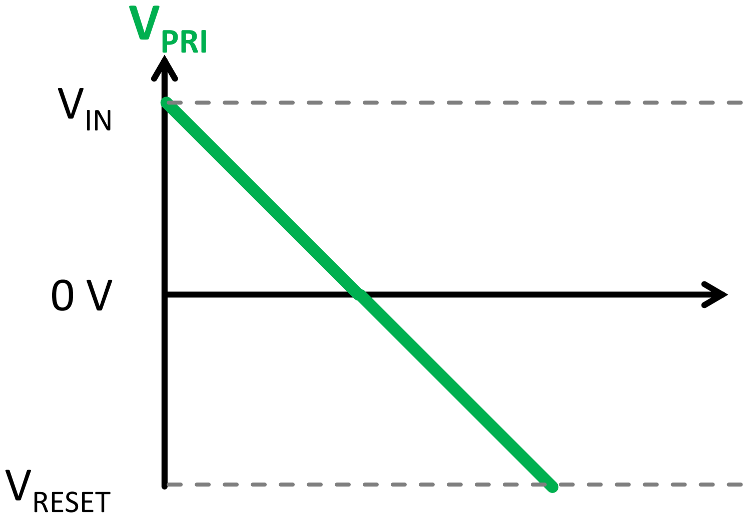
$V_{RESET} = (N_P/N_S)\cdot(V_{OUT} + V_{DIODE})$

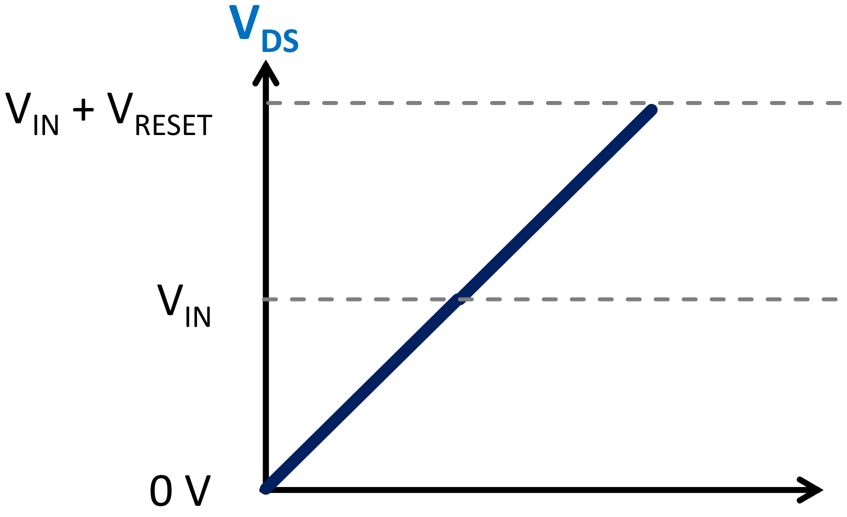


Just After MOSFET Turned Off

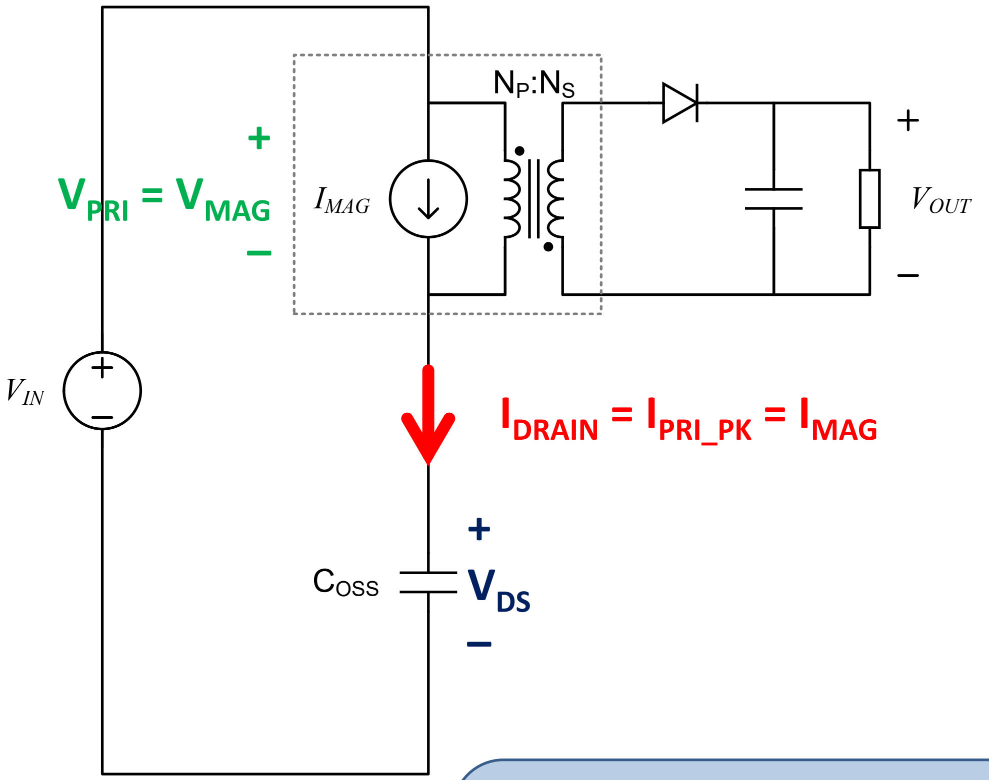


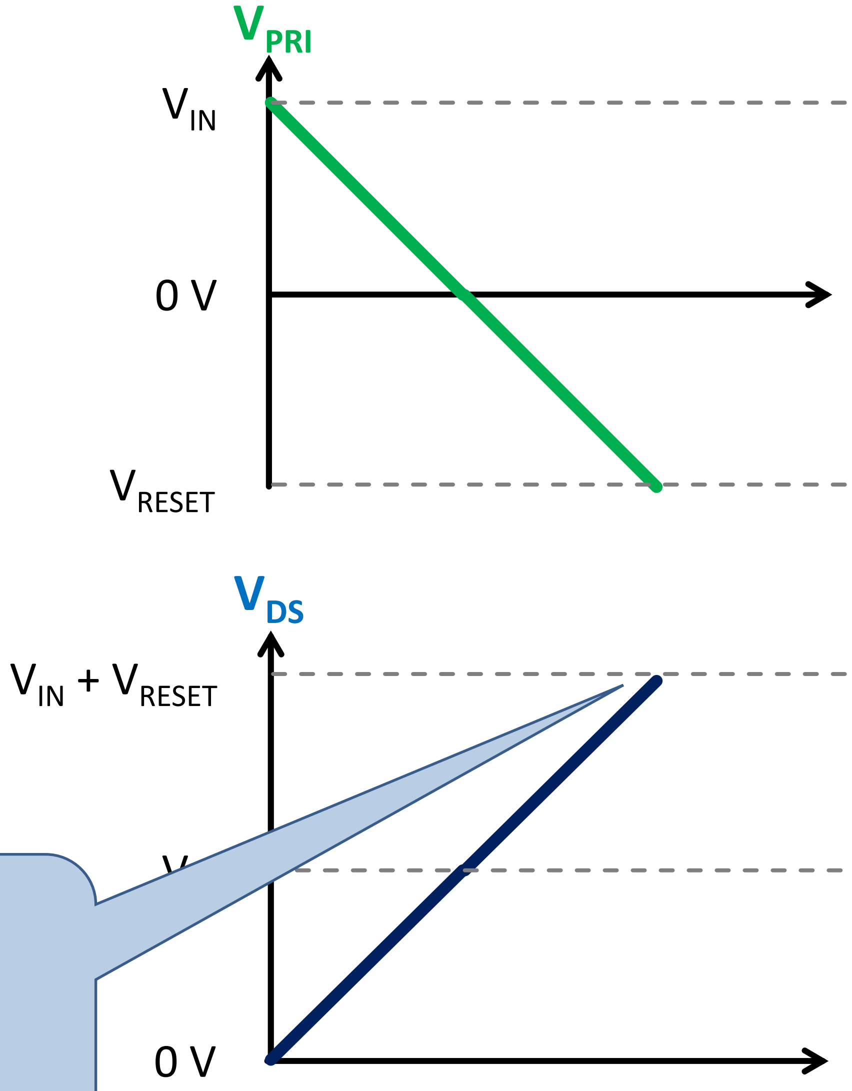
$$V_{RESET} = (N_P/N_S)\cdot(V_{OUT} + V_{DIODE})$$





Just After MOSFET Turned Off

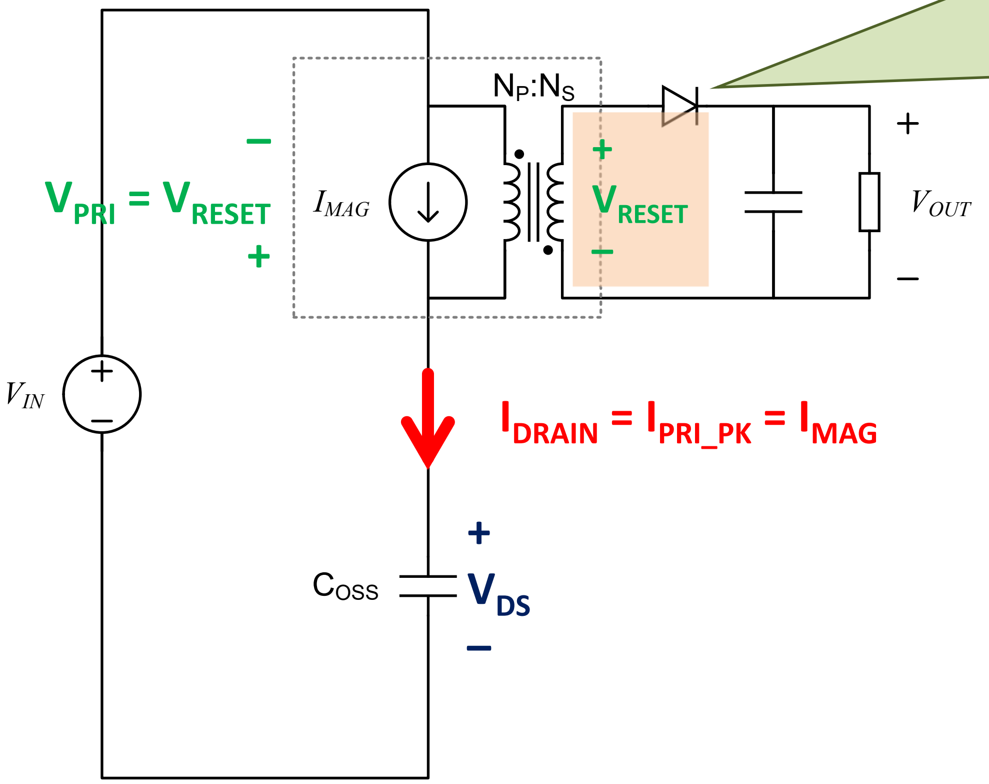




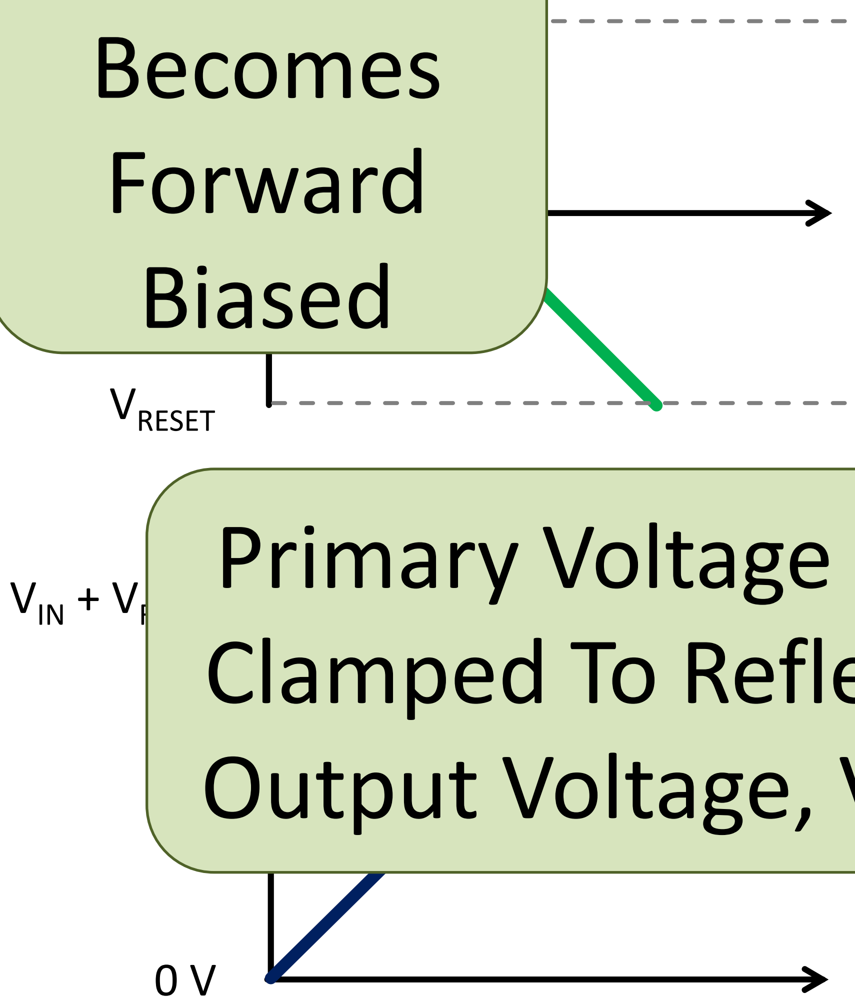
$V_{RESET} = (N_P/N_S)\cdot(V$

Now What Happens?

Just After MOSFET Turned Off



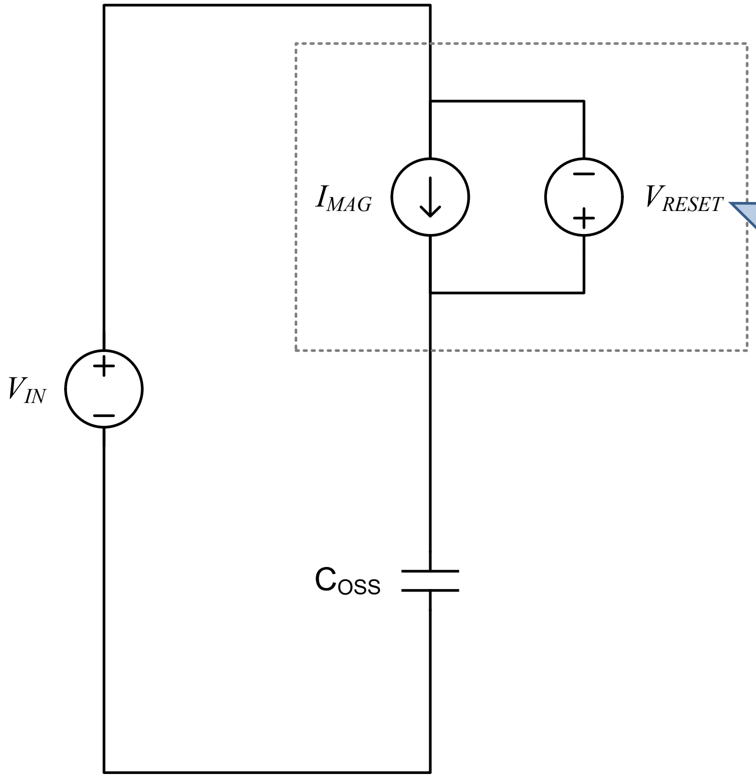
$V_{RESET} = (N_P/N_S)\cdot(V_{OUT} + V_{DIODE})$



Diode Becomes Forward Biased

Primary Voltage Now Clamped To Reflected Output Voltage, V_{RESET}

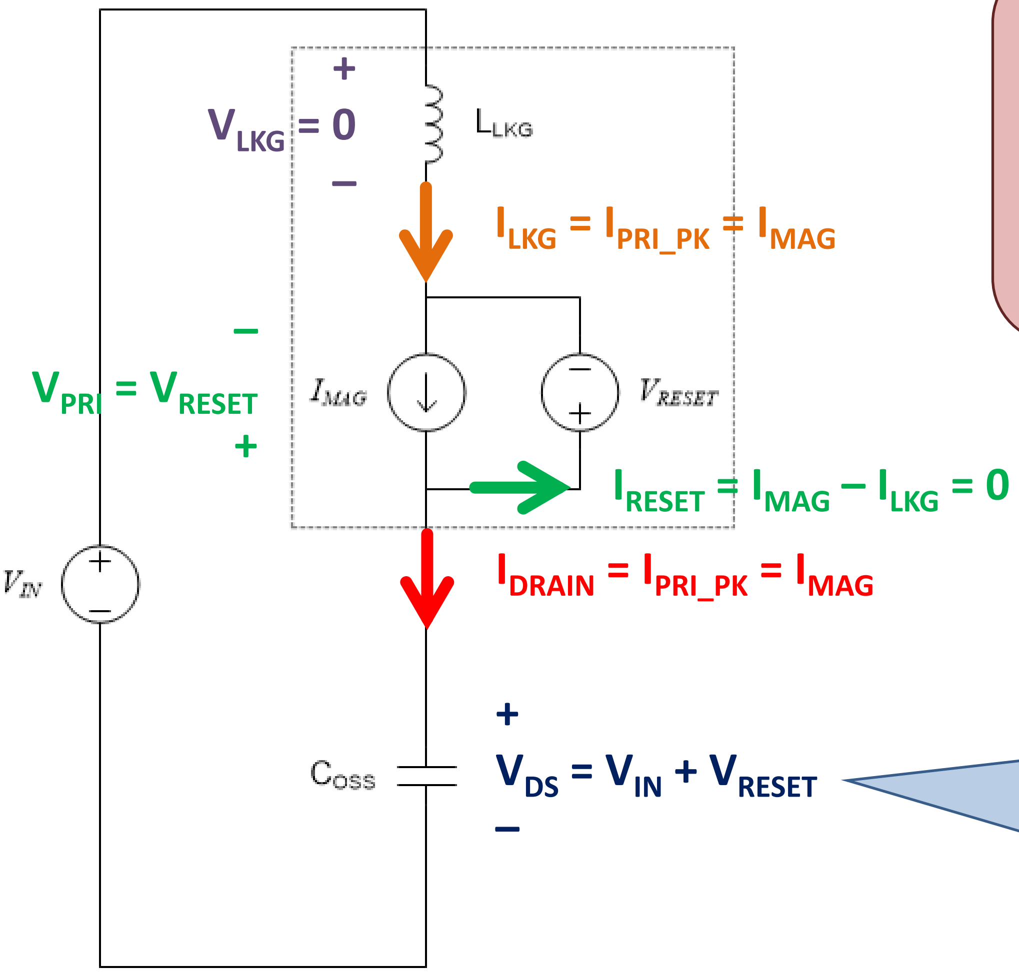
Just After MOSFET Turned Off



Modeling Voltage Reflected From Secondary As Constant

But Wait! There's More!

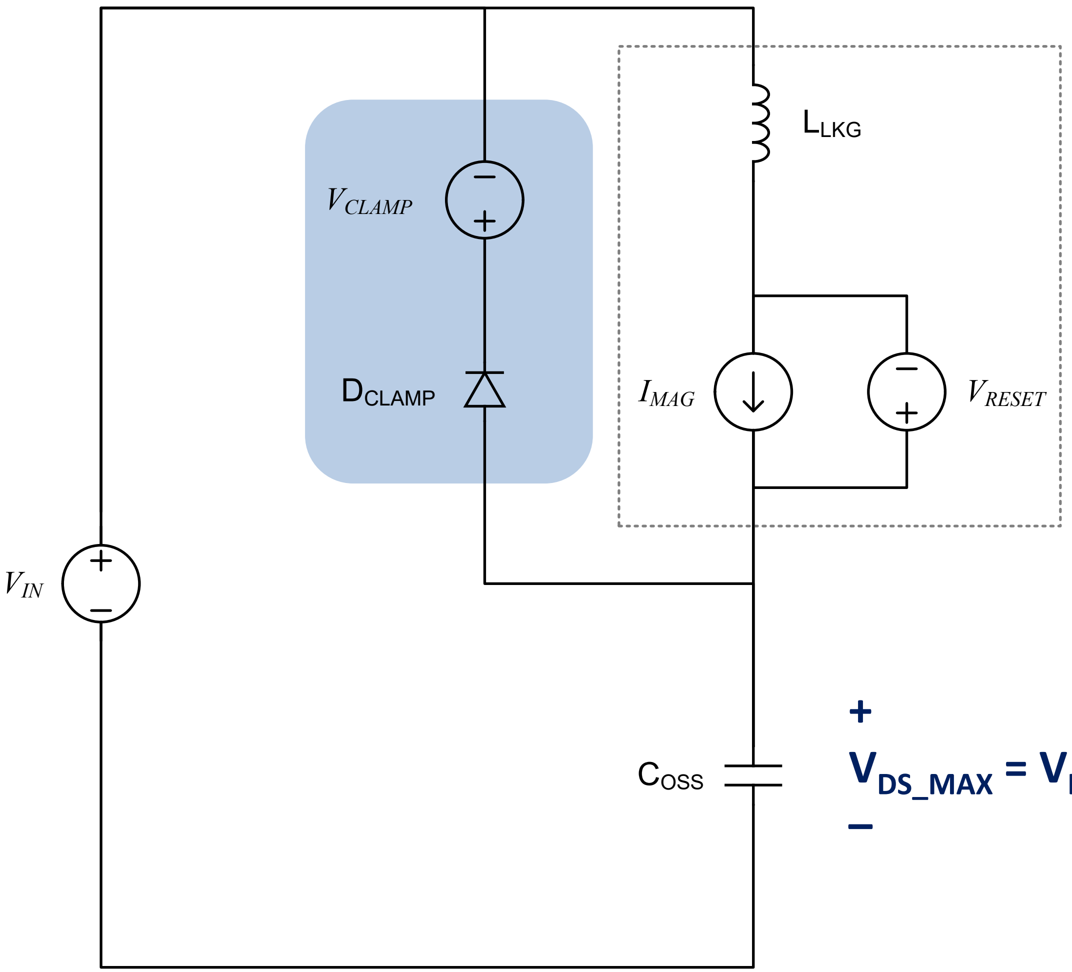
Just After MOSFET Turned Off



What Limits The Maximum Drain Voltage?

C_{OSS} Continues To Charge

Voltage Clamp

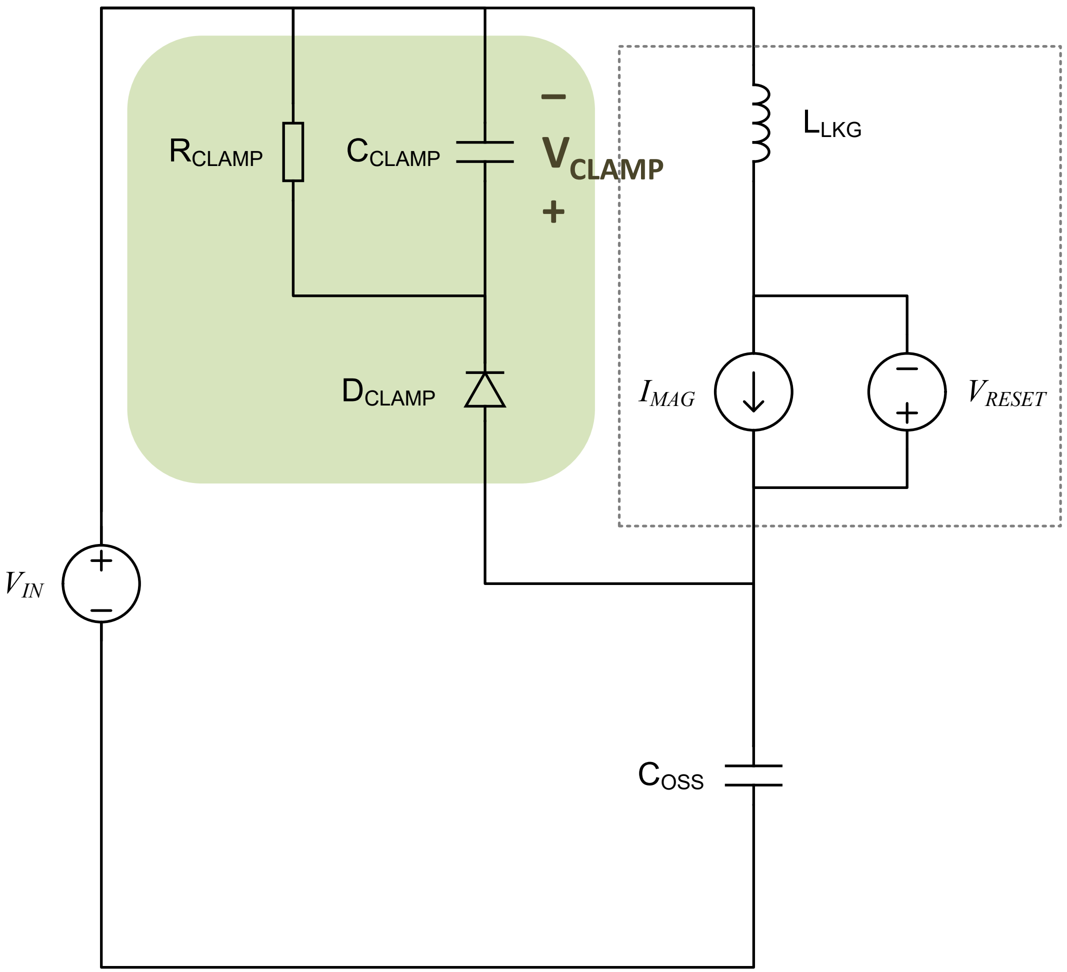


+

$V_{DS\_MAX} = V_{IN} + V_{CLAMP}$

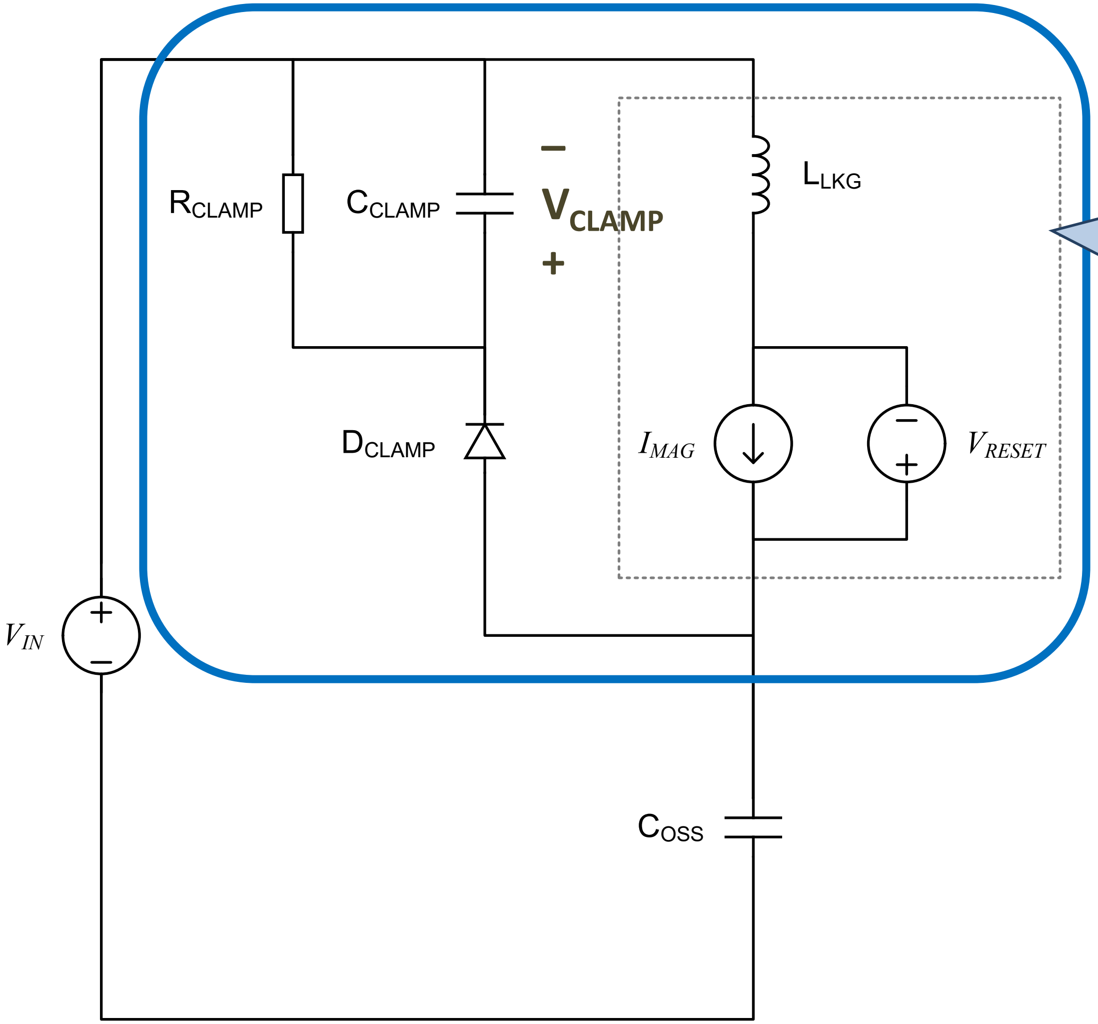
–

RCD Voltage Clamp



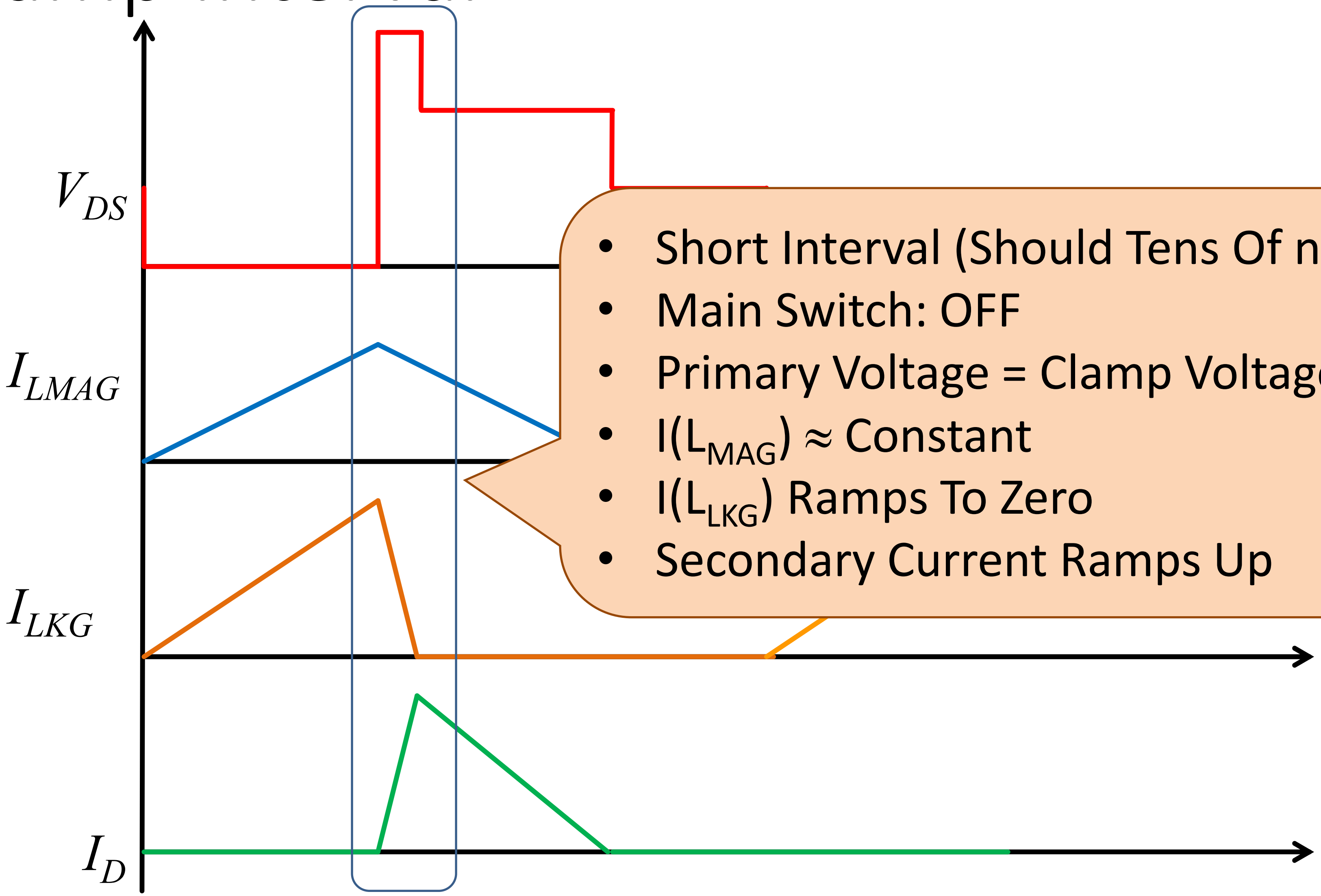
Shown Later C_{CLAMP} Is Large Enough That V_{CLAMP} Can Be Considered Constant

Clamp Interval



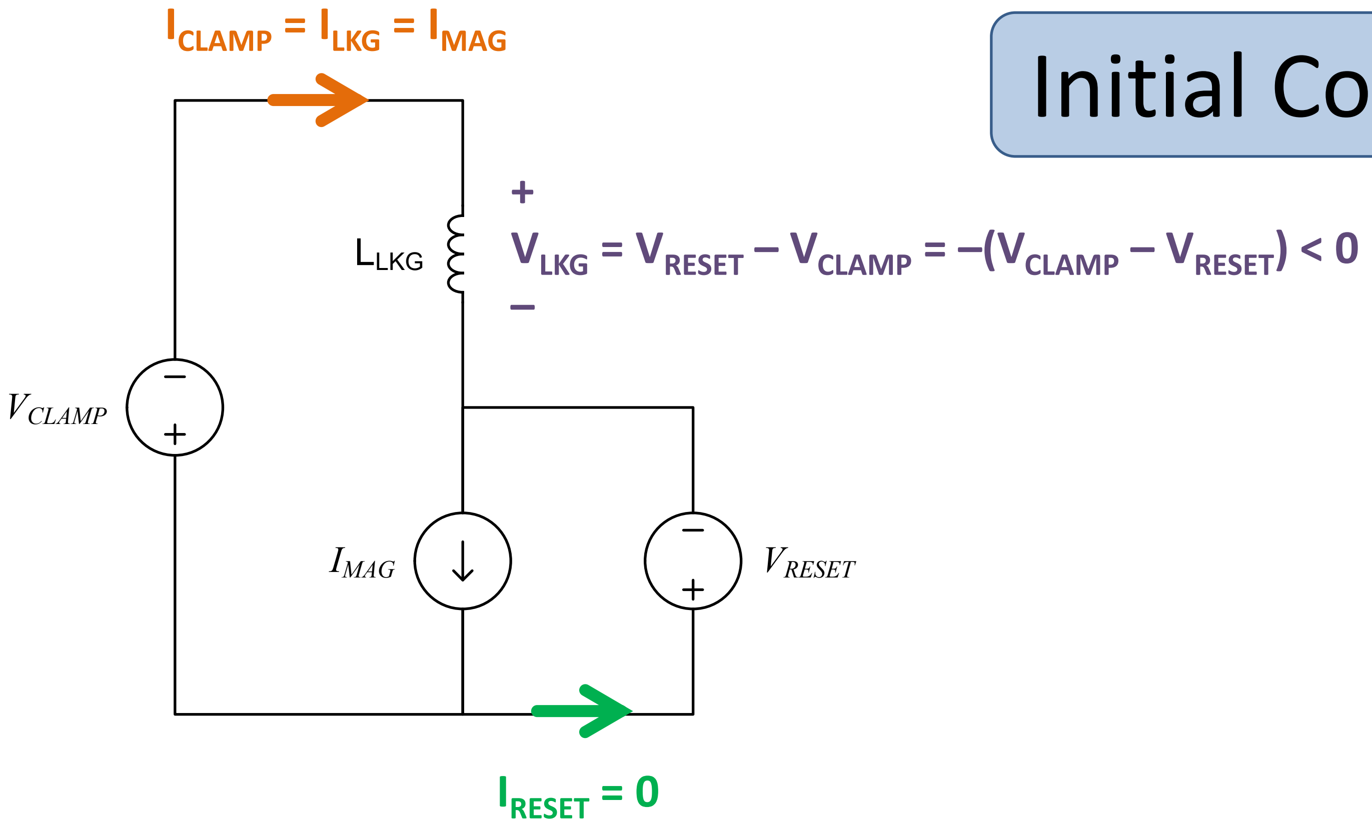
Focus Just On This Part Of The Circuit To Analyze What Happens During The Clamp Interval

DCM: Clamp Interval



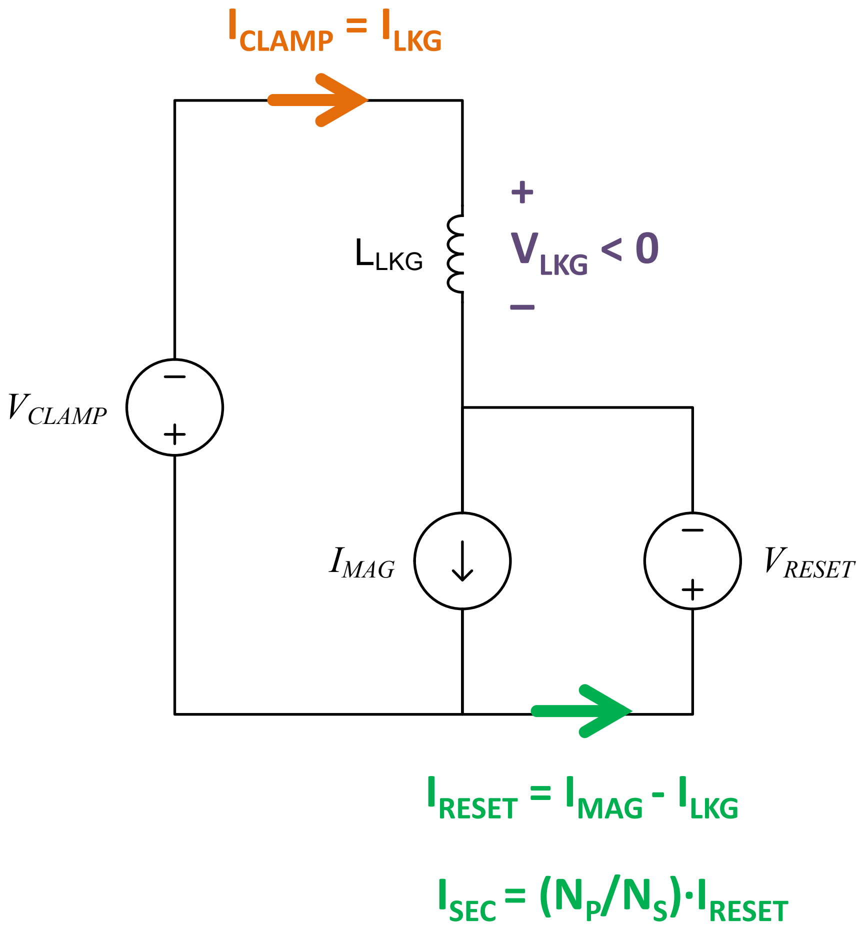
- Short Interval (Should Tens Of ns)
- Main Switch: OFF
- Primary Voltage = Clamp Voltage
- $I(L_{MAG}) \approx$ Constant
- $I(L_{LKG})$ Ramps To Zero
- Secondary Current Ramps Up

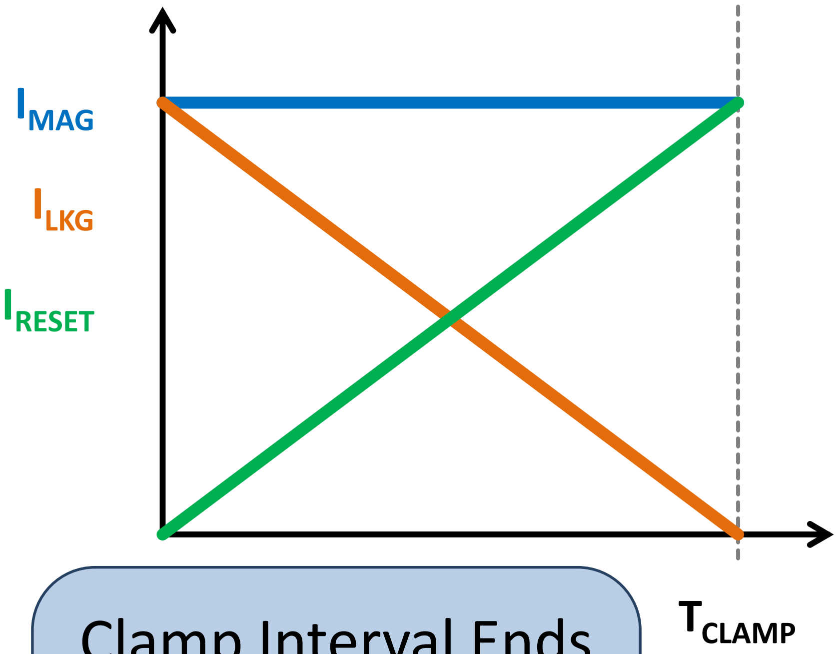
Clamp Interval Circuit Model



Initial Conditions

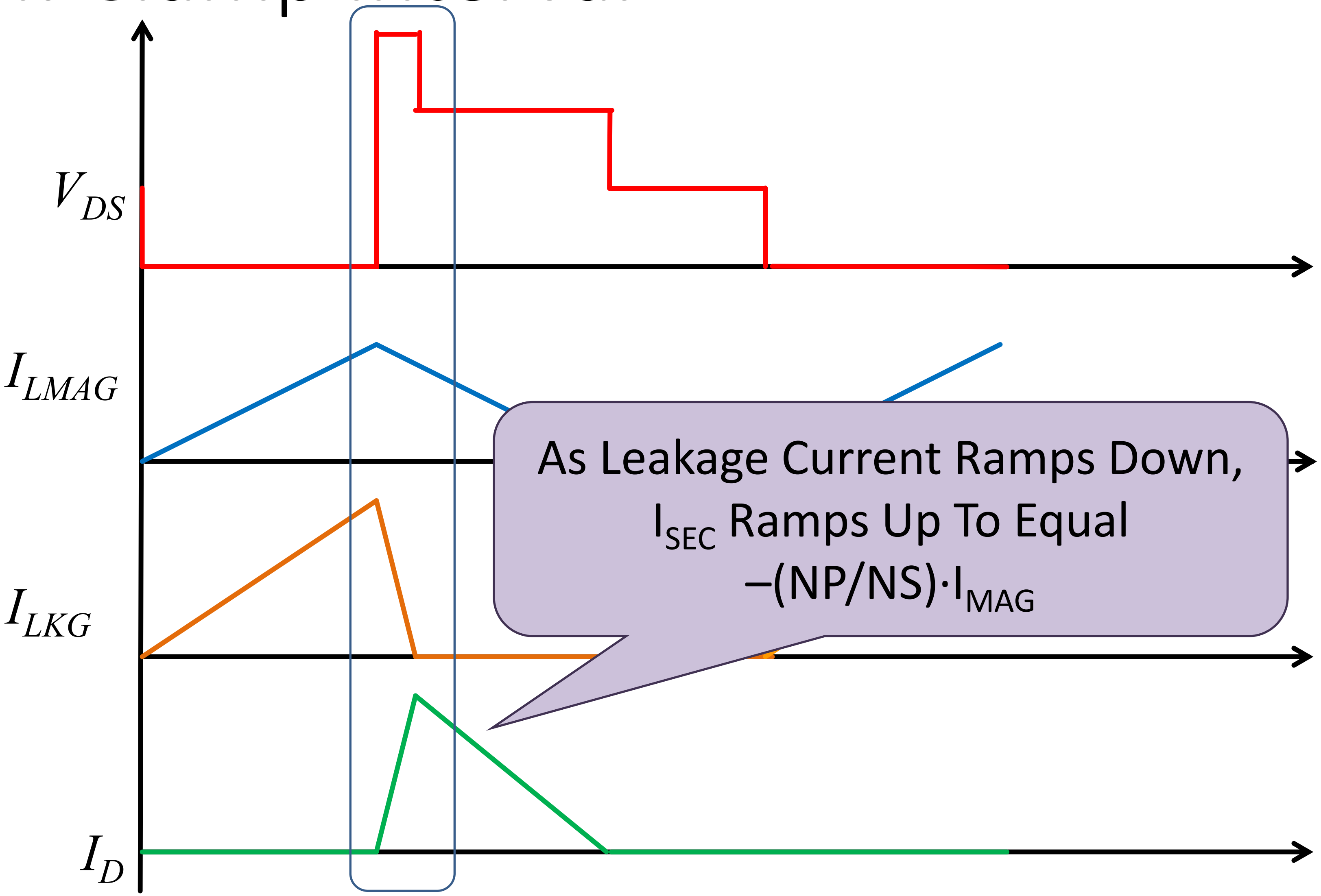
Clamp Interval Current



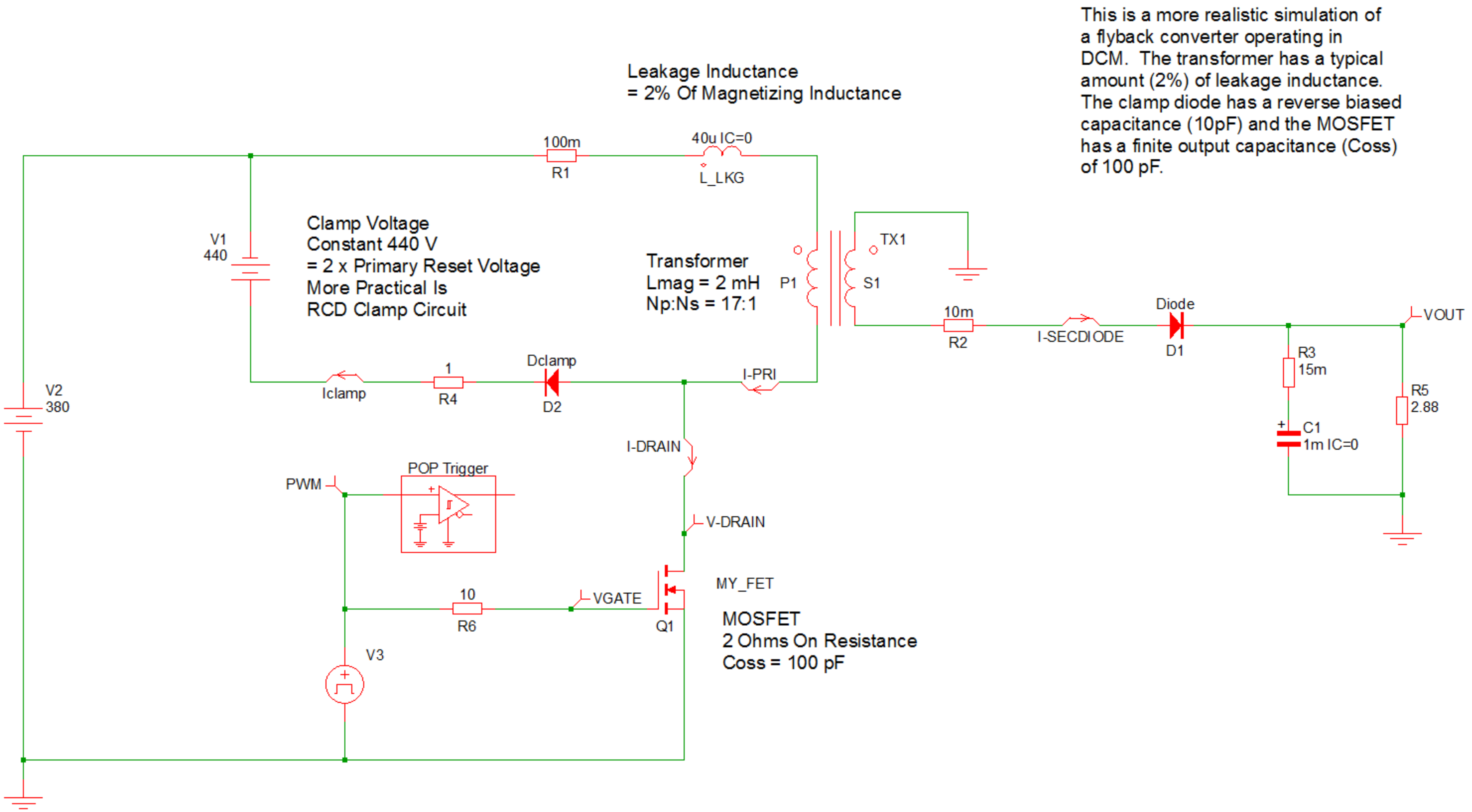


Clamp Interval Ends When Current In Leakage Inductance Goes To Zero

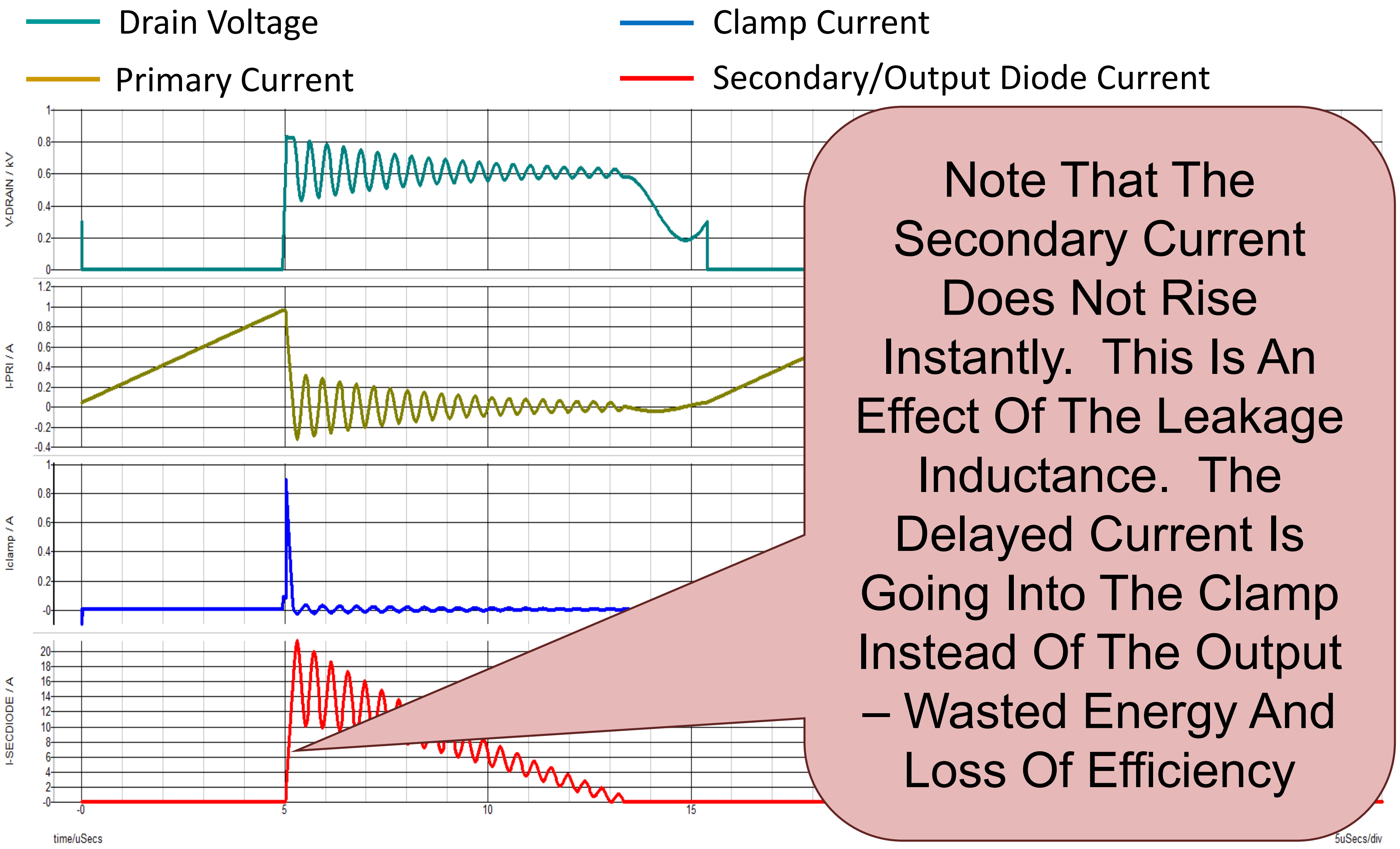
DCM: Clamp Interval



Discontinuous Conduction Mode
More Realistic Flyback Simulation

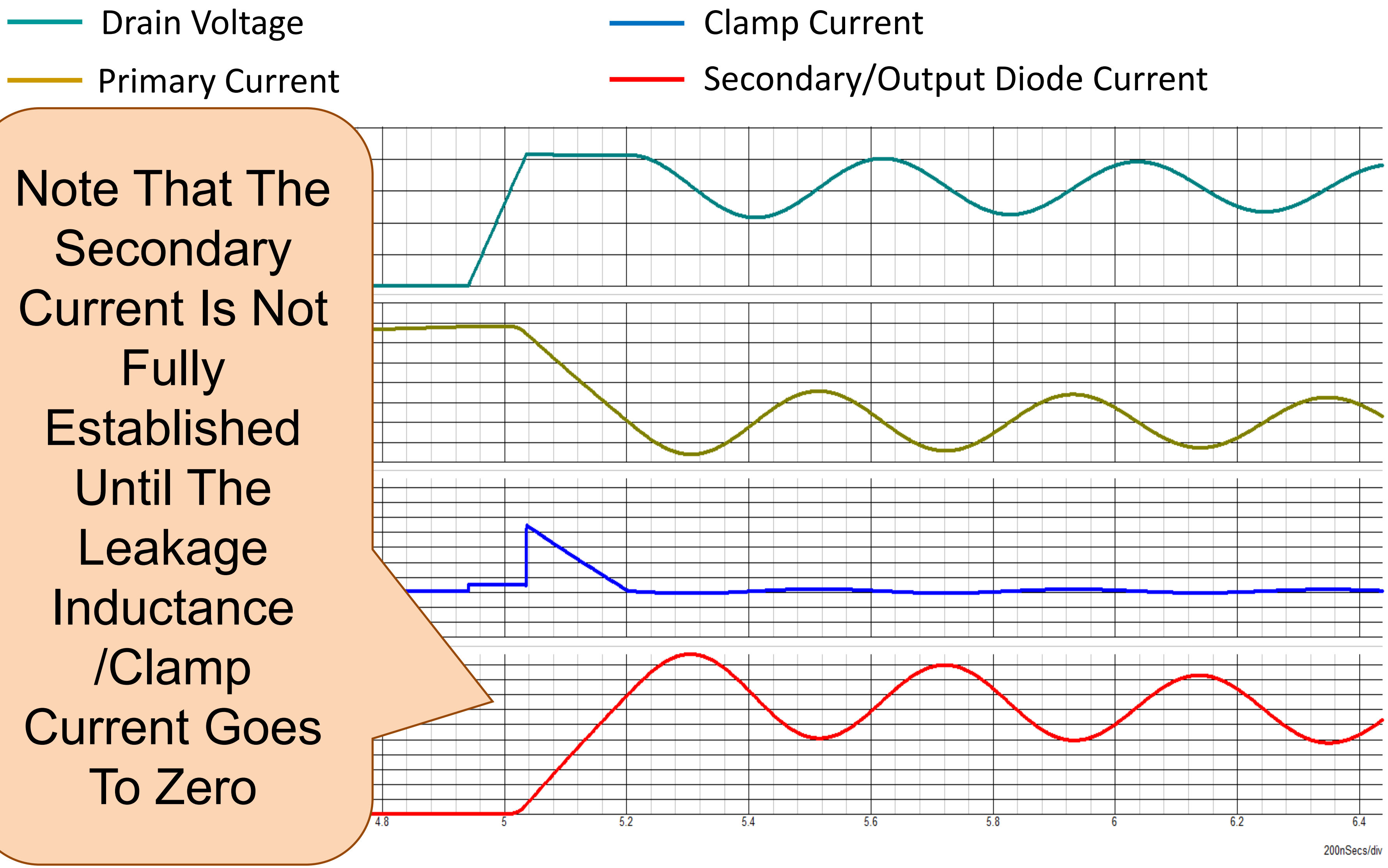


Discontinuous Conduction Mode
More Realistic Flyback Simulation



Note That The Secondary Current Does Not Rise Instantly. This Is An Effect Of The Leakage Inductance. The Delayed Current Is Going Into The Clamp Instead Of The Output – Wasted Energy And Loss Of Efficiency

Discontinuous Conduction Mode
Details Of The Clamp Time



Note That The Secondary Current Is Not Fully Established Until The Leakage Inductance /Clamp Current Goes To Zero

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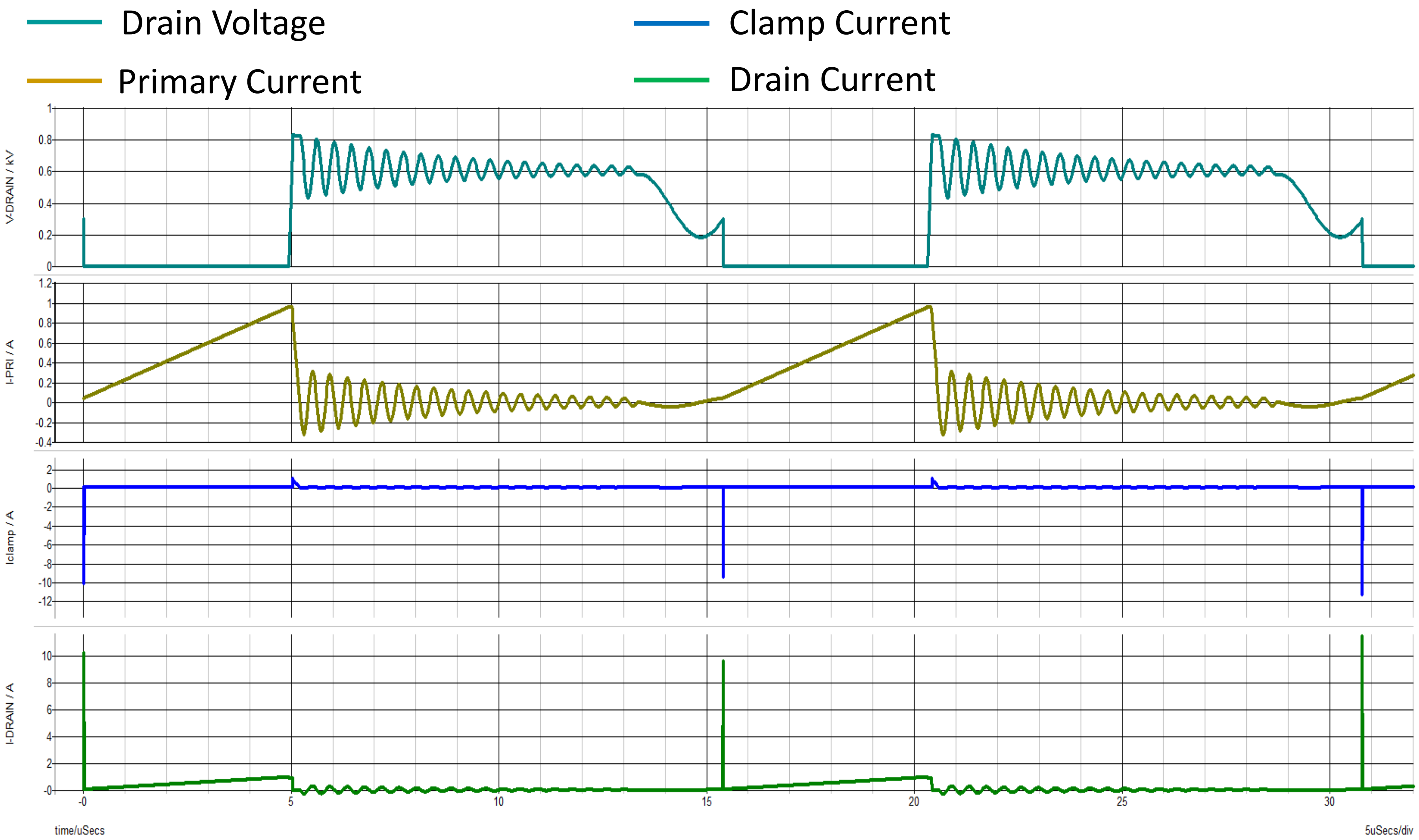
Discontinuous Conduction Mode
More Realistic Flyback Simulation



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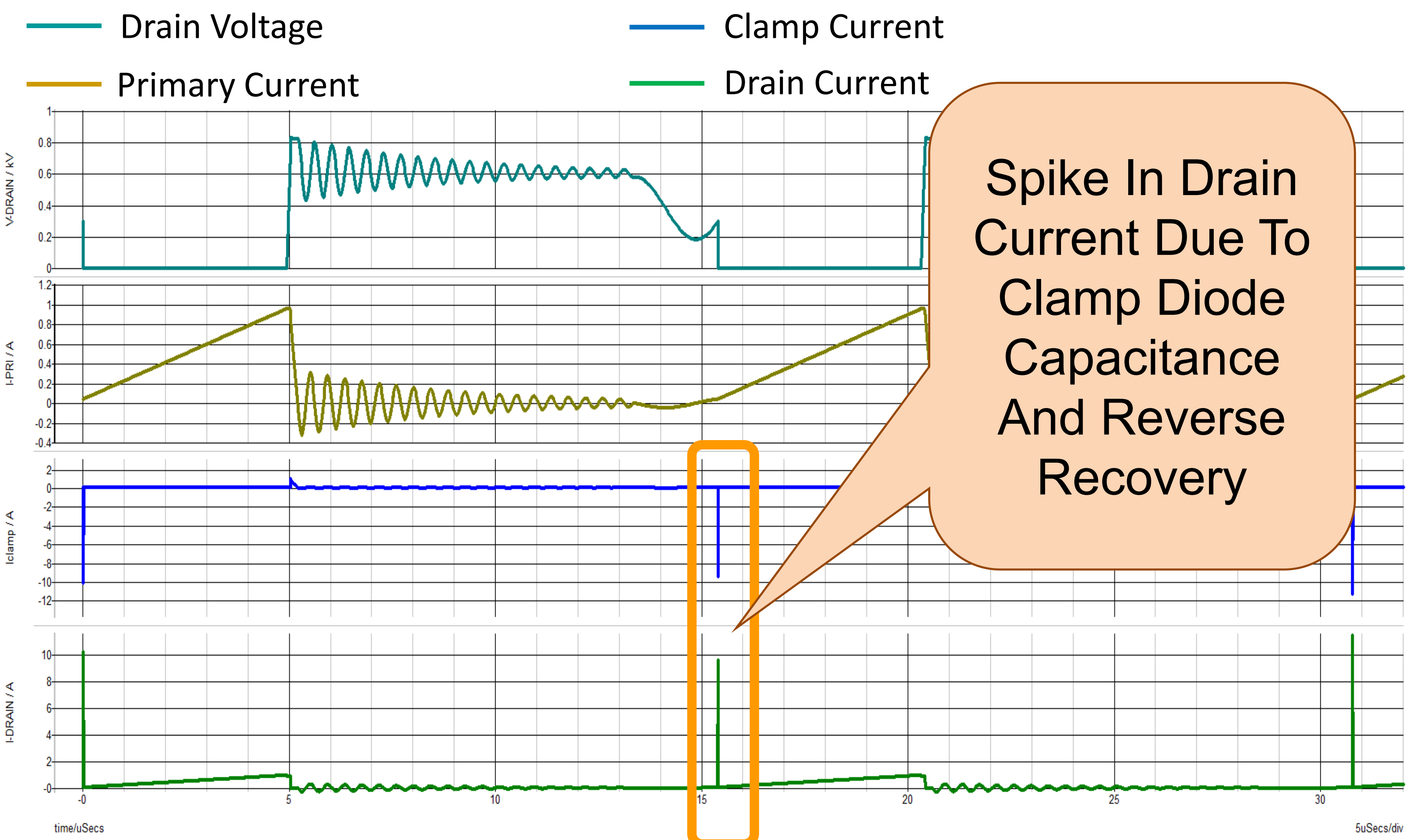
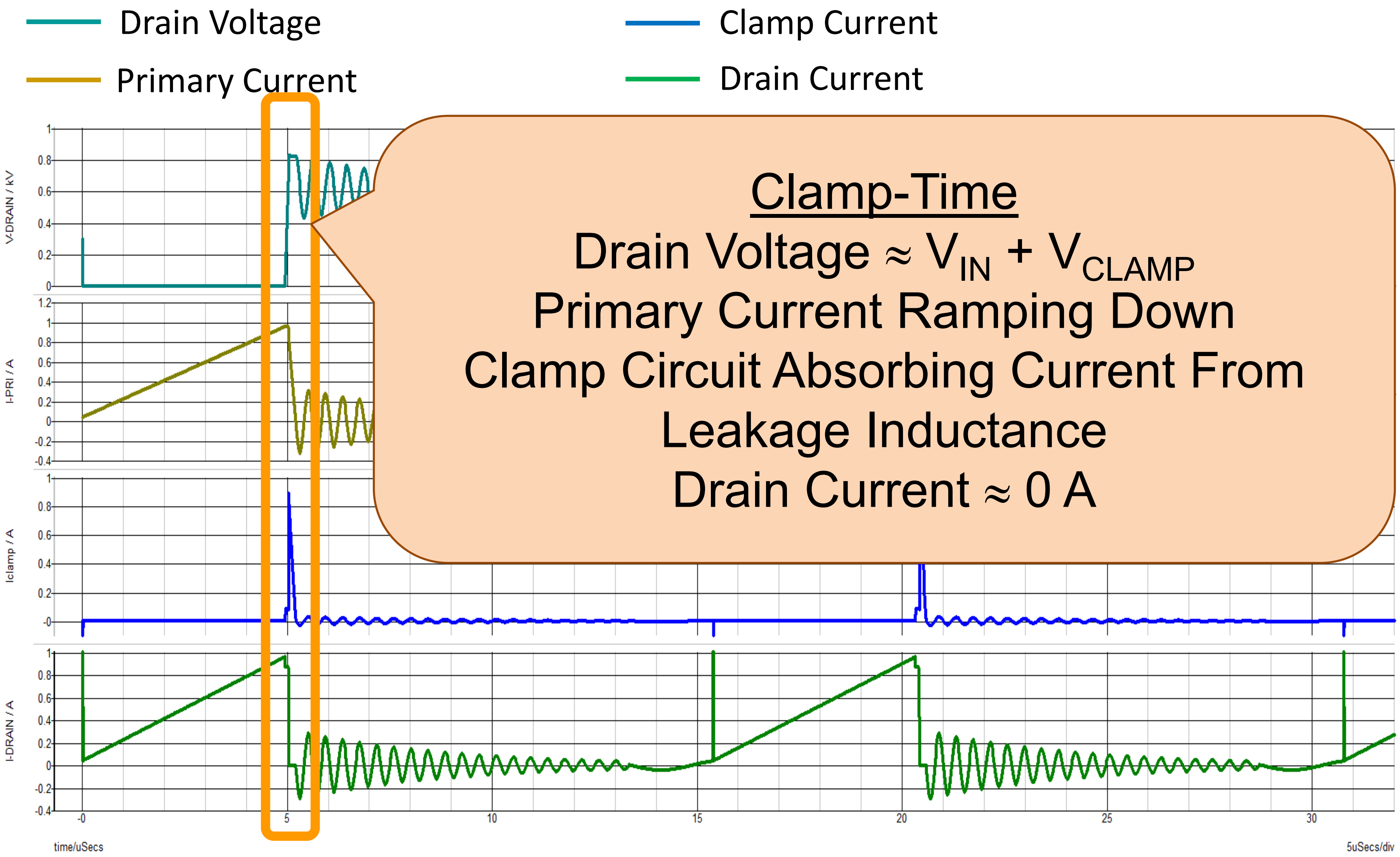
Discontinuous Conduction Mode
More Realistic Flyback Simulation



Discontinuous Conduction Mode
More Realistic Flyback Simulation



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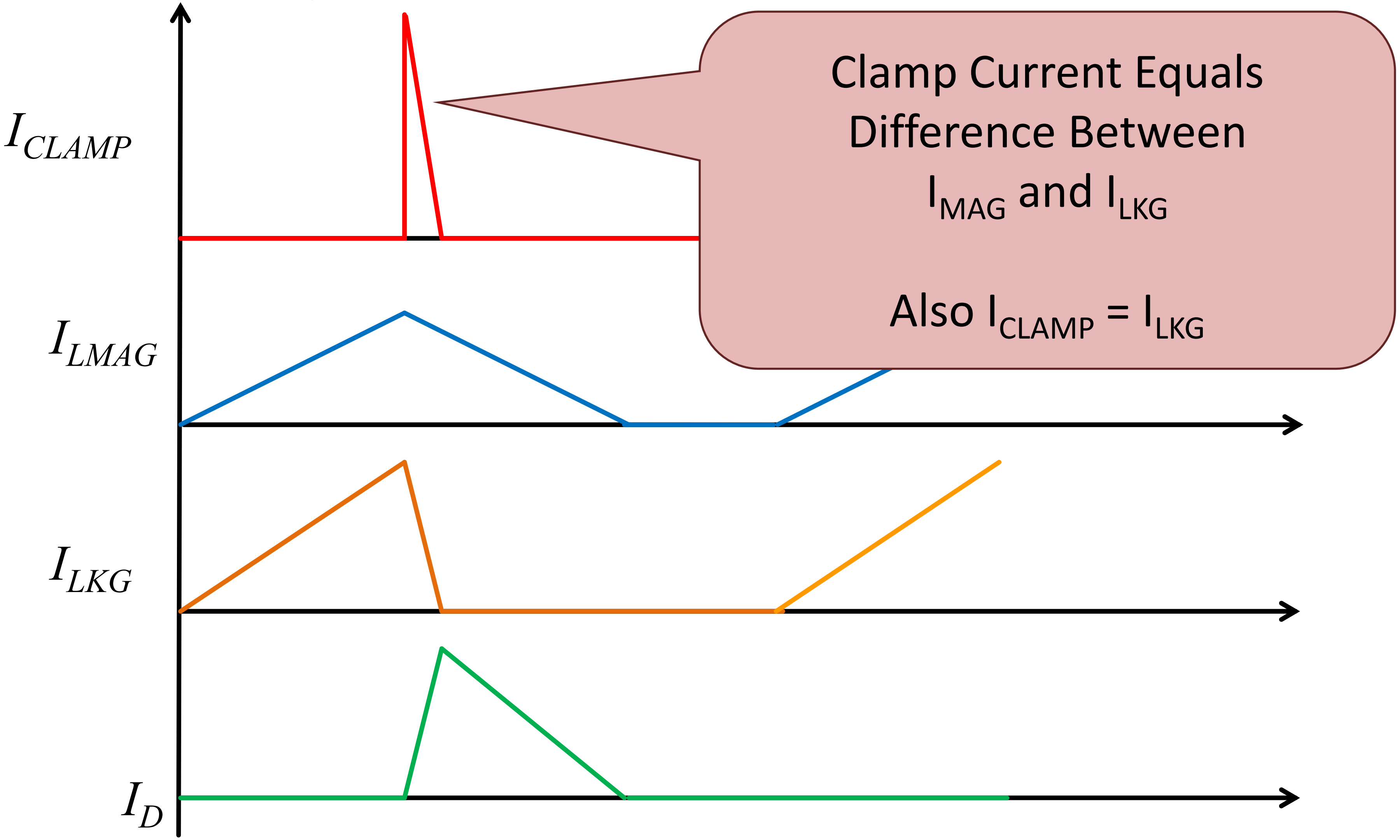
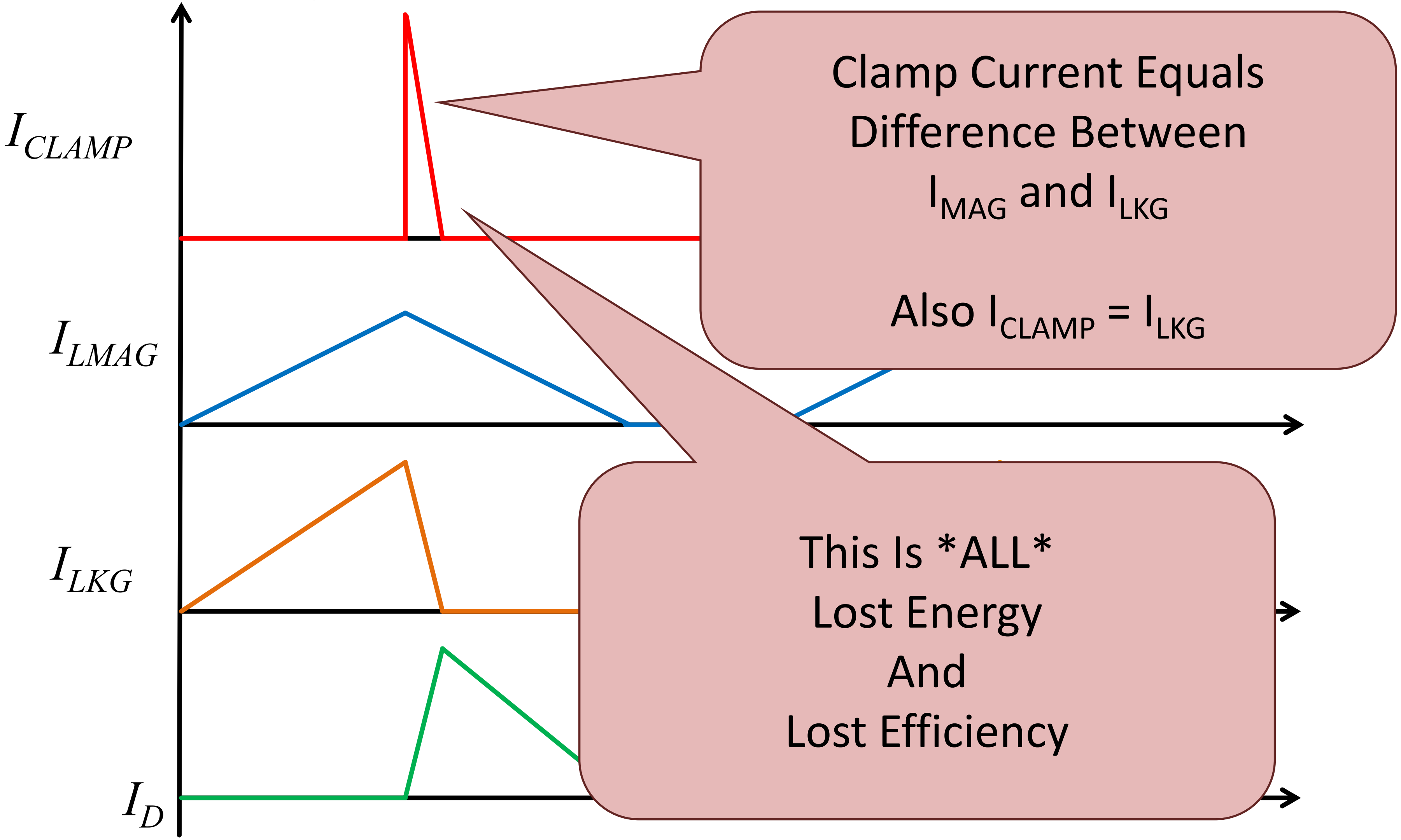
DCM: Clamp Interval

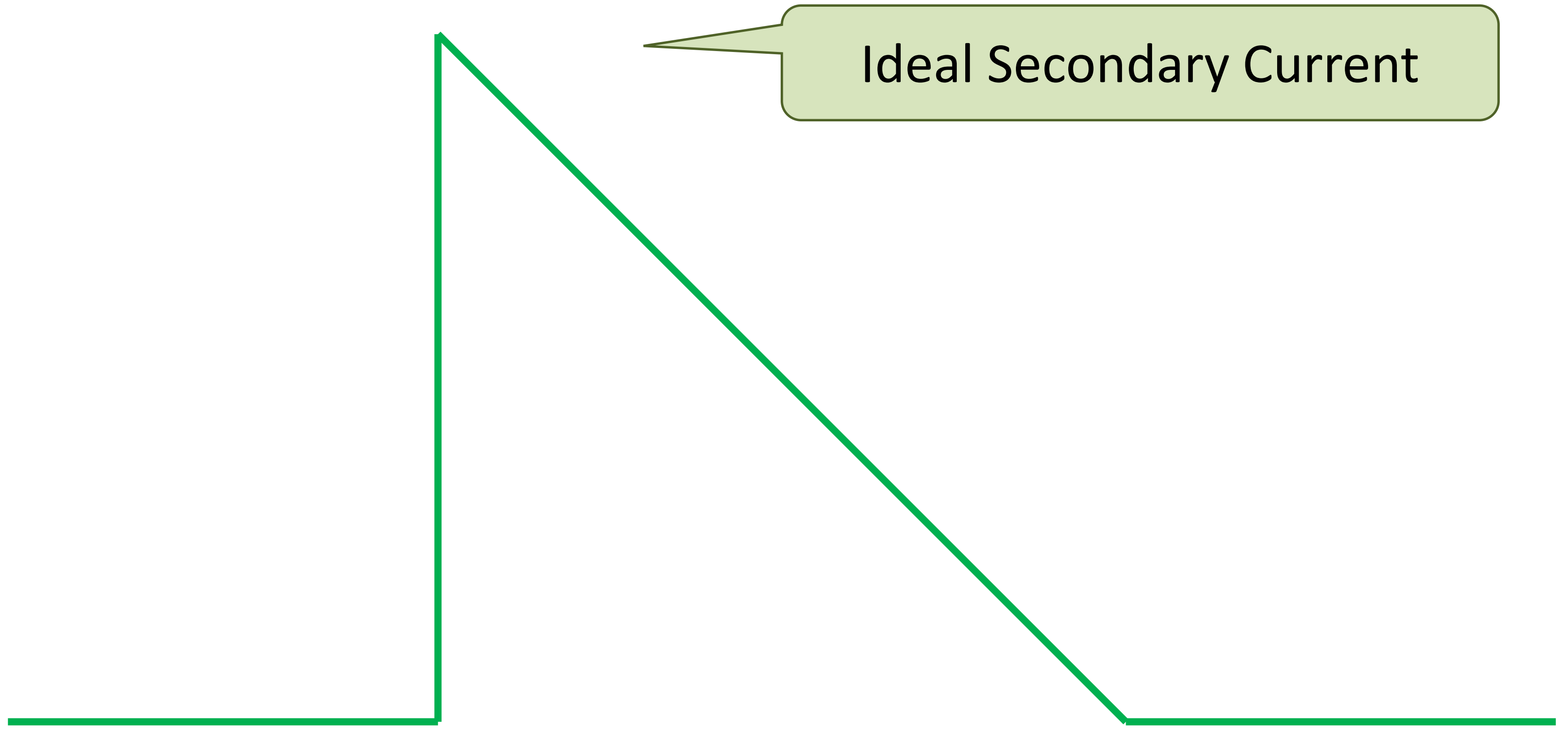


EMBEDDED POWER LABS

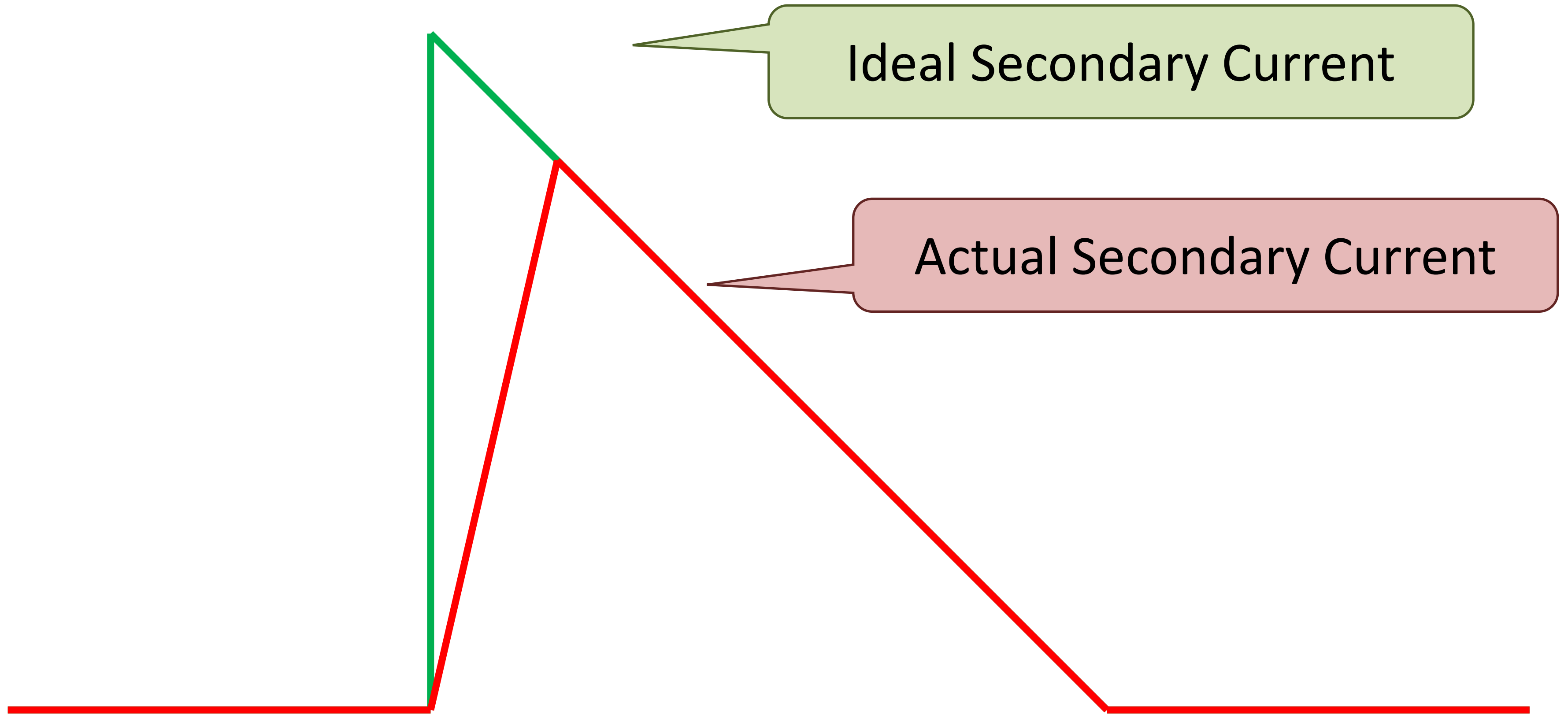
DCM: Clamp Interval



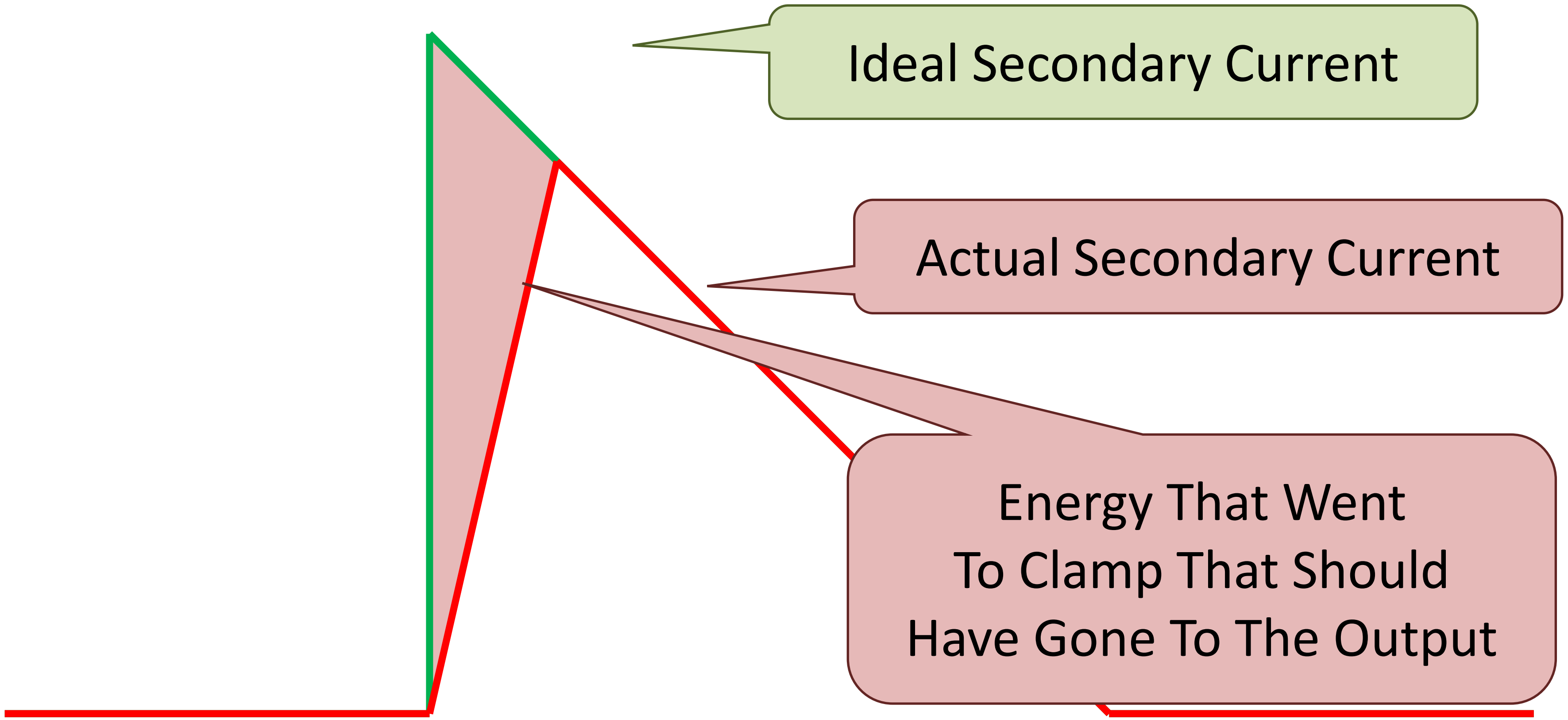
Leakage Loss



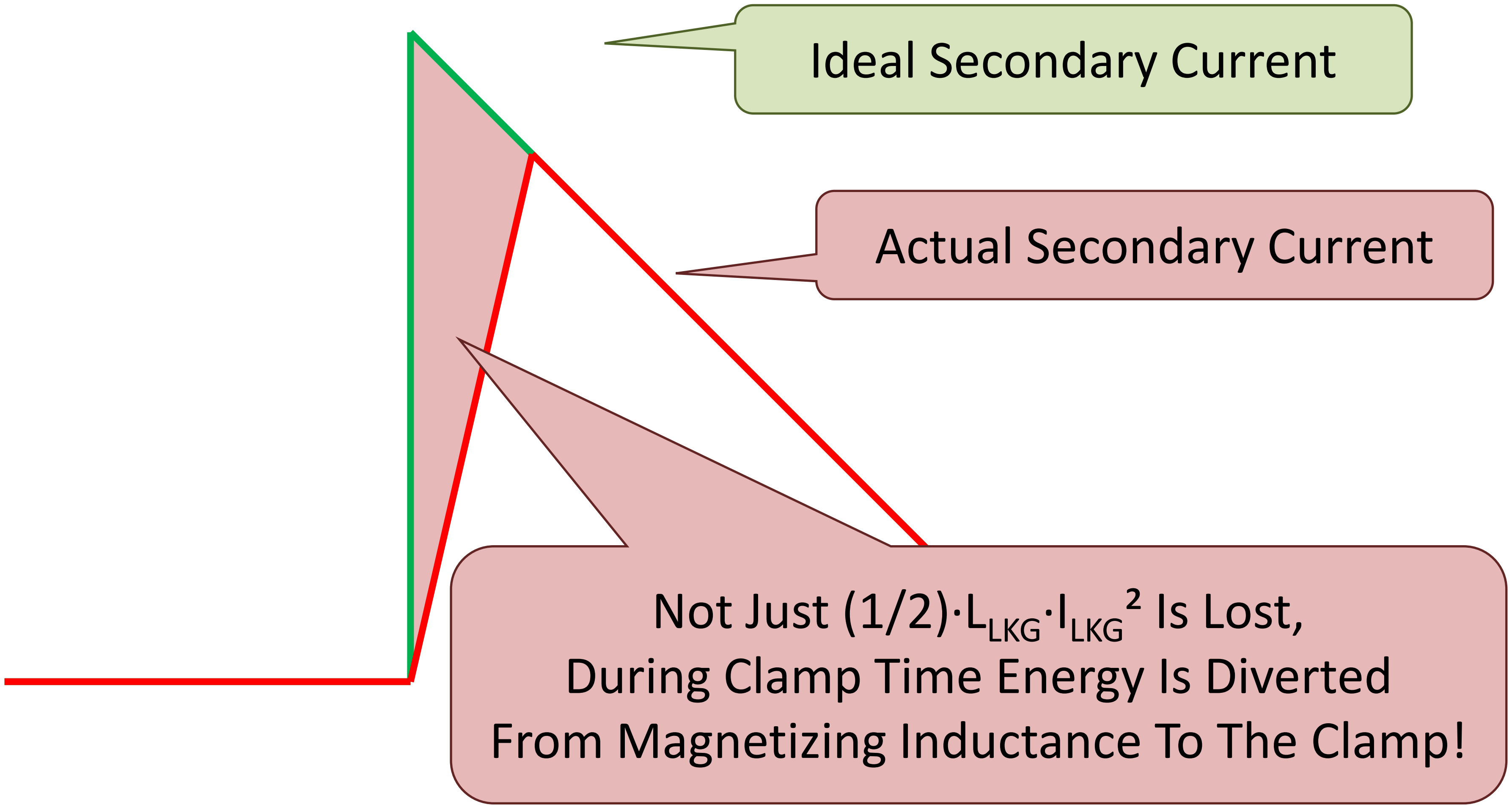
Leakage Loss



Leakage Loss



Leakage Loss



Clamp Loss

$$i_{LKG}(t=0) = i_{MAG}(t=0) = I_{PK}$$

$$\frac{di_{LKG}(t)}{dt} = \frac{V_{CLAMP} - V_{RESET}}{L_{LKG}}$$

$$V_{RESET} = \frac{N_P}{N_S} \cdot (V_{OUT} + V_{DIODE})$$

$$i_{LKG}(t) = I_{PK} - \left(\frac{V_{CLAMP} - V_{RESET}}{L_{LKG}}\right) \cdot t$$

Clamp Loss

$$i_{LKG}\left(T_{LKG}\right)=0$$

At End Of Clamp Interval,
Leakage Inductance Current = 0

$$I_{PK}-\left(\frac{V_{CLAMP}-V_{RESET}}{L_{LKG}}\right)\cdot T_{LKG}=0$$

$$T_{LKG}=\frac{I_{PK}}{\frac{V_{CLAMP}-V_{RESET}}{L_{LKG}}}=\frac{L_{LKG}\cdot I_{PK}}{V_{CLAMP}-V_{RESET}}$$

Clamp Loss

$$i_{CLAMP}(t) = i_{LKG}(t)$$

$$p_{CLAMP}(t) = v_{CLAMP}(t) \cdot i_{CLAMP}(t) \approx V_{CLAMP} \cdot i_{CLAMP}(t)$$

$$P_{CLAMP} = \frac{1}{T_{SW}} \cdot \int_0^{T_{SW}} p_{CLAMP}(t)\,dt = \frac{1}{T_{SW}} \cdot \int_0^{T_{LKG}} p_{CLAMP}(t)\,dt$$

$$= \frac{1}{T_{SW}} \cdot \int_0^{T_{LKG}} \left(V_{CLAMP} \cdot i_{CLAMP}(t)\right)dt = \frac{V_{CLAMP}}{T_{SW}} \cdot \int_0^{T_{LKG}} i_{CLAMP}(t)\,dt$$

Clamp Loss

$$P_{CLAMP} = \frac{V_{CLAMP}}{T_{SW}} \cdot \int_0^{T_{LKG}} \left(I_{PK} - \left(\frac{V_{CLAMP} - V_{RESET}}{L_{LKG}} \right) \cdot t \right) dt$$

$$= \frac{V_{CLAMP} \cdot I_{PK} \cdot T_{LKG}}{T_{SW}} - \frac{V_{CLAMP}}{T_{SW}} \cdot \int_0^{T_{LKG}} \left(\frac{V_{CLAMP} - V_{RESET}}{L_{LKG}} \right) \cdot t \, dt$$

$$= \frac{V_{CLAMP} \cdot I_{PK} \cdot T_{LKG}}{T_{SW}} - \frac{V_{CLAMP}}{T_{SW}} \cdot \left(\frac{V_{CLAMP} - V_{RESET}}{L_{LKG}} \right) \cdot \int_0^{T_{LKG}} t \, dt$$

$$= \frac{V_{CLAMP} \cdot I_{PK} \cdot T_{LKG}}{T_{SW}} - \frac{V_{CLAMP}}{T_{SW}} \cdot \left(\frac{V_{CLAMP} - V_{RESET}}{L_{LKG}} \right) \cdot \frac{T_{LKG}^2}{2}$$

Clamp Loss

$$P_{CLAMP} = \frac{V_{CLAMP} \cdot I_{PK} \cdot T_{LKG}}{T_{SW}} - \frac{1}{2} \cdot \frac{V_{CLAMP}}{T_{SW}} \cdot \left(\frac{V_{CLAMP} - V_{RESET}}{L_{LKG}} \right) \cdot \left(\frac{L_{LKG} \cdot I_{PK}}{V_{CLAMP} - V_{RESET}} \right)^2$$

$$= \frac{V_{CLAMP} \cdot I_{PK}}{T_{SW}} \cdot \frac{L_{LKG} \cdot I_{PK}}{V_{CLAMP} - V_{RESET}} - \frac{1}{2} \cdot \frac{V_{CLAMP}}{T_{SW}} \cdot \frac{L_{LKG} \cdot I_{PK}^2}{V_{CLAMP} - V_{RESET}}$$

$$= \frac{V_{CLAMP}}{T_{SW}} \cdot \frac{L_{LKG} \cdot I_{PK}^2}{V_{CLAMP} - V_{RESET}} - \frac{1}{2} \cdot \frac{V_{CLAMP}}{T_{SW}} \cdot \frac{L_{LKG} \cdot I_{PK}^2}{V_{CLAMP} - V_{RESET}}$$

$$= \frac{1}{2} \cdot \frac{V_{CLAMP}}{T_{SW}} \cdot \frac{L_{LKG} \cdot I_{PK}^2}{V_{CLAMP} - V_{RESET}}$$

$$P_{CLAMP} = \frac{V_{CLAMP}}{V_{CLAMP} - V_{RESET}} \cdot \frac{1}{2} \cdot L_{LKG} \cdot I_{PK}^2 \cdot F_{SW}$$

Clamp Loss

$$P_{CLAMP} = \frac{V_{CLAMP}}{V_{CLAMP} - V_{RESET}} \cdot \frac{1}{2} \cdot L_{LKG} \cdot I_{PK}^2 \cdot F_{SW}$$

$$= \frac{V_{CLAMP}}{V_{CLAMP} - V_{RESET}} \cdot E_{LKG} \cdot F_{SW}$$

Energy
Stored In
Leakage Inductance

Clamp Loss

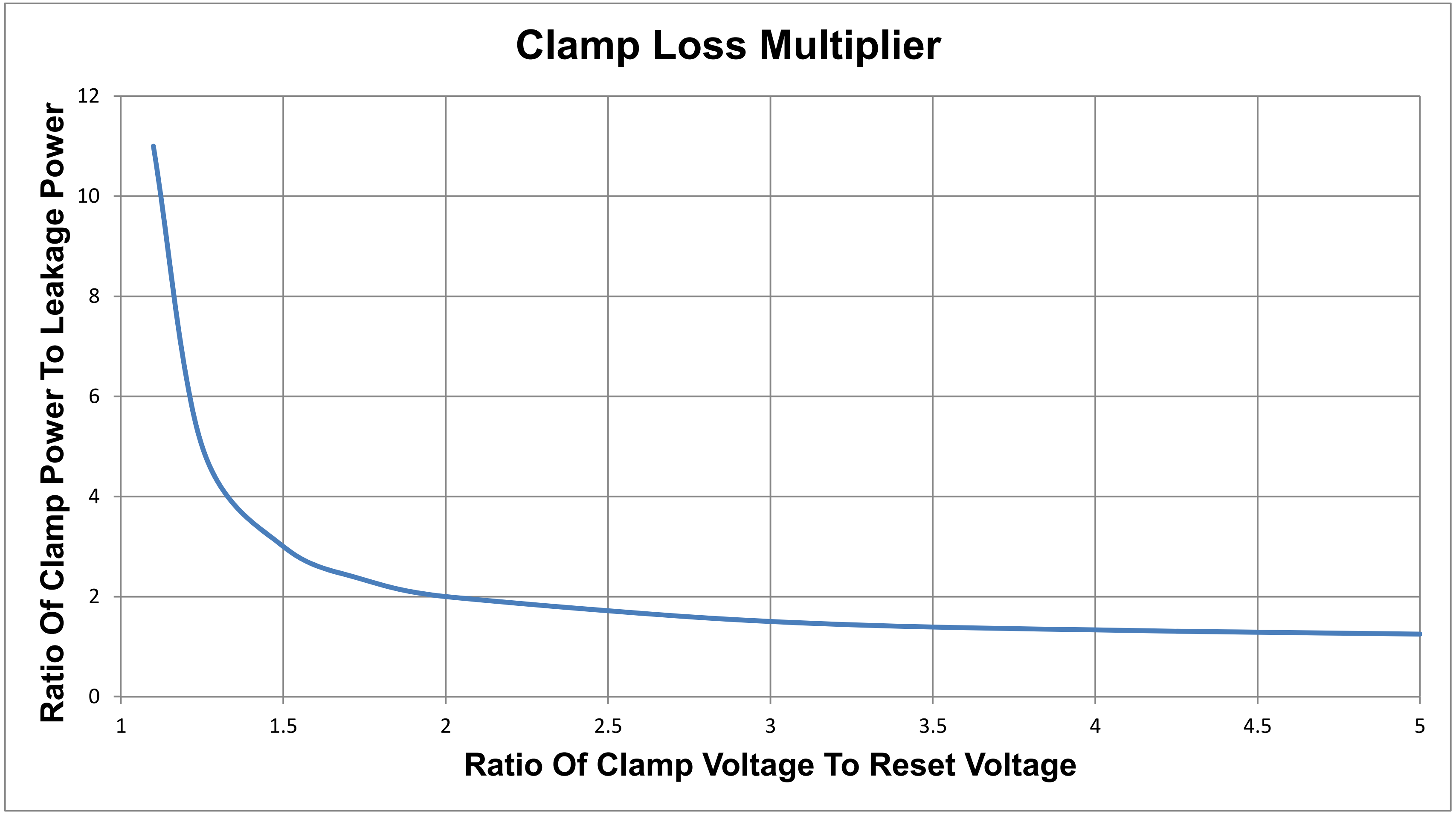
$$P_{CLAMP} = \frac{V_{CLAMP}}{V_{CLAMP} - V_{RESET}} \cdot \frac{1}{2} \cdot L_{LKG} \cdot I_{PK}^2 \cdot F_{SW}$$

$$= \frac{V_{CLAMP}}{V_{CLAMP} - V_{RESET}} \cdot E_{LKG} \cdot F_{SW}$$

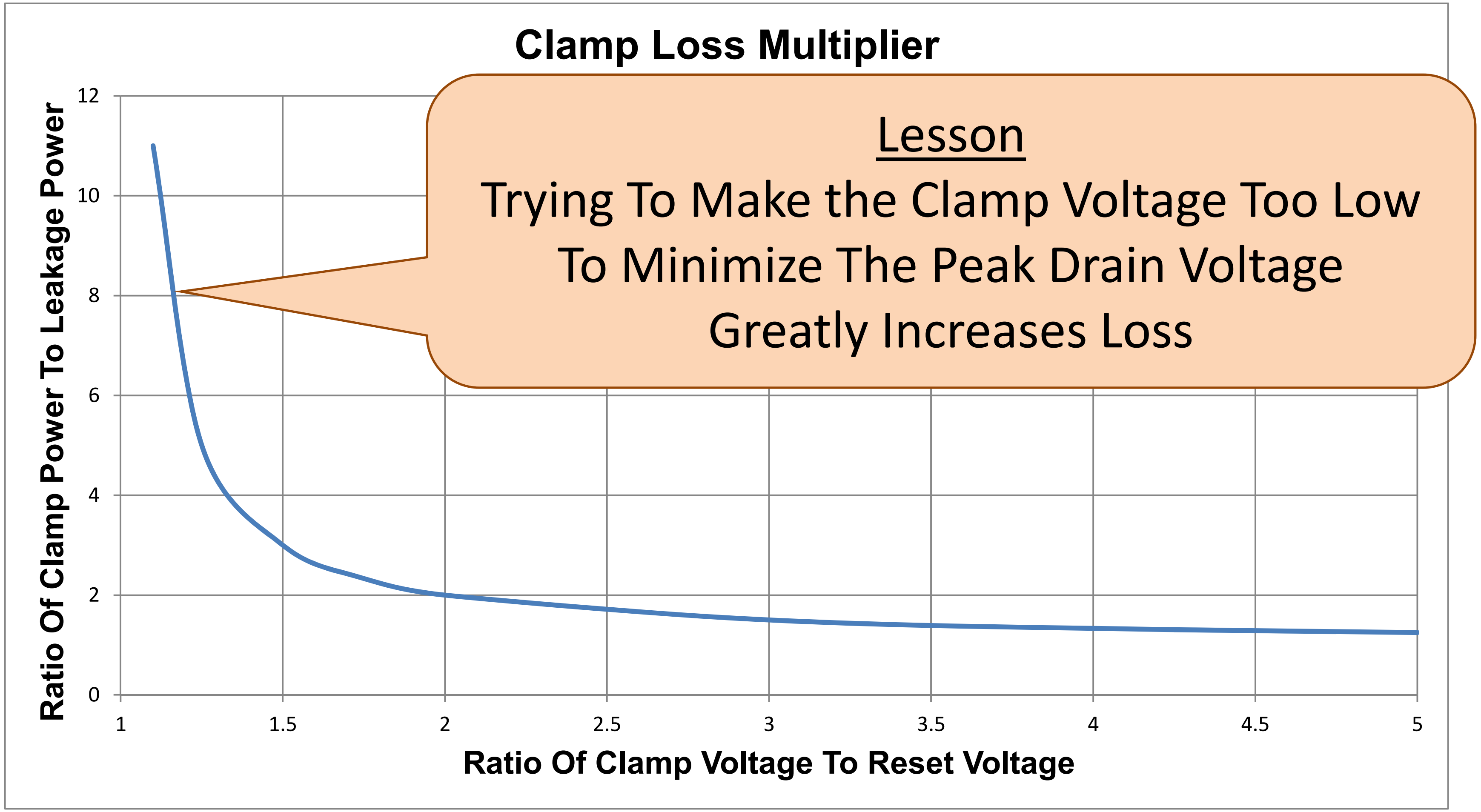
Loss Multiplier!

Energy Stored In The Leakage Inductance

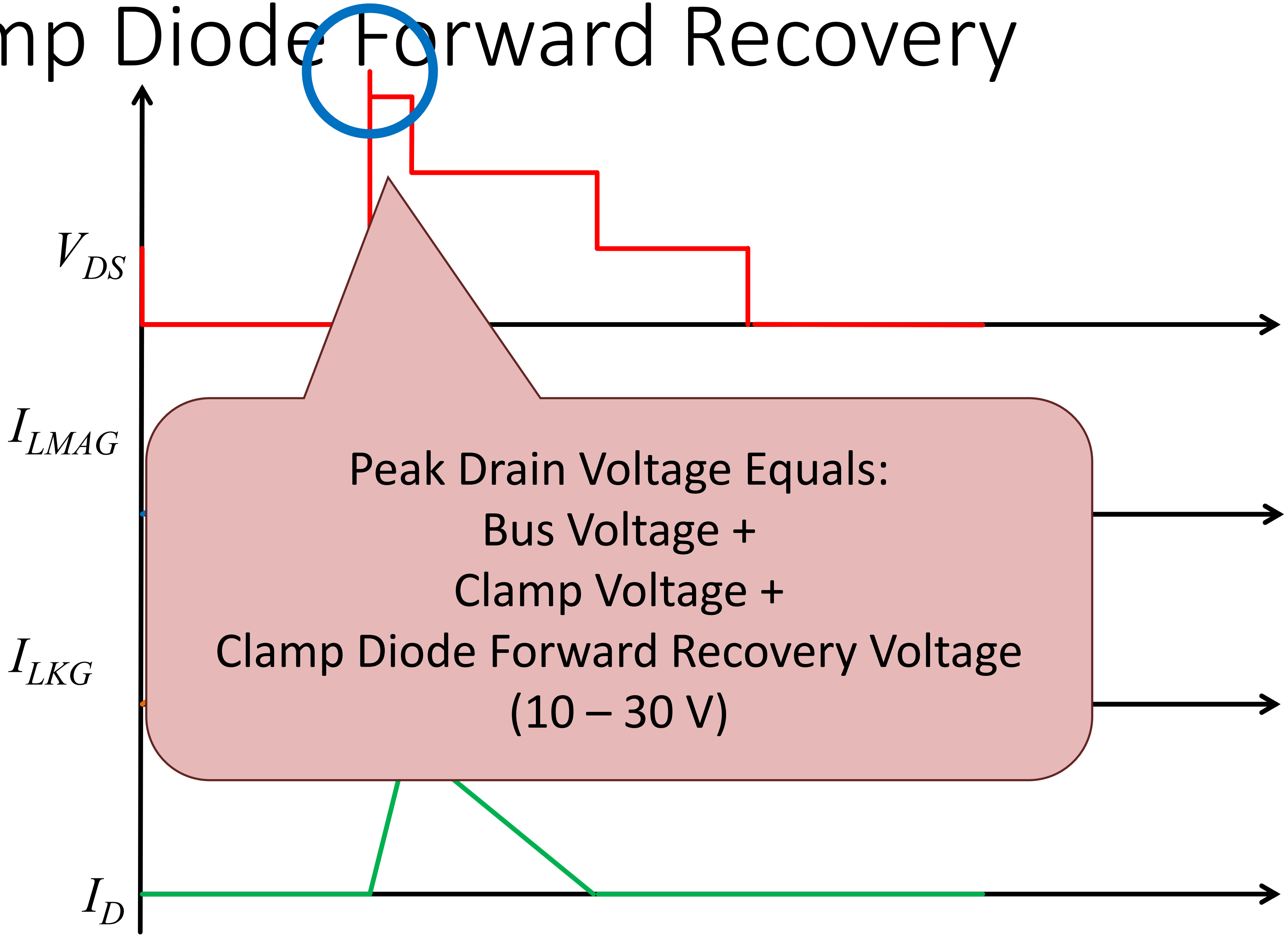
Clamp Loss


Clamp Loss Multiplier
Ratio Of Clamp Power To Leakage Power
Ratio Of Clamp Voltage To Reset Voltage
0
2
4
6
8
10
12
1
1.5
2
2.5
3
3.5
4
4.5
5

Clamp Loss

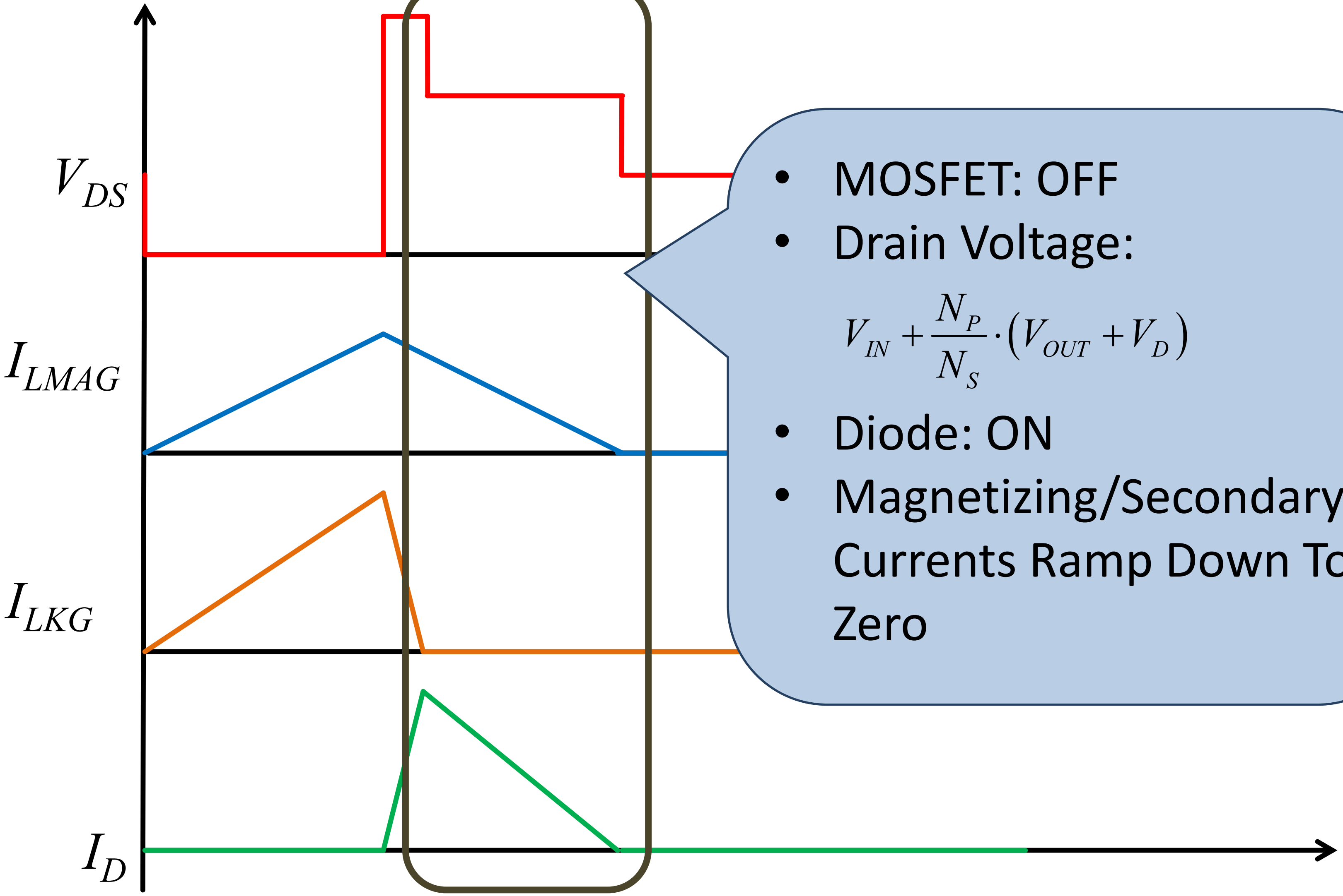

Clamp Loss Multiplier
Lesson
Trying To Make the Clamp Voltage Too Low
To Minimize The Peak Drain Voltage
Greatly Increases Loss
Ratio Of Clamp Power To Leakage Power
Ratio Of Clamp Voltage To Reset Voltage
0
2
4
6
8
10
12
1
1.5
2
2.5
3
3.5
4
4.5
5

Clamp Diode Forward Recovery



DCM Reset Time

DCM Reset Time

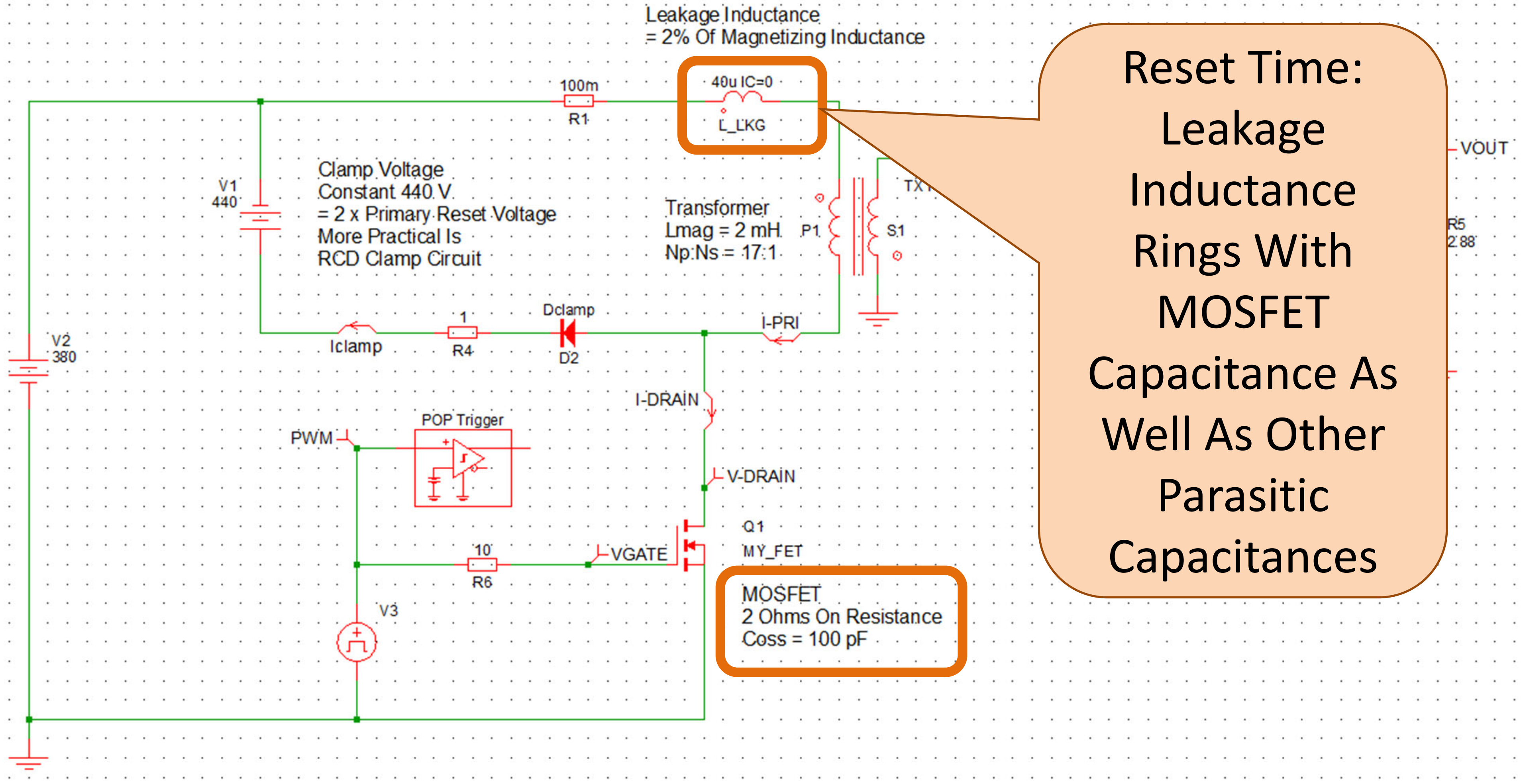


- MOSFET: OFF
- Drain Voltage:

$$V_{IN} + \frac{N_P}{N_S} \cdot \left(V_{OUT} + V_D\right)$$

- Diode: ON
- Magnetizing/Secondary Currents Ramp Down To Zero

Discontinuous Conduction Mode
More Realistic Flyback Simulation



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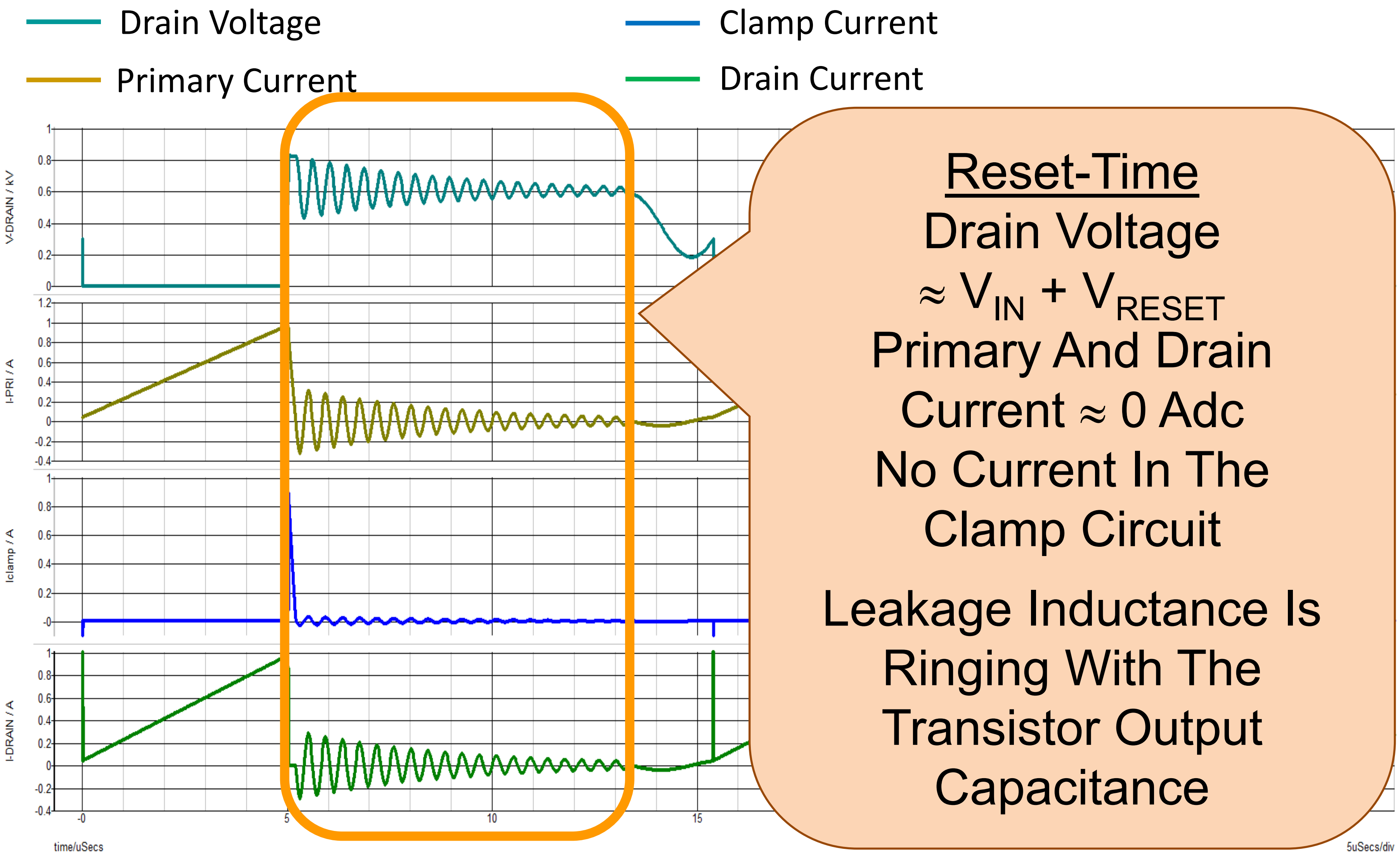
Discontinuous Conduction Mode
More Realistic Flyback Simulation



Reset-Time

Drain Voltage $\approx V_{IN} + V_{RESET}$

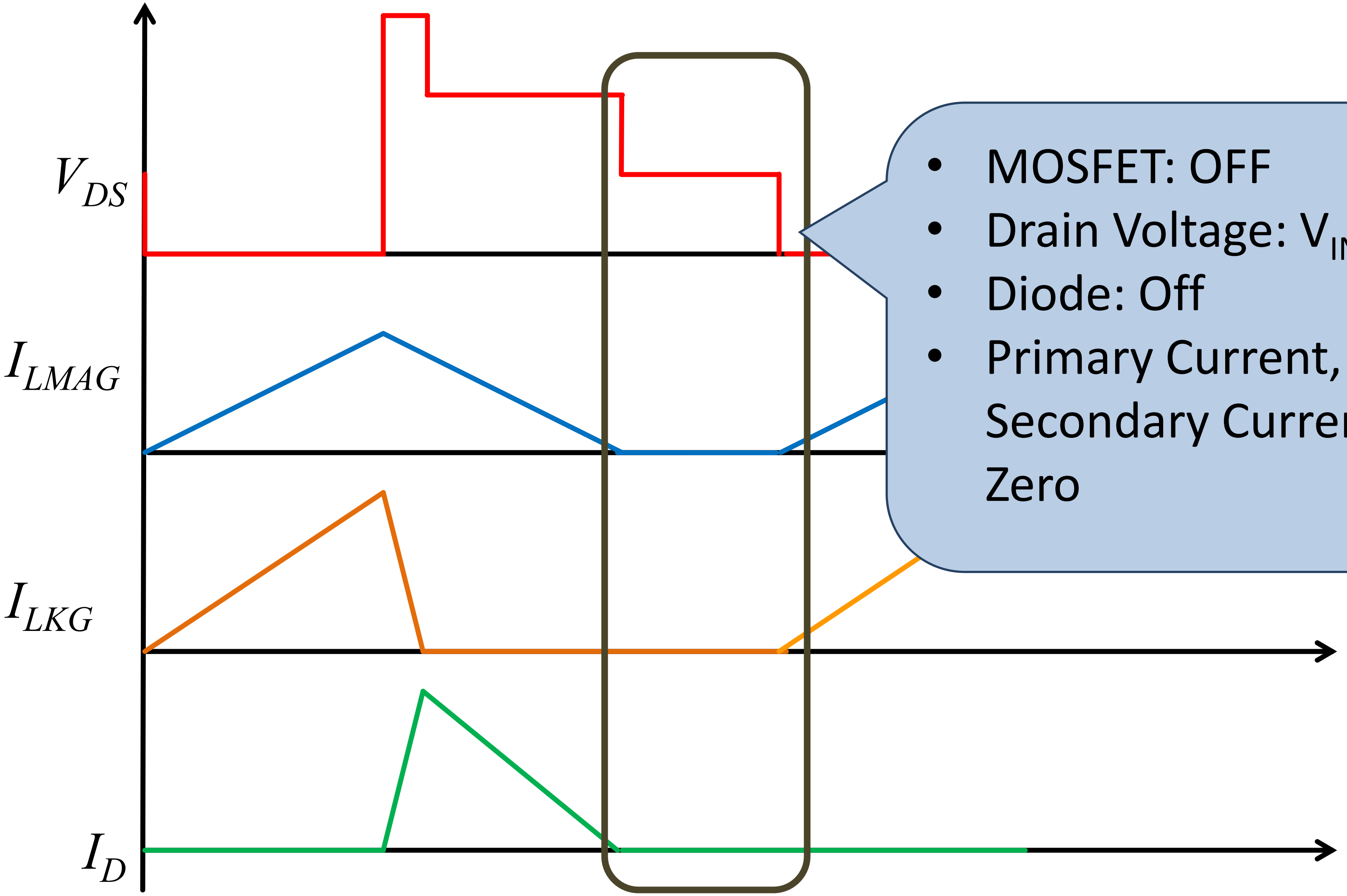
Primary And Drain Current ≈ 0 Adc

No Current In The Clamp Circuit

Leakage Inductance Is Ringing With The Transistor Output Capacitance

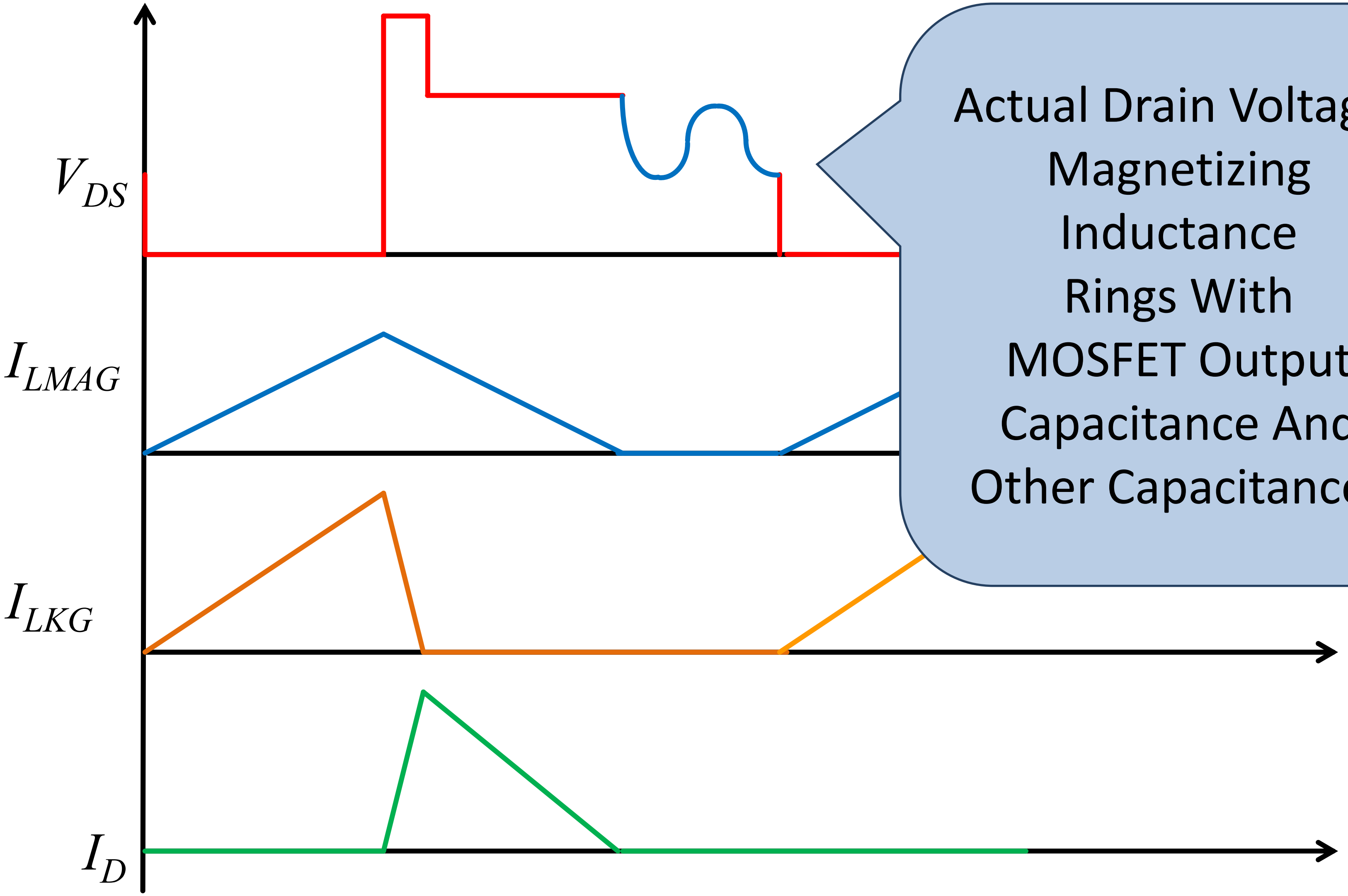
DCM Idle Time

DCM Idle Time



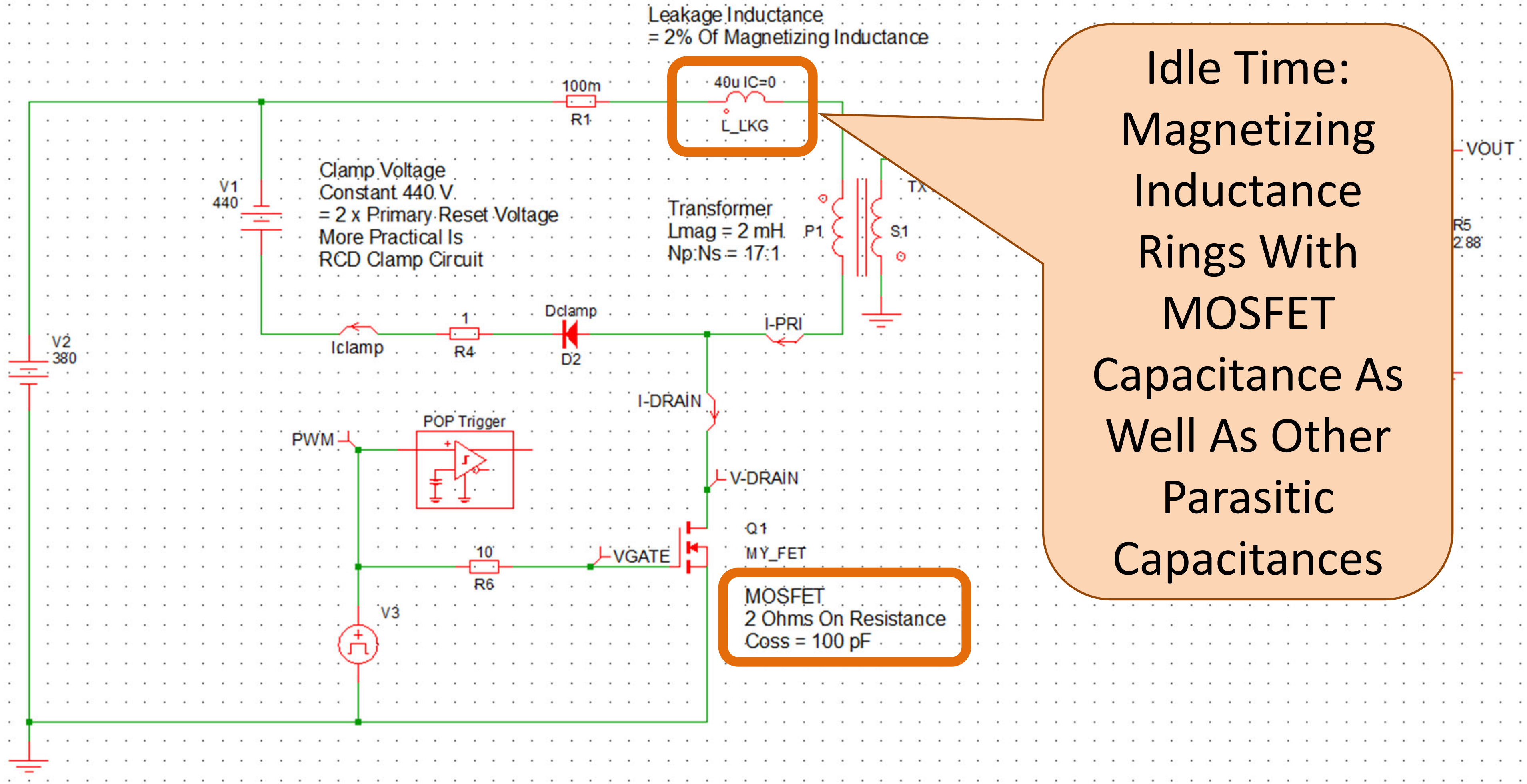
- MOSFET: OFF
- Drain Voltage: V_{IN}
- Diode: Off
- Primary Current, Secondary Current: Zero

DCM Idle Time

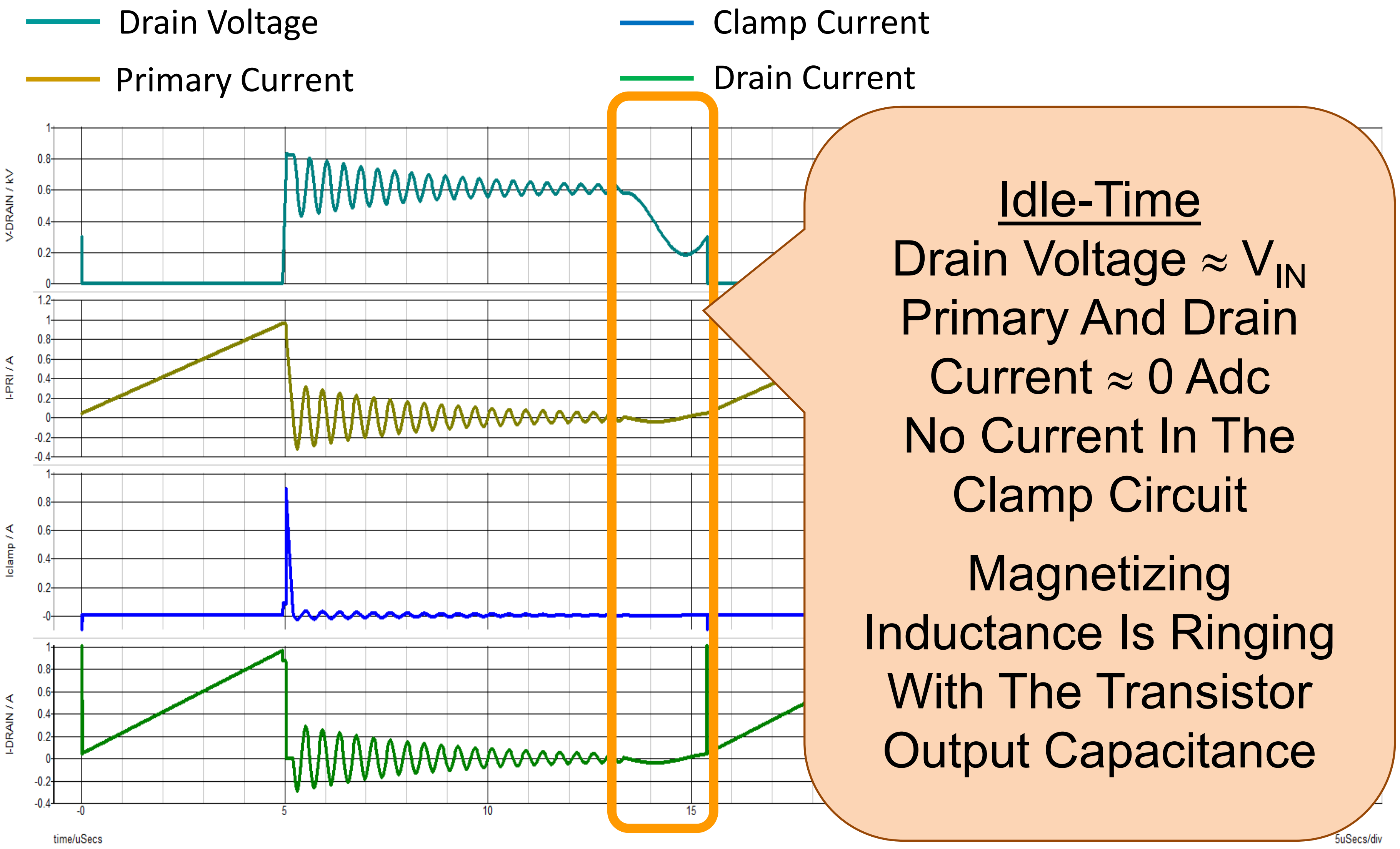


Actual Drain Voltage: Magnetizing Inductance Rings With MOSFET Output Capacitance And Other Capacitances

Discontinuous Conduction Mode
More Realistic Flyback Simulation



Discontinuous Conduction Mode
More Realistic Flyback Simulation



<u>Idle-Time</u>

Drain Voltage ≈ V_{IN}

Primary And Drain Current ≈ 0 Adc

No Current In The Clamp Circuit

Magnetizing Inductance Is Ringing With The Transistor Output Capacitance

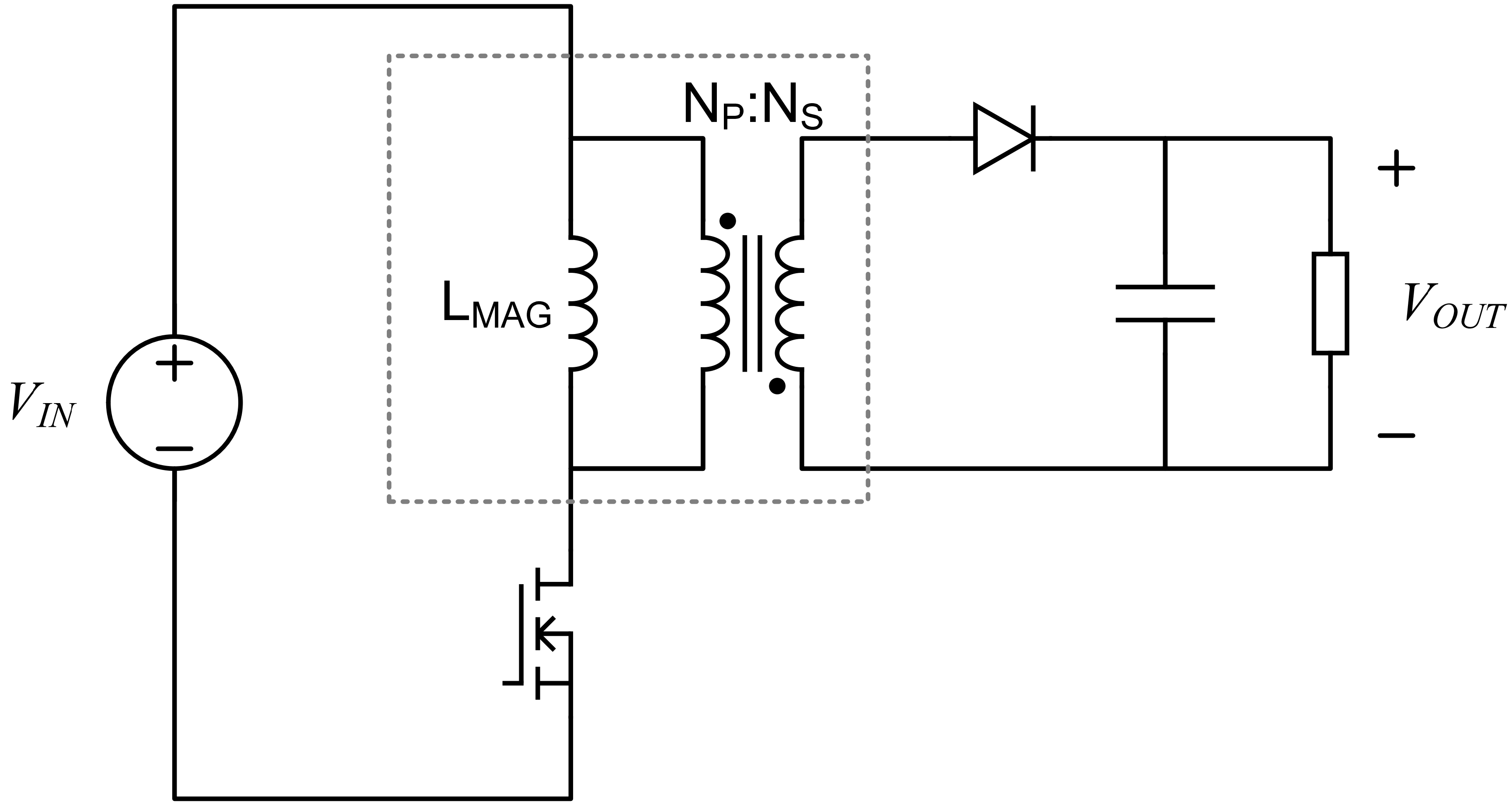
EMBEDD

DM Flyback Deeper Dive Summary

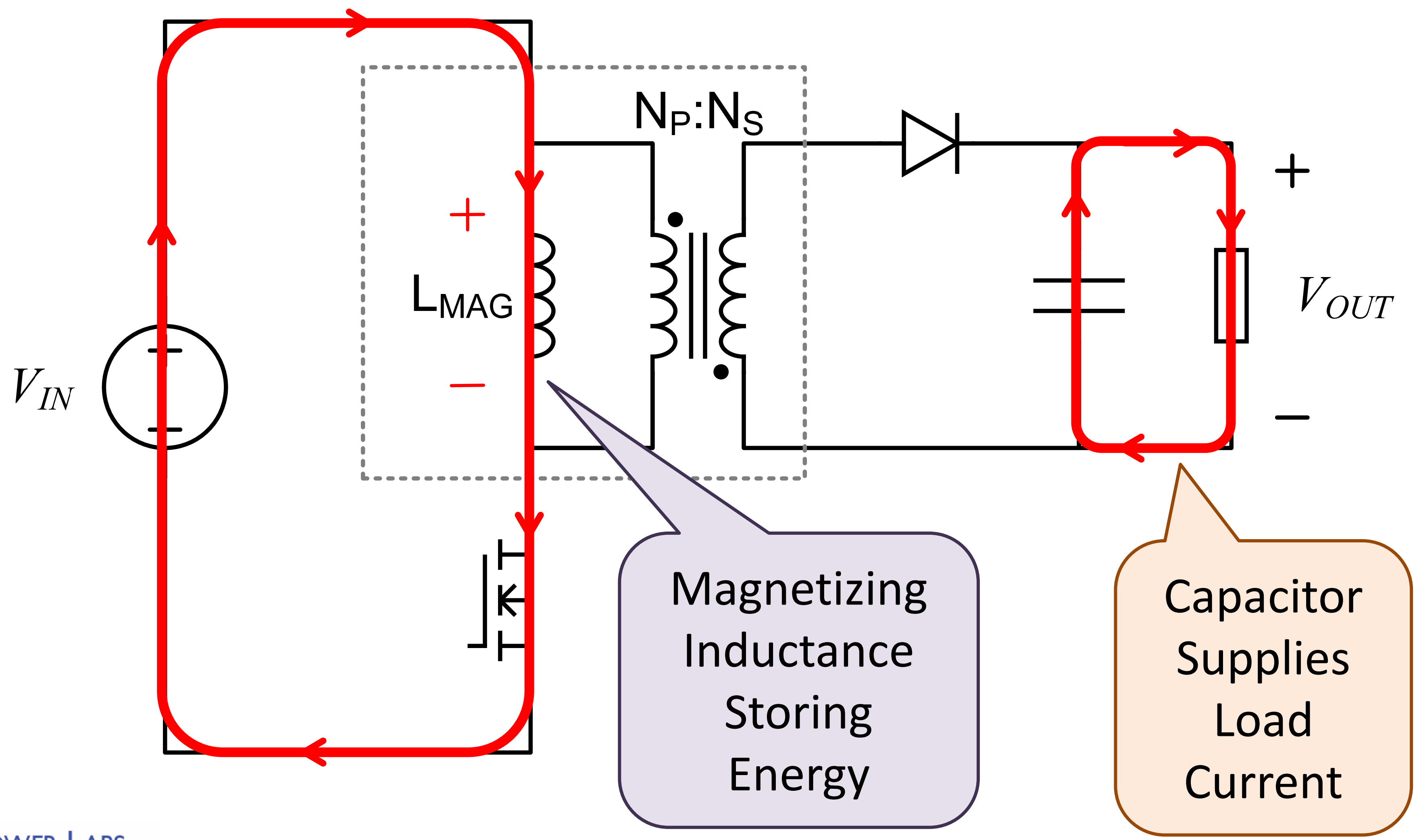
- Leakage Inductance Can Be Minimized But Not Eliminated
- Leakage Inductance Has A Profound Affect On DCM Flyback
  - High Frequency Ringing Causing EMI Problems
  - High Voltage On The Main Switch
  - Increased Losses
- Clamp Circuits Generally Used To Limit Voltage
- Choice Of Clamp Voltage Has Large Affect On Clamp Losses

APPENDIX VIII.
Flyback Converter
Continuous Conduction Mode

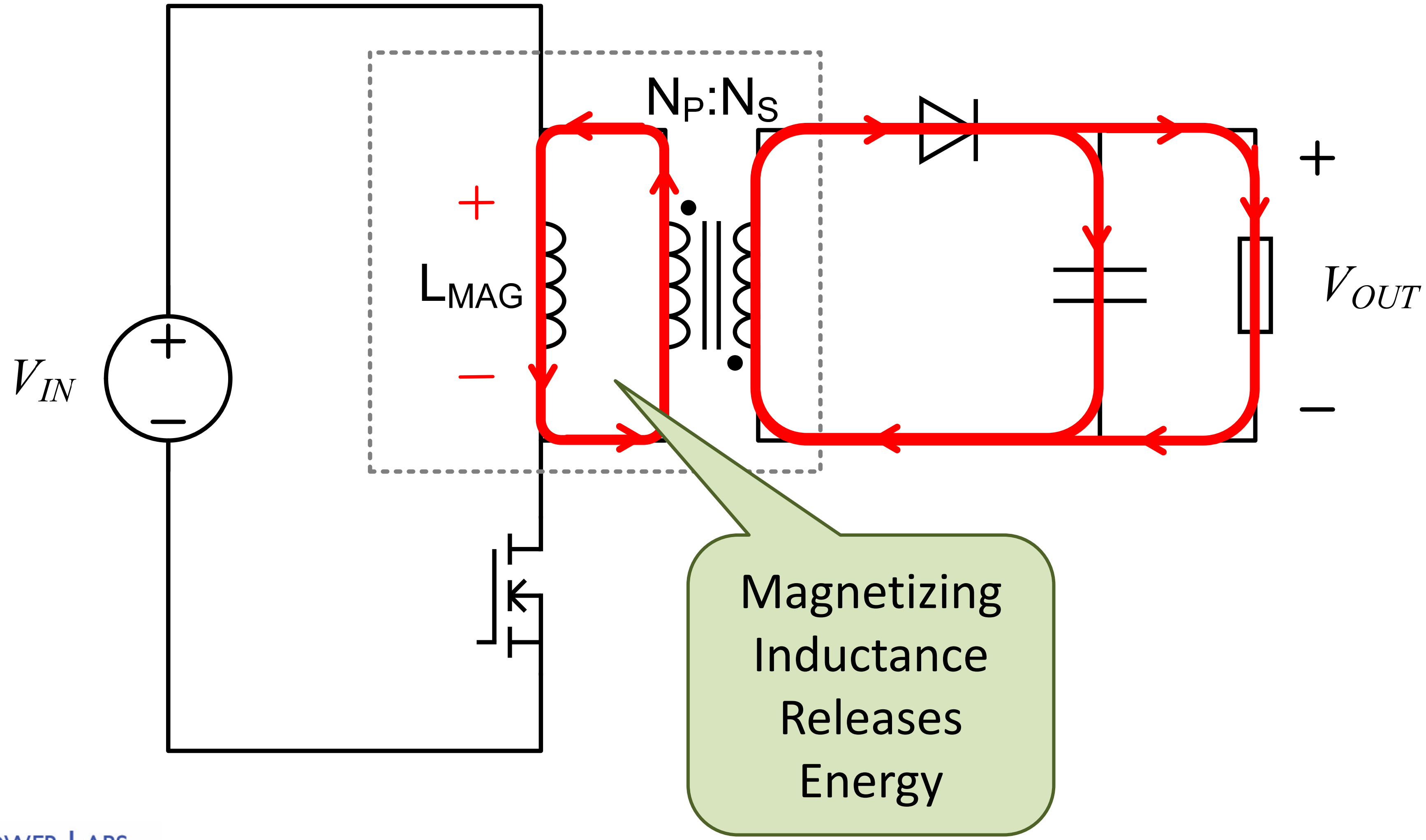
Flyback Converter



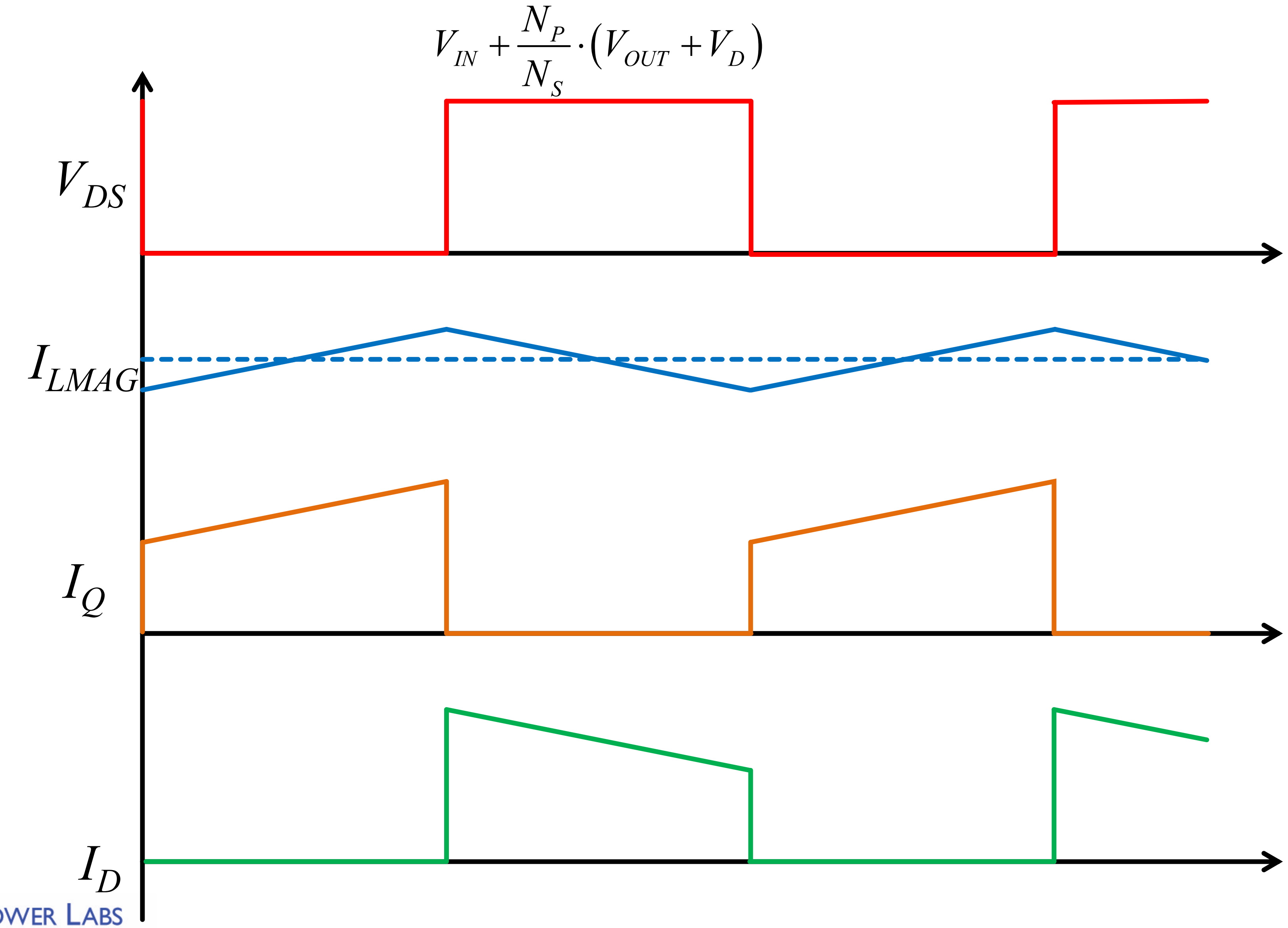
Flyback Converter On Time



Flyback Converter Off Time



Flyback Converter Waveforms



Flyback Conversion Ratio

$$V_{LMAG}(T_{ON}) \cdot T_{ON} + V_{LMAG}(T_{OFF}) \cdot T_{OFF} = 0$$

$$V_{IN} \cdot D + \left(-\frac{N_P}{N_S} \cdot (V_{OUT} + V_D) \right) \cdot D' = 0$$

Not Quite Ideal, Diode Voltage Included

$$D \cdot V_{IN} - D' \cdot \frac{N_P}{N_S} \cdot V_{OUT} - D' \cdot \frac{N_P}{N_S} \cdot V_D = 0$$

$$D' \cdot \frac{N_P}{N_S} \cdot V_{OUT} = D \cdot V_{IN} - D' \cdot \frac{N_P}{N_S} \cdot V_D$$

$$V_{OUT} = \frac{1}{D'} \cdot \frac{N_S}{N_P} \cdot \left(D \cdot V_{IN} - D' \cdot \frac{N_P}{N_S} \cdot V_D \right)$$

$$V_{OUT} = \frac{N_S}{N_P} \cdot \frac{D}{D'} \cdot V_{IN} - V_D$$

Flyback Conversion Ratio

$$V_{LMAG}\left(T_{ON}\right)\cdot T_{ON} + V_{LMAG}\left(T_{OFF}\right)\cdot T_{OFF} = 0$$

$$V_{IN}\cdot D + \left(-\frac{N_P}{N_S}\cdot\left(V_{OUT}+V_D\right)\right)\cdot D' = 0$$

$$D\cdot V_{IN} - D'\cdot\frac{N_P}{N_S}\cdot V_{OUT} - D'\cdot\frac{N_P}{N_S}\cdots$$

$$\cdots = D\cdot V_{IN} - D'\cdot\frac{N_P}{\cdots}\cdots$$

$$V_{OUT} = \frac{1}{D'}\cdot\frac{N_S}{N_P}\cdot\left(D\cdot V_{IN} - D'\cdot\frac{N_P}{N_S}\cdot V_D\right)$$

$$V_{OUT} = \frac{N_S}{N_P}\cdot\frac{D}{D'}\cdot V_{IN} - V_D$$

Transformer Turns Ratio

Standard Buck-Boost D/D' Conversion Ratio

Diode Voltage

Continuous Conduction Mode Flyback

Advantages

- Simpler Topology Than Forward Converter
- Can Process More Power Than DCM Flyback
- 100-250 W Possible

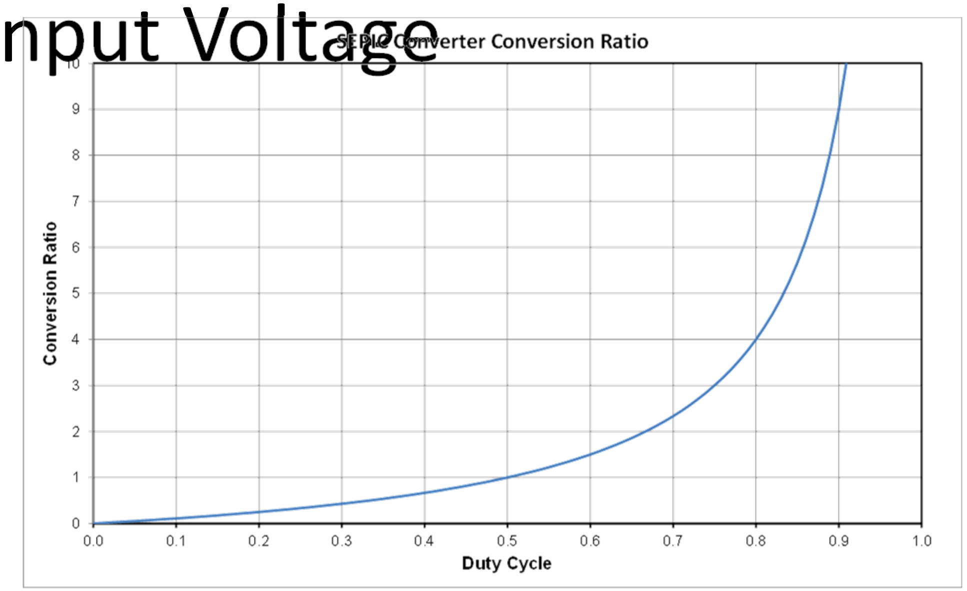
Disadvantages

- Control Characteristics Make It Hard To Stabilize And Get Good Transient Response
  - Load Dependent "Right Half Plane" Zero In Control-To-Output Transfer Function

Appendix IX
Four Switch Noninverting Buck Boost Converter

Noninverting Buck-Boost Converter

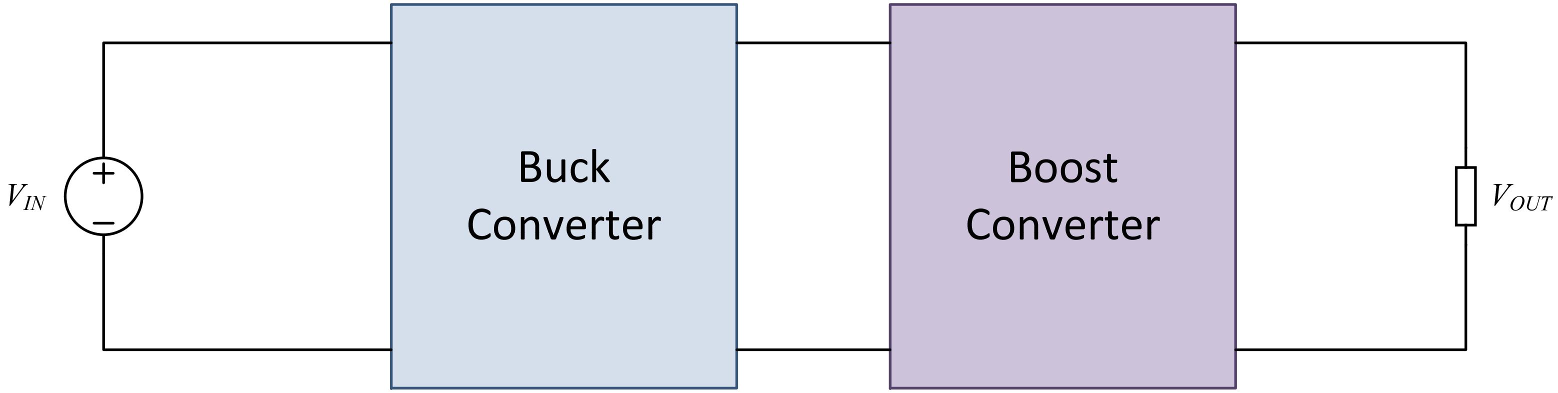
- The SEPIC Is A Noninverting Boost Converter
  - Input And Output Voltage Both Positive
- The D/(1 – D) Conversion Ratio Allows A Smooth And Continuous Variation Of The Output Voltage From Less Than To Greater Than The Input Voltage
- Another Noninverting Buck-Boost Converter Is The Four Switch Buck-Boost



Embedded Power Labs

Four-Switch Buck Boost

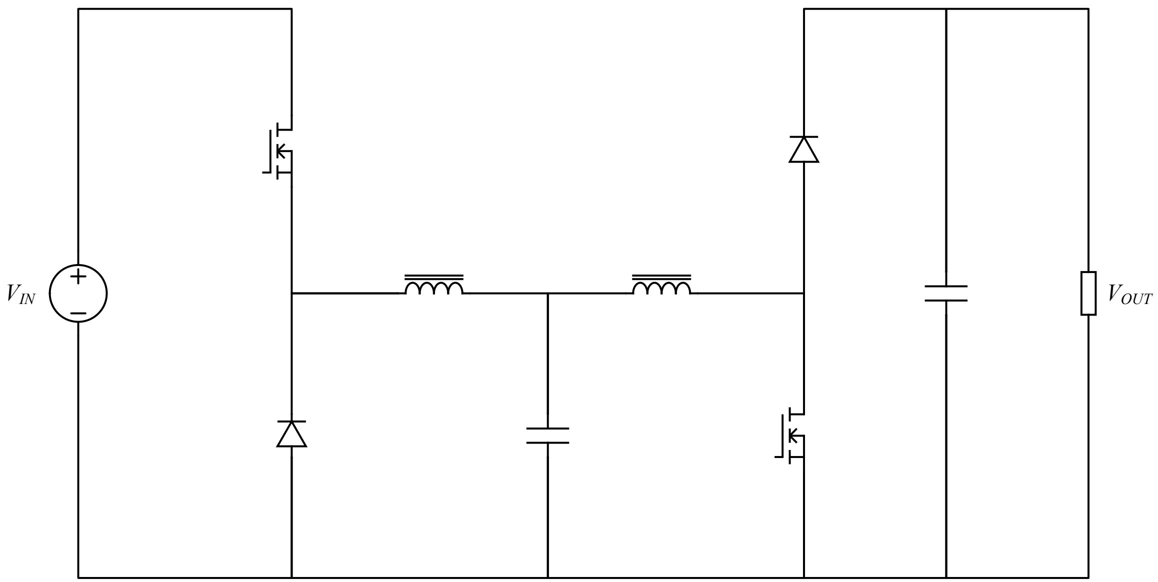
Consider The Cascade Of A Buck Converter And A Boost Converter



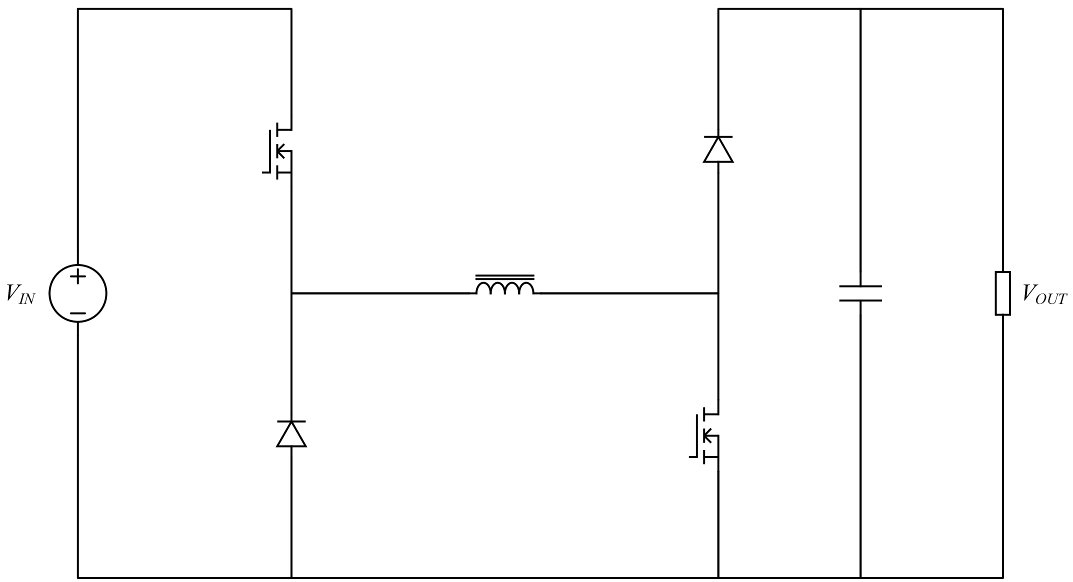
Conversion Ratio Is The Desired Noninverting Buck-Boost: $V_{OUT} = \frac{1}{1-D} \cdot D \cdot V_{IN}$

Four-Switch Buck Boost

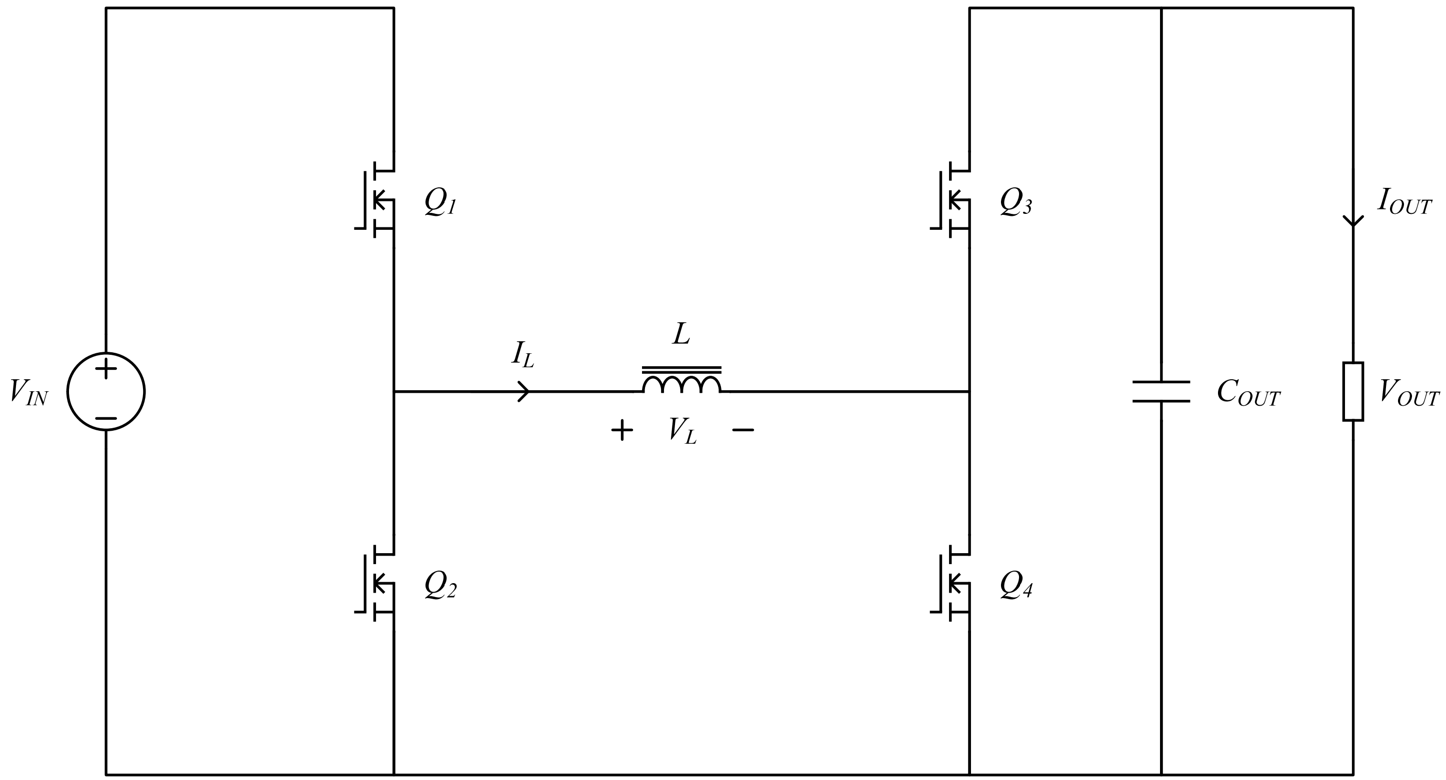
Cascade Of Buck And Boost Converters



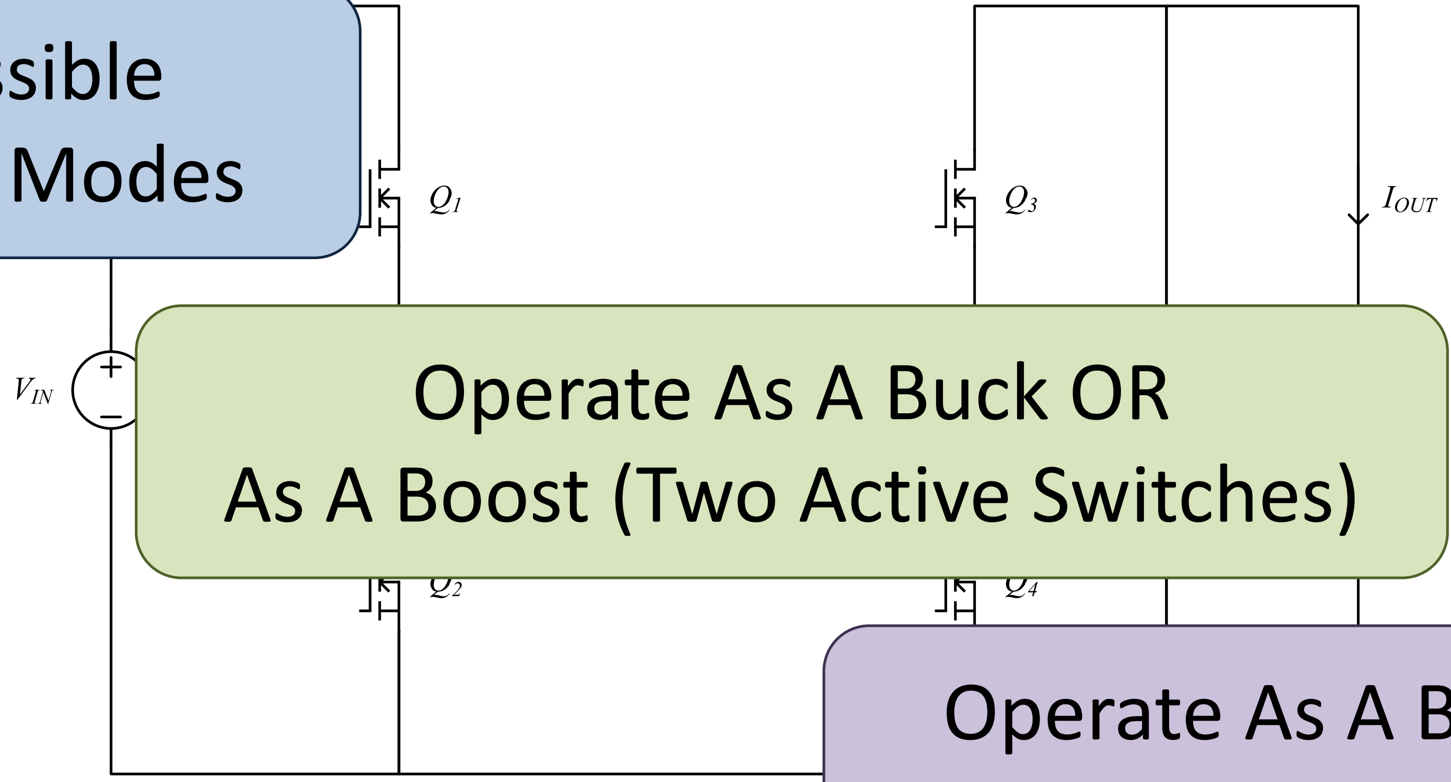
Simplify By Eliminating Common Capacitor And One Inductor



Synchronous Four Switch Buck Boost



Synchronous Four Switch Buck Boost



Two Possible Operating Modes

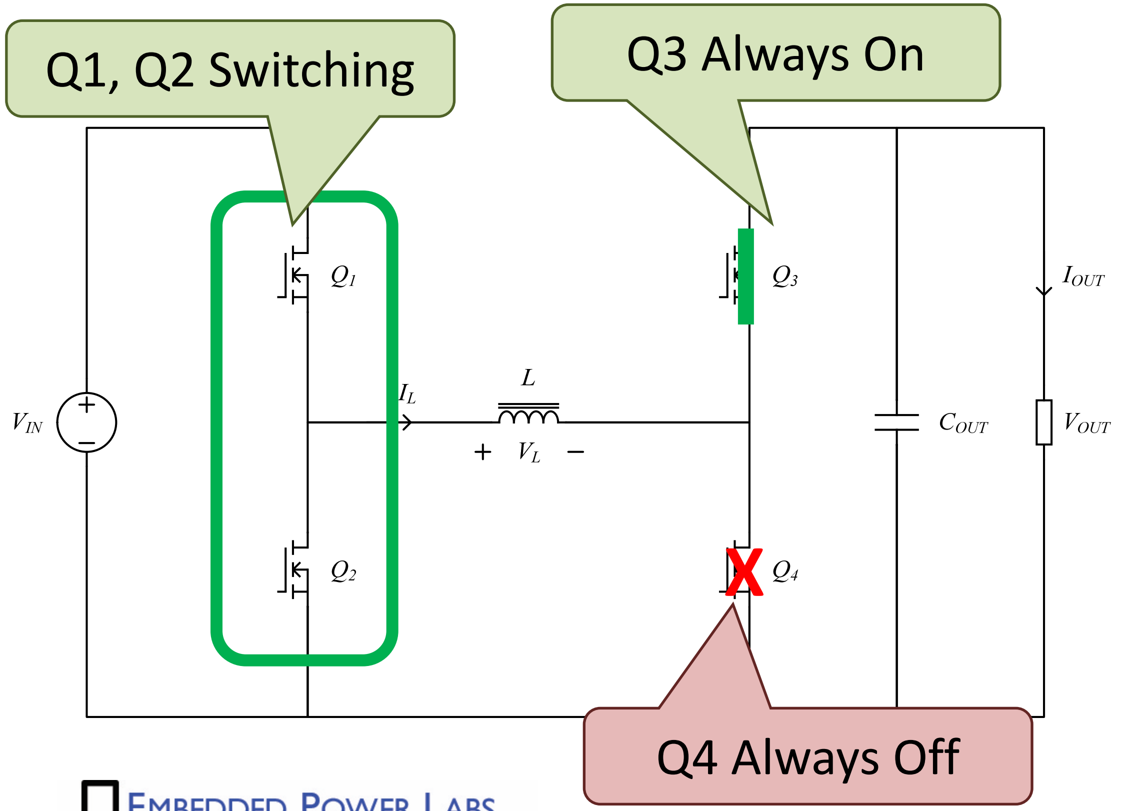
Operate As A Buck OR As A Boost (Two Active Switches)

Operate As A Buck-Boost (Four Active Switches)

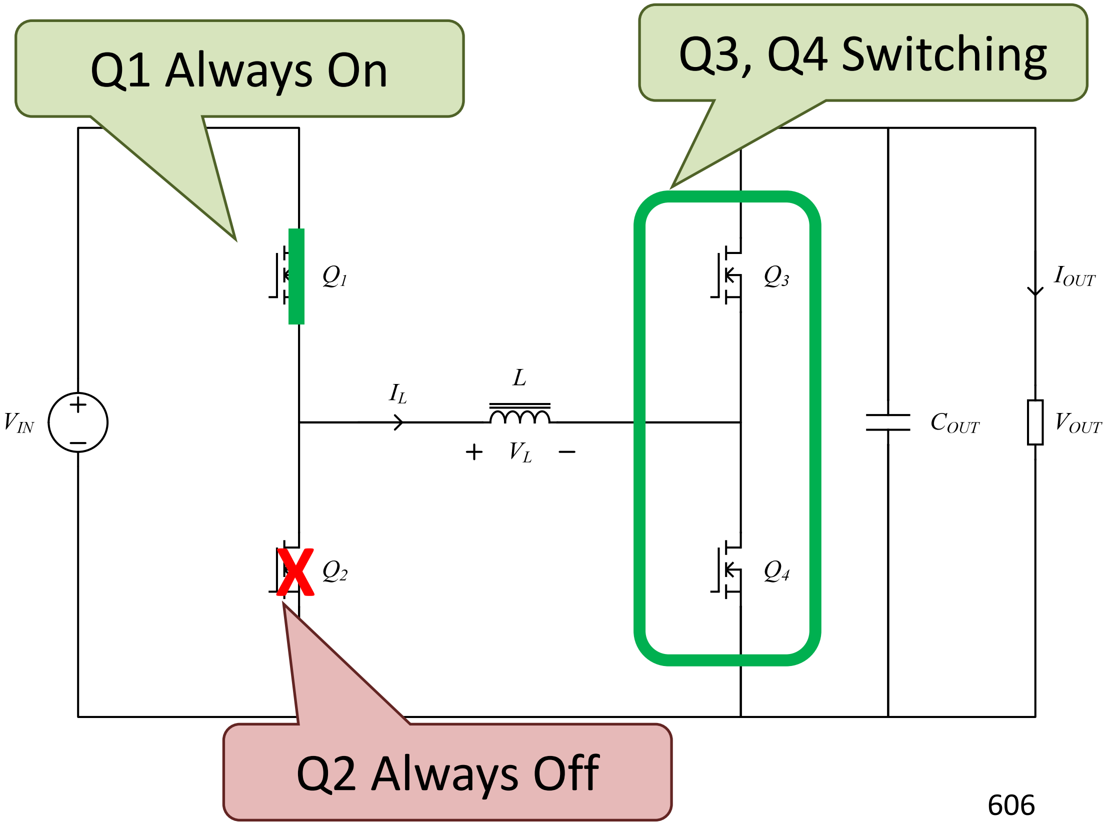
Synchronous Four Switch Buck-Boost

Operate As Either Buck OR Boost With Only Two Switches Active

Buck Mode

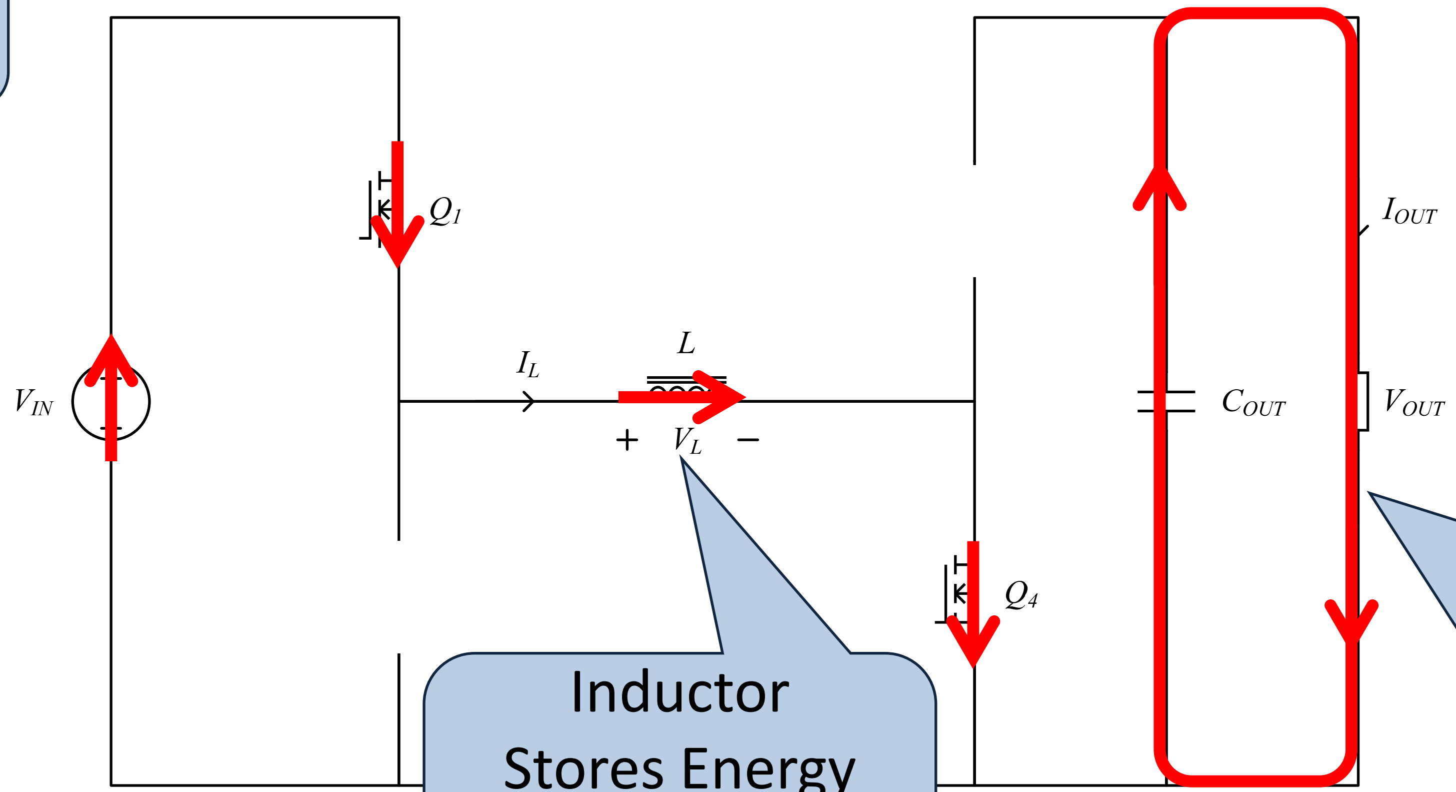


Boost Mode



Four Active Switches Mode – On Time

Q1, Q4: On
Q2, Q3: Off

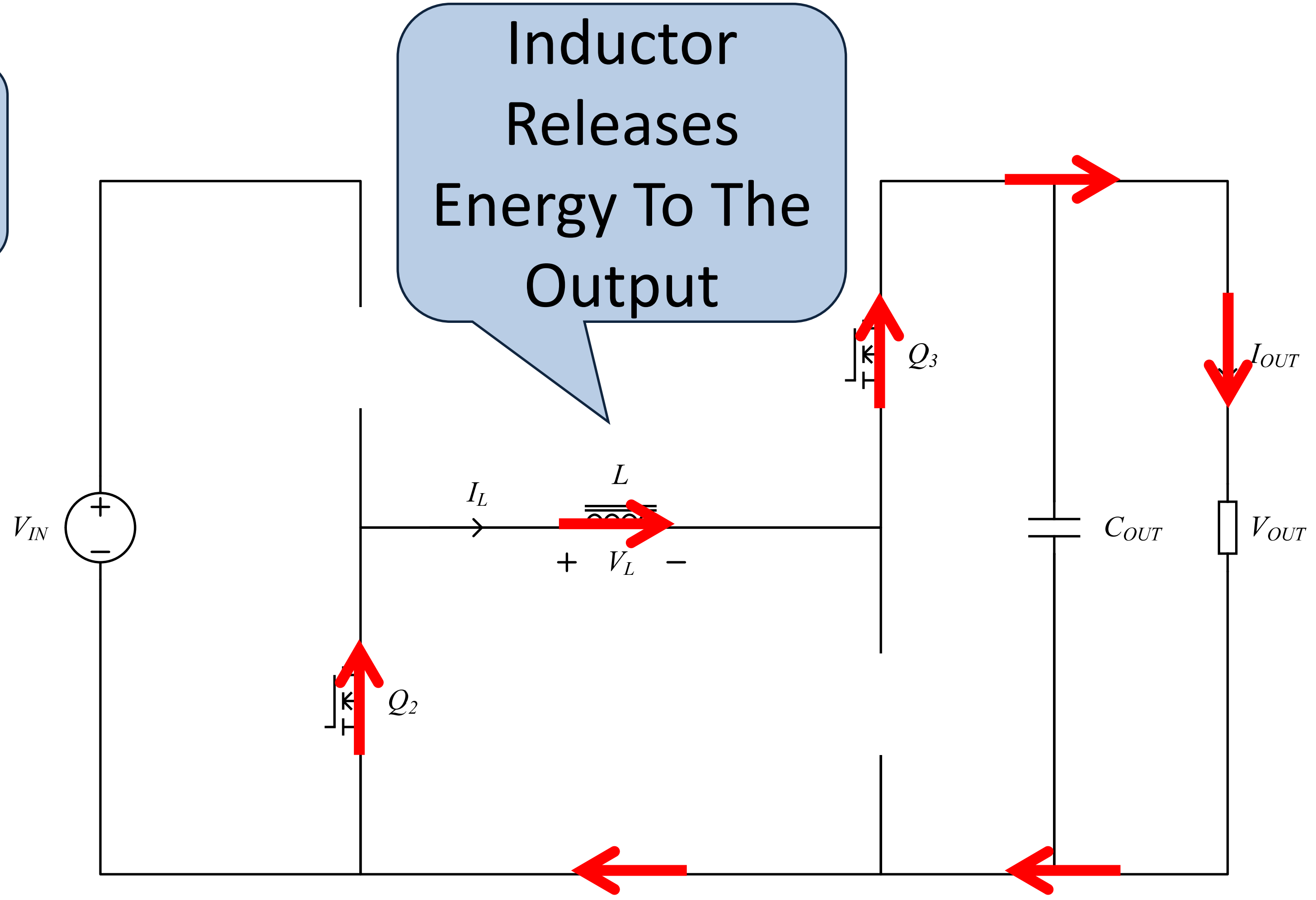


Inductor Stores Energy From Input Supply

Capacitor Supplies Energy To The Output

Four Active Switches Mode – Off Time

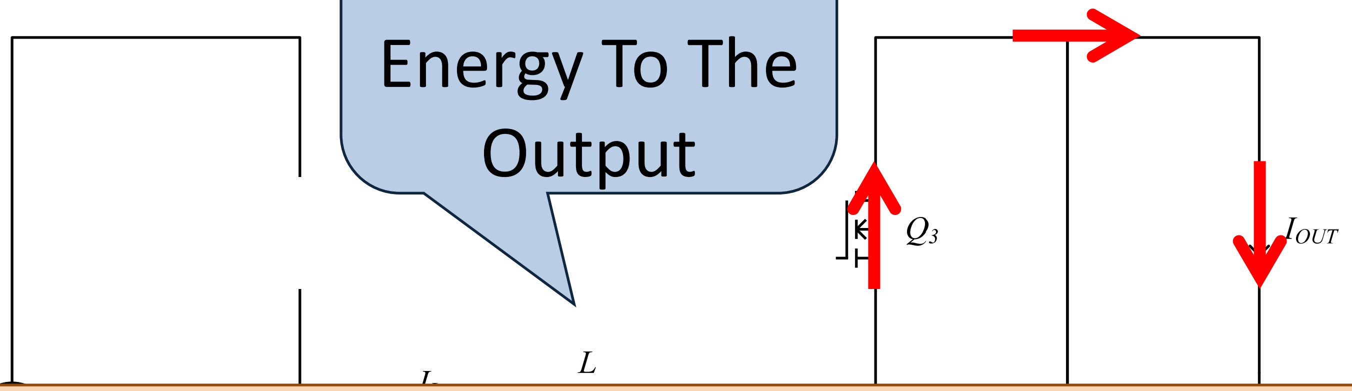
Q1, Q4: Off
Q2, Q3: On



Four Active Switches Mode – Off Time

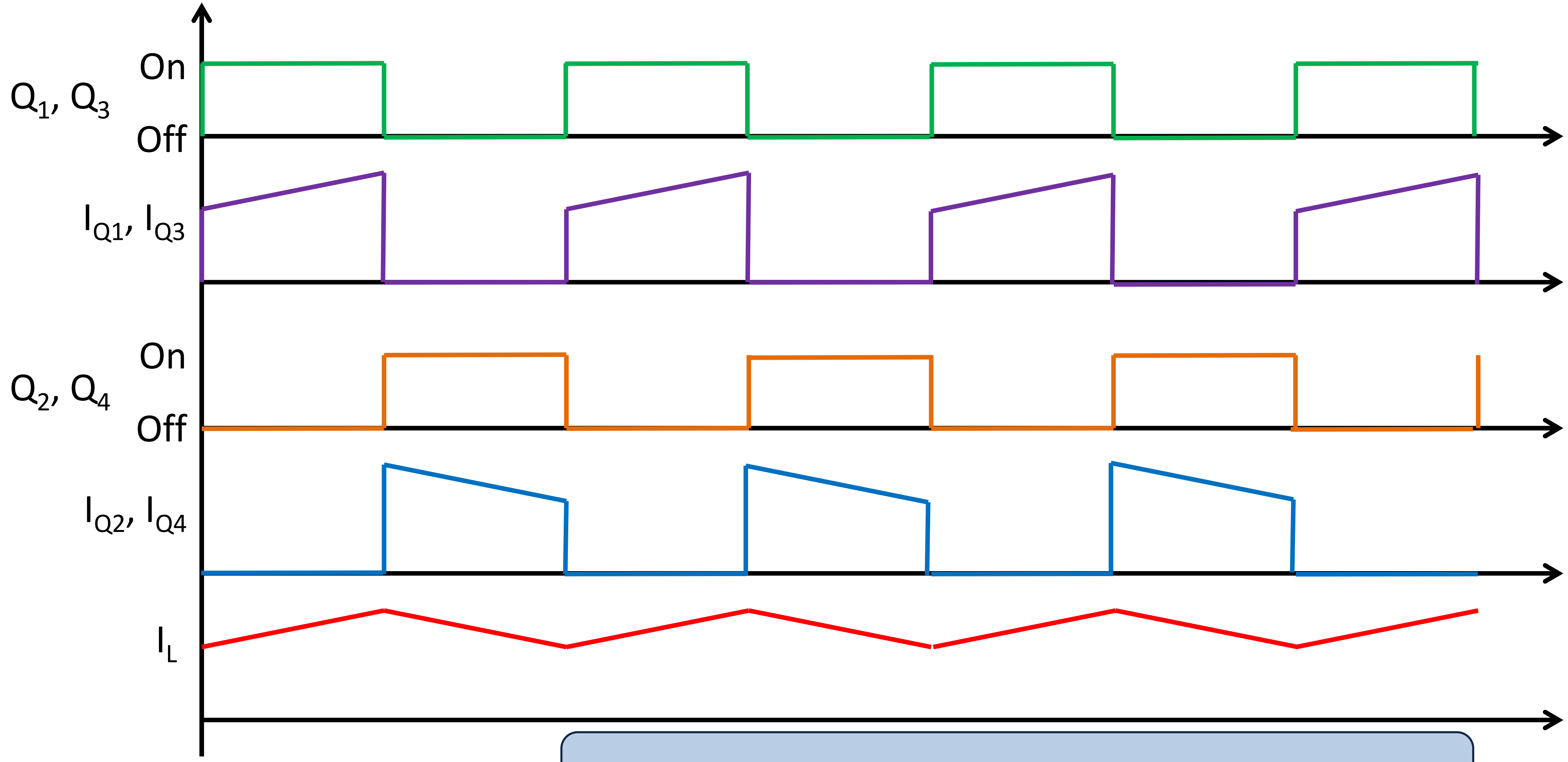
Q1, Q4: Off
Q2, Q3: On

Inductor Releases Energy To The Output



Note That Like The Inverting Buck-Boost
No Energy Is Transferred Directly From The Input To Output During Either The On-Time Or Off-Time

Four Active Switches Mode Waveforms - CCM



D = 0.5 $V_{OUT} = V_{IN}$ $I_{L\_AVE} = 2 \cdot I_{OUT}$

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Four Active Switches Mode Conversion Ratio

Output Voltage Conversion Ratio

Inductor Volt-Second Balance

$$V_{IN} \cdot T_{ON} + \left(-V_{OUT}\right) \cdot T_{OFF} = 0$$

$$V_{IN} \cdot D \cdot T_{SW} - V_{OUT} \cdot \left(1-D\right) \cdot T_{SW} = 0$$

$$V_{IN} \cdot D - V_{OUT} \cdot \left(1-D\right) = 0$$

$$V_{OUT} \cdot \left(1-D\right) = V_{IN} \cdot D$$

$$V_{OUT} = \frac{D}{1-D} \cdot V_{IN} = \frac{D}{D'} \cdot V_{IN}$$

Noninverting Buck-Boost Conversion Ratio

Average Inductor Current

Capacitor Charge Balance

$$-I_{OUT} \cdot T_{ON} + \left(I_L - I_{OUT}\right) \cdot T_{OFF} = 0$$

$$-I_{OUT} \cdot D \cdot T_{SW} + \left(I_L - I_{OUT}\right) \cdot \left(1-D\right) \cdot T_{SW} = 0$$

$$-I_{OUT} \cdot D + \left(I_L - I_{OUT}\right) \cdot \left(1-D\right) = 0$$

$$-I_{OUT} \cdot D + I_L \cdot \left(1-D\right) - I_{OUT} \cdot \left(1-D\right) = 0$$

$$-I_{OUT} + I_L \cdot \left(1-D\right) = 0$$

$$I_L = \frac{1}{1-D} \cdot I_{OUT}$$

Operating Mode Comparison

Buck Or Boost

- Highest Efficiency As Only Two Active Switches
- Hard To Control Transition As Output Voltage Increases Or Decreases Past The Input Voltage
  - Consider Cases When $V_{OUT} = 0.99 \cdot V_{IN}$ Or $V_{OUT} = 1.01 \cdot V_{IN}$

Four Active Switches Mode

- Smooth And Easy Control As Output Voltage Increases Or Decreases Past The Input Voltage
- Lower Efficiency With Four Active Switches

Four Switch Buck-Boost Summary/SEPIC Comparison

Four Switch Buck Boost

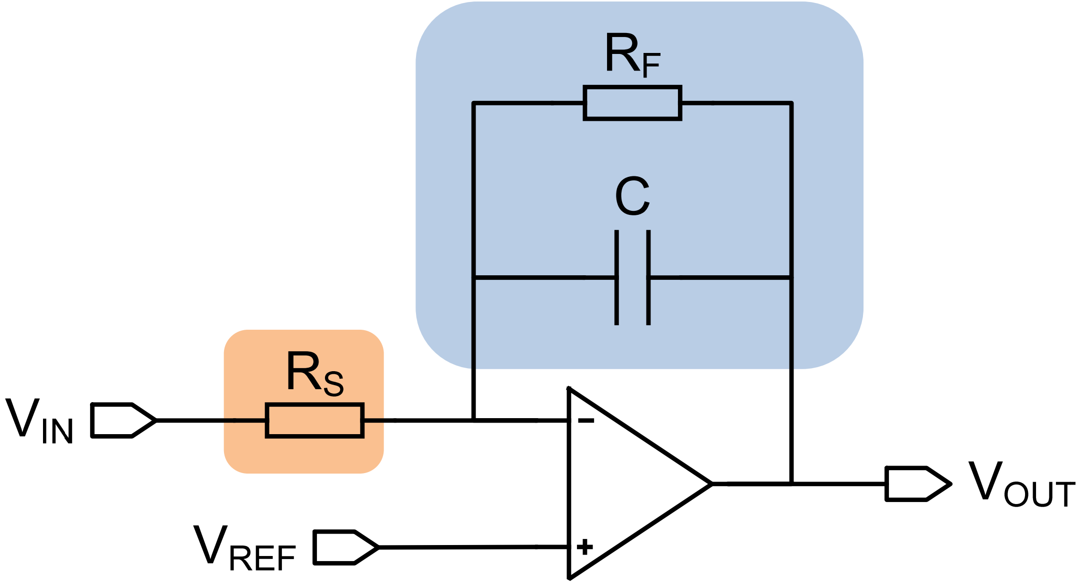
- Topology
  - Four Switches
  - One Simple Inductor
- More Complex Switch Drive
  - Two Or Four Controlled Switches
- Input-To-Output Shoot-Through Failure Mode
- Simpler Control Characteristic

SEPIC

- Topology
  - One Controlled Switch
  - Two Inductors Or One Coupled Inductor
- Simple Single Switch Drive
- No Input-To-Output Shoot-Through Failure Mode
- Complex Control Characteristic

Appendix X
Simple Pole And Zero Transfer Functions

Simple Pole Transfer Function $H_P(s)$



$$H_P(s) = -\frac{Z_F}{Z_S}$$

$$Z_F = R_F \parallel \frac{1}{s \cdot C}$$

$$= \frac{R_F \cdot \frac{1}{s \cdot C}}{R_F + \frac{1}{s \cdot C}} \cdot \frac{s \cdot C}{s \cdot C}$$

$$= \frac{R_F}{1 + s \cdot R_F \cdot C}$$

$$Z_S = R_S$$

Simple Pole Transfer Function

$$H_P(s) = -\frac{Z_F}{Z_S}$$

$$H_P(s) = -\frac{\dfrac{R_F}{1 + s \cdot R_F \cdot C}}{R_S}$$

$$= -\frac{R_F}{R_S} \cdot \frac{1}{1 + s \cdot R_F \cdot C}$$

$$H_P(s) = G_0 \cdot \frac{1}{1 + \dfrac{s}{\omega_P}}$$

Standard Form With DC Gain Term And Frequency Dependent Term

$$G_0 = -\frac{R_F}{R_S}$$

DC Gain

$$\omega_P = 2 \cdot \pi \cdot f_P = \frac{1}{R_F \cdot C}$$

$$f_P = \frac{1}{2 \cdot \pi \cdot R_F \cdot C}$$

Pole Frequency

Simple Pole Transfer Function: Magnitude

$$\left|H_P(j\cdot\omega)\right|^2 = H_P(j\cdot\omega)\cdot H_P^*(j\cdot\omega)$$
$$= H_P(j\cdot\omega)\cdot H_P(-j\cdot\omega)$$
$$= \left(G_0\cdot\frac{1}{1+\frac{j\cdot\omega}{\omega_P}}\right)\cdot\left(G_0\cdot\frac{1}{1+\frac{-j\cdot\omega}{\omega_P}}\right)$$
$$= G_0^2\cdot\frac{1}{1+\frac{j\cdot\omega}{\omega_P}}\cdot\frac{1}{1-\frac{j\cdot\omega}{\omega_P}}$$
$$= G_0^2\cdot\frac{1}{1+\left(\frac{\omega}{\omega_P}\right)^2}$$

$$\left|H_P(j\cdot\omega)\right| = G_0\cdot\frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_P}\right)^2}}$$
$$= G_0\cdot\frac{1}{\sqrt{1+\left(\frac{f}{f_P}\right)^2}}$$

Consider The Case When $f = f_P$

$$\left|H_P(j\cdot\omega_P)\right| = G_0\cdot\frac{1}{\sqrt{1+\left(\frac{f_P}{f_P}\right)^2}} = G_0\cdot\frac{1}{\sqrt{1+1^2}}$$
$$= G_0\cdot\frac{1}{\sqrt{2}} = G_0\cdot\frac{\sqrt{2}}{2} = -3\,\text{dB}$$

Simple Pole Transfer Function: Phase

$$\angle H_P(j\cdot\omega) = \tan^{-1}\left(\frac{\operatorname{Im}\left(H_P(j\cdot\omega)\right)}{\operatorname{Re}\left(H_P(j\cdot\omega)\right)}\right)$$

$$\operatorname{Im}\left(H_P(j\cdot\omega)\right) = \operatorname{Im}\left(G_0\cdot\frac{1}{1+\frac{j\cdot\omega}{\omega_P}}\right) = \operatorname{Im}\left(G_0\cdot\frac{1}{1+\frac{j\cdot\omega}{\omega_P}}\cdot\frac{1-\frac{j\cdot\omega}{\omega_P}}{1-\frac{j\cdot\omega}{\omega_P}}\right)$$

$$= \operatorname{Im}\left(G_0\cdot\frac{1-\frac{j\cdot\omega}{\omega_P}}{1+\left(\frac{\omega}{\omega_P}\right)^2}\right)$$

$$= G_0\cdot\frac{-\frac{\omega}{\omega_P}}{1+\left(\frac{\omega}{\omega_P}\right)^2}$$

$$\operatorname{Re}\left(H_P(j\cdot\omega)\right) = \operatorname{Re}\left(G_0\cdot\frac{1}{1+\frac{j\cdot\omega}{\omega_P}}\right) = \operatorname{Re}\left(G_0\cdot\frac{1}{1+\frac{j\cdot\omega}{\omega_P}}\cdot\frac{1-\frac{j\cdot\omega}{\omega_P}}{1-\frac{j\cdot\omega}{\omega_P}}\right)$$

$$= \operatorname{Re}\left(G_0\cdot\frac{1-\frac{j\cdot\omega}{\omega_P}}{1+\left(\frac{\omega}{\omega_P}\right)^2}\right)$$

$$= G_0\cdot\frac{1}{1+\left(\frac{\omega}{\omega_P}\right)^2}$$

Simple Pole Transfer Function: Phase

$$\angle H(j\cdot\omega) = \tan^{-1}\left(\frac{\mathrm{Im}\left(H(j\cdot\omega)\right)}{\mathrm{Re}\left(H(j\cdot\omega)\right)}\right)$$

$$= \tan^{-1}\left(\frac{G_0\cdot\dfrac{-\dfrac{\omega}{\omega_P}}{1+\left(\dfrac{\omega}{\omega_P}\right)^2}}{G_0\cdot\dfrac{1}{1+\left(\dfrac{\omega}{\omega_P}\right)^2}}\right)$$

$$= \tan^{-1}\left(-\frac{\omega}{\omega_P}\right) = \tan^{-1}\left(-\frac{f}{f_P}\right)$$

Consider The Case When f = f_P

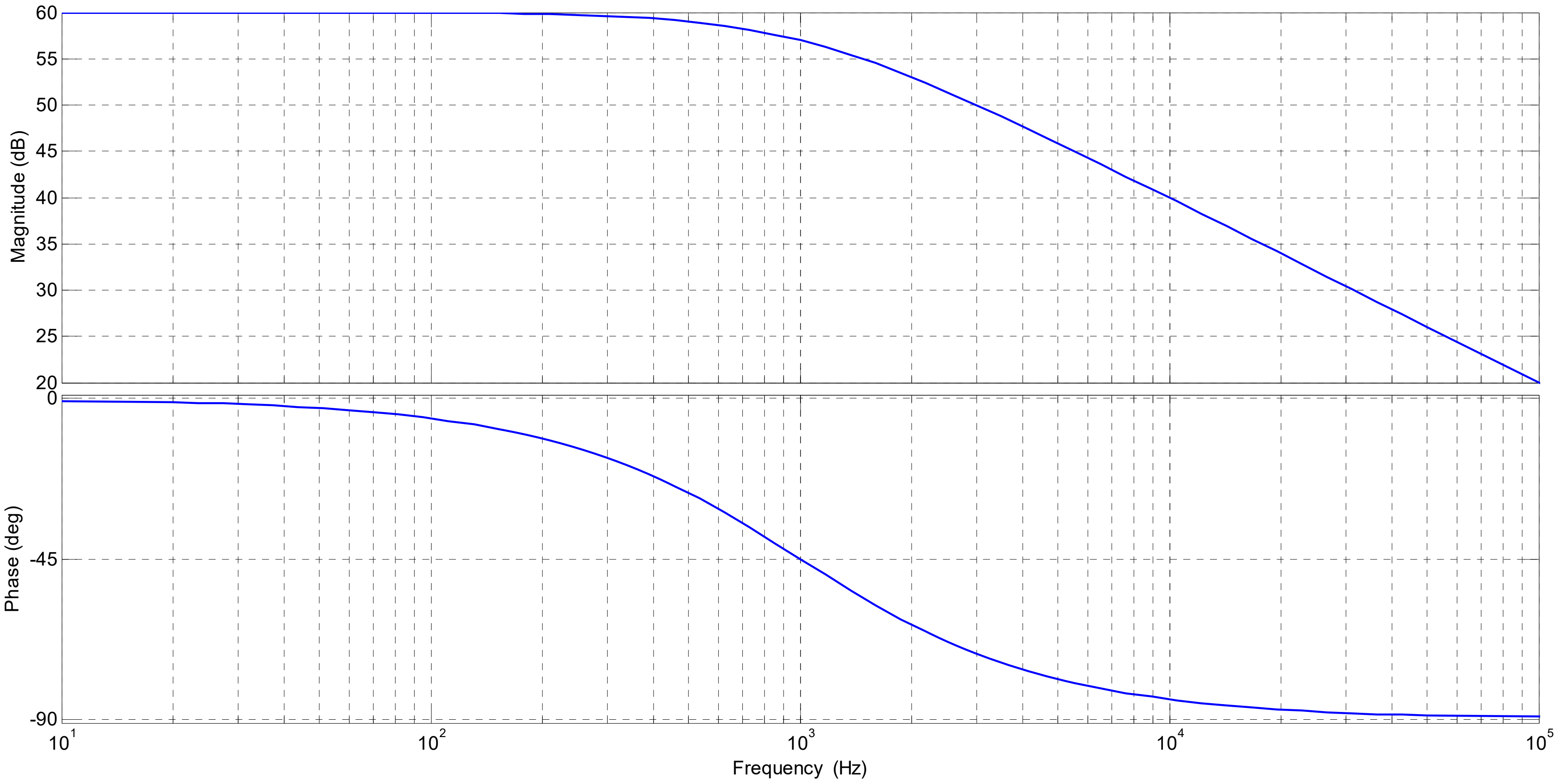
$$\angle H(j\cdot\omega_P) = \tan^{-1}\left(-\frac{f_P}{f_P}\right)$$

$$= \tan^{-1}(-1)$$

$$= -\frac{\pi}{4} = -45^\circ$$

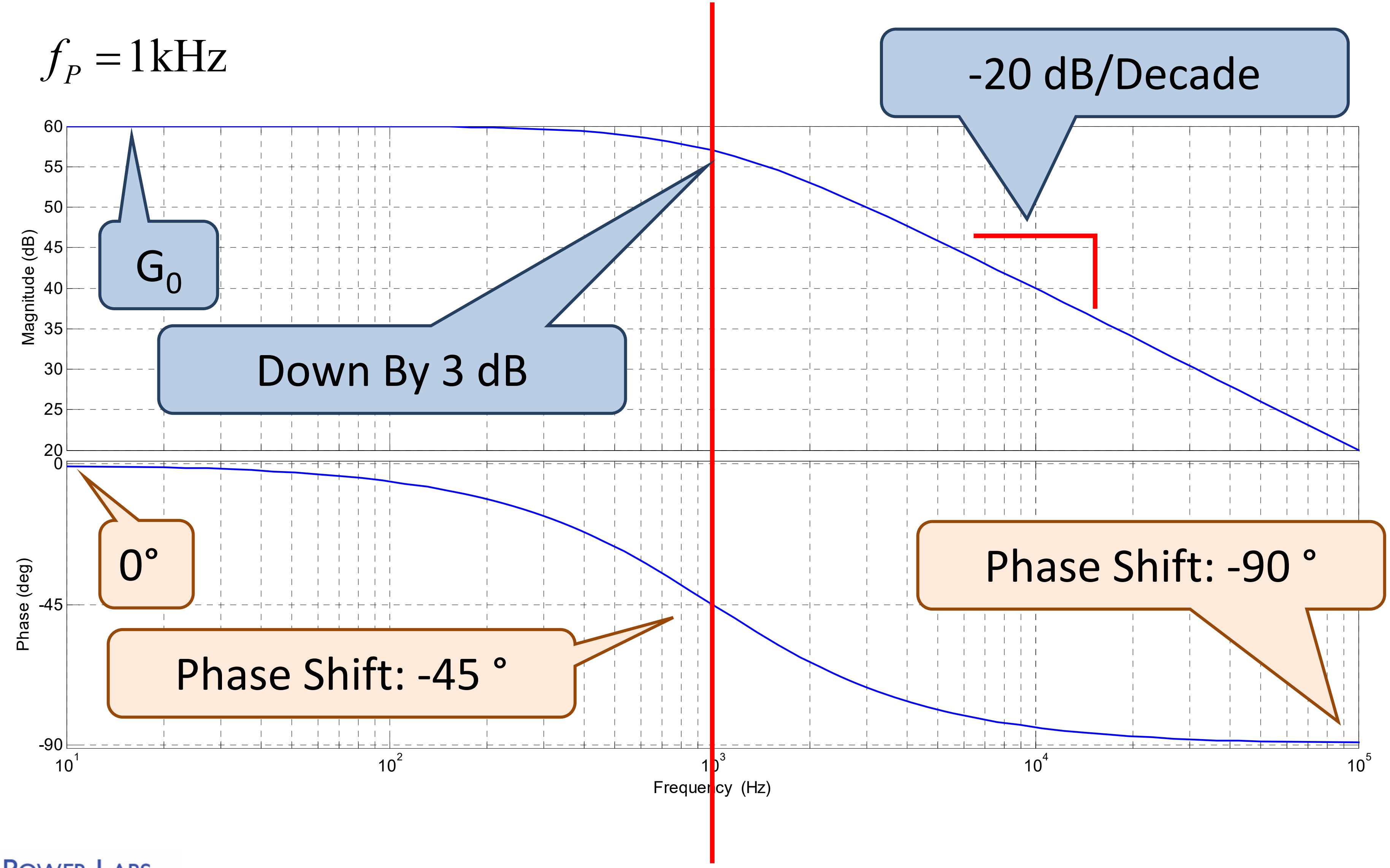
Simple Pole Bode Plot

$f_P = 1\text{kHz}$

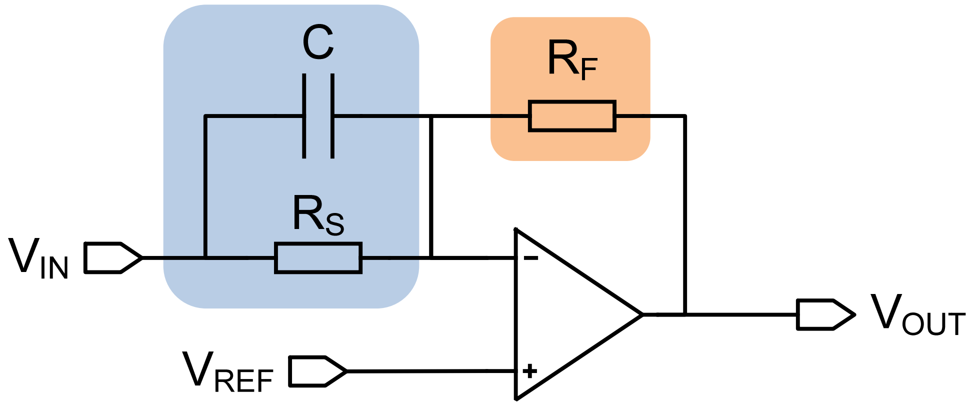


Simple Pole Bode Plot

$$f_P = 1\text{kHz}$$



Simple Zero Transfer Function $H_Z(s)$



$$H_Z(s) = -\frac{Z_F}{Z_S}$$

$$Z_S = R_S \parallel \frac{1}{s \cdot C}$$

$$= \frac{R_S \cdot \frac{1}{s \cdot C}}{R_S + \frac{1}{s \cdot C}} \cdot \frac{s \cdot C}{s \cdot C}$$

$$= \frac{R_s}{1 + s \cdot R_s \cdot C}$$

$$Z_F = R_F$$

Simple Zero Transfer Function

$$H_Z(s) = -\frac{Z_F}{Z_S}$$

$$= -\frac{R_F}{\dfrac{R_s}{1 + s \cdot R_s \cdot C}}$$

$$= -\frac{R_F}{R_S} \cdot (1 + s \cdot R_S \cdot C)$$

$$H_Z(s) = G_0 \cdot \left(1 + \frac{s}{\omega_Z}\right)$$

$$G_0 = -\frac{R_F}{R_S}$$

$$\omega_Z = 2 \cdot \pi \cdot f_Z = \frac{1}{R_S \cdot C}$$

$$f_z = \frac{1}{2 \cdot \pi \cdot R_S \cdot C}$$

Simple Zero Transfer Function: Magnitude

$$\left|H_Z(j\cdot\omega)\right|^2 = H_Z(j\cdot\omega)\cdot H_Z^*(j\cdot\omega)$$
$$= H_Z(j\cdot\omega)\cdot H_Z(-j\cdot\omega)$$
$$= G_0\cdot\left(1+\frac{j\cdot\omega}{\omega_Z}\right)\cdot G_0\cdot\left(1+\frac{-j\cdot\omega}{\omega_Z}\right)$$
$$= G_0^2\cdot\left(1+\left(\frac{\omega}{\omega_Z}\right)^2\right)$$

$$\left|H_Z(j\cdot\omega)\right| = G_0\cdot\sqrt{1+\left(\frac{\omega}{\omega_Z}\right)^2}$$
$$= G_0\cdot\sqrt{1+\left(\frac{f}{f_Z}\right)^2}$$

Consider The Case When $f = f_Z$

$$\left|H_Z(j\cdot\omega_Z)\right| = G_0\cdot\sqrt{1+\left(\frac{f_Z}{f_Z}\right)^2}$$
$$= G_0\cdot\sqrt{1+1^2}$$
$$= G_0\cdot\sqrt{2} \Rightarrow +3\,\mathrm{dB}$$

Simple Zero Transfer Function: Phase

$$\angle H(j\cdot\omega) = \tan^{-1}\left(\frac{\mathrm{Im}(H(j\cdot\omega))}{\mathrm{Re}(H(j\cdot\omega))}\right)$$

$$\mathrm{Im}(H(j\cdot\omega)) = \mathrm{Im}\left(G_0\cdot\left(1+\frac{j\cdot\omega}{\omega_Z}\right)\right)$$

$$= G_0\cdot\frac{\omega}{\omega_Z}$$

$$\mathrm{Re}(H(j\cdot\omega)) = \mathrm{Re}\left(G_0\cdot\left(1+\frac{j\cdot\omega}{\omega_Z}\right)\right)$$

$$= G_0$$

$$\angle H(j\cdot\omega) = \tan^{-1}\left(\frac{G_0\cdot\frac{\omega}{\omega_Z}}{G_0}\right)$$

$$= \tan^{-1}\left(\frac{\omega}{\omega_Z}\right) = \tan^{-1}\left(\frac{f}{f_Z}\right)$$

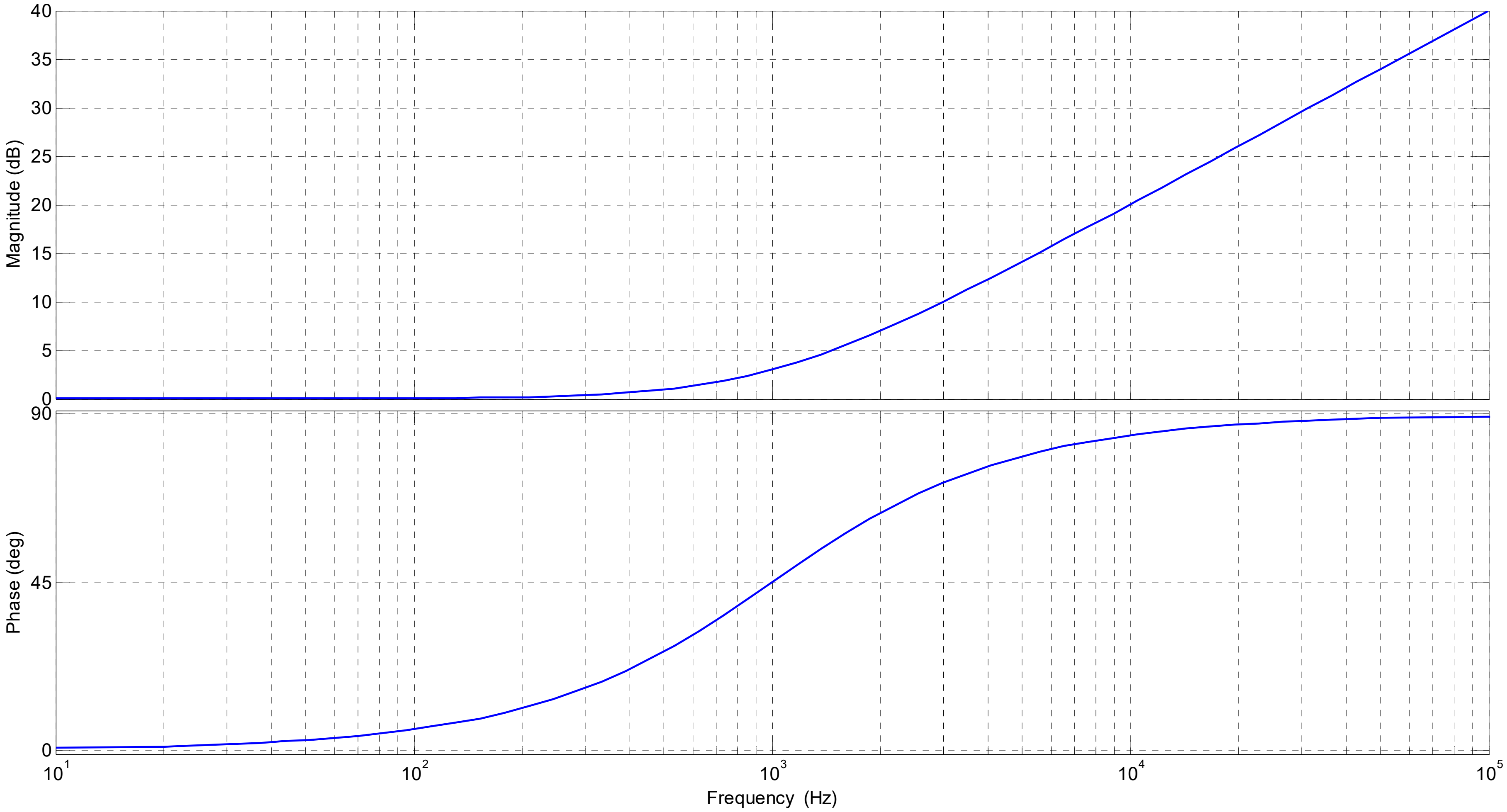
Consider The Case When $f = f_Z$

$$\angle H(j\cdot\omega_Z) = \tan^{-1}\left(\frac{f_Z}{f_Z}\right) = \tan^{-1}(1)$$

$$= \frac{\pi}{4} = 45°$$

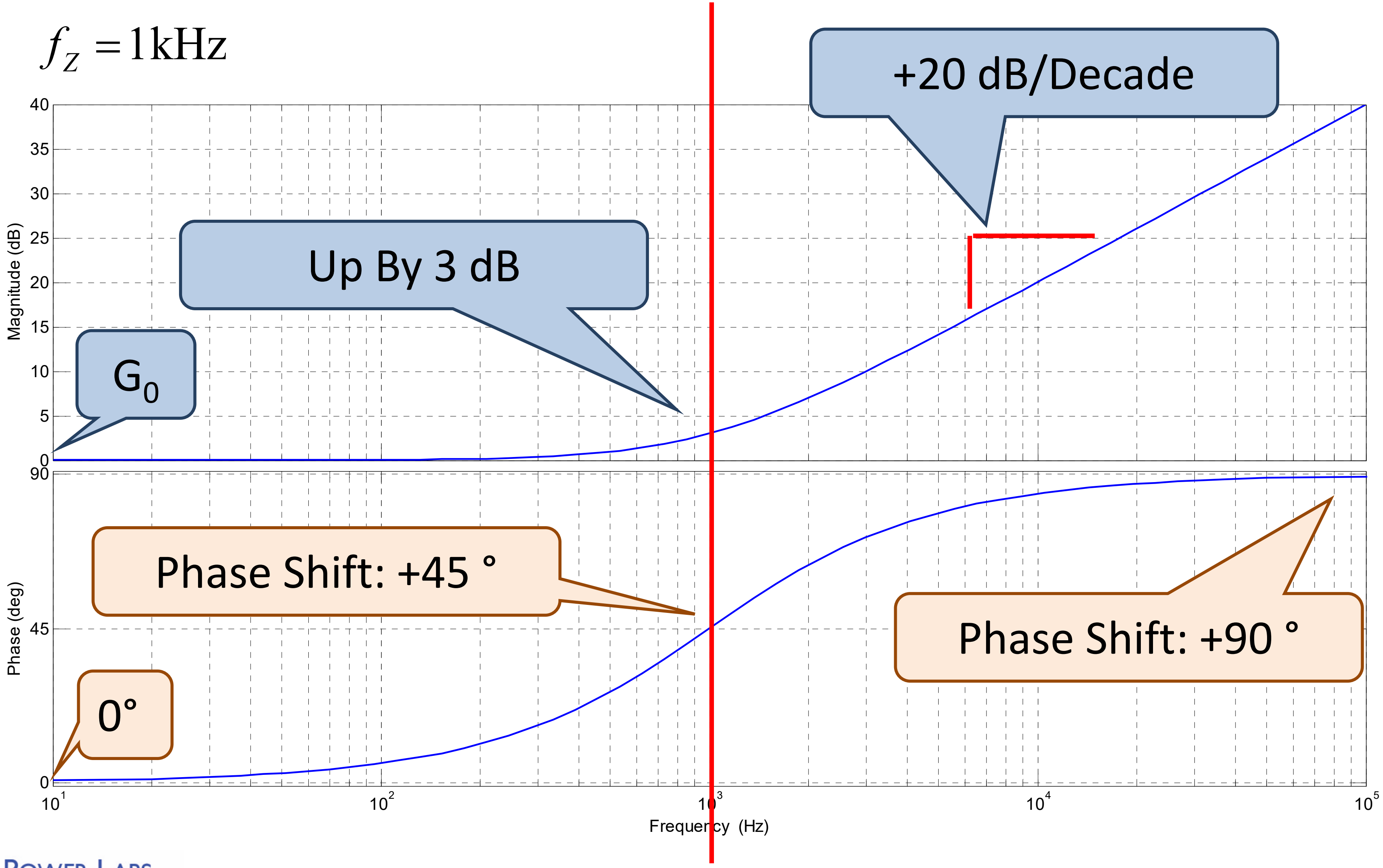
Simple Zero Bode Plot

$$f_Z = 1\text{kHz}$$



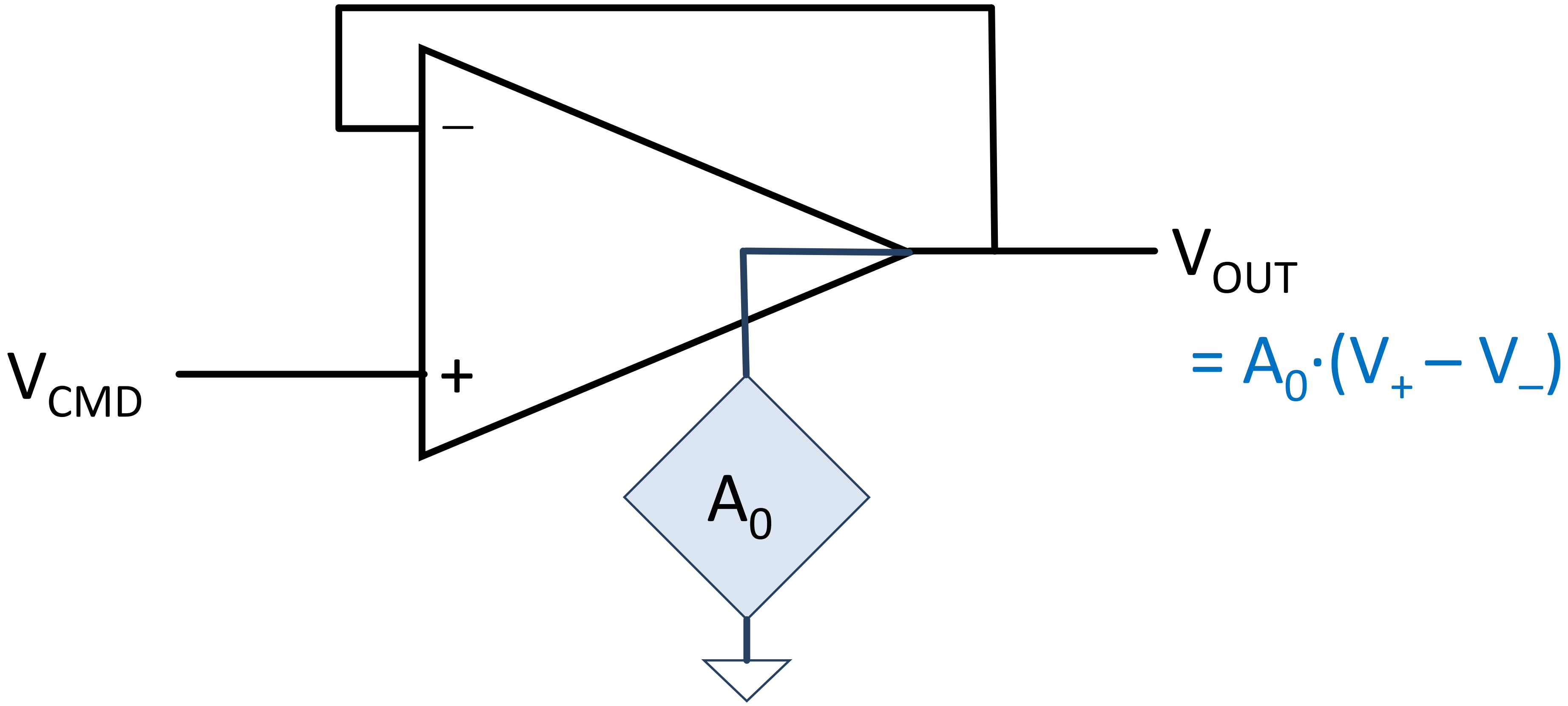
Simple Zero Bode Plot

$f_Z = 1\text{kHz}$



Appendix XI
Loop Gain And Tracking

Op-Amp Buffer/Follower



Op-Amp Buffer/Follower

$$v_{out}(s) = A(s)\cdot\left(v_+(s) - v_-(s)\right)$$

$$v_-(s) = v_{out}(s)$$

$$v_+(s) = v_{cmd}(s)$$

$$v_{out}(s) = A(s)\cdot\left(v_{cmd}(s) - v_{out}(s)\right)$$

$$= A(s)\cdot v_{cmd}(s) - A(s)\cdot v_{out}(s)$$

$$v_{out}(s) + A(s)\cdot v_{out}(s) = A(s)\cdot v_{cmd}(s)$$

$$v_{out}(s) = \frac{A(s)}{1 + A(s)}\cdot v_{cmd}(s)$$

$$|A(s)| \gg 1$$

Let The
Op-amp Gain
Be Very Large

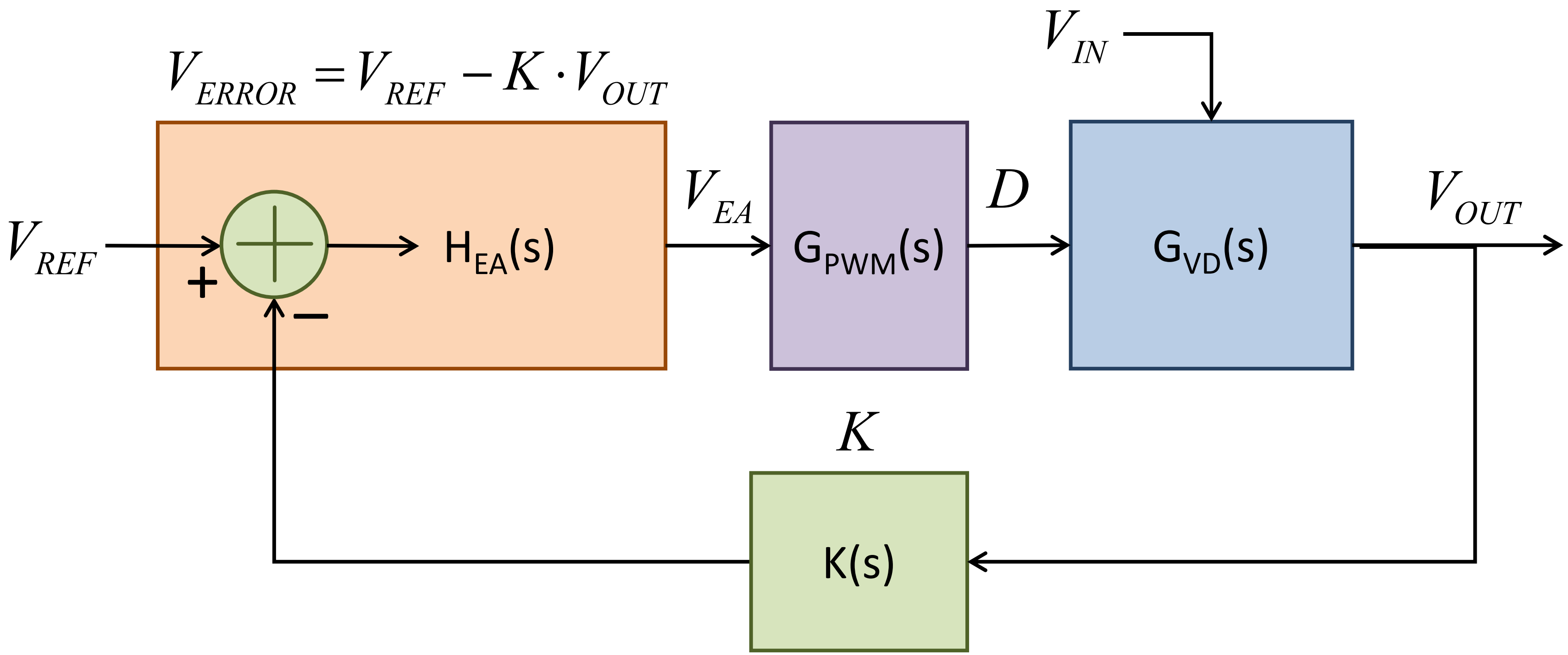
$$v_{out}(s) \approx \frac{A(s)}{\cancel{1} + A(s)}\cdot v_{cmd}(s)$$

$$\approx \frac{A(s)}{A(s)}\cdot v_{cmd}(s)$$

$$v_{out}(s) \approx v_{cmd}(s)$$

With Very Large Op-amp Gain
The Output Tracks The
Command Voltage

Loop Gain And Tracking



Loop Gain And Tracking

$$V_{OUT}(s) = G_{VD}(s)\cdot D(s) = G_{VD}(s)\cdot G_{PWM}(s)\cdot V_{EA}(s)$$

$$V_{EA}(s) = H_{EA}(s)\cdot\left(V_{REF} - K(s)\cdot V_{OUT}(s)\right)$$

$$V_{OUT}(s) = G_{VD}(s)\cdot G_{PWM}(s)\cdot H_{EA}(s)\cdot\left(V_{REF} - K(s)\cdot V_{OUT}(s)\right)$$

$$= G_{VD}(s)\cdot G_{PWM}(s)\cdot H_{EA}(s)\cdot V_{REF}$$

$$-K(s)\cdot G_{VD}(s)\cdot G_{PWM}(s)\cdot H_{EA}(s)\cdot V_{OUT}(s)$$

$$V_{OUT}(s) + K(s)\cdot G_{VD}(s)\cdot G_{PWM}(s)\cdot H_{EA}(s)\cdot V_{OUT}(s) = G_{VD}(s)\cdot G_{PWM}(s)\cdot H_{EA}(s)\cdot V_{REF}$$

$$V_{OUT}(s) = \frac{G_{VD}(s)\cdot G_{PWM}(s)\cdot H_{EA}(s)}{1 + K(s)\cdot G_{VD}(s)\cdot G_{PWM}(s)\cdot H_{EA}(s)}\cdot V_{REF}$$

$$= \frac{1}{K(s)}\cdot\frac{K(s)\cdot G_{VD}(s)\cdot G_{PWM}(s)\cdot H_{EA}(s)}{1 + K(s)\cdot G_{VD}(s)\cdot G_{PWM}(s)\cdot H_{EA}(s)}\cdot V_{REF}$$

$$V_{OUT}(s) = \frac{1}{K(s)}\cdot\frac{T(s)}{1 + T(s)}\cdot V_{REF}$$

Loop Gain And Tracking

$$V_{OUT}(s) = \frac{1}{K(s)} \cdot \frac{T(s)}{1+T(s)} \cdot V_{REF}$$

$$|T(s)| \gg 1$$

Let The Loop Gain Be Large

$$V_{OUT}(s) \approx \cdot \frac{1}{K(s)} \cdot \frac{T(s)}{\cancel{1} + T(s)} V_{REF}$$

$$\approx \frac{1}{K(s)} \cdot \frac{T(s)}{T(s)} \cdot V_{REF}$$

$$V_{OUT}(s) \approx \frac{1}{K(s)} \cdot V_{REF}$$

$$K(s) = K$$

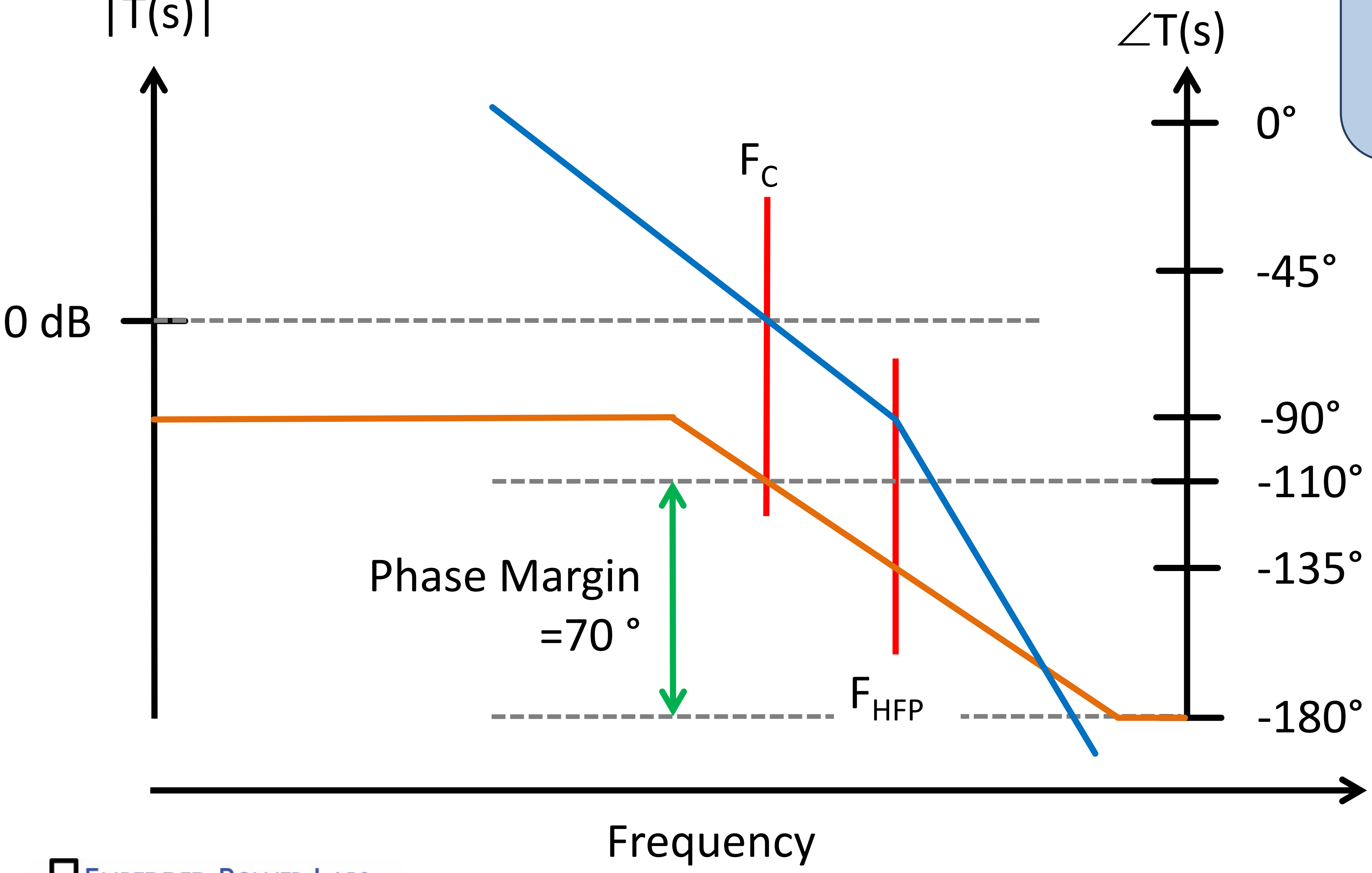
Feedback Divider Usually Constant (Just Resistors)

$$V_{OUT}(s) \approx \frac{1}{K} \cdot V_{REF}$$

With Large Loop Gain Output Voltage Tracks The Reference Voltage/Input

Appendix XII
Ideal Loop Calculations

Better T(s)



$$T(s) = \frac{\omega_0}{s} \cdot \frac{1}{1 + \frac{s}{\omega_{HFP}}}$$

To Have The Desired Phase Margin, What Is The High Frequency Pole (F_{HFP}) Frequency?

Calculating F_{HFP}

- Given Desired Phase Margin: PM
- Total Loop Phase Lag At Crossover Equals:
  Phase_Lag = -180° + PM
  - e.g. PM = 70°, Phase_Lag = -180° + 70° = -110°
- Integrator Pole Contributes -90° Phase Shift
- Phase Lag At Crossover Due To High Frequency Pole (FHFP):
  - Phase_Lag_HFP = -180° - (-90°) + PM
    = -90° + PM
  - Example: PM = 70°
    Phase_Lag_HFP = -90° + 70° = -20°

Calculating F_{HFP}

- Phase Shift As A Function Of Frequency For a Simple Pole (See Previous Example):

$$\angle H_P(j \cdot \omega) = \tan^{-1}\left(-\frac{\omega}{\omega_P}\right)$$

$$\angle H_P(j \cdot 2 \cdot \pi \cdot f) = \tan^{-1}\left(-\frac{f}{f_P}\right)$$

- Solving For F_{HFP}

$$Phase\_Lag\_HFP(F_C) = \tan^{-1}\left(-\frac{F_C}{F_{HFP}}\right)$$

$$-\frac{F_C}{F_{HFP}} = \tan\left(Phase\_Lag\_HFP(F_C)\right)$$

$$F_{HFP} = -\frac{F_C}{\tan\left(Phase\_Lag\_HFP(F_C)\right)}$$

- Example

$$Phase\_Lag\_HFP(F_C) = -20°$$

$$F_{HFP} = -\frac{F_C}{\tan(-20°)} = 2.75 \cdot F_C$$

Calculating ω_0

- Ideal Loop Transfer Function

$$T(s) = \frac{\omega_0}{s} \cdot \frac{1}{1 + \frac{s}{\omega_{HFP}}}$$

- At Crossover We Want:

$$\left| T(j \cdot \omega_C) \right| = 1$$

- Using Identity For Square Of The Magnitude:

$$\left| T(j \cdot \omega_C) \right|^2 = \left(\frac{\omega_0}{j \cdot \omega_C} \cdot \frac{1}{1 + \frac{j \cdot \omega_C}{\omega_{HFP}}} \right) \cdot \left(\frac{\omega_0}{-j \cdot \omega_C} \cdot \frac{1}{1 - \frac{j \cdot \omega_C}{\omega_{HFP}}} \right)$$

Calculating ω_0

$$\left|T\left(j\cdot\omega_C\right)\right|^2 = \left(\frac{\omega_0}{j\cdot\omega_C}\cdot\frac{1}{1+\frac{j\cdot\omega_C}{\omega_{HFP}}}\right)\cdot\left(\frac{\omega_0}{-j\cdot\omega_C}\cdot\frac{1}{1-\frac{j\cdot\omega_C}{\omega_{HFP}}}\right)$$

$$= \left(\frac{\omega_0}{\omega_C}\right)^2\cdot\frac{1}{1+\left(\frac{\omega_C}{\omega_{HFP}}\right)^2} = 1$$

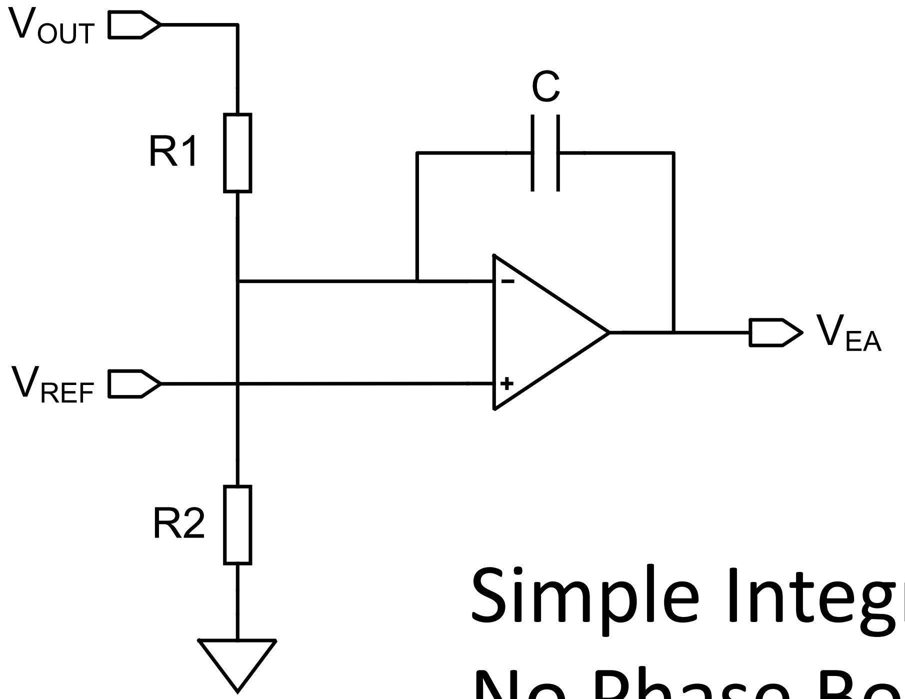
$$\left(\frac{\omega_0}{\omega_C}\right)^2 = 1+\left(\frac{\omega_C}{\omega_{HFP}}\right)^2$$

$$\frac{\omega_0}{\omega_C} = \sqrt{1+\left(\frac{\omega_C}{\omega_{HFP}}\right)^2}$$

$$\omega_0 = \omega_C\cdot\sqrt{1+\left(\frac{\omega_C}{\omega_{HFP}}\right)^2}$$

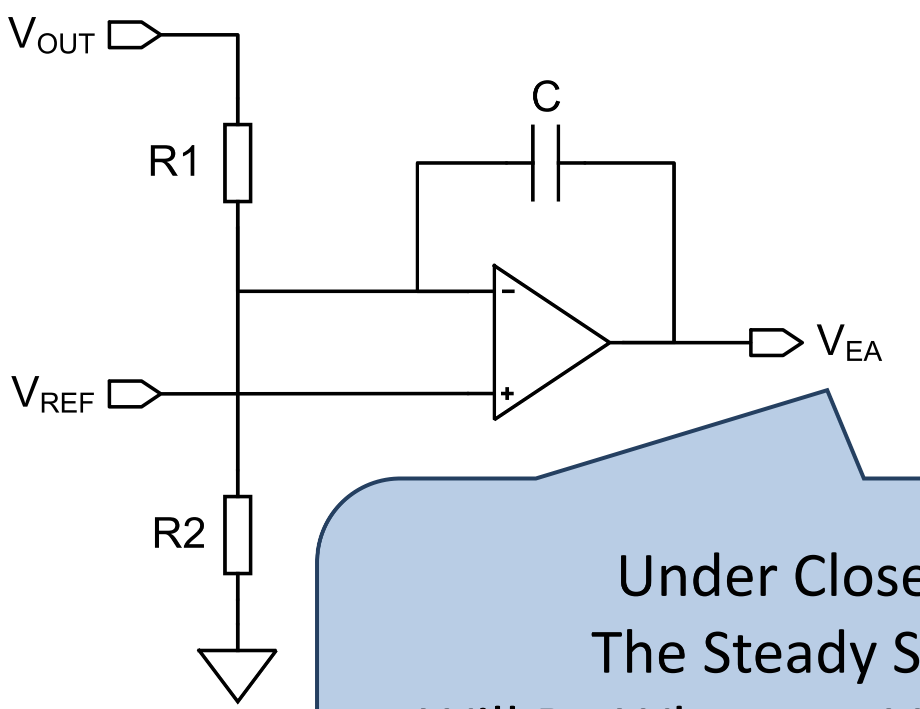
Appendix XIII
Analog Compensators

Type I Compensator



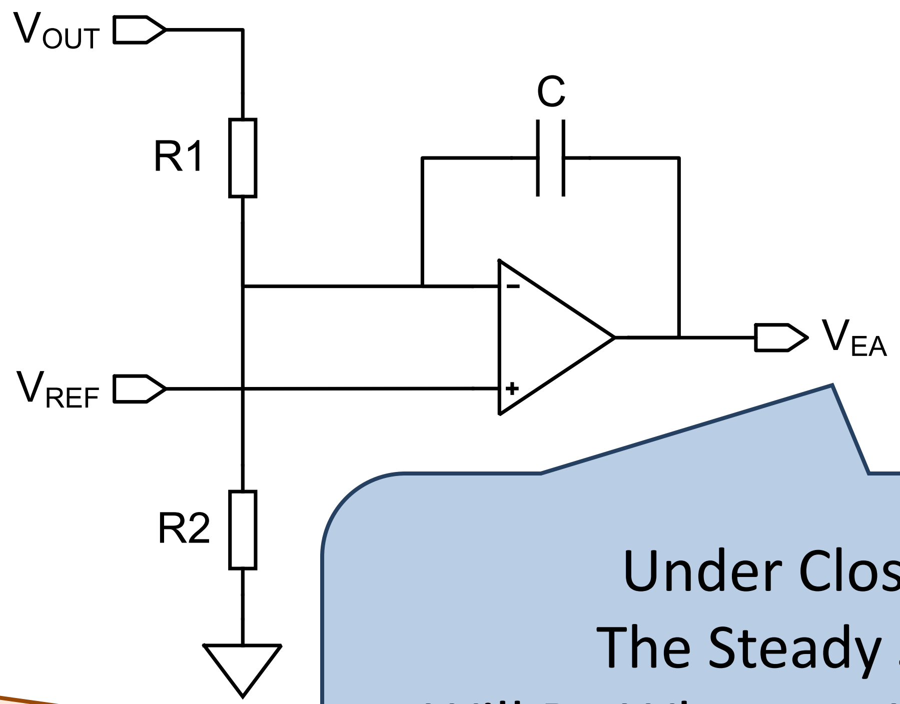
Simple Integrator
No Phase Boost

Type I Compensator



Under Closed Loop Control.
The Steady State Value Of V_{EA}
Will Be Whatever Value Is Needed To Drive
V_{OUT} Such That The Op-Amp Differential Input
Voltage (V_+ - V_-) Is Essentially Zero

Type I Compensator



True For All Compensators That Include An Integrator

Under Closed Loop Control.
The Steady State Value Of V_{EA} Will Be Whatever Value Is Needed To Drive V_{OUT} Such That The Op-Amp Differential Input Voltage ($V_+ - V_-$) Is Essentially Zero

Type I Compensator: DC

$$V_{EA} = A_{OL} \cdot (V_{+} - V_{-})$$

$$V_{-} = V_{+} - \frac{1}{A_{OL}} \cdot V_{EA}$$

$$V_{+} = V_{REF}$$

$$V_{-} = V_{REF} - \frac{1}{A_{OL}} \cdot V_{EA}$$

$$A_{OL} \approx 10^4 \Rightarrow \frac{1}{A_{OL}} \cdot V_{EA} \approx 50\mu\text{V}$$

$$V_{-} = V_{REF} - 50\mu\text{V} \approx V_{REF}$$

Op-Amp Output Voltage At DC
Is The Differential Input Voltage (V_{+} - V_{-})
Times The DC Open Loop Gain (A_{OL})

Typical Op-Amp Open Loop Gain Is 40 dB (10,000)
And Assuming Error Amp Output At 5V

Type I Compensator: DC

$$I(R_1) = I(R_2) + I(V_-) \approx I(R_2)$$

$$\frac{V_{OUT} - V_-}{R_1} = \frac{V_{OUT} - V_{REF}}{R_1} = \frac{V_-}{R_2} = \frac{V_{REF}}{R_2}$$

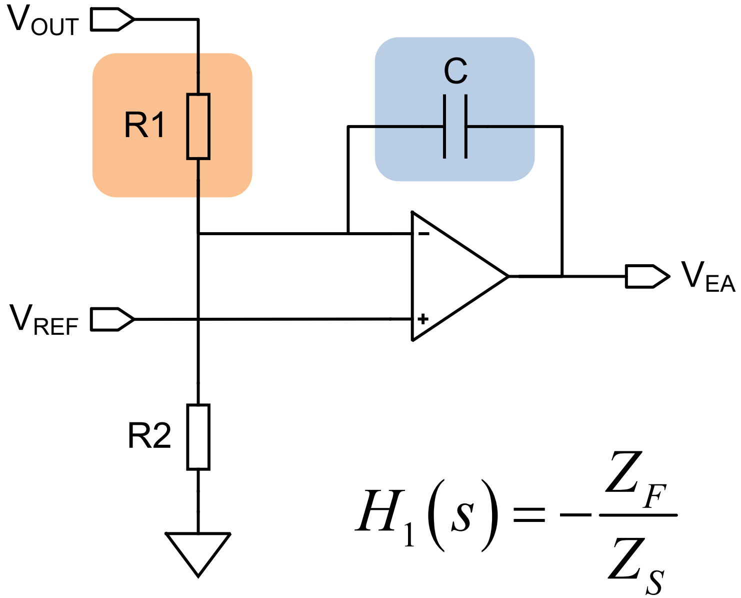
$$\frac{V_{OUT} - V_{REF}}{R_1} = \frac{V_{REF}}{R_2}$$

$$\frac{V_{OUT}}{R_1} = \frac{V_{REF}}{R_2} + \frac{V_{REF}}{R_1} = \frac{R_1 + R_2}{R_1 \cdot R_2} \cdot V_{REF}$$

$$V_{OUT} = \frac{R_1 + R_2}{R_2} \cdot V_{REF}$$

This Relationship Is True For All Compensators With An Integrator

Type I Compensator Transfer Function $H_1(s)$



$$H_1(s) = -\frac{Z_F}{Z_S}$$

$$Z_F = \frac{1}{s \cdot C}$$

$$Z_S = R$$

$$H_1(s) = -\frac{Z_F}{Z_S} = -\frac{\frac{1}{s \cdot C}}{R} = -\frac{1}{s \cdot R \cdot C}$$

$$H_1(s) = -\frac{1}{\frac{s}{\omega_P}} = -\frac{\omega_P}{s}$$

$$\omega_P = 2 \cdot \pi \cdot f_P = \frac{1}{R \cdot C}$$

$$f_P = \frac{1}{2 \cdot \pi \cdot R \cdot C}$$

Type I Compensator Transfer Function

$$\left|H_1(j\cdot\omega)\right| = \left|-\frac{\omega_P}{j\cdot\omega}\right|$$

$$= \sqrt{\mathrm{Re}\left(H(j\cdot\omega)\right)^2 + \mathrm{Im}\left(H(j\cdot\omega)\right)^2}$$

$$= \sqrt{0^2 + \left(-\frac{\omega_P}{\omega}\right)^2}$$

$$= \frac{\omega_P}{\omega} = \frac{2\cdot\pi\cdot f_P}{2\cdot\pi\cdot f}$$

$$= \frac{f_P}{f}$$

Gain = 1 (0 dB)
When f = f_P

$$\angle H_1(j\cdot\omega) = \tan^{-1}\left(\frac{\mathrm{Im}\left(H(j\cdot\omega)\right)}{\mathrm{Re}\left(H(j\cdot\omega)\right)}\right)$$

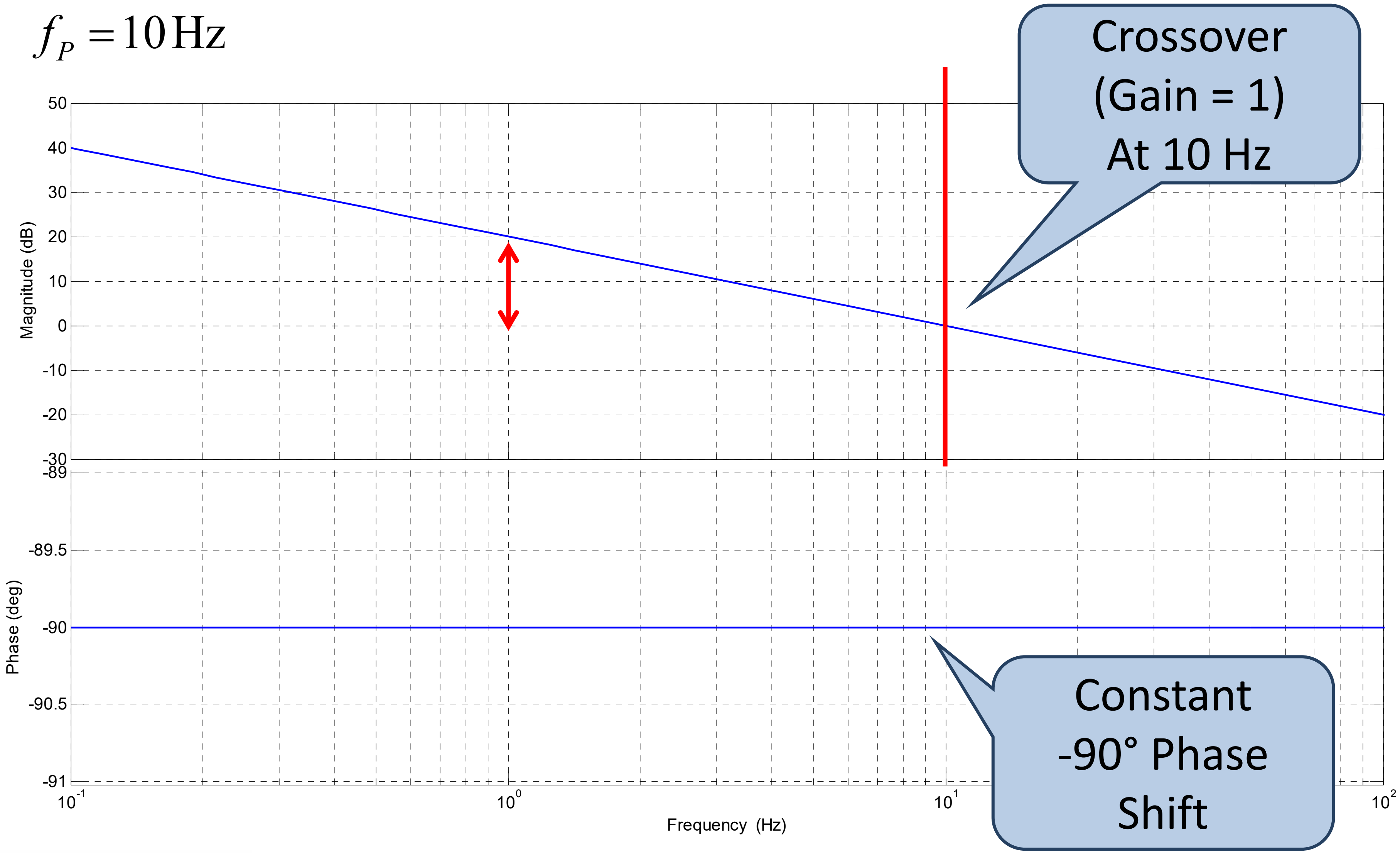
$$= \tan^{-1}\left(\frac{-\frac{\omega_P}{\omega}}{0}\right) = \tan^{-1}(-\infty)$$

$$= -\frac{\pi}{2} = -90°$$

Constant -90°
Phase Shift Independent
Of Frequency

Type I Compensator Bode Plot

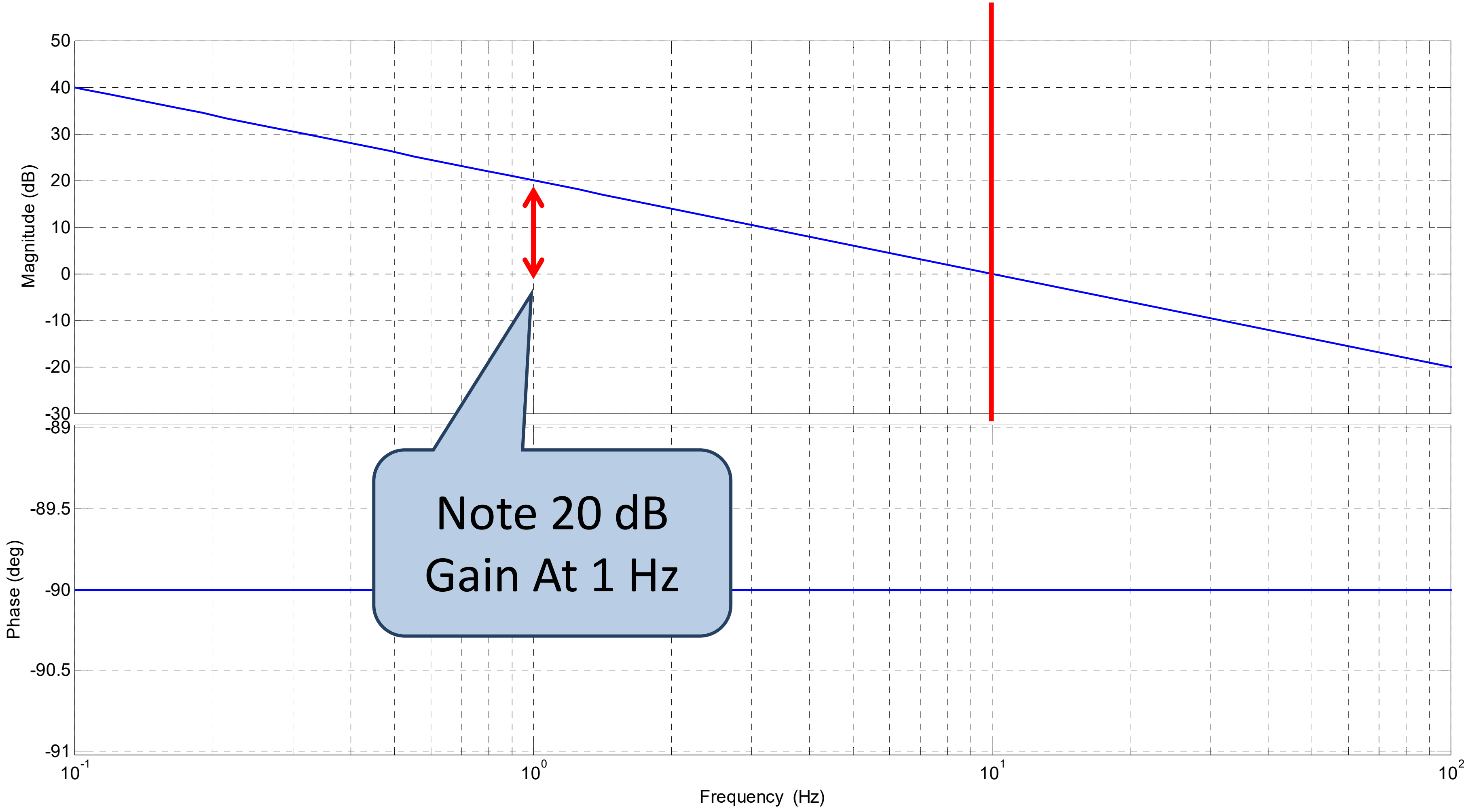
$f_P = 10\,\text{Hz}$



EMBEDDED POWER LABS

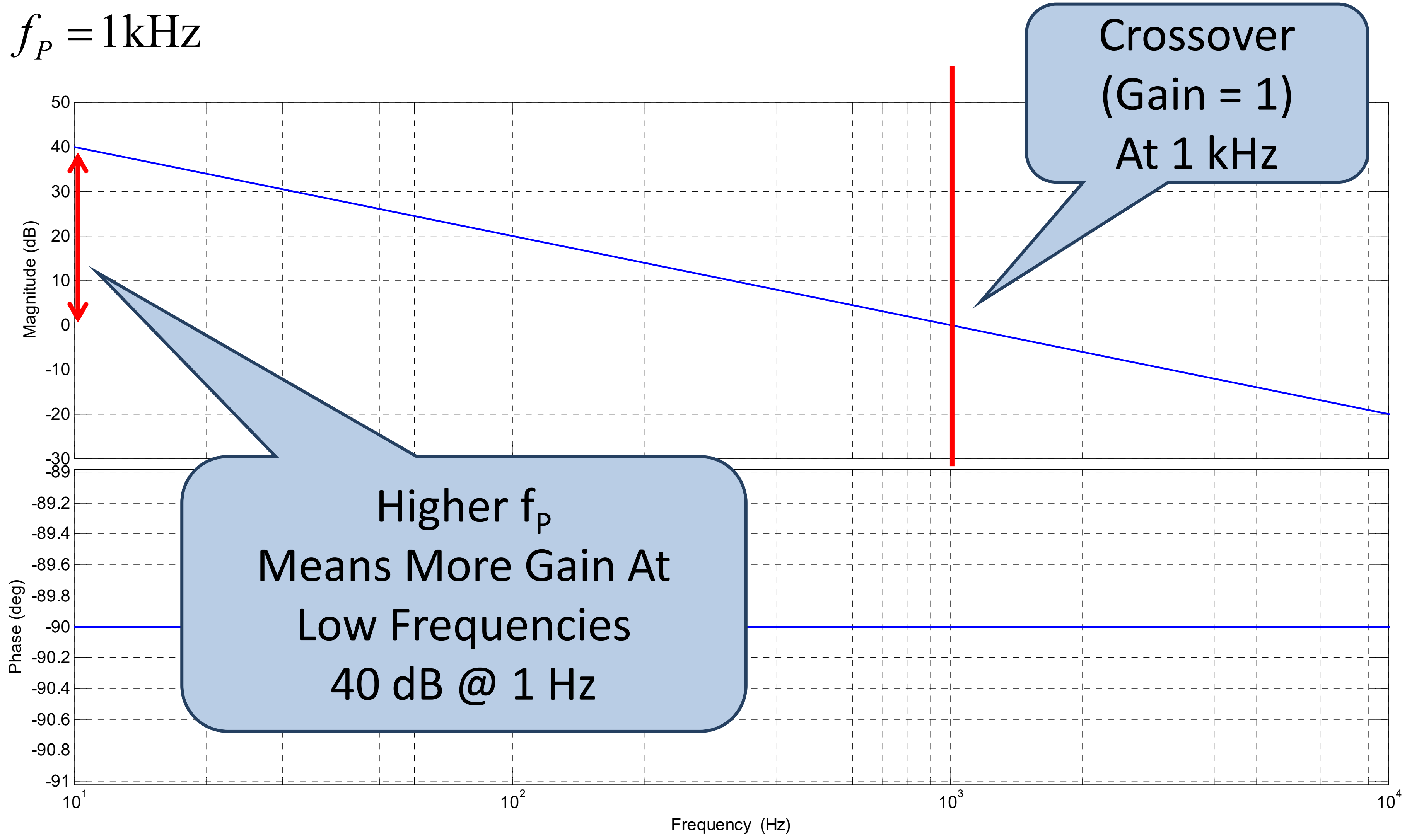
Type I Compensator Bode Plot

$f_P = 10\,\text{Hz}$



Type I Compensator Bode Plot

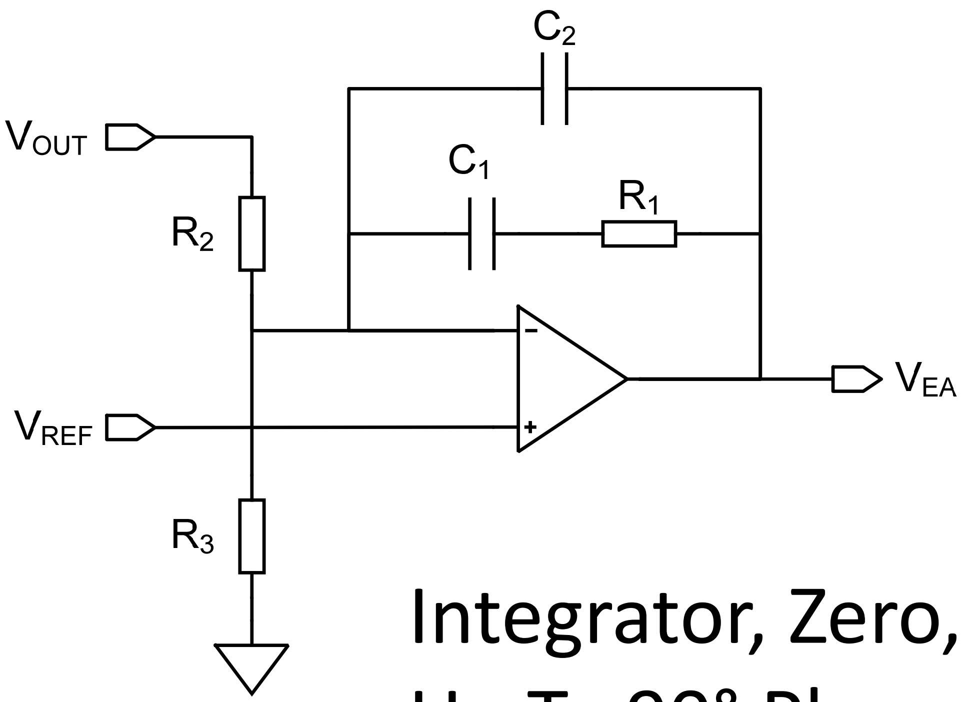
$f_P = 1\text{kHz}$



Type I For Debugging

- If Having Problems With Stability, Change Compensator To Type I
- Set Compensator Pole Frequency To No More Than 1/10th Of The Output Filter Frequency
  - 1/100th (2 Decades Lower) Is Better
- Loop Should Be Stable So You Can Debug
- Transient Response Will Be Poor
  - Input Ripple Rejection May Also Be Poor

Type II Compensator



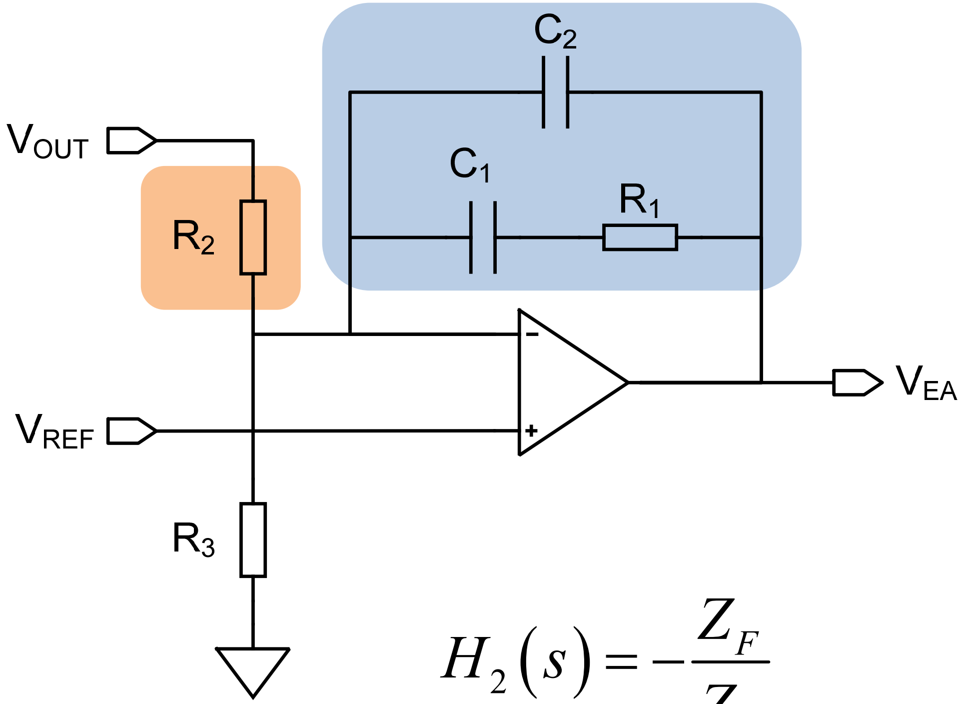
Integrator, Zero, Pole

Up To 90° Phase Boost

Type II Compensator

- Type II Compensator Used With Power Stages That Have Only -90° Phase Shift
  - Current Mode Control
  - Converters Operating In Discontinuous Mode

Type II Compensator Transfer Function $H_2(s)$



$$H_2(s) = -\frac{Z_F}{Z_S}$$

$$Z_F = \frac{1}{s \cdot C_2} \parallel \left(R_1 + \frac{1}{s \cdot C_1} \right) = \frac{1}{s \cdot C_2} \parallel \frac{1 + s \cdot R_1 \cdot C_1}{s \cdot C_1}$$

$$= \frac{\frac{1}{s \cdot C_2} \cdot \frac{1 + s \cdot R_1 \cdot C_1}{s \cdot C_1}}{\frac{1}{s \cdot C_2} + \frac{1 + s \cdot R_1 \cdot C_1}{s \cdot C_1}} \cdot \frac{s \cdot C_1}{s \cdot C_1} \cdot \frac{s \cdot C_2}{s \cdot C_2}$$

$$= \frac{1 + s \cdot R_1 \cdot C_1}{s \cdot C_1 + s \cdot C_2 \cdot (1 + s \cdot R_1 \cdot C_1)}$$

$$Z_S = R_2$$

Type II Compensator Transfer Function $H_2(s)$

$$H_2(s) = -\frac{Z_F}{Z_S}$$

$$= -\frac{\dfrac{1}{s\cdot(C_1+C_2)}\cdot\dfrac{1+s\cdot R_1\cdot C_1}{1+s\cdot R_1\cdot\dfrac{C_1\cdot C_2}{C_1+C_2}}}{R_2}$$

$$= -\frac{1}{s\cdot R_2\cdot(C_1+C_2)}\cdot\frac{1+s\cdot R_1\cdot C_1}{1+s\cdot R_1\cdot\dfrac{C_1\cdot C_2}{C_1+C_2}}$$

Zero

Integrator Pole

Second Pole

EMBEDDED POWER LABS

Type II Compensator Transfer Function $H_2(s)$

$$H_2(s) = -\frac{1}{\frac{s}{\omega_{P0}}}\frac{1+\frac{s}{\omega_Z}}{1+\frac{s}{\omega_{P1}}}$$

$$= -\frac{\omega_{P0}}{s}\frac{1+\frac{s}{\omega_Z}}{1+\frac{s}{\omega_{P1}}}$$

$$\omega_{P0} = 2\cdot\pi\cdot f_{P0} = \frac{1}{R_2\cdot(C_1+C_2)}$$

$$f_{P0} = \frac{1}{2\cdot\pi\cdot R_2\cdot(C_1+C_2)}$$

$$\omega_Z = 2\cdot\pi\cdot f_Z = \frac{1}{R_1\cdot C_1}$$

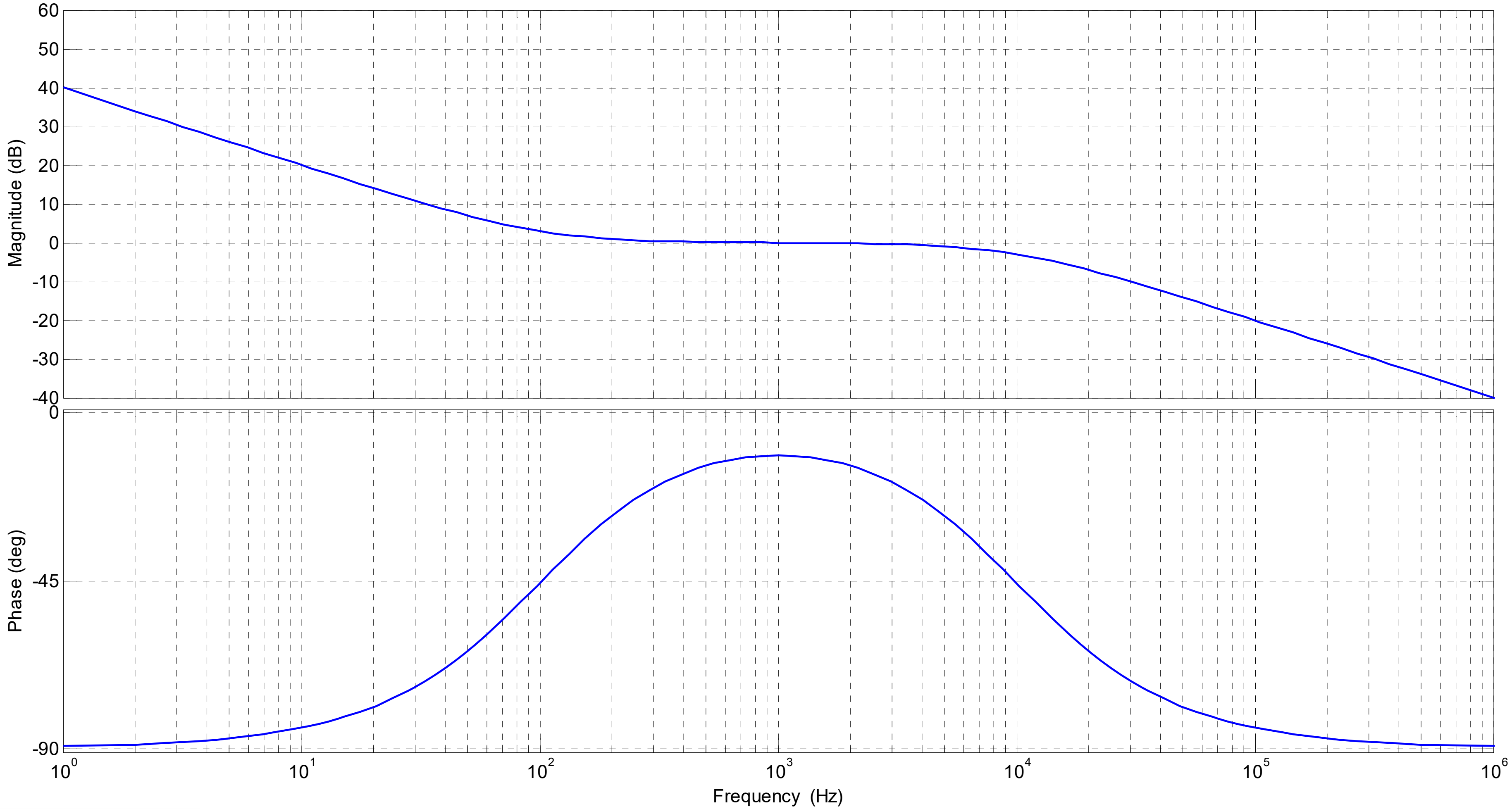
$$f_Z = \frac{1}{2\cdot\pi\cdot R_1\cdot C_1}$$

$$\omega_{P1} = 2\cdot\pi\cdot f_{P1} = \frac{1}{R_1\cdot\frac{C_1\cdot C_2}{C_1+C_2}}$$

$$f_{P1} = \frac{1}{2\cdot\pi\cdot R_1\cdot\frac{C_1\cdot C_2}{C_1+C_2}}$$

Type II Compensator Bode Plot

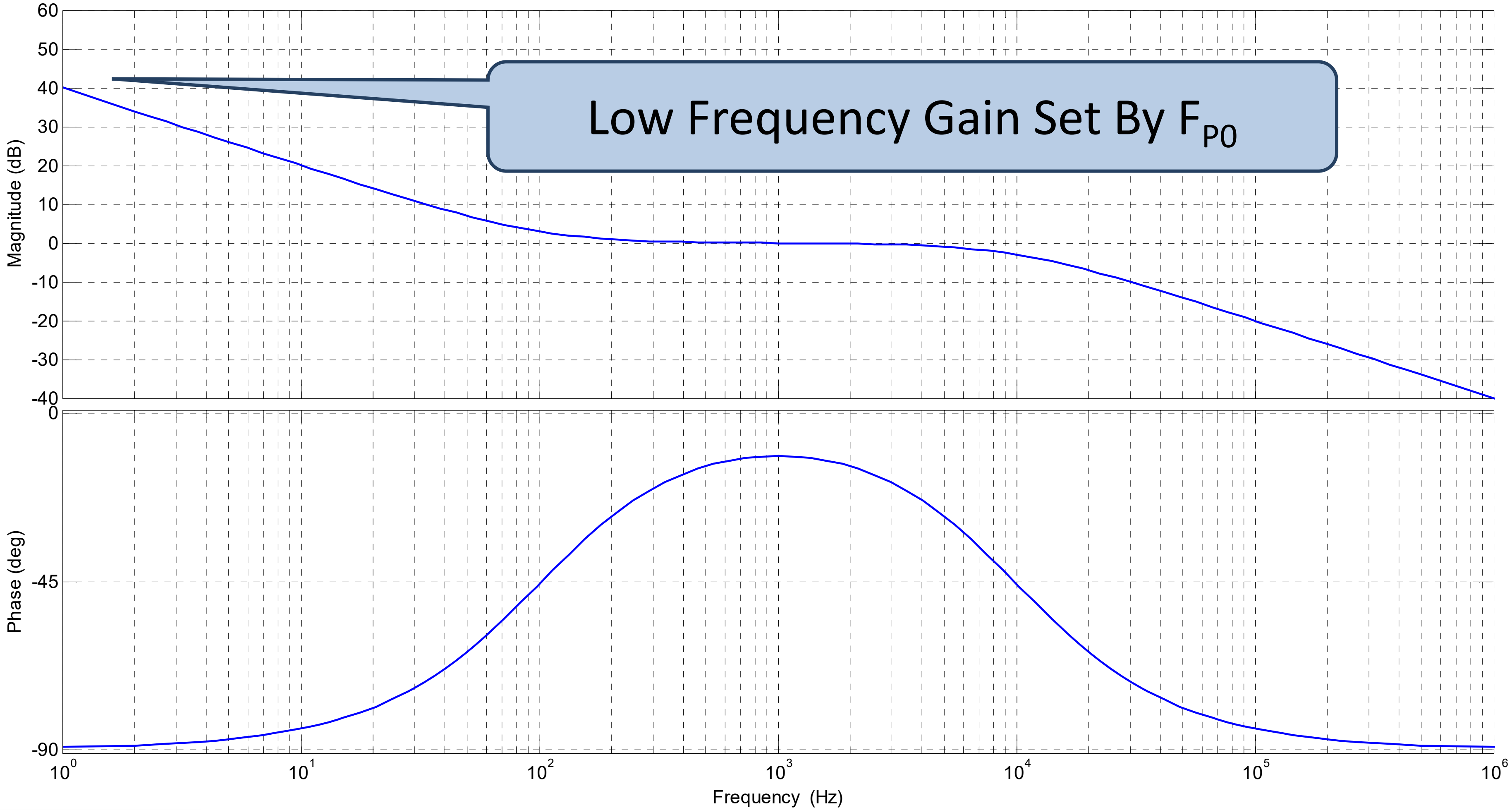
$f_{P0} = 100\,\text{Hz}$ $f_Z = 100\,\text{Hz}$ $f_{P1} = 10\,\text{kHz}$



EMBEDDED POWER LABS

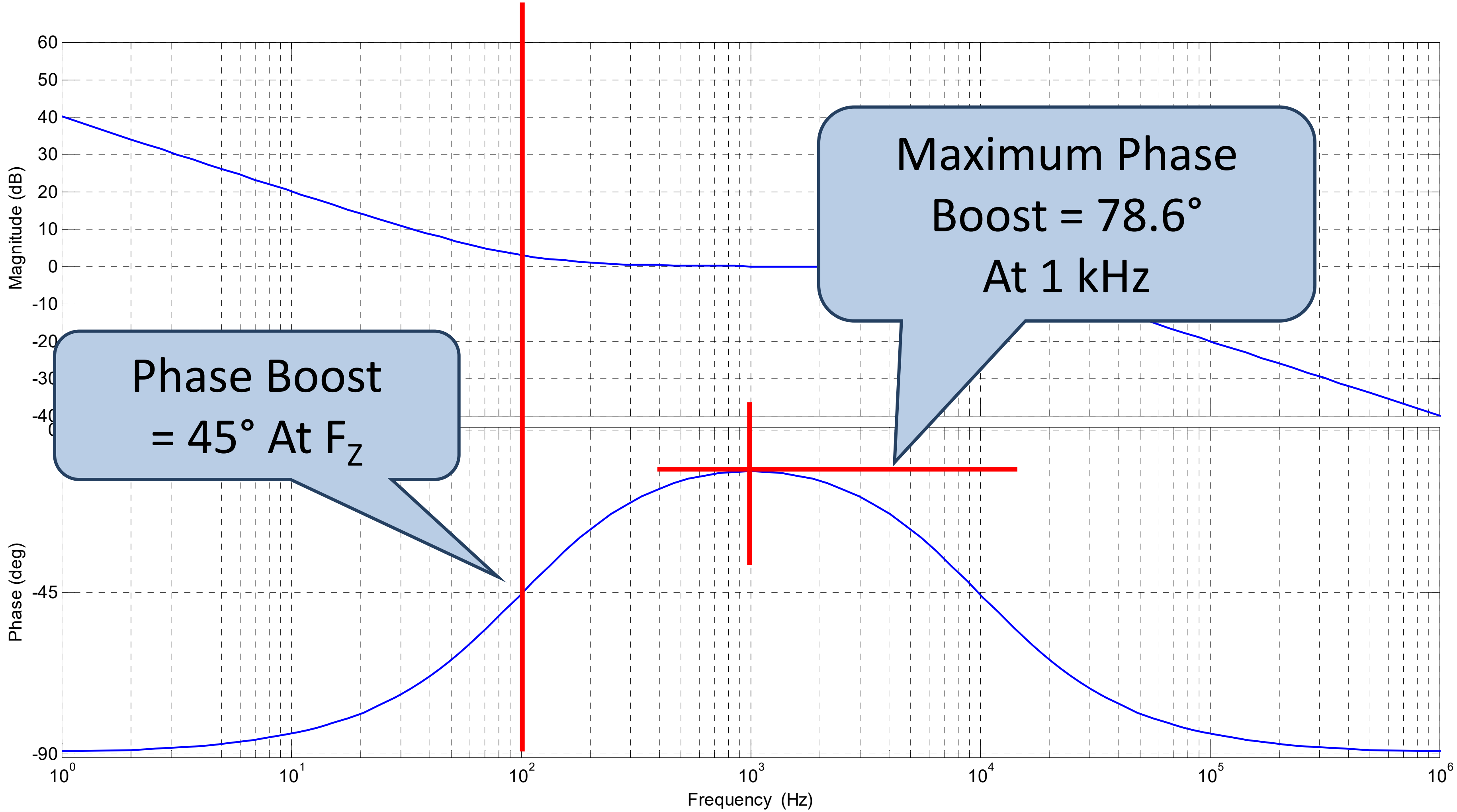
Type II Compensator Bode Plot

$f_{P0} = 100\,\text{Hz}$ $\quad f_Z = 100\,\text{Hz}$ $\quad f_{P1} = 10\,\text{kHz}$



Type II Compensator Bode Plot

$f_{P0} = 100\,\text{Hz}$ $f_Z = 100\,\text{Hz}$ $f_{P1} = 10\,\text{kHz}$

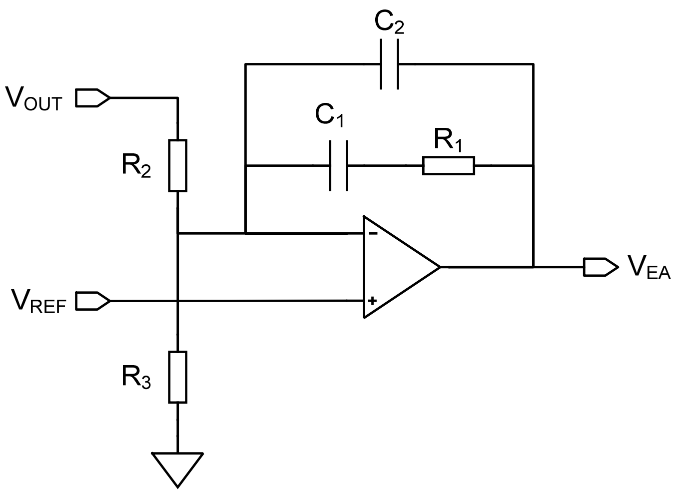


Calculating Component Values

- If $V_{OUT} > V_{REF}$ Choose Current (I_{BIAS}) Through R2 And R3
  - 100 µA – 1 mA Typical

$$R_3 = \frac{V_{REF}}{I_{BIAS}}$$

$$R_2 = \frac{V_{OUT} - V_{REF}}{V_{REF}} \cdot R_3$$



- If $V_{OUT} = V_{REF}$:
  - R3 Not Used
  - Choose Convenient Value For R2 (10 kΩ)

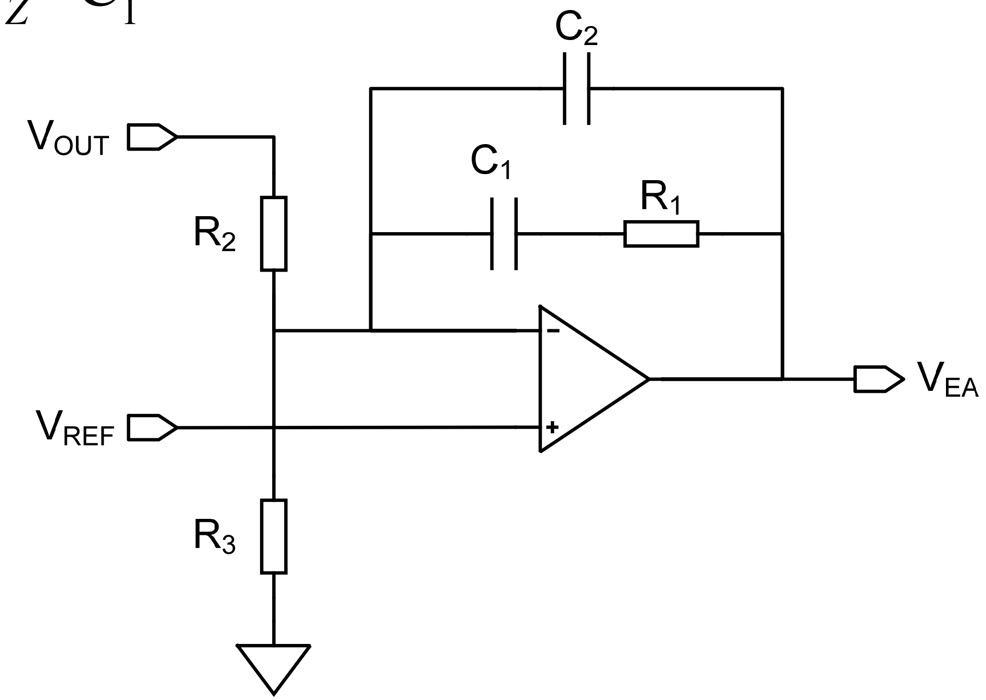
Calculating Component Values

Knowing The Desired Pole And Zero Frequencies Of The Compensator
The Individual Component Values Can Easily Be Calculated

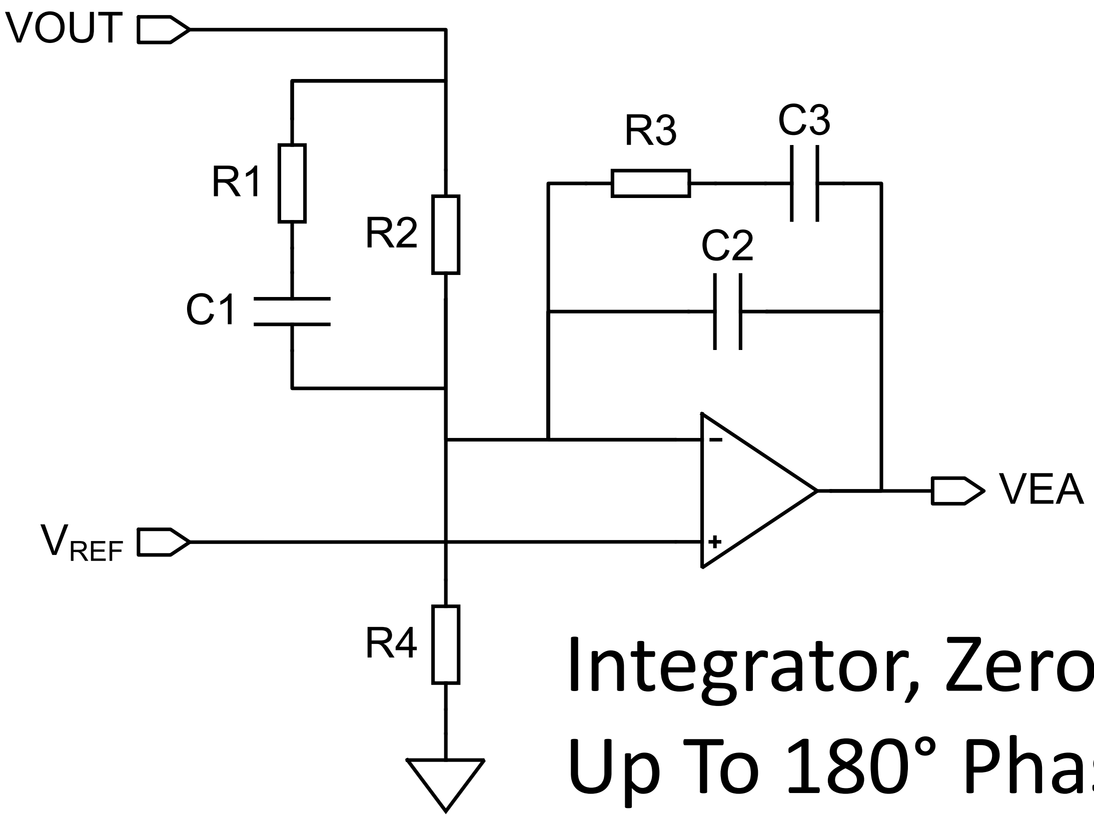
$$C_2 = \frac{f_Z}{2 \cdot \pi \cdot R_2 \cdot f_{P1} \cdot f_{P0}} \qquad R_1 = \frac{1}{2 \cdot \pi \cdot f_Z \cdot C_1}$$

$$C_1 = \frac{1}{2 \cdot \pi \cdot R_2 \cdot f_{P0}} - C_2$$

$$= \frac{1}{2 \cdot \pi \cdot R_2 \cdot f_{P0}} \cdot \left(1 - \frac{f_Z}{f_{P1}}\right)$$



Type III Compensator



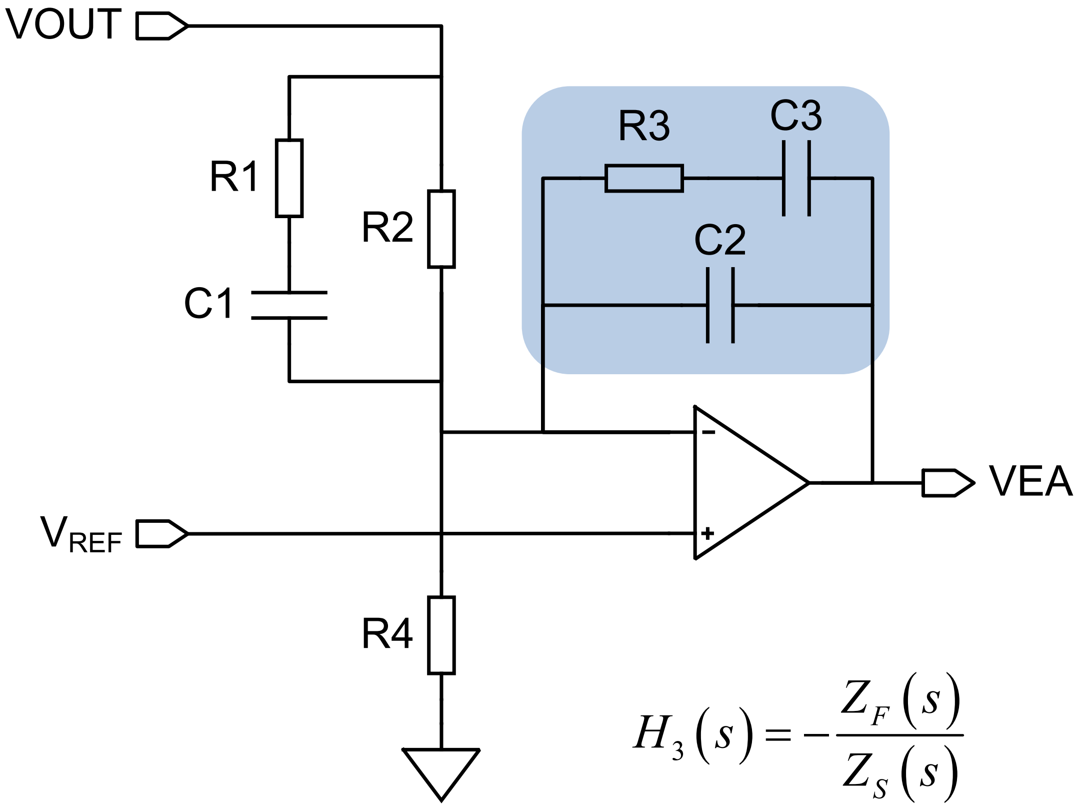
Integrator, Zero, Zero , Pole, Pole

Up To 180° Phase Boost

Type III Compensator

- Type III Compensator Used With Power Stages That Have More Than -90° Phase Shift
  - e.g. Buck Converter With Voltage Mode Control

Type III Compensator Transfer Function $H_3(s)$



$$H_3(s) = -\frac{Z_F(s)}{Z_S(s)}$$

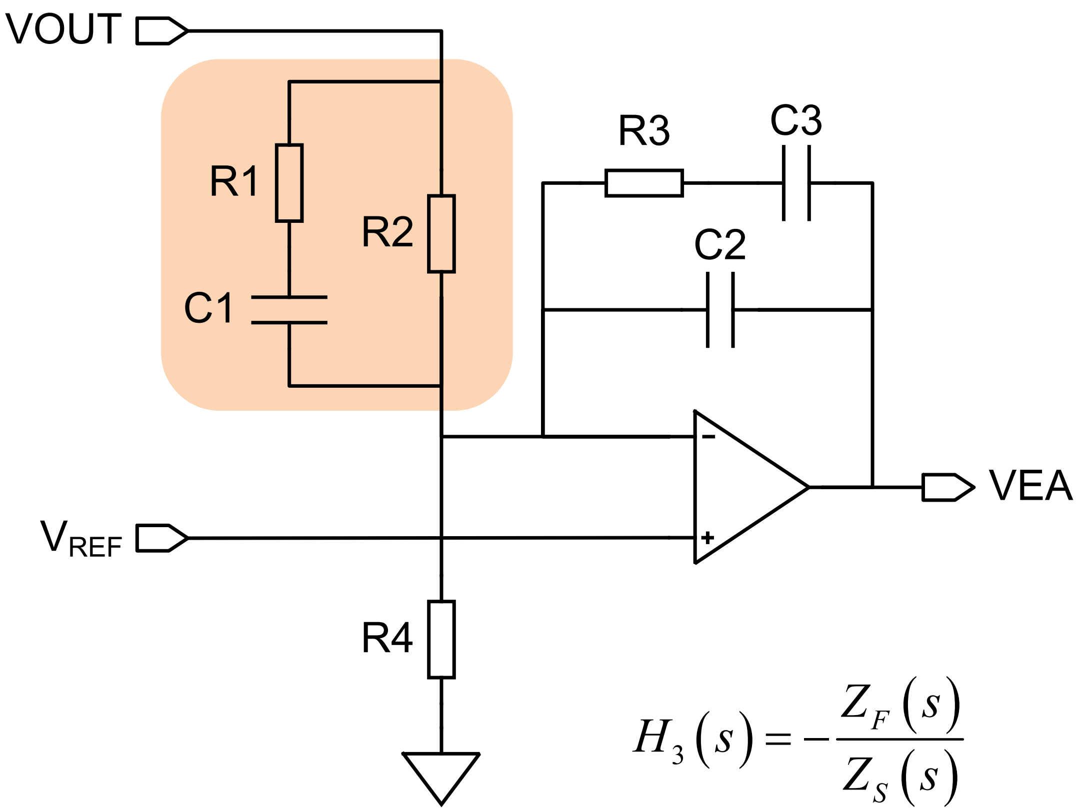
$$Z_F(s) = \frac{1}{s \cdot C_2} \parallel \left(R_3 + \frac{1}{s \cdot C_3} \right) = \frac{1}{s \cdot C_2} \parallel \left(\frac{1 + s \cdot R_3 \cdot C_3}{s \cdot C_3} \right)$$

$$= \frac{\frac{1}{s \cdot C_2} \cdot \frac{1 + s \cdot R_3 \cdot C_3}{s \cdot C_3}}{\frac{1}{s \cdot C_2} + \frac{1 + s \cdot R_3 \cdot C_3}{s \cdot C_3}} \cdot \frac{s \cdot C_2}{s \cdot C_2} \cdot \frac{s \cdot C_3}{s \cdot C_3}$$

$$= \frac{1 + s \cdot R_3 \cdot C_3}{s \cdot C_3 + s \cdot C_2 \cdot (1 + s \cdot R_3 \cdot C_3)}$$

$$Z_F(s) = \frac{1}{s} \cdot \frac{1 + s \cdot R_3 \cdot C_3}{C_3 + C_2 \cdot (1 + s \cdot R_3 \cdot C_3)}$$

Type III Compensator Transfer Function $H_3(s)$



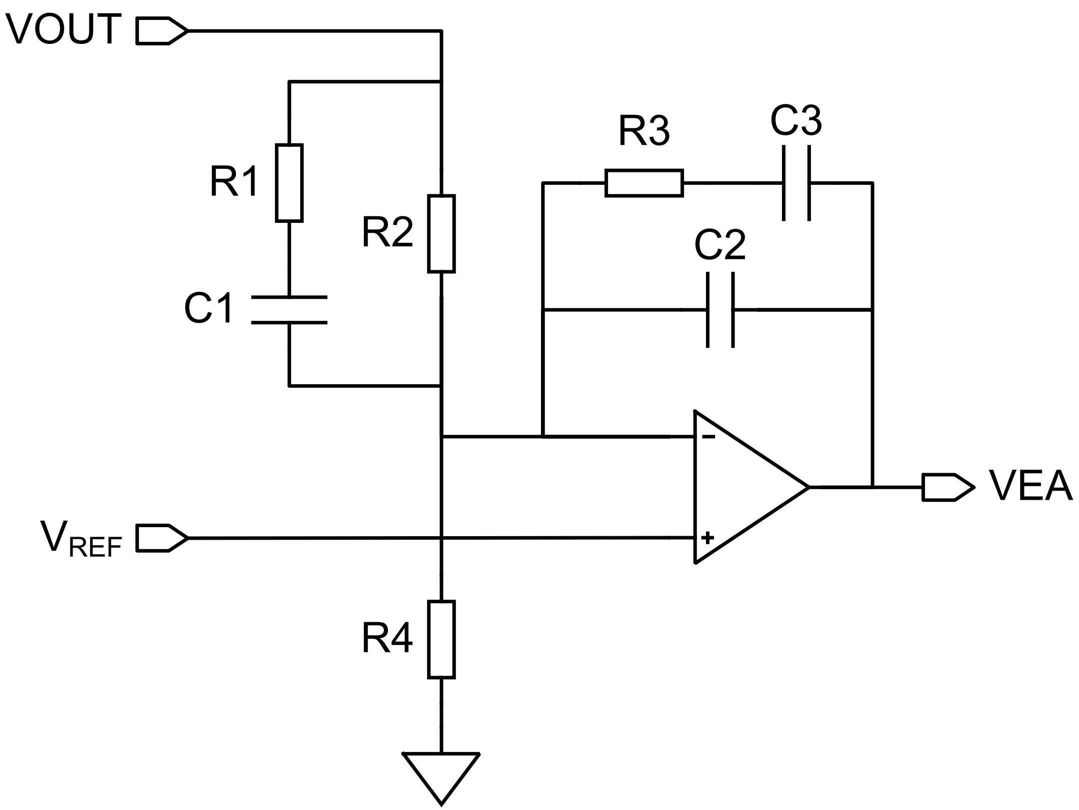
$$H_3(s) = -\frac{Z_F(s)}{Z_S(s)}$$

$$Z_S(s) = R_2 \parallel \left(R_1 + \frac{1}{s \cdot C_1} \right) = R_2 \parallel \frac{1 + s \cdot R_1 \cdot C_1}{s \cdot C_1}$$

$$= \frac{R_2 \cdot \left(\frac{1 + s \cdot R_1 \cdot C_1}{s \cdot C_1} \right)}{R_2 + \left(\frac{1 + s \cdot R_1 \cdot C_1}{s \cdot C_1} \right)} = R_2 \cdot \frac{1 + s \cdot R_1 \cdot C_1}{1 + s \cdot R_1 \cdot C_1 + s \cdot R_2 \cdot C_1}$$

$$= R_2 \cdot \frac{1 + s \cdot R_1 \cdot C_1}{1 + s \cdot (R_1 + R_2) \cdot C_1}$$

Type III Compensator Transfer Function $H_3(s)$



$$H_3(s) = -\frac{Z_F(s)}{Z_S(s)}$$

$$= -\frac{\frac{1}{s\cdot(C_2+C_3)}\cdot\frac{1+s\cdot R_3\cdot C_3}{1+s\cdot R_3\cdot\frac{C_2\cdot C_3}{C_2+C_3}}}{R_2\cdot\frac{1+s\cdot R_1\cdot C_1}{1+s\cdot(R_1+R_2)\cdot C_1}}$$

$$H_3(s) = -\frac{1}{s\cdot R_2\cdot(C_2+C_3)}\cdot\frac{1+s\cdot R_3\cdot C_3}{1+s\cdot R_1\cdot C_1}\cdot\frac{1+s\cdot(R_1+R_2)\cdot C_1}{1+s\cdot R_3\cdot\frac{C_2\cdot C_3}{C_2+C_3}}$$

EMBEDDED POWER LABS

Type III Compensator Transfer Function $H_3(s)$

$$H_3(s) = -\frac{\omega_{P0}}{\omega} \cdot \frac{\left(1+\frac{\omega}{\omega_{Z1}}\right)\cdot\left(1+\frac{\omega}{\omega_{Z2}}\right)}{\left(1+\frac{\omega}{\omega_{P1}}\right)\cdot\left(1+\frac{\omega}{\omega_{P2}}\right)}$$

$$\omega_{P0} = \frac{1}{R_2 \cdot (C_2 + C_3)}$$

$$f_{P0} = \frac{1}{2\cdot\pi\cdot R_2 \cdot (C_2 + C_3)}$$

$$\omega_{Z1} = \frac{1}{R_3 \cdot C_3}$$

$$f_{Z1} = \frac{1}{2\cdot\pi\cdot R_3 \cdot C_3}$$

$$\omega_{Z2} = \frac{1}{(R_1 + R_2)\cdot C_1}$$

$$f_{Z2} = \frac{1}{2\cdot\pi\cdot(R_1 + R_2)\cdot C_1}$$

$$\omega_{P1} = \frac{1}{R_1 \cdot C_1}$$

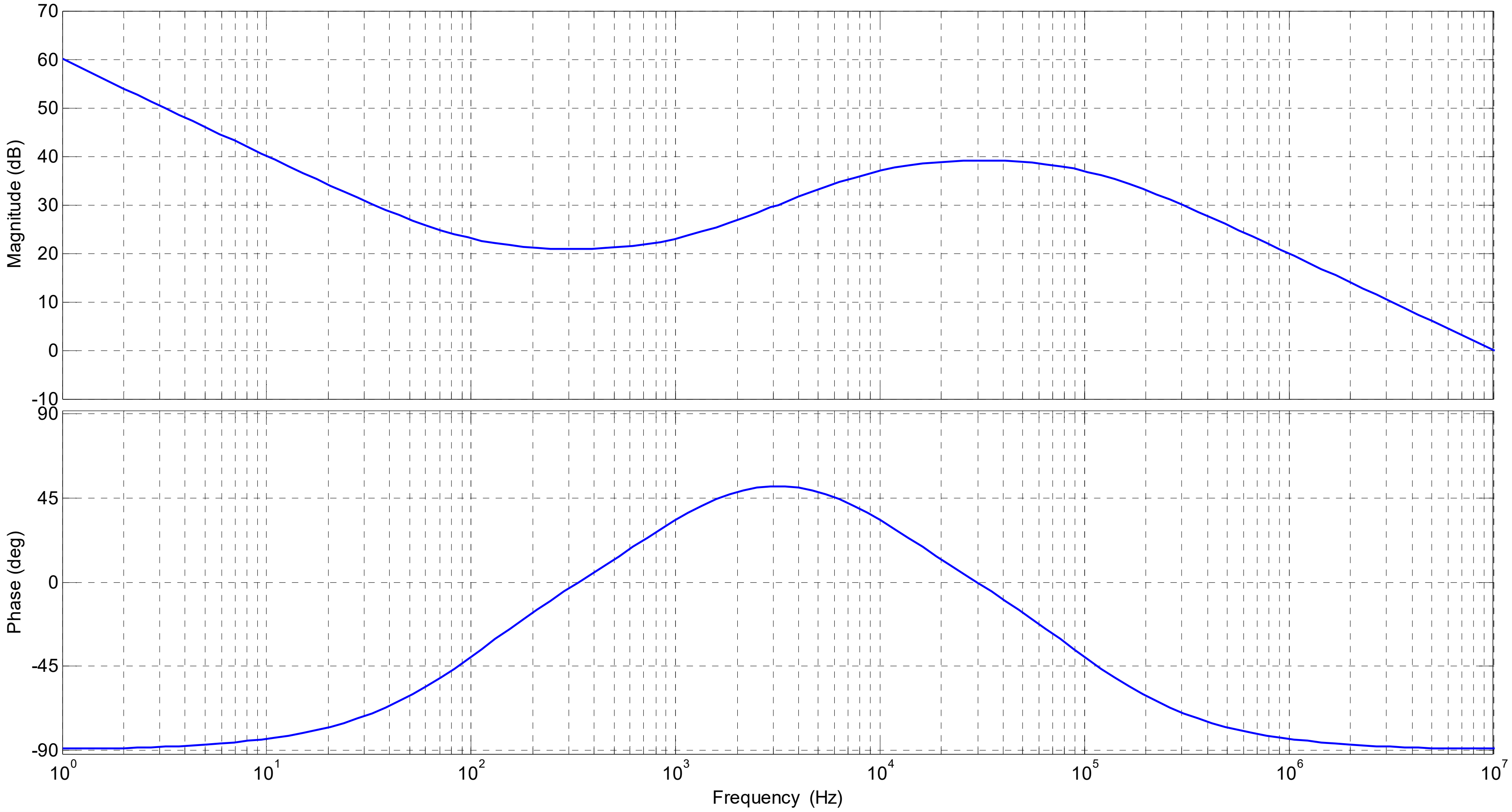
$$f_{P1} = \frac{1}{2\cdot\pi\cdot R_1 \cdot C_1}$$

$$\omega_{P2} = \frac{1}{R_3 \cdot \frac{C_2 \cdot C_3}{C_2 + C_3}}$$

$$f_{P2} = \frac{1}{2\cdot\pi\cdot R_3 \cdot \frac{C_2 \cdot C_3}{C_2 + C_3}}$$

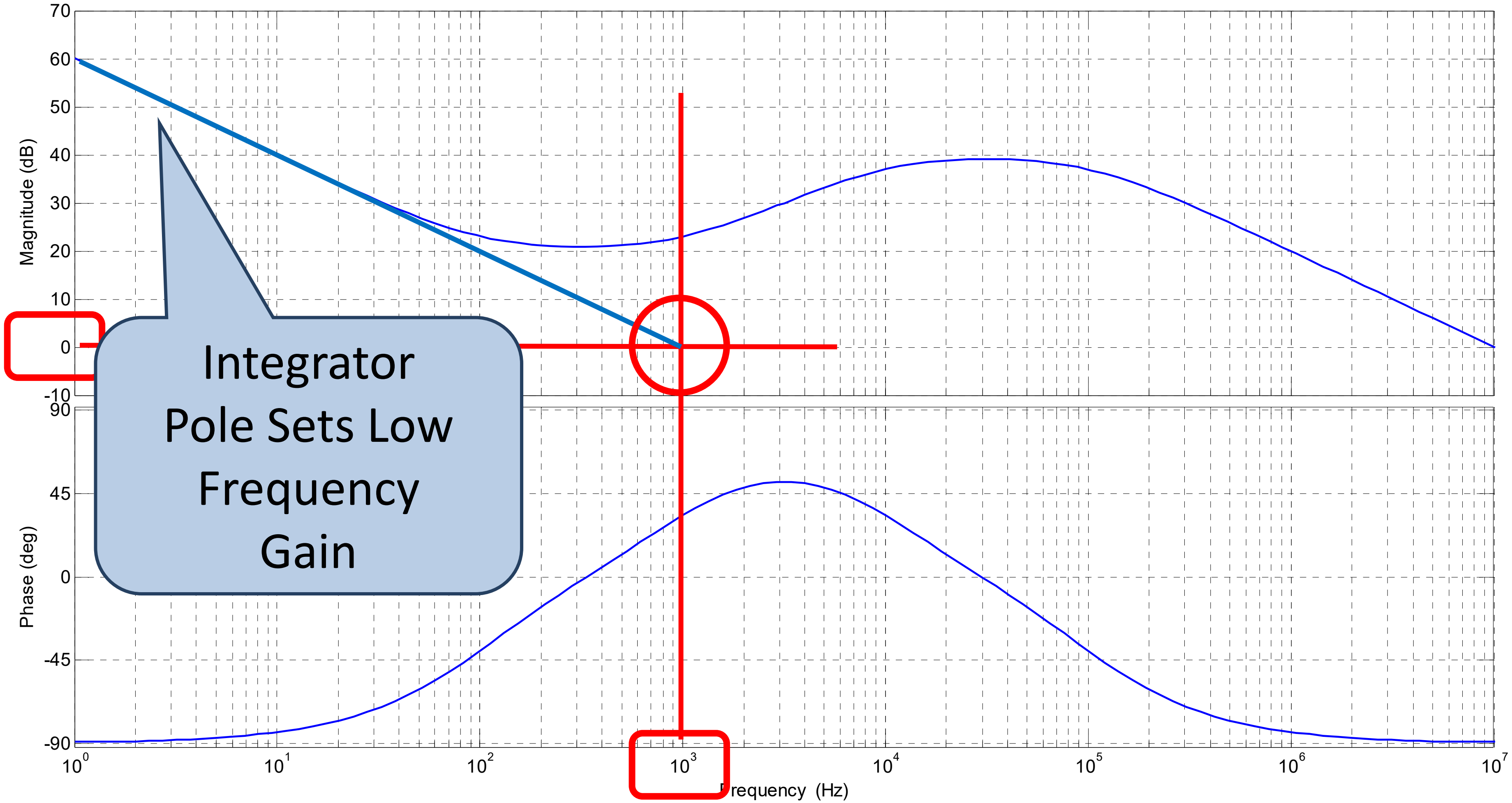
Type III Bode Plot

$$f_{P0} = 1000\,\text{Hz} \quad f_{Z1} = 100\,\text{Hz} \quad f_{Z2} = 1000\,\text{Hz} \quad f_{P1} = 10\,\text{kHz} \quad f_{P2} = 100\,\text{kHz}$$



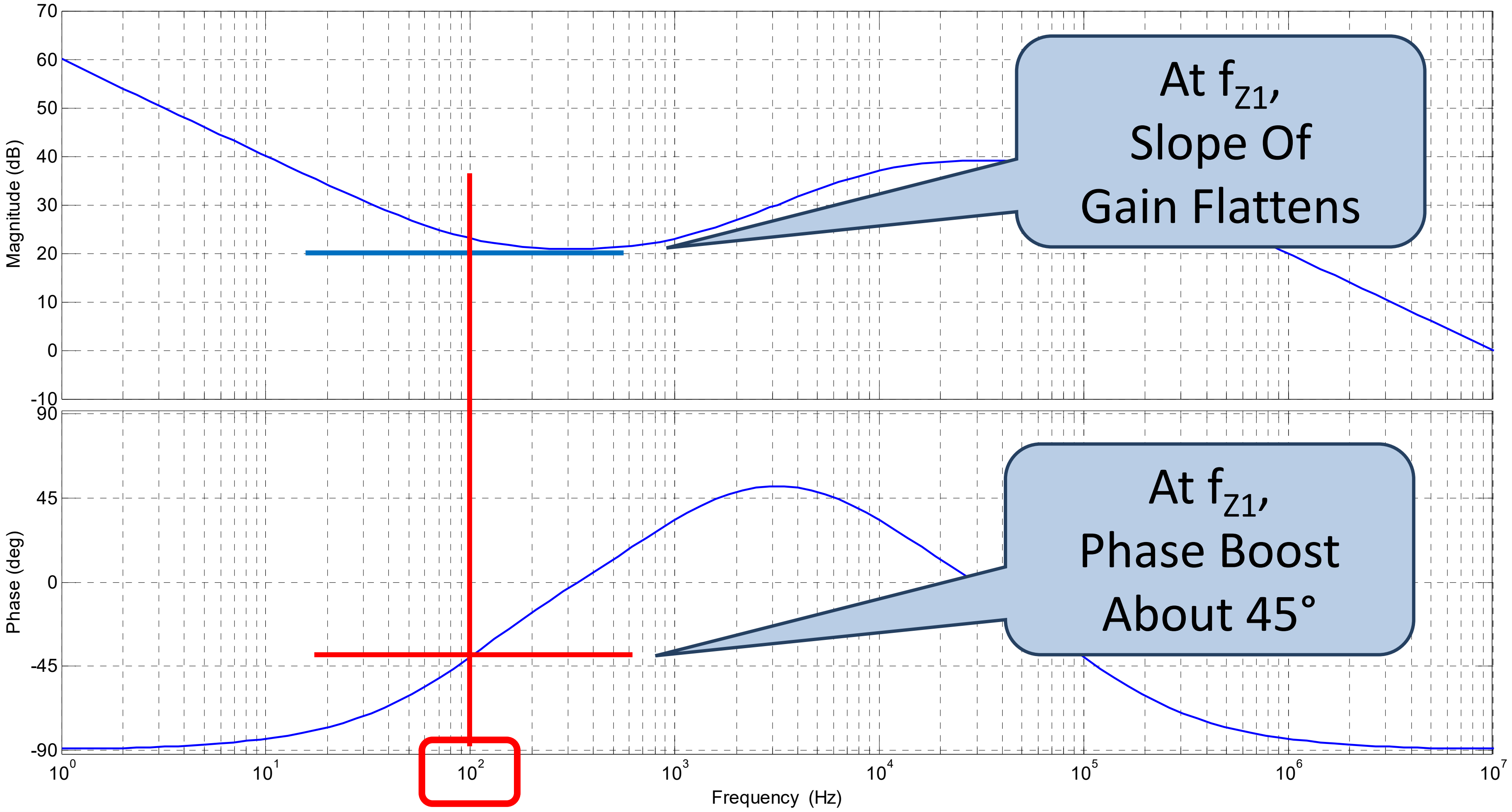
Type III Bode Plot

$f_{P0} = 1000\,\mathrm{Hz}$ $\quad f_{Z1} = 100\,\mathrm{Hz}$ $\quad f_{Z2} = 1000\,\mathrm{Hz}$ $\quad f_{P1} = 10\,\mathrm{kHz}$ $\quad f_{P2} = 100\,\mathrm{kHz}$



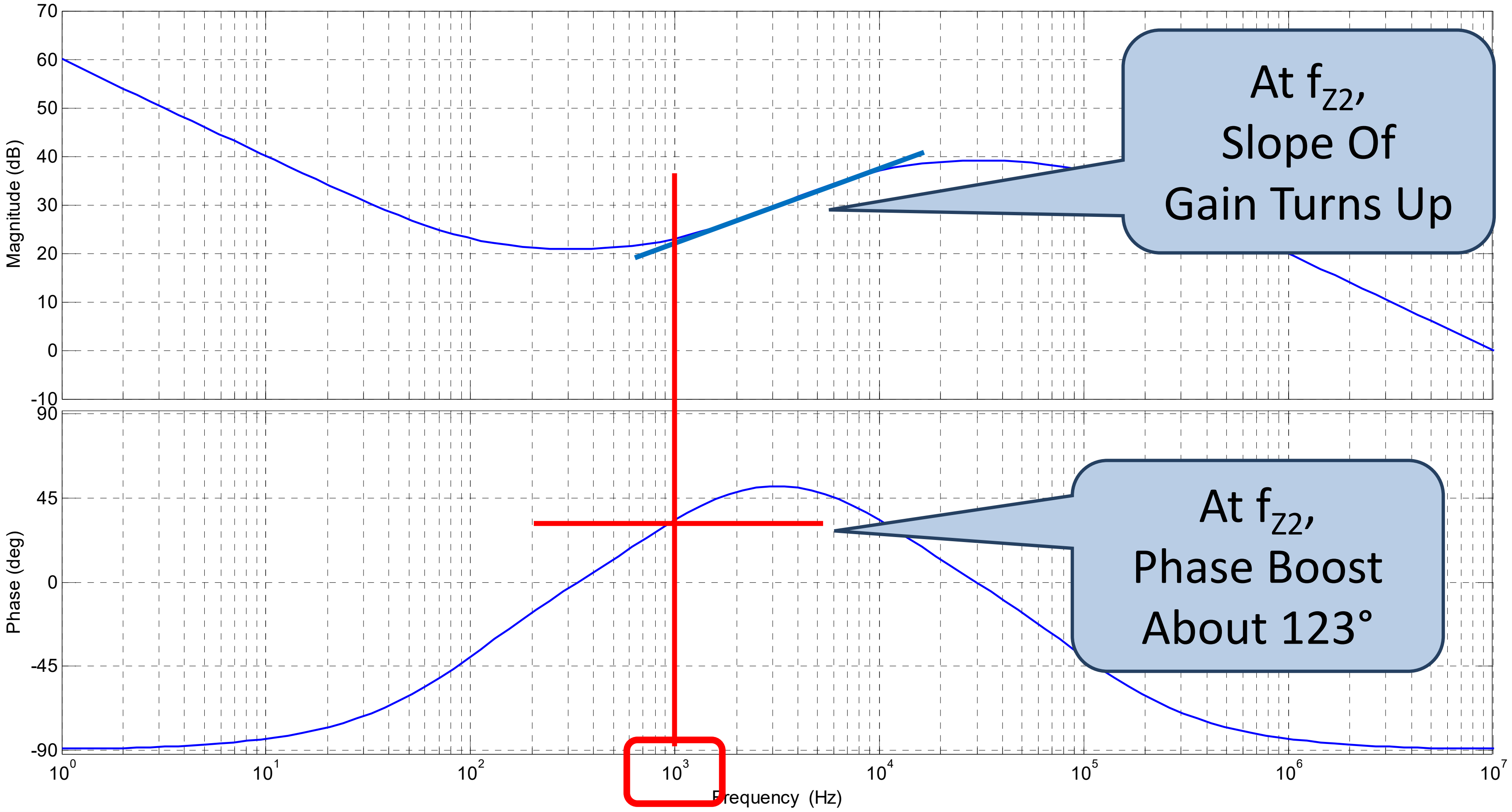
Type III Bode Plot

$f_{P0} = 1000\,\text{Hz}$ $f_{Z1} = 100\,\text{Hz}$ $f_{Z2} = 1000\,\text{Hz}$ $f_{P1} = 10\,\text{kHz}$ $f_{P2} = 100\,\text{kHz}$



Type III Bode Plot

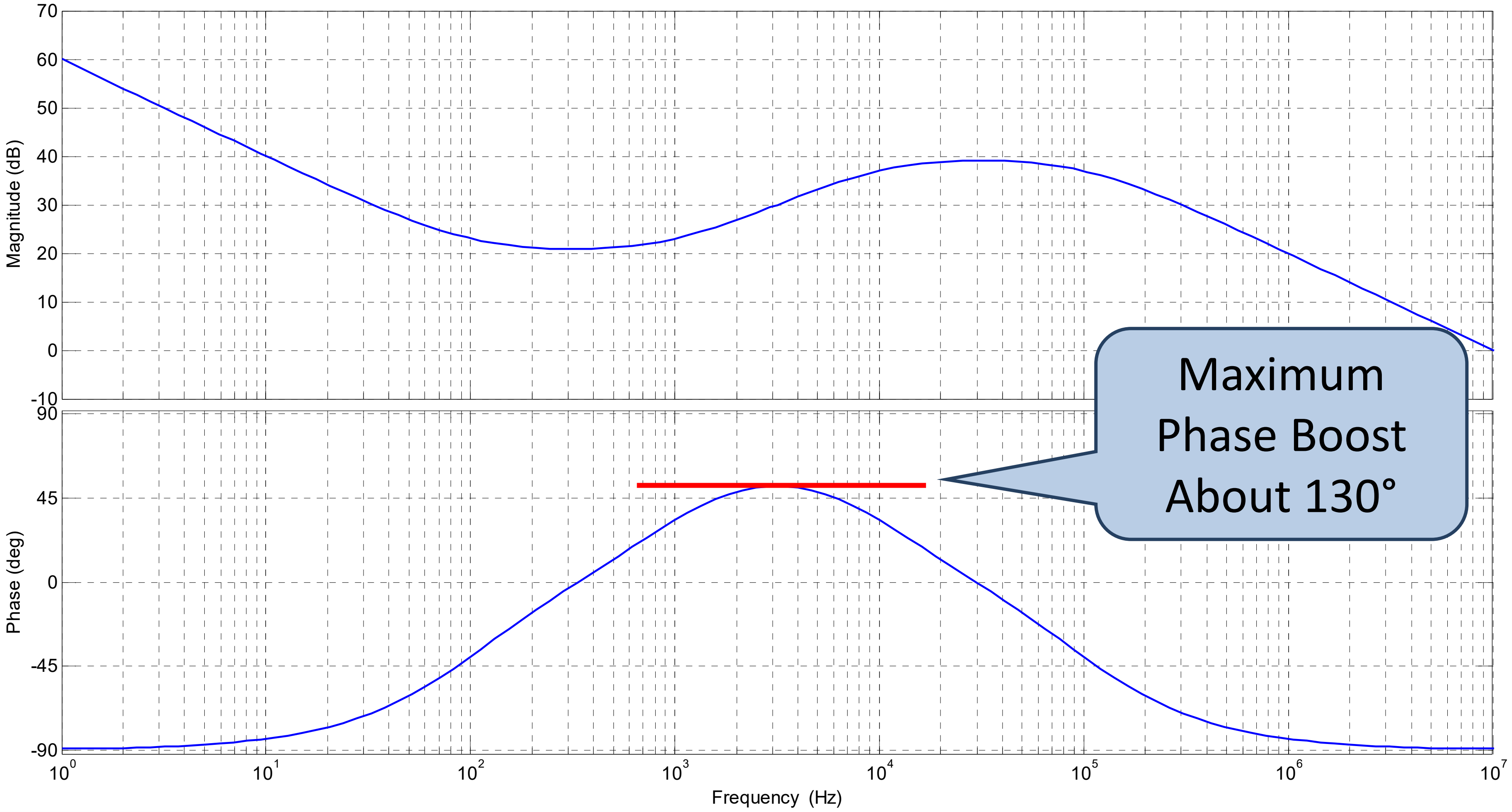
$f_{P0} = 1000\,\text{Hz}$ $f_{Z1} = 100\,\text{Hz}$ $f_{Z2} = 1000\,\text{Hz}$ $f_{P1} = 10\,\text{kHz}$ $f_{P2} = 100\,\text{kHz}$



EMBEDDED POWER LABS

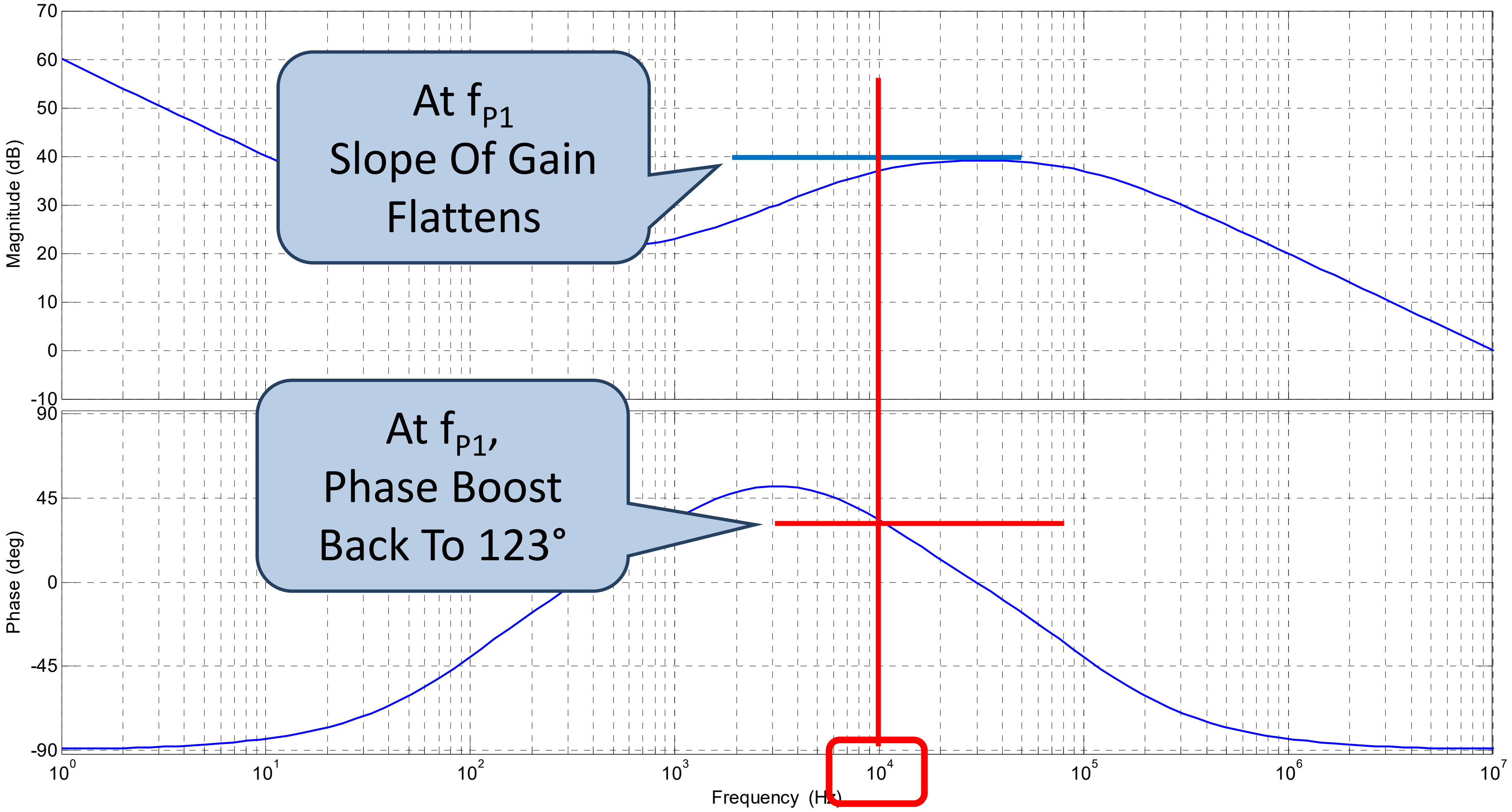
Type III Bode Plot

$$f_{P0} = 1000\,\text{Hz} \quad f_{Z1} = 100\,\text{Hz} \quad f_{Z2} = 1000\,\text{Hz} \quad f_{P1} = 10\,\text{kHz} \quad f_{P2} = 100\,\text{kHz}$$



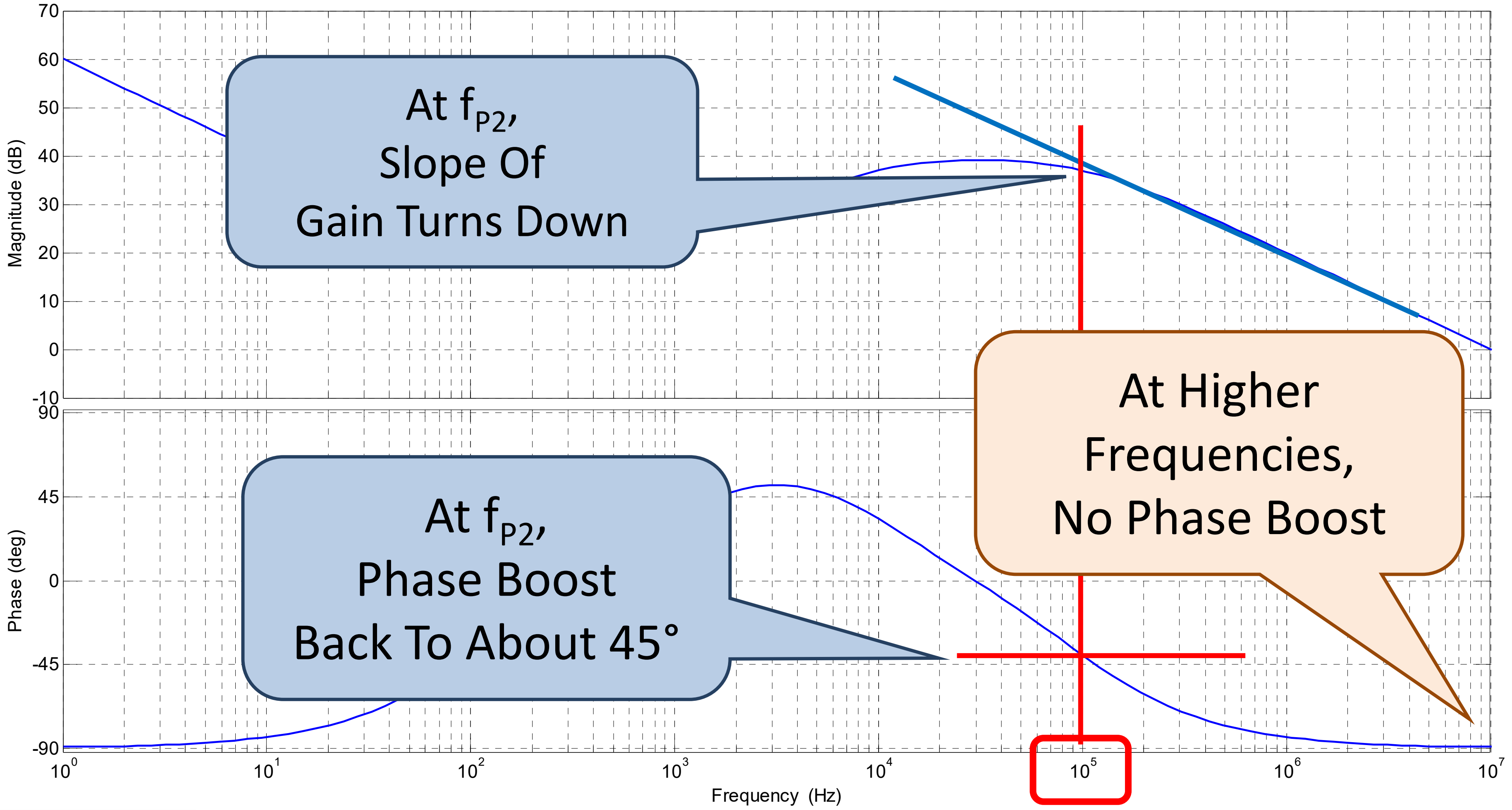
Type III Bode Plot

$f_{P0} = 1000\,\text{Hz}$ $f_{Z1} = 100\,\text{Hz}$ $f_{Z2} = 1000\,\text{Hz}$ $f_{P1} = 10\,\text{kHz}$ $f_{P2} = 100\,\text{kHz}$



Type III Bode Plot

$f_{P0} = 1000\,\text{Hz} \quad f_{Z1} = 100\,\text{Hz} \quad f_{Z2} = 1000\,\text{Hz} \quad f_{P1} = 10\,\text{kHz} \quad f_{P2} = 100\,\text{kHz}$



Choosing Compensator Poles And Zeroes

Given

- F_{SW}
- F_{LC}
- F_{ESR}
- 70° Phase Margin

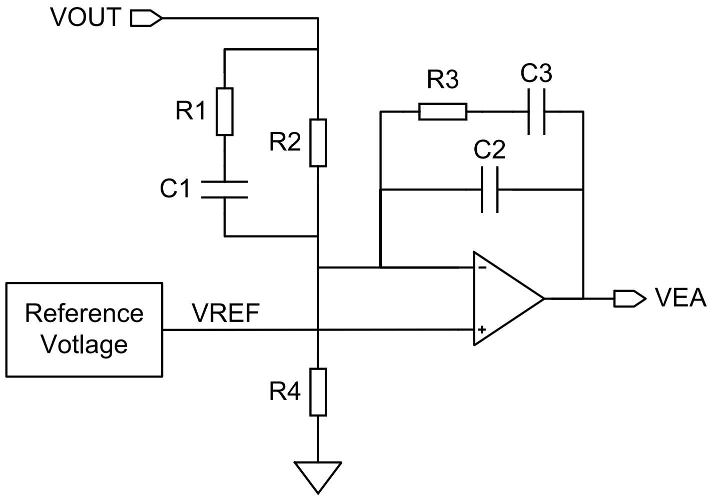
Choose

- $F_C = F_{SW}/10$
- $F_{Z1} = F_{LC}$
- $F_{Z2} = 0.75 \times F_{LC}$
- $F_{Z\_ESR} = F_{ESR}$
- $F_{HFP} = 2.7 \times F_C$

This Will Give You A Good Starting Point With Confidence Loop Will Be Stable

Calculating Component Values

- If $V_{OUT} > V_{REF}$ Choose Current (I_{BIAS}) Through R2 And R4
  - 100 µA - 1 mA Typical



$$R_4 = \frac{V_{REF}}{I_{BIAS}}$$

$$R_2 = \frac{V_{OUT} - V_{REF}}{V_{REF}} \cdot R_4$$

- If $V_{OUT} = V_{REF}$:
  - R4 Not Used
  - Choose Convenient Value For R2 (10 kΩ)

Calculating Component Values

Knowing The Desired Pole And Zero Frequencies Of The Compensator
The Individual Component Values Can Easily Be Calculated

$$R_1 = \frac{f_{Z2}}{f_{P1} - f_{Z2}} \cdot R2$$

$$C_3 = \frac{1}{2 \cdot \pi \cdot f_{P0} \cdot R_2} - C_2$$

$$C_1 = \frac{1}{2 \cdot \pi \cdot f_{P1} \cdot R_1}$$

$$R_3 = \frac{1}{2 \cdot \pi \cdot f_{Z1} \cdot C_3}$$

$$C_2 = \frac{f_{Z1}}{2 \cdot \pi \cdot f_{P2} \cdot f_{P0} \cdot R_2}$$